

# Design Method of a Direct Drive Permanent Magnet Vernier Generator for a Wind Turbine System

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**Abstract**—In this study, a novel design method of a direct-drive (DD) permanent magnet vernier generator (PMVG) is proposed for a gearless, high power and lightweight wind turbine system. Once the maximum power requirement is given, the base and maximum speeds of the generator are firstly obtained from aerodynamic characteristics of wind turbine blade. Then the necessary electrical circuit parameters of the generator are scoped with consideration of the maximum torque per ampere (MTPA) control scheme of a PM generator. To determine the generator geometries satisfying the scoped parameters, the correlations between the parameters and geometries of a PMVG with concentrated windings are deduced. Using the deduced correlations and the obtained parameters, a systematic design procedure of a PMVG is proposed. For a case study, a 5kW DD-PMVG with outer rotor is designed through the proposed method. The performance characteristics of the designed generator are analyzed with finite element (FE) simulations and compared with analytically predicted results. Finally, the experimental results are provided.

**Keywords**—direct drive, generator, vernier machine, wind turbine

## I. INTRODUCTION

Recently, the interest in renewable energies is very high, and the research and the development on them are more active than ever. Among them, wind-powered generators operate in every size range, from small turbines for battery charging at isolated residences to large wind farms that provide electricity to national electric transmission systems. In particular, wind power generation is being transferred from the ground to the sea, and at the same time, the capacity of generation is growing to the level of gigawatts. These tendencies cause the following compromise; when a mechanical component such as a gear box in an offshore wind turbine breaks down, the maintenance procedure is very complicated and expensive. Therefore, it is beneficial to adopt a direct-drive (DD) generator with no gears, but in this case, the weight of the generator becomes excessively large [1-4]. Hence, to solve these two problems at once, it is necessary to use a DD-generator having much higher power density.

Thus far, various unique machines for the low speed-high torque applications have been presented such as magnetic gears, vernier machines, flux reversal machines and flux switching machines etc. [4-25], and then it is proven that all these machines are classified into the modulation flux machinery which utilizes the magnetic gear effects, alternatively called the flux modulation effects [5].

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Especially, it was proven that the permanent magnet (PM) vernier motor, which is a magnetically coupled structure of the magnetic gear and the PM motor, has theoretically much higher output power density because it uses the main PM flux at the same time as the modulation flux [6, 7].

Therefore, several research studies on these kinds of machines have been carried out for generating systems [1, 2], [8-13]. In particular, a magnetic geared machine called a pseudo DD machine was proposed for a wind generator [8] and optimized which has higher torque than the transverse flux machine [1]. In addition, the high possibility of vernier PM machine with dual gap for the DD generator was shown [11, 12], because the structure has the advantage of higher power factor as well as higher power density [14, 15].

On the other hands, the design procedures commonly used for conventional PM machines are not proper for the design of these flux modulation machines. For an example, the most popular design procedure of a conventional PM machine is as follows; The volume of the machine can be initially assumed by using the  $D^2L$ -based power equation with the value of torque per rotor volume (TRV), which is actually the air gap volume, obtained from various theoretical predictions and empirical results [16]. Then using the pre-set magnetic and electric loadings, the geometries such as the air gap diameter and the stack length are detailed. From the geometries of the initially designed machine, the circuit parameter values are estimated. The performance characteristics are predicted by solving the equivalent circuit or by using the numerical methods such as FEM. This procedure is repeatedly performed until the desired characteristics are satisfied, commonly coupled with optimization algorithm [26-29]. The effectiveness of this long-established procedure is expected to be quite high for the classical machines, especially for the fixed speed applications. However, the procedure above is not adequate for the flux modulation machines having substantially different TRV values due to unique operating principles. Thus, some studies of design of these machines start with the TRV value chosen from insufficient theoretical prediction [1], [3], [30], which apparently leads to much more repeated adjusting procedures due to the inaccuracy of the TRV value. Furthermore, if the variable speed performances are needed, the design procedure becomes much more complicated and increases uncertainty because the characteristics of both the reactance and the back EMF should be considered simultaneously [31].

In this paper, a novel method for a vernier PM machine design is proposed to avoid the complexity due to repeated design and performance computations and to reduce the uncertainty in the design results. The proposed method is applied to a 5kW DD-PMVG with outer rotor for the laboratory experiments. In the proposed design method, first,

by considering the machine control schemes of MTPA and the aerodynamic properties of wind power, the equivalent circuit parameter values of the generator meeting torque requirements are scoped. Then, the correlations between the circuit parameters and the geometries of the vernier machine are derived. Using several design constraints such as the maximum surface and volume current densities, the functions expressing the machine volume and the capacity of power converter can be obtained, which suggests the most adequate circuit parameter values. Consequently, the detailed mechanical geometries and electrical specifications of the generator can be determined directly from the suggested circuit parameter values by using the correlations. Finally, the designed outer-rotor, vernier generator is analyzed through finite element (FE) analysis and experimented with a actually produced machine, and the results are compared with analytically predicted ones.

## II. DESIGN CONSIDERATION FOR A DIRECT-DRIVE WIND POWER GENERATOR

### A. Calculation of Rotational Speed and Torque

The power transferred to the turbine rotor from wind can be expressed by

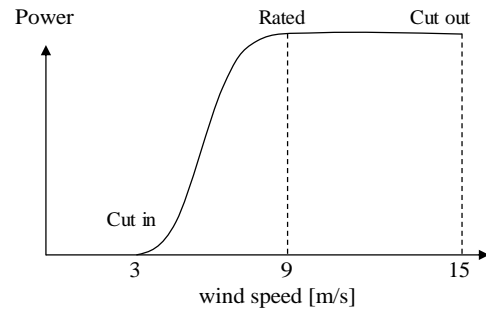
$$P_{turbine} = C_p P_{air} = \frac{1}{2} C_p \rho v_w^3 A_{blade} \quad (1)$$

$$= \frac{1}{2} \pi C_p \rho v_w^3 R_{blade}^2$$

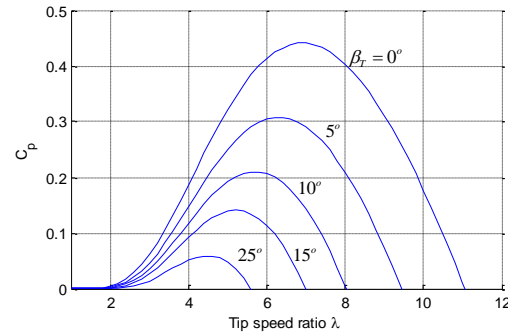
where  $P_{air}$  is the kinetic energy of moving air,  $C_p$  is the power coefficient less than 0.59 of Betz limit,  $A_{blade}$  is the rotor blade area,  $R_{blade}$  is its radius,  $\rho$  is the air density approximately  $1.225\text{kg/m}^3$ , and  $v_w$  is the wind velocity. In the wind turbine system, the cut-in, the rated and the cut-out speeds of wind are specified with consideration of the capacity of wind turbine. In this study, they are set as 3m/s, 9m/s and 15m/s, respectively, demonstrated in Fig. 1(a). The power generation begins at the cut-in speed and increases with a cube of wind speed. After reaching the rated speed, the power is controlled to be constant up to the cut-out speed. In practice, it is convenient to use a tip-speed ratio,  $\lambda$  of (2) to estimate the rotational speed of turbine rotor, where  $\omega_{blade}$  is the angular speed of the blade rotor.

