Math.Comput.Sci.
DOI 10.1007/s11786-016-0262-4
Mathematics in Computer Science

Implementation of Markovian Queueing Network Model with Multiple Closed Chains

K. Sivaselvan · C. Vijayalakshmi

Received: 23 June 2015 / Revised: 28 January 2016 / Accepted: 22 February 2016

Abstract Mathematical strategy portrays the performance evaluation of computer and communication system and it deals with the stochastic properties of the multiclass Markovian queueing system with class-dependent and server-dependent service times. An algorithm is designed where the job transitions are characterized by more than one closed Markov chain. Generating functions are implemented to derive closed form of solutions and product form solution with the parameters such as stability, normalizations constant and marginal distributions. For such a system with N servers and L chains, the solutions are considerably more complicated than those for the systems with one sub-chain only. In Multi-class queueing network, a job moves from a queue to another queue with some probability after getting a service. A multiple class of customer could be open or closed where each class has its own set of queueing parameters. These parameters are obtained by analyzing each station in isolation under the assumption that the arrival process of each class is a state-dependent Markovian process along with different service time distributions. An algorithmic approach is implemented from the generating function representation for the general class of Networks. Based on the algorithmic approach it is proved that how open and closed sub-chain interact with each other in such system. Specifically, computation techniques are provided for the calculation of the Markovian model for multiple chains and it is shown that these algorithms converge exponentially fast.

Keywords Transition · Markov chain · Marginal distribution · Closed and open sub chain · Queueing network

1 Introduction

Representation and analysis of multiprocessor Queueing system have drawn more attention in the recent years. By product-form algorithms the network parameters are computed and approximating solution is also considered in some cases. The network performance has to be characterized by a Markov chain and its large size of the state space, time intricacy is a drawback for solving linear systems. The computational methods for basic equilibrium distribution of jobs with exponential servers and the marginal distribution are also derived in the closed queueing

K. Sivaselvan
Department of Mathematics, Jeppiaar Engineering College, Chennai, India
e-mail: sivajpr@gmail.com

C. Vijayalakshmi (✉)
SAS, Mathematics Division, VIT University, Chennai, India
e-mail: vijusesha2002@yahoo.co.in

Published online: 08 April 2016

network. Finite number of jobs only can pass through the network, new arrivals and departures are not permitted to enter into the closed queueing network. The product form algorithm is derived in the case of multiple closed sub chains and computational algorithm is presented for general classes of queueing networks. The routing transition is characterized by a Markov chain which is decomposable into multiple sub chains. In limited cases, network with closed sub chain is introduced for choosing an open network. Several aggregate states and their marginal distributions are discussed in the conclusion.

2 Literature Survey

The Distributed solving of Markov chains for computer network models had been developed by [1]. Chen et al. [2], has explained the reliable shortest path problems in stochastic time-dependent networks. Bhattacharjee et al. [3] has envisaged the statistical analysis of network traffic inter-arrival. Ching et al. [4] had explained the Markov chains models, algorithms and applications. Stability criterion of a general multiserver multiclass queueing system has elaborated by [5]. Graham et al. [6] had explained the concept of interacting multi-class transmissions in large Stochastic Networks. Domańska et al. [7] has clearly explained a few investigations of long-range dependence in network traffic. Smith et al. [8] has enlightened the system capacity and performance modelling of finite buffer queueing networks. van Woense Frederico et al. [9] had envisaged the optimal routing in general finite multi-server queueing networks modeling functionality. Tadj et al. [10] had explained optimal design and control of queues. H.C. Tijms [11] had approached algorithmic procedure on stochastic models. Valakevicius et al. [12] had created an algorithm for Markovian models of complex systems. Dai et al. [13] has analysed the stability of the shortest queueing networks and Harchol-Balter et al. [14], has discussed about the multi-server queueing systems with multiple priority classes. Khalid et al. [15] has designed a discrete event simulation model for evaluating the performances of an M/G/C/C state dependent queuing system. Manitz [16] had made an analysis of assembly/disassembly queueing networks with blocking after service and Morozov et al. [17] has analysed about the Stability analysis of multi-server discrete-time queueing systems with renewal-type server interruptions. Nogueira et al. [18] has implemented the Markovian model for Internet Traffic. Network performance engineering.

Notations

$P_{ir.(i'r')}$	:	probability of transition from state (i, r) to state (i', r')
$\lambda_l(n_l)$	:	independent Poisson arrival streams rate,
N_l	:	equilibrium means queue size of service center l,
P_r	:	routing matrix of chain r
N_r	:	population size of chain r
$N_1, N_2, \ldots, N_R$	:	population vector
$S(r)$	:	service center passes by a chain r
$R(l)$	:	set of chains which pass service center
$\omega_{r,l}$	:	equilibrium mean waiting time
e_r	:	R-dimensional unit vector in direction r

2.1 Scrutiny and Delineation

The queueing systems have the following parameters are arrival fashion, service rate dispensation, queue restraint in channels, routing probabilities, and system composition. The following Fig. 1 shows the general structure of Markovian closed queueing networks.

For K service centers, jobs transitions are ordained from one state to another by a first-order Markov chain M in R different classes. The transition matrix is $KR \times KR$ and its elements $P_{ir.(i'r')}$ is the probability of transition

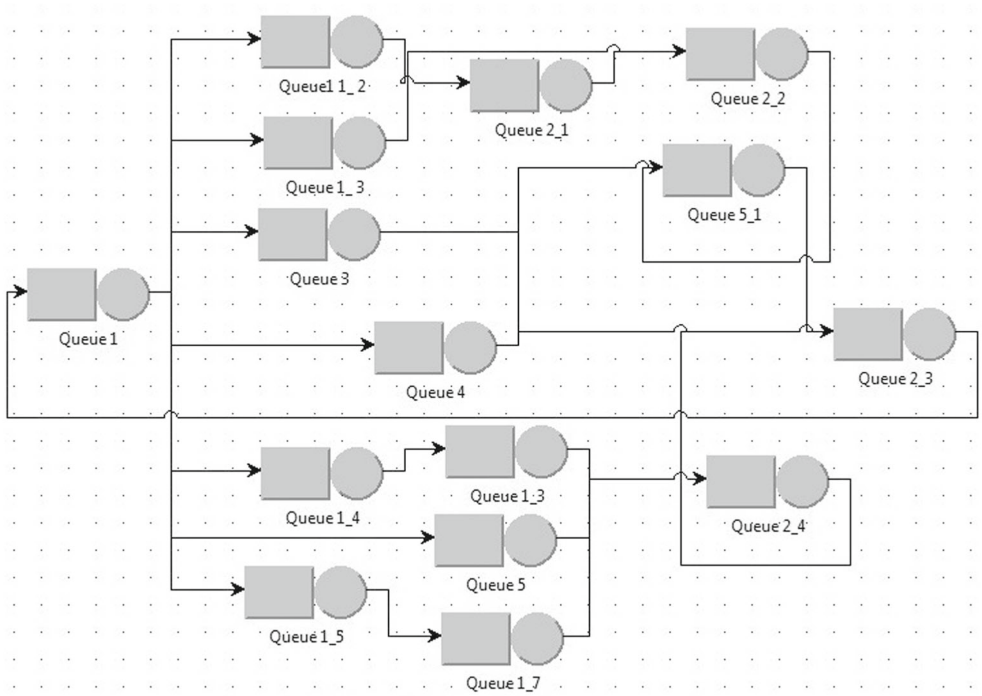


Fig. 1 Multistage Markovian of closed queueing networks

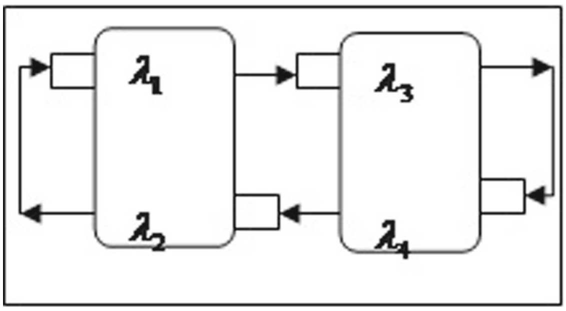


Fig. 2 Two stations closed queueing network

from state (i, r) to state (i', r') in M, namely, the probability that a job of class r which completes service at the center i will go to service center i' and changes to class r'.

Then the Markov chain M is decomposable into irreducible multiple sub chains $M_1, M_2, \ldots, M_L$ which arrant it may either open are closed. The sub chains are driven by L independent Poisson arrival streams with rate $\lambda_l(n_l)$, where n_l is the total number of job in subchain M_l at a given system. An afresh inward jobs out of a stream l will first join the station i with class identification r whose probability is $P_{l(ir)}$. A job class of r' completing service center i' departs the network with probability $P_{(i'r'),l}$. In a closed sub chain M_l, the number of jobs is held at constant at n_l. This situation is realized by choosing $P_{l(ir)} = P_{(ir)l} = 0$ for all $(ir) \in M_l$. The following example has clearly explained the two station closed queueing network, in which each station has a single server with the two job of same class circulate in the model. The service rates are clearly mentioned as $\psi = \frac{\mu_1}{\mu_2}$ and showed in the following Fig. 2.