$$\lambda = \omega_{blade} \frac{R_{blade}}{v_w} \quad (2)$$

The typical characteristics of  $\lambda$ - $C_p$  curve of two-bladed rotor is given by Fig. 1(b), showing the maximum  $C_p$  about 0.45 is obtained when the ratio  $\lambda$  is around 7 [32]. Therefore, for a given maximum power 5kW,  $A_{blade}$  in (1) is calculated with  $C_p$  of 0.45, and thus the radius  $R_{blade}$  of blade rotor is also obtained. Finally, the base speed  $\omega_{t,base}$  of turbine rotor can be obtained by using (2) and  $\lambda$  of 7, consequently providing the torque of the turbine at each speed, which are given in table I. It should be noted that since the DD-generator is considered, the speed and torque of turbine are same to those of the generator.



(a) Wind speed vs power characteristics



(b) Tip-speed ratio vs  $C_p$  characteristics with two bladed-rotor  
Fig. 1. Power characteristics of wind power. ( $\beta_r$ : blade pitch angle)

TABLE I. REQUIRED SPECIFICATIONS OF WIND POWER GENERATOR

	Speed (RPM)	Power (kW)	Torque (Nm)
Rated ( $\omega_{base}$ )	213.8	5kW ( $=P_{max}$ )	236.2
Cut-out ( $\omega_{max}$ )	356.3	5kW	138.6

### B. Scoping of Circuit Parameters of PM Generator Considering MTPA Control Scheme

Fig. 2 represents the schematic of a wind turbine system with non-salient PM generator which is represented by the per-phase equivalent circuit consisting of the back electro-motive force (EMF)  $E_b$ , the synchronous reactance  $X_{syn}$  and the winding resistance  $R$ . Neglecting  $R$  for convenience, the voltage equation of the generator in DQ-frame can be expressed as (3), and it is depicted as a circle in the DQ-current axes of Fig. 3 whose center,  $C_v$  is  $E_b/X_{syn}$  of (3). On the other hand, there is the maximum allowable current depending on the thermal capacity of an electrical machine or the power capacity of the converter, and it is represented by a circle with radius  $I_{max}$  around the origin. For a non-salient electrical machine, only the  $q$ -axis current  $I_q$  contribute to torque production, as shown by (4).

$$\left( \frac{V_{ph}}{X_{syn}} \right)^2 = \left( I_d - \frac{E_b}{X_{syn}} \right)^2 + I_q^2 \quad (3)$$

$$I_{max}^2 = I_d^2 + I_q^2 \quad (4)$$

$$T = \frac{P_m}{\omega_m} = 3I_q \frac{E_b}{\omega_m} \quad (4)$$

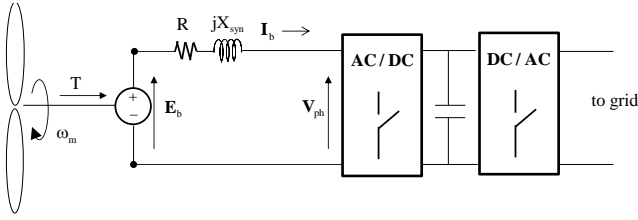


Fig. 2. Wind turbine system with a PM generator

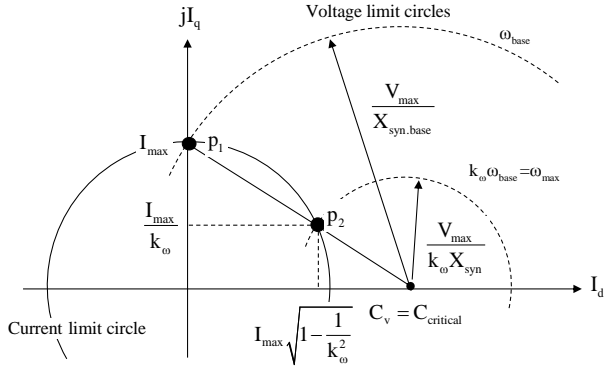


Fig. 3. Operating points for maximum torque production

As is well known, in the maximum torque per ampere (MTPA) control scheme, the radius of the circle  $V_{ph}/X_{sys}$  is kept constant as  $V_{max}/X_{syn.base}$  to achieve the conditions of  $I_d=0$ , that is,  $I_q = I_{max}$  until the base speed  $\omega_{base}$ , so the voltage circle meets with the current circle at  $p_1$  in Fig. 3, and one gets the maximum power  $P_{max} = 3I_{max}E_{b.base}$  from (4) at  $\omega_{base}$ . Above  $\omega_{base}$ , the radius of voltage limit circle shrinks proportionally as frequency increases, and the voltage circle meets with the current circle at  $p_2$  in Fig.3 at the maximum speed  $\omega_{max}$ . Because the maximum power  $P_{max}$  should be kept from  $\omega_{base}$  to  $\omega_{max}$ , it is obvious that  $I_q$  at  $p_2(\omega_{max})$  should be larger than  $I_{max}/k_\omega$  in which  $k_\omega$  is  $\omega_{max}/\omega_{base}$ . These relations can be expressed by (5).

$$I_q^2(\omega_{max}) = I_{max}^2 - \left( \frac{1}{2C_v} \left( C_v^2 + I_{max}^2 - \left( \frac{V_{max}}{k_\omega X_{syn.base}} \right)^2 \right) \right)^2 \geq \left( \frac{I_{max}}{k_\omega} \right)^2 \quad (5)$$

From the right-angled triangle with hypotenuses with  $p_1$ , the following (6) is obtained.

$$C_v^2 + I_{max}^2 = \left( \frac{V_{max}}{X_{syn.base}} \right)^2 \quad (6)$$

Using (6),  $(V_{max}/k_\omega X_{syn.base})^2$  in (5) can be replaced with  $(C_v^2 + I_{max}^2)/k_\omega^2$ , and then it can be solved for  $C_v/I_{max}$  which is denoted as  $\gamma$  and it is given by (7).

$$\gamma = \frac{C_v}{I_{max}} \leq \sqrt{(k_\omega + 1)/(k_\omega - 1)} = \gamma_{max} \quad (7)$$

Hence, the available range of  $\gamma$  can be obtained with the both speeds in table I, which gives  $\gamma \leq \gamma_{max} = 2$ .