The system states have been portrayed in triplet form (n, l_1, l_2) where $l_1, l_2 \in \{1, 2\}$ denotes the phase of the job's service at station 1 and $n \in \{0, 1, 2\}$ denotes the number of job at station 1. A total of 8 states generated and its infinitesimal generator matrix is given by

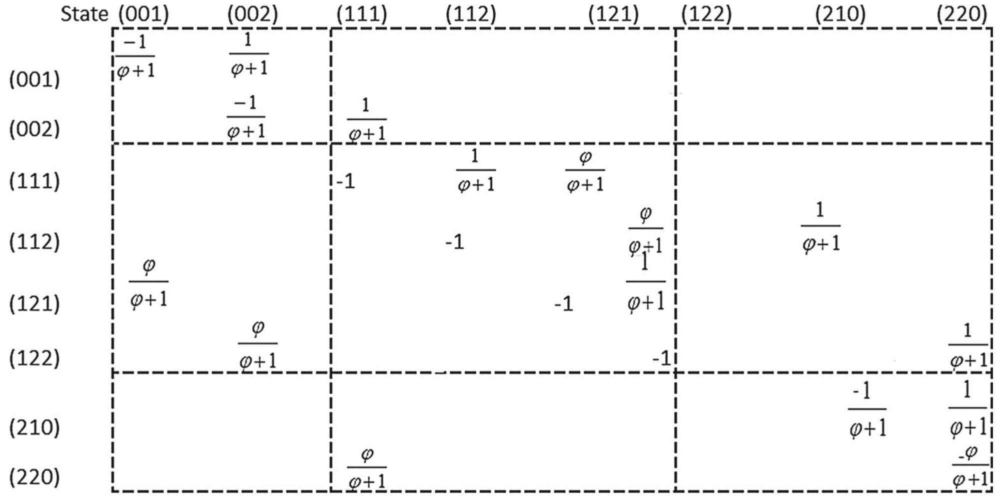


Fig. 3 State block diagram

The stationary probability distribution vector ν is attained by solving $\nu^t T = 0$; $\nu^t e = 1$, where e denotes a vector whose elements are 1 and ν^t is the transpose of the vector ν. Thus the vector $(\phi^3(\phi+1), \phi^3(\phi+3), \phi(\phi+1)^2, \phi(\phi+1), \phi^2(\phi+1), 2\phi^2, (\phi+1), (3\phi+1))^t$ satisfies $\nu^t T = 0$ but not $\nu^t e = 1$.

The state transition block diagram is depicted in Fig. 3.

2.2 Agglomeration Algorithm

Step 1: Form $(Q^{(n)})_{ij} = U^{(n)} T_{ij} e_j$, where

$$U = \begin{pmatrix} b_1^t & 0 & \cdot\cdot & \cdot & 0 \\ 0 & b_2^t & \cdot\cdot & \cdot & 0 \\ \cdot & \cdot\cdot & \cdot\cdot & \cdot & \cdot \\ 0 & \cdot & \cdot\cdot & b_{n-1}^t & 0 \\ 0 & 0 & \cdot\cdot & \cdot & b_n^t \end{pmatrix} \text{ and } T_{ij} e_j = \begin{pmatrix} 0 & 0 & 0 \\ \frac{-1}{\phi+1} & \frac{1}{\phi+1} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{-1}{\phi+1} & \frac{1}{\phi+1} \\ \frac{\phi}{\phi+1} & \frac{-\phi}{\phi+1} & 0 \\ \frac{\phi}{\phi+1} & -1 & \frac{1}{\phi+1} \\ 0 & 0 & 0 \\ 0 & \frac{\phi}{\phi+1} & \frac{\phi}{\phi+1} \end{pmatrix}$$