From the relation (6),  $X_{syn.base}$  is given as (8) using  $\gamma$ , and the maximum power  $P_{max}$  can be expressed in various form as (9) by using the relations  $E_{b.base} = C_v X_{syn.base}$  and  $C_v = \gamma I_{max}$ . Furthermore, replacing  $X_{syn.base}$  in (9) with (8),  $I_{max}$  can be obtained as (10).

$$X_{syn.base} = \frac{V_{max}}{I_{max} \sqrt{1 + \gamma^2}} \quad (8)$$

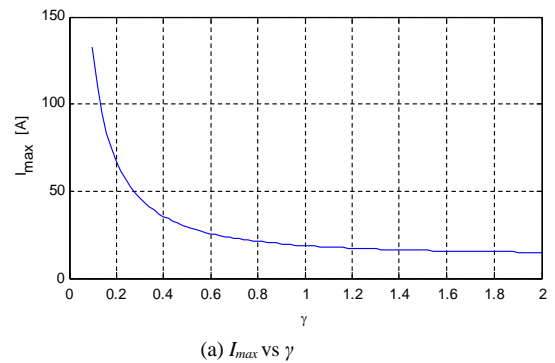
$$P_{max} = 3I_{max} E_{b.base} = 3\gamma I_{max}^2 X_{syn.base} \quad (9)$$

$$I_{max} = \frac{1}{3} \frac{P_{max}}{V_{max}} \frac{\sqrt{1 + \gamma^2}}{\gamma} \quad (10)$$

The maximum phase voltage  $V_{max}$  is determined by the DC voltage behind the AC/DC converter in Fig. 2 and is set to  $220/\sqrt{3}$  considering the general line voltage. Now, with the voltage  $V_{max}$ , the required maximum phase current  $I_{max}$  can be obtained using (10) for available  $\gamma$  of (7), and are depicted in Fig. 4. It shows that  $I_{max}$  increases as  $\gamma$  decreases, alternatively implying that higher capacity of the power converter is needed as the center of voltage circle is closer to the origin, causing poorer power factor.

In addition, using the obtained  $I_{max}$  for the given  $\gamma$ , the parameter  $C_v$  is calculated,  $X_{syn.base}$  and  $E_{b.base}$  are acquired by using (8) and (9) which are given in Fig. 4. It means that there are various available combinations of the maximum current, the back EMF and the synchronous reactance to satisfy the adjustable torque requirements, and it is necessary to find the most suitable combination among them. Furthermore, using (11), the power factor at the base speed under MTPA operation can be estimated and shown in Fig. 4(c). It demonstrates the power factor gets less than 0.5 which is too low when  $\gamma$  is less than 0.6.

$$\cos \theta \approx \frac{E_{b.base}}{\sqrt{E_{b.base}^2 + (X_s I_{max})^2}} \quad (11)$$



(a)  $I_{max}$  vs  $\gamma$

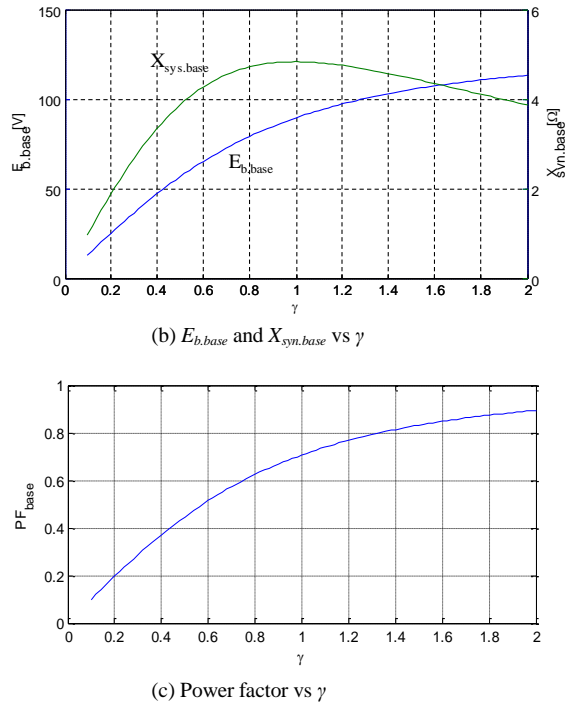


Fig. 4. Necessary maximum current  $I_{max}$ , back EMF  $E_{b,base}$ , synchronous reactance  $X_{syn,base}$ , and power factor for variation of  $\gamma$  at the base speed

### III. CORRELATIONS BETWEEN CIRCUIT CONSTANTS AND GEOMETRIES OF A PM VERNIER GENERATOR WITH CONCENTRATED WINDINGS

Once the geometries and electrical specifications of a general PM machine are provided, it is not difficult to calculate the circuit constant values of the machines such as the back EMF and inductances using the classical formula expressed in terms of the geometry of the machine. On the contrary, it is not easy to find the geometries from the circuit constants because the relationship between the two is not a one-to-one correspondence. That is, the number of geometric variables are much more than those of the circuit constants. Moreover, unlike the conventional PM machine, the circuit parameters for the vernier machine has not yet been systematized in the common forms.

In this study, it will be shown the geometric shape of vernier PM generator can be determined from the circuit constants, where several design constraints are used considering current densities and demagnetization of magnet.

To this aim, however, the formula of circuit constants for the PM vernier machine expressed with the geometries are required first. In a PM vernier machine of Fig. 5, the phase coils are wound in  $Q_s$  main slots with concentrated winding manner. Since 3 slots in the non-overlapping concentrated winding configurations make up 1 winding pole pair, and thus the winding pole pair  $p_w$  is  $Q_s/3$ . Each main tooth is divided into  $n_{split}$  auxiliary teeth, and then the total number of slots  $Q_{fmp}$  which is called the flux modulation poles becomes  $n_{split}Q_s$ . To get vernier effects, the general condition which is  $p_m - Q_{fmp} = -p_w$  should be satisfied where  $p_m$  is the magnet pole pairs. Solving this condition for  $p_m$ , one gets (12) where  $G_r$  is commonly called the gear ratio.

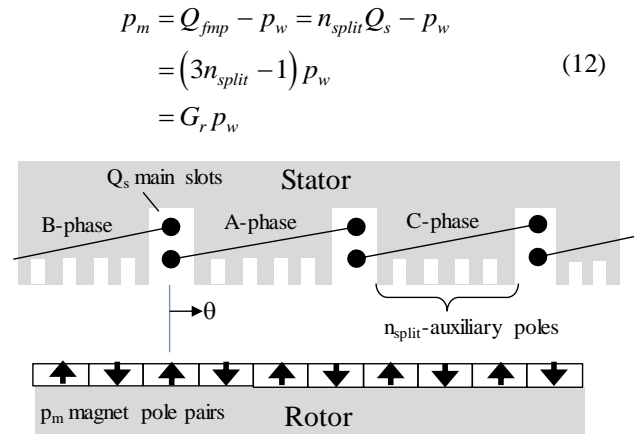


Fig. 5. Schematic of a vernier machine with concentrated windings

#### A. Back EMF Equation

When the magnetomotive force (MMF) from the  $p_m$  magnet pole pairs and the airgap permeance due to  $Q_{fmp}$  slots are expressed as

$$F_m \approx F_{1m} \cos p_m (\theta - \theta_m) \quad (13)$$

$$P_g \approx P_0 - P_1 \cos Q_{fmp} \theta \quad (14)$$

where  $F_{1m}$  is  $4B_r g_m / \pi \mu_r \mu_0$  and  $B_r$  is the residual flux density of PM,  $\mu_0$  is permeability of vacuum,  $\mu_r$  is the recoil permeability of PM,  $g_m$  is the magnet thickness. It was revealed for vernier machines, it is desirable to set around 0.5 for  $c_0$  which is the ratio between the slot open width  $o$  and the slot pitch [6][15], and in this case, the coefficients  $P_0$  and  $P_1$  of (13) and (14) are given by (15) and (16) respectively in which the coefficient  $\beta$  is given as (17), and  $g_{m+a}$  is the effective air gap length, normally the sum of magnet thickness  $g_m/\mu_r$  and air gap length  $g_a$ .