Step 2: Solve $w^{(n)} Q^{(n)} = 0$ with $w^{(n)} e = 1$

Step 3: For $k = 1, 2, \ldots, n$, calculate $b_k^{(n)}$

(a) Form $Q_k^{(n)}$ use $b_l^{(n)}$ $l \leq k-1$ and $b_k^{(n)}$ $l \geq k-1$

(b) Get $(p_k^t, 1-\omega_k)$ from $(p_k^t, 1-\omega_k) Q_k^{(n)} = 0$

(c) Form $b_k^{(n)} = \frac{1}{\omega_k} p_k$

Step 4: Convergence test: if satisfied, stop
else
go to Step (1)

where b_k^t denotes the conditional stationary probabilities of the states of block k, ω_k is the probability of being in one of the states of block k, r is the number of blocks, z_i number of states in block i thus $\sum_{i=1}^{r} z_i = z$, e_i is the vector of unit length z_i with unit elements. The generator matrix $T_{ij} \in \Re^{zi \times zj}$ and the elements of the matrix Q gives the rate of transition between blocks.

2.3 Equilibrium State Probabilities

A solution of equilibrium state probabilities must satisfy the balance equations for the system. $\forall$ states, S_i, $\sum_{\text{all states } S_j} P(S_i)$ [rate of flow from S_i to S_j] = $p(S_i)$ $\lfloor$ rate of flow out $S_j \rfloor$. Representation of general service time distribution in the method of stages is, only one stage can accommodate a job at a given time. In FCFS discipline, a job waiting at the head of the line is not allowed to enter the first stage until the job currently in service has completed its last stage and departs from this center. That is the entrance stage is blocked as longs as a job exists in the same stage. The steady state distribution provides a solution in a product form when it is not blocked. In the situation of blocking, the solution is complicated for the queueing system. Hence the service center is assumed to be a queue dependent exponential server in FCFS discipline.

In Processor sharing (PS) queue discipline, the problem of blocking on the exponential server could not exist. In a multi server queue, there are many servers available than jobs and no waiting line is formed, thus blocking is non existent. In an infinite server queue, where the service rate is lowered according to the number of jobs in the center at a given time. In FCFS, when a new job enters, the first stage of the server is provided to it. If a new job is entered prior which has been served by its own server. If a job is staying at any stage restart the service among those remaining in the system. When a new job entered the service center without blocking, this leads to provide a product form solution. In Processor Sharing (PS) and Last Come First Serve (LCFS), the service stages are specified in the system.

To discuss the solution of a network S, where all service centers are First come First Serve (FCFS) service center and the service time distributions are exponential. The completion of service rate is common for all classes. The state of such system is represented by a vector $\overline{S} = [S_1, S_2, \ldots, S_i, \ldots, S_N]$, where S_i it is a vector of FCFS stack at center i: $S_i = [r_i(1), r_i(2), \ldots, r_i(n_i)]$, in which $r_i(j)$ is the class of jet job at service center I and n_i is the total number of jobs. Let $P[\overline{S}]$ denotes the equilibrium probability of state $\overline{S}$ and M_l be the number of closed sub chain. The state $\overline{S}([\overline{ir}])$ which is the same as $\overline{S}$ except the last entry is missing. Thus a transition of $\overline{S}([\overline{ir}])$ to state $\overline{S}$ take place upon i with a class r.

2.4 Algorithm for Mean Value Analysis of a Closed Multichain Queueing Networks

Consider a closed multi chain queuing networks, which have a product-form solution. Assume R closed routing chains and K service centers. According to Markov chain, each chain has a finite number of jobs ensue through the sub service centers. Service Center should take up one of the following methods:

(i) FCFS: Processes are dispatched according to their arrival time on the ready queue. Being a non-preemptive discipline, once a process has a CPU, it runs to completion.
(ii) PS: Each job receiving an equal share of the service time.
(iii) D: Delayed jobs at the service center.

Algorithm I: Single Server Case

Step 1: $ni^*(0) \leftarrow 1$ for all $i = 1, 2, \ldots, K$
Step 2: for $i_1 = 0, 1, \ldots, N_1; l_2 = 0, 1, \ldots, N_2; \ldots; l_R = 0, 1, \ldots, N_R$ perform step 3 to 5
Step 3: $\omega^*_{r,l} = \begin{cases} \rho_{r,l}\delta(i_r) & \text{if service centre is } (iii) \\ \rho_{r,l}ni^*(i - e_r) & \text{otherwise} \end{cases}$
For all $r = 1, 2, \ldots, K$ and $l \in S(r)$
Step 4: Little's equation for chain $\lambda_r = \frac{l_r}{\sum_{l \in S(r)} \omega^*_{r,l}}$ for all $r = 1, 2, \ldots, K$
Step 5: Little's equation for service center $n^*_i(i) \leftarrow 1 + \sum_{r \in R(l)} \lambda_r \omega^*_{r,l}$ for all $l = 1, 2, \ldots, L$

Algorithm II: Multi Server Case

Step 1: $ni^*(0) \leftarrow 1$ $p_l(0,0) \leftarrow 1$, $p_l(1,0) gets 0$ for all $i = 1, 2, \ldots, K, l = 1, 2, \ldots, M_{i-1}$

Step 2: $\omega^*_{r,l} \leftarrow \dfrac{\rho_{r,l}}{M_l}\left[ni^*(i-e_r) + \displaystyle\sum_{j=0}^{M_{r-2}} (M_l - 1 - j) p_l(j, i-e_r)\right]$

for $r = 1, 2, \ldots, R$ and each multi server FCFS service center $l \in S(r)$

Step 3: Littles equation for chain yields λ_r as Algorithm I.

Step 4: Littles equation for service center yields $n_i^*(i)$ as Algorithm I.

Step 5: For each multi server FCFS service center l and $j = 1, 2, \ldots, M_{i-1}$

$$p_l(j,i) \leftarrow \frac{1}{j} \sum_{r \in R(l)} \lambda_r \rho_{r,l} p_l(y-1, i-e_r) u_l \leftarrow \sum_{r \in R(l)} \lambda_r \rho_{r,l}$$

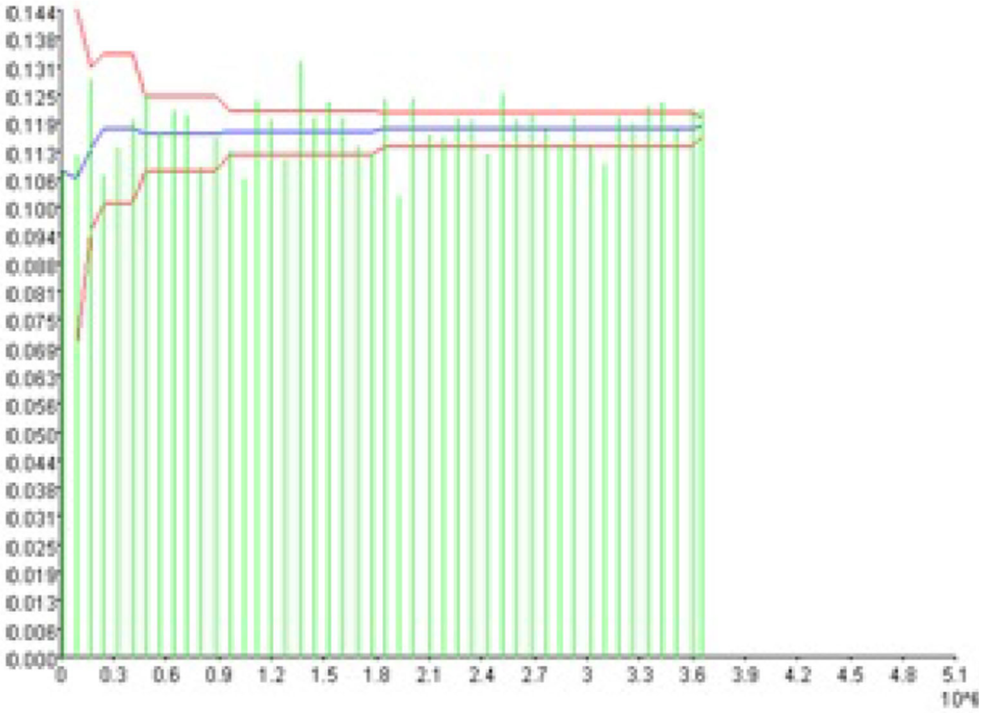
$$p_l(0,i) \leftarrow 1 - \frac{1}{M_i}\left[u_l + \sum_{j=1}^{M_i - 1} (M_i - j) p_l(j,i)\right]$$