$$P_0 = \frac{\mu_0}{g_{m+a}} (1 - 1.6\beta c_0) \quad (15)$$

$$= \frac{\mu_0}{g_{m+a}} (1 - 0.8\beta) \quad (\text{when } c_0 = 0.5)$$

$$P_1 = \frac{\mu_0}{g_{m+a}} \frac{2\beta}{\pi} \left( \frac{0.39}{0.39 - c_0^2} \right) \sin(1.6\pi c_0) \quad (16)$$

$$\approx 1.04 \frac{\mu_0}{g_{m+a}} \beta \quad (\text{when } c_0 = 0.5)$$

$$\beta = \frac{1}{2} - \left\{ \sqrt{4 + \left( \frac{o}{g_{m+a}} \right)^2} \right\}^{-1} \quad (17)$$

$$= \frac{1}{2} - \left\{ \sqrt{4 + \left( \frac{\pi D_g}{2 Q_{fmp} g_{m+a}} \right)^2} \right\}^{-1} \quad (\text{when } c_0 = 0.5)$$

Especially, the characteristic of  $\beta$  vs  $o/g_{m+a}$  is depicted in Fig. 6. It shows that  $\beta$  increases up to 1/2 as the magnet thickness decreases. However, it is realistic to take around 3~5 for  $o/g_{m+a}$  considering the demagnetization of magnet.

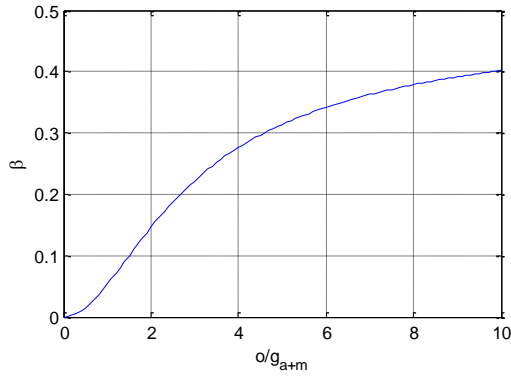


Fig. 6. The characteristic of  $\beta$  vs  $o/g_{a+m}$

Multiplying  $F_m$  of (13) and  $P_g$  of (14) and neglecting the terms having the small magnitude and speed, the airgap flux density is approximated as (18).

$$B_g \approx F_{1m} P_0 \cos p_m (\theta - \theta_m) - \frac{F_{1m} P_1}{2} \cos(p_w \theta + p_m \theta_m) \quad (18)$$

Applying Faraday's law with (18), the back EMF of the concentrated winding with  $N_{ph}$  turns per phase can be obtained as

$$e_b = \frac{N_{ph} D_g l_{stk}}{2} \frac{d}{dt} \int_0^{2\pi} B_g d\theta \quad (19)$$

$$= \sqrt{2} E_b \cos(\omega_e t + \varphi)$$

where  $D_g$  is the airgap diameter, and  $l_{stk}$  is the core stack length and the rms value  $E_b$  is given as

$$E_b = \sin\left(\frac{1}{3}\pi\right) \frac{\omega_e N_{ph} D_g l_{stk}}{p_m \sqrt{2}} F_{1m} \left(P_0 + \frac{G_r}{2} P_1\right) \quad (20)$$

From (20), it is seen that  $P_1$  due to the slot harmonic contributes back EMF production, which is called the vernier effects. Using (15) and (16), the back EMF of (20) is rearranged as

$$E_b = \frac{\sqrt{6}}{\pi} \frac{B_r}{\mu_r} \frac{g_m}{g_{m+a}} N_{ph} D_g l_{stk} \{1 + (0.52G_r - 0.8)\beta\} \omega_m \quad (21)$$

which shows well the vernier effects telling that as  $G_r$  and  $\beta$  are larger, the back EMF increases.

### B. Synchronous Reactance Equation

The synchronous reactance  $X_{syn}$  consists of both the air gap and the leakage reactance, and the inductance of the vernier machine is fundamentally same to that of conventional machine. The air gap inductance  $L_g$  per phase is dependent on the number of winding turns  $N_{ph}$  and the air gap reluctance. The gap inductance of the winding on a tooth can be given as

$$L_{tooth} = \left(\frac{N_{ph}}{Q_s}\right)^2 \left(\frac{3}{2} \mathcal{R}_{g.slot}\right)^{-1} \quad (22)$$

in which  $\mathcal{R}_{tooth}$  is the magnetic reluctance of airgap under one slot pitch area and is given as

$$\mathcal{R}_{tooth} = \frac{g_{m+g}}{\mu_0} \frac{Q_s}{\pi D_g l_{stk}} \quad (23)$$

Now the air gap inductance per phase for a machine with concentrated windings is given by [33]

$$L_{gap} = \frac{Q_s}{3} L_{tooth} = 2\pi\mu_0 \left(\frac{N_{ph}}{Q_s}\right)^2 \frac{D_g l_{stk}}{g_{m+g}} \quad (24)$$

The major part of leakage inductance of PM machines is the slot leakage and it depends on the slot geometries. For an instance, the slot shape of vernier machine with  $n_{split}=2$  and the leakage flux are illustrated in Fig. 7, and the slot leakage inductance per phase is given as

$$L_{slot} \approx 4\mu_0 \left(\frac{N_{ph}}{Q_s}\right)^2 l_{stk} Q_s \left(\frac{d}{o} + \frac{h}{3w}\right) \quad (25)$$

It is realistic to take the ratio of  $h$  and  $w$  in Fig.7 as 2 for a PM machine with concentrated windings. Since the slot open width  $o$  is  $0.5\pi D_g/(n_{split}Q_s)$ , the width  $w$  can be taken as  $n_{split}o$  for the room of stator windings, and the height  $h$  becomes  $\pi D_g/Q_s$ . Assuming the height of slot shoe  $d$  is same to the slot opening  $o$ , the slot leakage inductance per phase of (25) becomes

$$L_{slot} \approx \pi\mu_0 \left(\frac{N_{ph}}{Q_s}\right)^2 \left(\frac{10}{3} \frac{D_g}{n_{split}o}\right) l_{stk} \quad (26)$$

As mentioned above,  $o/g_{m+a}$  is around 3~5, in that case, the slot inductance with  $n_{split}=2$  or 3 is roughly estimated as  $L_{slot} \approx 1/3 L_{gap}$ . Then, the total leakage inductance considering end-turn leakage inductance  $L_{end}$  is approximated as

$$L_{leakage} = L_{slot} + L_{end} \approx \frac{1}{2} L_{gap} \quad (27)$$

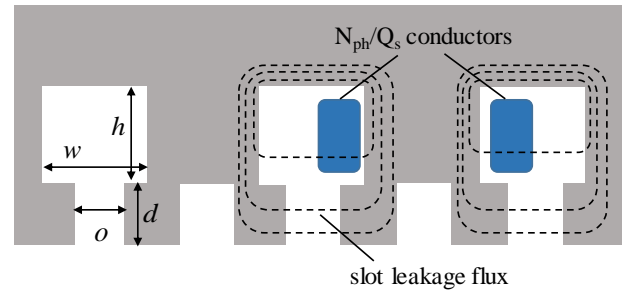


Fig. 7. The sketch of slot leakage with concentrated windings

Consequently, the synchronous reactance is approximated as

$$X_{syn} \approx \frac{3}{2} \omega_e (L_{gap} + L_{leakage}) \quad (28)$$

$$= \frac{9\pi}{2\mu_0} \left(\frac{N_{ph}}{Q_s}\right)^2 \frac{D_g l_{stk}}{g_{m+g}} p_m \omega_m$$

It should be noted that  $X_{syn}$  of (28) includes  $p_m$ , and  $p_m$  is proportional to  $G_r$  from (12), resulting that  $X_{syn}$  is directly proportional to  $G_r$ . Comparing the reactance  $X_{syn}$  with the back EMF  $E_b$  of (21), it is apparent that  $X_{syn}$  increases much more than  $E_b$  as  $G_r$  increases, causing worse power factor. Alternatively, due to the relation  $G_r = n_{split} - 1$ , it is common the power factor gets excessively poor when  $n_{split}$  is larger than 3 [6][23].



### C. Additional Relations Considering Surface Current Density limitation

The center of the voltage limit  $C_v$  which is  $E_b/X_{syn}$  is obtained by dividing (21) by (28), and is expressed as

$$C_v = K_{EX} \frac{B_r}{\mu_r} \frac{g_m}{N_{ph}} \frac{P_w}{(n_{split} - 1/3)} \{1 + (0.52G_r - 0.8)\beta\} \quad (29)$$

where the coefficient  $K_{EX}$  is  $2\sqrt{6}/(3\pi^2\mu_0)$ .

Due to the allowable temperature rise of the electrical machines, it is common to set the proper maximum surface current density  $K_s$  depending on the cooling capacity. The relation between  $K_s$  and the maximum current  $I_{max}$  is as follow.

$$I_{max} = \frac{\pi D_g K_s}{6N_{ph}} \quad (30)$$

Therefore, the variable  $\gamma$  which is  $C_v/I_{max}$  is also given by

$$\gamma = \left( \frac{6}{\pi} K_{EX} \frac{B_r}{\mu_r} \frac{1}{K_s} \right) \frac{P_w}{(n_{split} - 1/3)} \frac{g_m}{D_g} \{1 + (0.52G_r - 0.8)\beta\} \quad (31)$$

### IV. DETERMINATION OF GEOMETRIES OF A PMVG

As described in Section II, in order to achieve the performance requirements of the wind turbine,  $\gamma \leq \gamma_{max}(=2)$  must be satisfied, and the range of circuit parameters such as  $X_{syn,base}$  and  $E_{b,base}$  for given  $\gamma$  were specified in Fig. 4. In Section III, the various correlations between the geometric variables and the circuit constants were provided. Especially, it should be noted from (31) that for a given  $\gamma$ , once the magnet material is determined and  $K_s$  is specified as a typical value, the variables left to be determined for design are  $D_g$ ,  $p_w$ ,  $g_m$ ,  $n_{split}$ , and  $g_a$  because  $G_r$  and  $\beta$  depend on  $D_g$ ,  $n_{split}$ , and  $g_a$ .

Thus, in this study, NdFeB is selected for the PM material whose residual flux density  $B_r$  is 1.1T and  $\mu_r$  is unity. In addition, empirically 30kA/m is chosen as the surface current density  $K_s$  [34]. However, for a given  $\gamma$ , the number of circuit constants which are  $X_{syn,base}$  and  $E_{b,base}$  is still much less than that of the geometric parameters which are  $D_g$ ,  $p_w$ ,  $g_m$ ,  $n_{split}$ , and  $g_a$ , it is impossible to directly determine the geometries from the electrical parameter value.

In this section, it will be shown that using reasonable mechanical and electrical constraints, the number of shape variables can be reduced, and the most favorable structure is determined among the various possible design candidates.

Since magnet material and surface current density are determined,  $\gamma$  of (31) can be reduced as (32) where  $K_\gamma$  is a constant and  $G_r$  is replaced with  $3n_{split}-1$ .

$$\gamma = K_\gamma \frac{\{1 + (1.56n_{split} - 1.32)\beta\}}{(n_{split} - 1/3)} \frac{p_w g_m}{D_g} \quad (32)$$

It is seen that  $\beta$  of (17) appears in (32). On the other hand, since the magnet thickness  $g_{m+a}$  in (17) is slightly greater than  $g_m$ , it can be replaced with  $g_m/0.9$ , and using  $Q_{fmp}=3n_{split}p_w$ ,  $\beta$  is represented as

$$\beta \approx \frac{1}{2} - \left\{ 4 + \left( \frac{3\pi}{20} \frac{1}{n_{split}} \frac{D_g}{p_w g_m} \right)^2 \right\}^{-1} \quad (33)$$

Now, it should be noted that if the term  $D_g/(p_w g_m)$  in (32) and (33) is set as  $X$ , the variable  $\gamma$  becomes the function of only  $n_{split}$  and  $X$ . Therefore,  $X(=D_g/(p_w g_m))$  can be calculated by solving (33) for given  $\gamma$  and  $n_{split}$ .

On the other hand, replacing the term  $N_{ph}$  in the back EMF of (21) with that of (30), the following (34) is obtained where  $K_E$  are the constant determined by  $B_r$  and  $K_s$ .

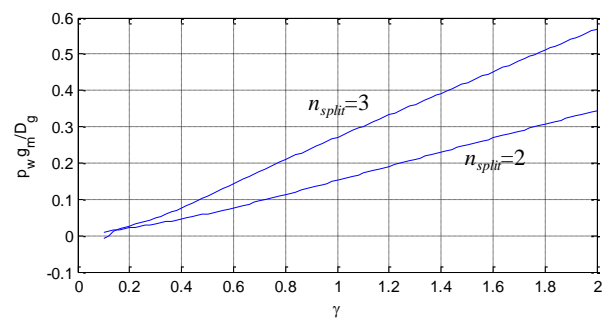
$$E_b = \frac{K_E}{I_{max}} D_g^2 l_{stk} \left\{ 1 + (1.56n_{split} - 1.32)\beta \right\} \omega_m \quad (34)$$

Also, as shown in Fig. 4, the back-EMF  $E_{b,base}$ , the required maximum current  $I_{max}$ , and the variable  $\beta$  as described above are all a function of  $\gamma$ . Therefore, the equation (34) can be arranged for  $D_g^2 l_{stk}$  meaning the air gap volume, as (35) which means the airgap volume can be determined by  $\gamma$ .

$$D_g^2 l_{stk}(\gamma) = \frac{E_{b,base} I_{max}}{K_E \{1 + (1.56n_{split} - 1.32)\beta\} \omega_{m,base}} \quad (35)$$

The above is summarized as follows;  $D_g/p_w g_m(=X)$  and  $\beta$  are obtained for available  $\gamma$  using the nonlinear equation (32), and it can be solved with a numerical method such as the simple iterative method [35]. Since  $E_{b,base}$  and  $I_{max}$  are known for a given  $\gamma$  as shown in Fig. 4, one can calculate  $D_g^2 l_{stk}$  using (35), which is very useful information in the design. Fig. 8 represents the calculation results of  $p_w g_m/D_g$  and  $D_g^2 l_{stk}$  in  $\gamma \leq \gamma_{max}=2$  through this method. It shows the necessary volume of the generator with  $n_{split}=2$  is less than that with  $n_{split}=3$  for the region of  $0.4 \leq \gamma$ . When  $\gamma$  is less than 0.4, the volume with  $n_{split}=3$  can be smaller, but the power factor will be severely poor as described in Fig. 4. Hence, in this study,  $n_{split}$  is determined as 2.