3 Numerical Calculation

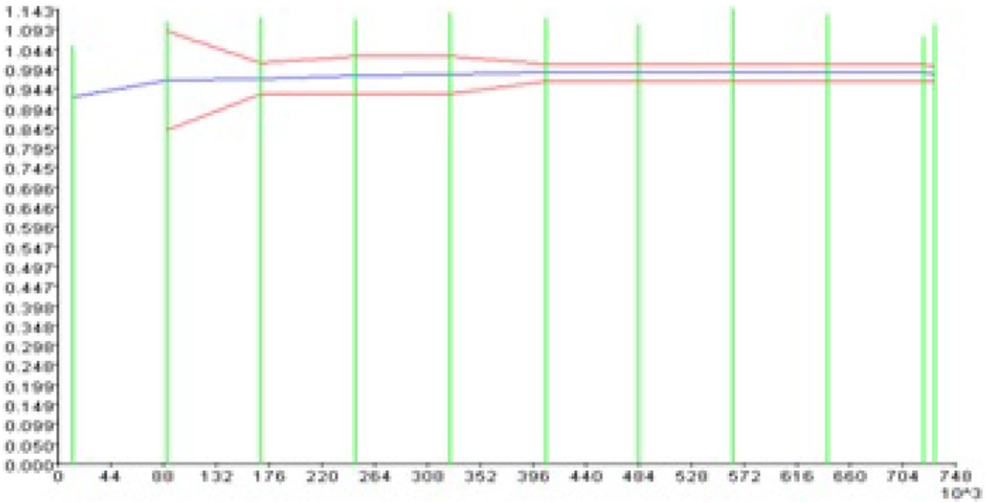
In Multi Server Queueing Network, different classes of customers may be coupled with various classes have been noticeable by their routing pattern within the network and their mean service demands at various centers. Decomposition method describes the estimation of throughput of a closed network that receives the input from various sub-networks. The effectuation of class aggregation technique have reduced the complexity of performance analysis in isolation and by load dependent service rate has avoids the recursive iteration. The convergence criteria the result is generalized to a queueing network in which the customer routing transitions are characterized by a Markov chain decomposable into multiple sub-chains. Networks with closed subchains are introduced as a limiting case of suitable chosen open network. The several aggregate states and their marginal distributions are introduced. An algorithmic approach is implemented from the generating function representation for the general class of Networks. Convergence criteria is tested and rate of transition between blocks are obtained. Based on the algorithmic approach it is proved that how open and closed sub-chain interact with each other in such system.

4 Graphical Representation

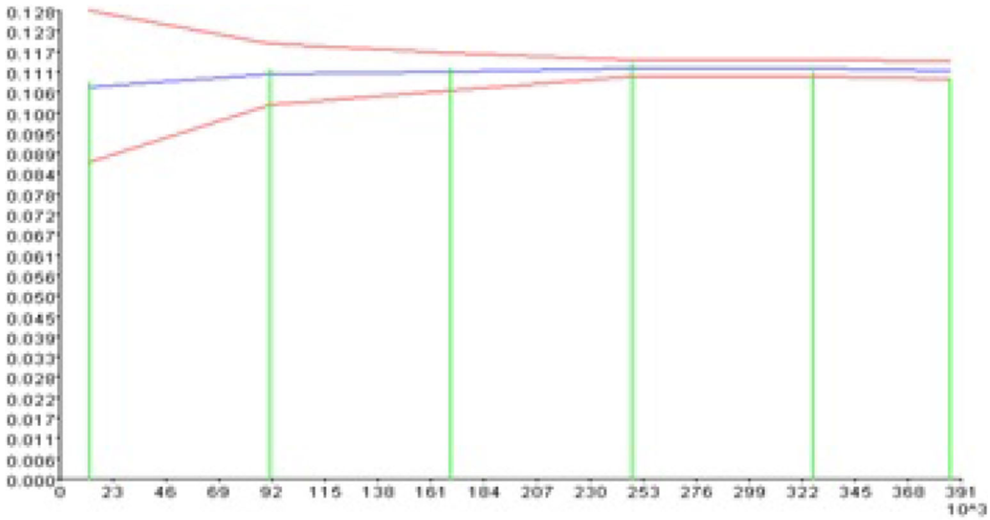
Queue time: Average queue time for each chosen class at each station.