From the volume characteristics of  $n_{split}=2$  in Fig. 8(b), it can be also seen that the smaller the value of  $\gamma$  is, the smaller the air gap volume is. This tendency is opposite to the characteristics of  $\gamma$  and the inverter capacity requirement shown in Fig. 4(a), which means that a compromise is required. In other words, the value of  $\gamma$  needs to be strategically determined according to various actual conditions such as the price of the converter and the volume of the generator. In this study,  $\gamma$  was set to 0.6, focusing on the volume of the wind turbine rather than the converter capacity.



(a)  $p_w g_m/D_g$  vs  $\gamma$

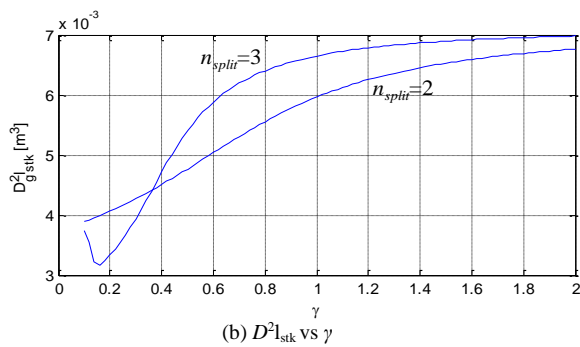
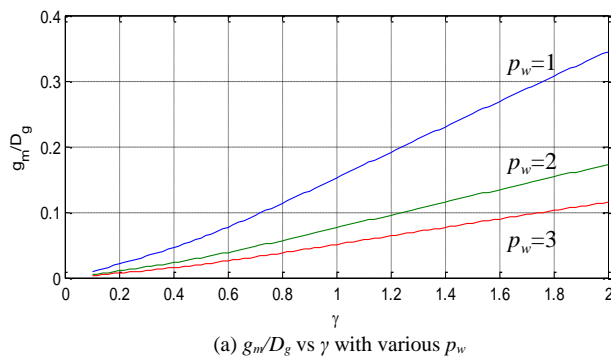
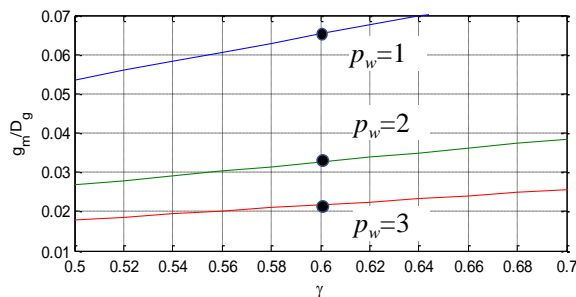


Fig. 8. Geometric variables with  $\gamma$

The characteristics of  $p_w g_m / D_g$  vs  $\gamma$  of Fig. 8(a) is also very useful in the machine design. Since the winding pole pair  $p_w$  is an integer, the value of  $g_m / D_g$ , that is, the ratio of the magnet thickness to the air gap diameter, can be obtained for  $p_w$  from 1 to 3, and the results are depicted in Fig. 9(a). From the results, it can be seen that the larger  $p_w$  is, the smaller the thickness of the magnet is, and especially when  $p_w$  is 1, the thickness of the magnet becomes excessively thick. Since  $\gamma$  is selected as 0.6, the characteristics of  $g_m / D_g$  around the value are redrawn in Fig. 9(b) for convenient investigation. As shown in the figure, when  $p_w$  is 3, the thickness of the magnet is the smallest, but when the magnet is excessively thin, there is a high possibility of demagnetization. Therefore, to avoid demagnetization,  $p_w$  is chosen as 2, and  $g_m / D_g$  becomes 1/30 from Fig. 9(b). At the same time, the value of  $D_g^2 l_{stk}$  is  $5 \cdot 10^{-3} \text{m}^3$  from Fig. 8(b), and the magnet pole pair  $p_m$  is 10 from (12).



(a)  $g_m / D_g$  vs  $\gamma$  with various  $p_w$



(b)  $g_m / D_g$  with various  $p_w$ , around  $\gamma=0.6$

Fig. 9. Ratio of  $g_m$  and  $D_g$  vs  $\gamma$  with various  $p_w$

For the DD machinery with outer rotor, it is beneficial to have a diameter longer than a stack length, and by setting  $l_{stk} / D_g$  as 1/2,  $D_g$  and  $l_{stk}$  can be determined respectively by using the obtained value of  $D_g^2 l_{stk}$ . Thus, the magnet thickness is obtained from the value of  $g_m / D_g$  and the airgap length is also from the selected value 0.9 for  $g_m / g_{m+a}$ . Furthermore, using (28), the number of turn per phase  $N_{ph}$  is calculated. Now, it can be said that the major geometries of the stator and

the rotor are almost determined. In table II, the circuit constants of the generator required to satisfy the specified performances when  $\gamma$  is 0.6, and the shape dimensions determined by the proposed method are summarized.

TABLE II. CALCULATED PARAMETER VALUES WHEN  $\Gamma=0.6$

Parameter	Value	Parameter	Value
Back EMF $E_{b,base}$	66.3V	Wind. pole pairs $p_w$	2
Syn. react $X_{sys,base}$	4.27 $\Omega$	Magnet pole pairs $p_m$	10
Max. current $I_{max}$	25.2A	Airgap diameter $D_g$	205mm
$n_{split}$	2	Stack length $l_{stk}$	103mm
Phase turns $N_{ph}$	64	Flux mod. pole $Q_{fmp}$	12
Main slots $Q_s$	6	Airgap length $g_a$	0.8mm
Mag. thickness $g_m$	6.8mm		

The shapes that have not yet been determined are only the stator slots and the yoke of the stator and rotor, and they can be easily designed using the classical methods. Regarding the shape of the slot, the cross-sectional area of the conductor by is calculated appropriately setting the volume current density of the conductor by the maximum volume current, and then multiplying the number of conductors per slot  $2N_{ph}/Q_s$ , and the slot occupancy. As mentioned above, since the slot opening ratio  $c_0$  is 0.5, the slot width is calculated from the airgap diameter, and the slot depth is obtained from the calculated slot area. To design the stator and rotor yoke, the magnetic flux density was calculated using the equivalent magnetic circuit of the determined generator, and then the thickness of the yoke of the outer rotor is determined to achieve the magnetic flux density of it as 1.2T. At this time, the influence of the main flux and modulation flux of the vernier motor on the yoke magnetic flux density are considered [24]. Finally, the shape of the determined outer rotor PM vernier generator is shown in Fig. 10.

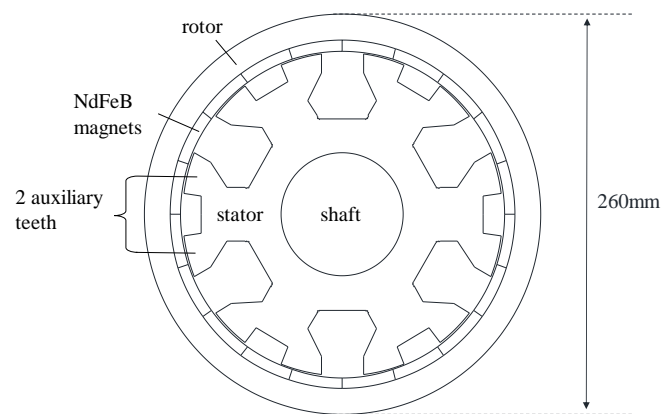


Fig. 10. Designed structure of outer rotor PM vernier generator

## V. SIMULATION AND EXPERIMENTAL RESULTS

To prove the validity of the proposed design procedure and the derived equations, the performance characteristics of the designed DD vernier generator of Fig. 10 are analyzed by FEM.

First, the flux distribution and the phase back EMF characteristics without armature currents are calculated at the base speed 214rpm and illustrated in Fig. 11. It shows that the

flux density of teeth is around 1.5T and the density of yokes is lower than 1.2T, meaning that the core parts are properly designed to avoid saturation. Fig. 12 depicts the back-EMF waveforms of three-phase windings, and the rms value estimated from the waveform is 70.5V which is very close to the expected value 66.3V

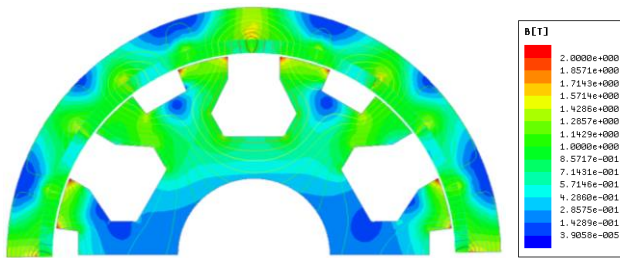


Fig. 11. Flux line and flux density without load currents

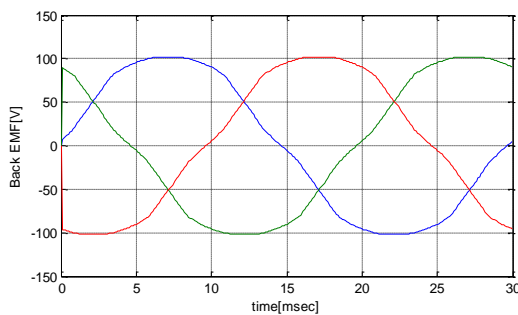


Fig. 12. Back EMF at 214 rpm (FEA results)

Second, to calculate the generator's synchronous reactance  $X_{syn}$ , the steady-state currents are calculated at the base speed by the FEM with three-phase windings shorted, which are depicted in Fig. 13. These short currents depends on the back EMF and the synchronous reactance, and to be exact, is given by the relation of  $I_{short} = E_b / X_s$ . The rms value of the steady state current estimated from the analysis results is about 17.7A. Recalling the back EMF already obtained from FEM is 70.5V, the synchronous reactance is estimated as  $4\Omega (=70.5V/17.7A)$  which is also almost the same as the predicted value of  $4.2\Omega$  shown in table II. As a result, it can be said that the proposed correlations between the geometries and parameters of the vernier PM machine are very accurate, and the machine design method using the relations are also valid.

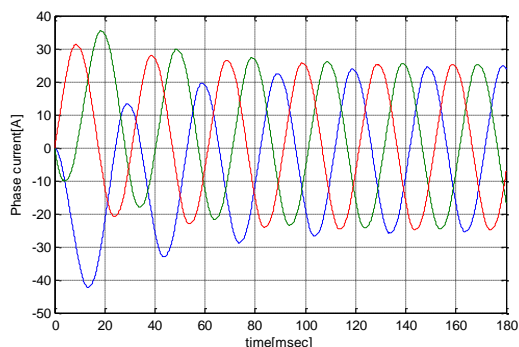
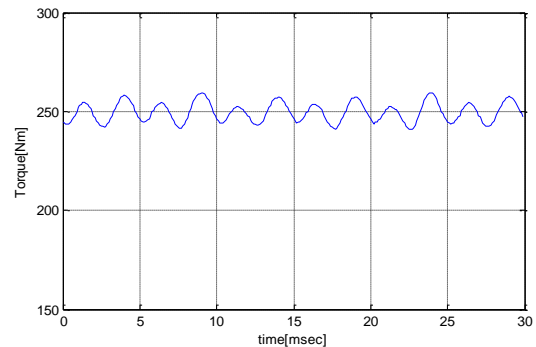


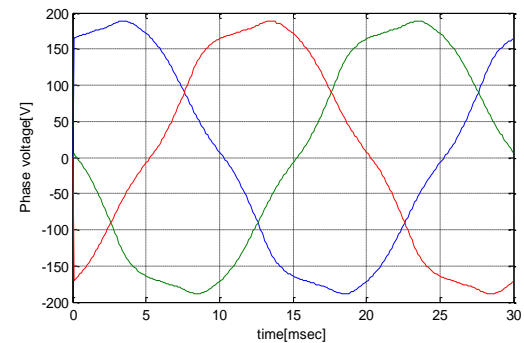
Fig. 13. Phase currents with shorted windings at 214rpm (FEA results)

Next, in order to check the output torque characteristics at the base speed, when the maximum phase current  $I_{max}$  25.2A in phase with the back EMF is supplied, the torque and the induced voltage of the winding are analyzed with finite element method. Fig. 14(a) shows the waveform of the torque

whose average value is about 249.6Nm close to the design target of 236 Nm for 5 kW. Furthermore, the torque ripple is about 5% which is good enough. Finally, Fig. 14(b) shows the voltage induced in the winding, which refers to the phase voltage to obtain the same current. The effective value of the voltage is 133.2V, which shows good agreement with the design target of  $220/\sqrt{3}$  ( $=127V$ ). It also means that even if the armature current is applied, the estimated values of the reactance and the back EMF are still valid with little saturation effects.



(a) Torque waveforms



(b) Induced voltage of phase windings

Fig. 14. Load torque and induced voltage characteristics of the designed PM vernier generator (FEA results)

In table III, the simulated values of back EMF, synchronous reactance, the phase voltage and average torque are compared with the predicted values in design procedure, showing very good agreements.