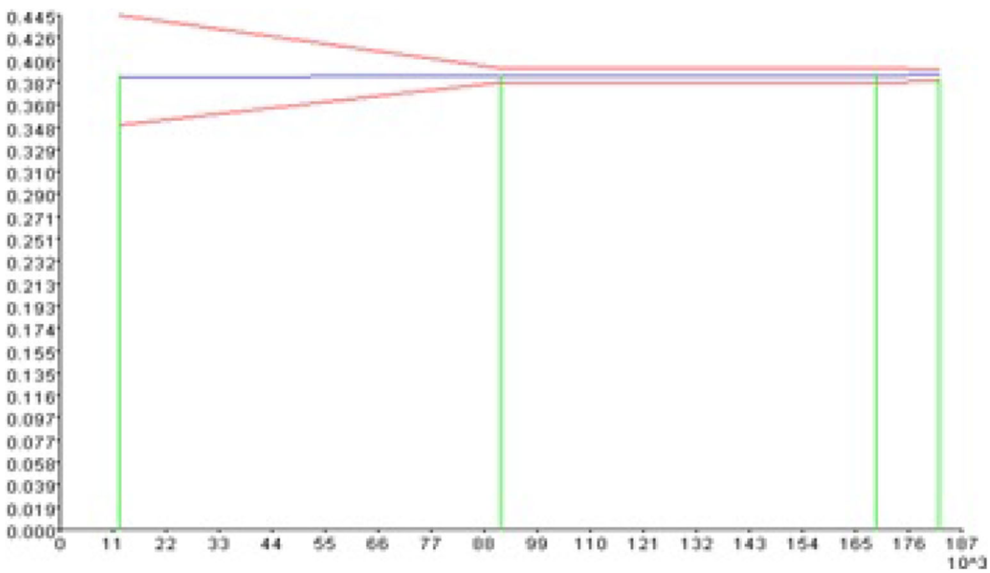
Residence time: Average residence time for each chosen class at each chosen station. (Residence Time = Number of visits * Response time).



Utilization: Utilization of a customer class at the selected station.



Throughput: Average throughput for each chosen class at each chosen station.



Throughput for each class at each stations are displayed in Table 1 and the average number of jobs for each class at each station is depicted in Table 2. Table 3 reflects the total time spent by each job class summed across all visits to a station and utilization of a job class at the selected station is shown in Table 4. Service demands are displayed in Table 5. Based on the above graphical representations, the average queue time is reduced by maximizing the utilization. An algorithm is designed for the efficiency of throughput for each chosen class and stations.

Table 1 Throughput for each class at each station

	Aggregate	Class 1	Class 2	Class 3	Class 4	Class 5
Aggregate	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830
Station 1	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830
Station 2	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830
Station 3	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830
Station 4	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830
Station 5	1.046828	1.000000	0.027198	0.011007	0.001793	0.006830

Table 2 Average number of jobs for each class at each station

	Aggregate	Class 1	Class 2	Class 3	Class 4	Class 5
Aggregate	209.986447	129.986447	20.000000	20.000000	20.000000	20.000000
Station 1	122.532916	101.739365	0.080990	0.698459	19.712993	0.301110
Station 2	11.652943	1.415555	2.663514	7.549281	0.019396	0.005197
Station 3	10.765444	8.090300	0.372593	0.002138	0.111893	2.188521
Station 4	0.906351	0.147929	0.029509	0.431160	0.152002	0.145751
Station 5	64.128793	18.593299	16.853394	11.318963	0.003716	17.359421

Table 3 Total time spent by each job class summed across all visits to a station

	Aggregate	Class 1	Class 2	Class 3	Class 4	Class 5
Aggregate	200.593071	129.986447	735.349582	1817.0273...	11,156.309...	2928.0962
Station 1	117.051620	101.739365	2.977782	63.455911	10,996.212...	44.083886
Station 2	11.131669	1.415555	97.930707	685.862451	10.819649	0.760929
Station 3	10.283871	8.090300	13.699292	0.194221	62.415610	320.409992
Station 4	0.865807	0.147929	1.084981	39.171474	84.788825	21.338631
Station 5	61.260104	18.593299	619.656920	1028.3432...	2.072786	2541.5027...

Table 4 Utilization of a job class at the selected station

	Aggregate	Class 1	Class 2	Class 3	Class 4	Class 5
Station 1	1.000000	0.823581	0.000655	0.005656	0.167670	0.002438
Station 2	0.979599	0.111876	0.208228	0.657574	0.001529	0.000392
Station 3	0.920261	0.687632	0.030572	0.000182	0.009506	0.192369
Station 4	0.476957	0.077598	0.015372	0.227603	0.079716	0.076667
Station 5	0.999999	0.285485	0.266054	0.173535	0.000057	0.274868

Table 5 Service demands

	Class 1	Class 2	Class 3	Class 4	Class 5
Station 1	0.8235810218	0.0240974379	0.5138413289	93.5289230950	0.3568942645
Station 2	0.1118755137	7.6560354436	59.7414818781	0.85307344684	0.0578922897
Station 3	0.6876323418	1.1240468267	0.0165524034	5.30270705330	28.1637318811
Station 4	0.0775979926	0.5652042121	20.6780599301	44.4670136778	11.2244901748
Station 5	0.2854850804	9.7821251641	15.7659119078	0.0318101352	40.2419571295

5 Conclusion

In this paper an iterative method is implemented for multiple closed chains. In disagglomeration steps involve the enumeration of all the states underlying Markov process. In Multi-class queueing network the transition behavior of the Markov chain is analysed and based on that it reveals the fact that a job completing the service at server i will go next to server j with probability p_{ij}. The performance measures are evaluated by the computational algorithm. Convergence criteria are tested and rate of transition between blocks is obtained. Based on the algorithmic approach it is proved that how open and closed sub-chain interact with each other in such system.