TABLE III. REQUIRED AND OBTAINED CHARACTERISTICS OF THE DESIGNED SPM VERNIER GENERATOR AT THE BASE SPEED(213RPM)

Characteristics	Required	FEA	Measured	Err. (%): Exp./Mes.
Torque	236.2Nm	249.6Nm	235.2Nm	5.6
Back EMF	66.3V	70.5V	70.6V	6.3
Syn. reactance	4.2Ω	4.0 Ω	3.91 Ω	4.7
Phase voltage	127V	133.2V	133.0V	4.7

The designed vernier generator with outer-rotor was actually manufactured. Fig. 15 shows the stator and the rotor assembled respectively, and Fig. 16 shows the set-up for experiment. First, the back EMF was measured at the base speed of 214 rpm and the phase voltage waveforms are represented in Fig. 17. The rms value of the back EMF measured is about 71 V, which is very similar to the design value of 66.3 V as well as the finite element analysis results



in Table 1. Next, to estimate the synchronous reactance of the vernier generator, the short-circuit current was measured at the base speed, and the measured current waveforms are shown in Fig. 18. The maximum value 25A of the measured currents is also similar to that of the finite element analysis in Fig. 13. As a result, the estimated synchronous reactance is almost the same as the design value of 4.2  $\Omega$ .

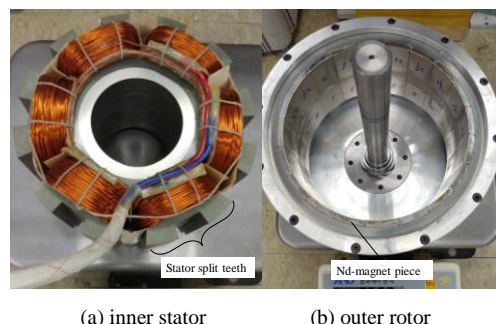


Fig. 15. Manufactured stator and rotor.

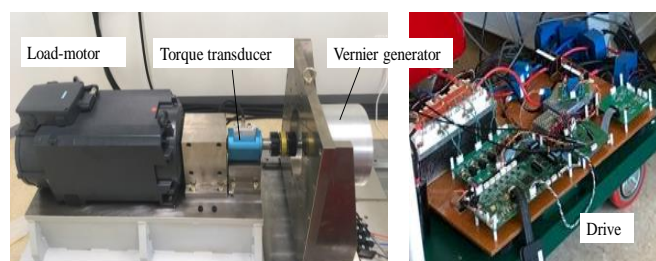


Fig. 16. Set-up for experimental measurement set



Fig. 17. Phase back EMF at the base speed 214rpm (experimental)

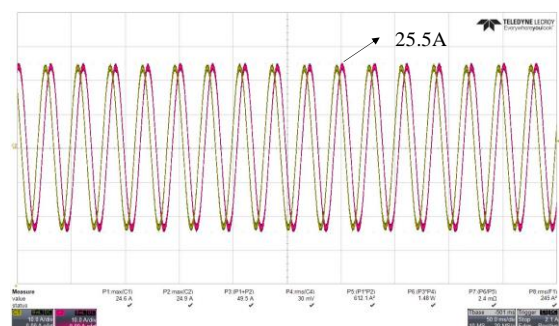


Fig. 18. Phase currents with shorted windings at 214rpm (experimental)

Finally, the maximum torque performances for various speeds were tested with the DC link voltage of 340V under MTPA control, as shown in Fig. 19. The torque values at the

base speed 213rpm and the maximum speed 356rpm are 235.2Nm and 125.0Nm, respectively. When the measured torque is compared with the design required value, about 10% error occurred at high speed due to mechanical friction and the voltage drop of the cable between the inverter and the generator, but it is considered to be in general good agreement. The efficiency is also excellent, about 90% at the base speed. In addition, it is noteworthy that the torque per air gap volume is very high, 69.2kNm/m<sup>3</sup>. Consequently, it is confirmed that the actual PM vernier generator with very high torque density was designed whose characteristics are very close to the predictions through the proposed method.

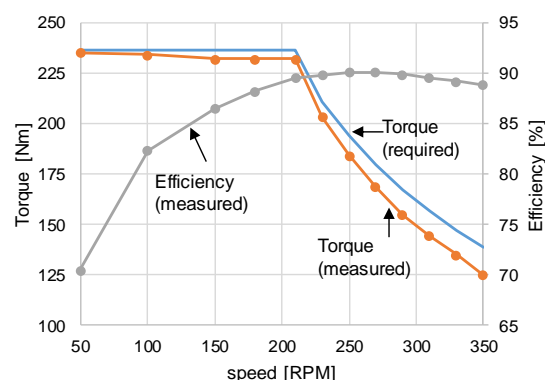


Fig. 19. Torque and efficiency characteristics (experimental)

## VI. CONCLUSION

To improve the problems of the repetitive performance calculation and the resulting inaccuracy in the conventional design procedure of variable-speed electrical machines, this paper proposes a novel design method which is mathematically clear and thus direct and accurate. As a case study, a 5kW direct-drive, outer-rotor PM vernier generator for wind turbine system was designed using the proposed method. The proposed method shows the direct design procedure from choosing the operational speeds of the rotor such as the base and the cut-off speeds to determining the geometries of the generator in details.

The results of this study can be summarized as follows. It has been shown that there are various combinations of circuit constants that satisfy the torque performance required for a wind turbine when a PM generator is represented by a regular electric equivalent circuit. In other words, there may be generators of various different structures. The circuit constant equations expressed in terms of the geometrical structure of the concentrated winding PM vernier generator were derived in consideration of the magnetic gear effects. We have successfully performed a reverse design to determine the shape from the circuit constants by assigning some constraints and reasonable assumptions such as the ratio of the surface current density, the ratio of the gap to the magnet thickness. Then, the reverse design to determine the geometries of a PM vernier generator from the circuit constants has been successfully performed by assigning some constraints and reasonable assumptions such as the ratio of the surface current density, the ratio of the gap to the magnet thickness.

From the design results of the 5 kW DD vernier generators, it was found that 1) the power factor sharply deteriorates as the generator volume decreases, and 2) the generator volume depends on the number of auxiliary teeth of the vernier

generators. In this study, it was found that two auxiliary teeth are advantageous for the generator volume. In addition, 3) The number of stator winding poles is closely related to the thickness of the magnets, and therefore the determination of the proper number of winding poles is needed to avoid demagnetization of PM.

By analyzing and testing the performance of the designed DD vernier generator, the following results were obtained. From the FE analysis results, it was confirmed that the back EMF and the synchronous reactance value agree with the predicted value within about 5% error range. It is found that the maximum torque and output of the generator at the base speed are almost similar to 5 kW under the maximum current supply condition. The accuracy of the proposed method was also verified by confirming that the generated voltage at this condition had an error within 5% comparing to the design specification. Finally, we measured the back EMF and the synchronous reactance by testing the vernier generators actually manufactured and confirmed that they are in good agreement with the predicted values at the design stage. In conclusion, it is expected that the proposed systematic design method connecting the circuit constant, required characteristics and geometrical shape of the machine can be very useful for designing various variable speed electrical machines including vernier generators.

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