Applications The proposed scheme can be implemented in multiple-tier Internet service systems. Self-Similarity in High-Speed Packet Traffic be analyzed and Ethernet Traffic Measurements can be modeled for the Performance analysis for future high speed networks.

References

1. Bylina, J.: Distributed solving of Markov chains for computer network models. Annales UMCS Informatica. Lublin. **1**, 15–20 (2003)
2. Chen, B.Y., Lam, W.H.K., Sumalee, A., Li, Q., Tam, M.L.: Reliable shortest path problems in stochastic time-dependent networks. J. Intell. Transp. Syst. Technol. Plan. Oper. **18**(2), 177–189 (2014)
3. Bhattacharjee, A., Nandi, S.: Statistical analysis of network traffic inter-arrival. In: Proceedings of the 12th International Conference on Advanced Communication Technology, ICACT'10, IEEE Press, pp. 1052–1057 (2010)
4. Ching, W.-K., Huang, X., Ng, M.K., Siu, T.K.: Markov chains : Models, algorithms and applications. International Series in Operations Research and Management Science, vol. 189, 2nd ed. ISBN: 978-1-4614-6311-5, 978-1-4614-6312-2 (2013)
5. Morozov, E.: Stability criterion of a general multiserver multiclass queueing system. In: 29th International Symposium on Computer and Information Sciences (ISCIS), pp. 229–238 (2014)
6. Graham, C.: Robert: interacting multi-class transmissions in large stochastic networks. Ann. Appl. Prob. **19**, 2334–2361 (2009)
7. Domańska, J., Domańska, A., Czachórski, T.: A few investigations of long-range dependence in network traffic. In: Czachorski, T., Gelenbe, E., Lent, R. (eds.) Information Science and Systems, pp. 137–144. Springer International Publishing, Switzerland (2014)
8. MacGregor Smith, J.: System capacity and performance modelling of finite buffer queueing networks. Int. J. Prod. Res. **52**(11), 3125–3163 (2014)
9. van Woense Frederico, T., Cruz, R.B.: Optimal routing in general finite multi-server queueing networks. doi:10.1371/journal.pone.0102075 (2014)
10. Tadj, L., Choudhury, G.: Optimal design and control of queues. Top **13**, 359–412 (2005)
11. Tijms HC. A first course in stochastic models. Weily, Chichester (2003). ISBN:0471498807
12. Valakevicius, E., Pranevicius, H.: An algorithm for creating Markovian models of complex systems. In: Proceedings of the 12th World Multi-Conference on Systemics, Cybernetics and Informatics, Orlando, USA, pp. 258–262 (2008)
13. Dai, J., Hasenbein, J., Kim, B.: Stability of join-the-shortest-queue networks. Queueing Syst. **57**, 129–145 (2007)
14. Harchol-Balter, M., Osogami, T., Scheller-Wolf, A., Wierman, A.: Multi-server queueing systems with multiple priority classes. Queueing Syst. Theory Appl. **51**, 331–360 (2005)
15. Khalid, R., Nawawi, M.K.M., Kawsar, L.A., Ghani, N.A., Kamil, A.A., et al.: A discrete event simulation model for evaluating the performances of an M/G/C/C state dependent queuing system. PLoS One **8**, e58402 (2013). doi:10.1371/journal.pone.0058402
16. Manitz, M.: Analysis of assembly/disassembly queueing networks with blocking after service and general service times. Ann. Oper. Res. **226**(1), 417–441 (2014)
17. Morozov, E., Fiems, D., Bruneel, H.: Stability analysis of multiserver discrete-time queueing systems with renewal-type server interruptions. Perform. Eval. **68**, 1261–1275 (2011)
18. Nogueira, A., Salvador, P., Valadas, R., Pacheco, A.: Markovian Modelling of Internet Traffic. In: Network Performance Engineering, pp. 98–124. Springer, Berlin (2011)