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Heart disease diagnosis based on mediative fuzzy logic

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ABSTRACT

Mediative fuzzy logic is an approach able to deal with inconsistent information providing a solution when contradiction exists. The aim of this paper is to design an expert system based on this type of fuzzy logic in order to diagnose a possible heart disease for a patient. Our proposed system is an extension of the standard Mamdani fuzzy logic controller and contains 44 rules of the type single input–single output. The system works with 11 variables as inputs and one variable as output.

1. Introduction

Uncertainty appears in different forms and affects decisions making. Frequently, information may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way [1]. Much of this uncertainty can be handle using Fuzzy logic type 1 [2], Fuzzy logic type 2 [3,4] or Intuitionistic fuzzy logic [5,6].

The theory of fuzzy logic provides a mathematical theory to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. The conventional approaches to knowledge representation lack the means for representing the meaning of fuzzy concepts. As a consequence, the approaches based on first order logic do not provide an appropriate conceptual framework for dealing with the representation of commonsense knowledge, since such knowledge is by its nature both lexically imprecise and non-categorical. The development of fuzzy logic was motivated in large measure by the need for a conceptual framework which can address the issue of lexical imprecision.

Intuitionistic fuzzy sets (IFSs), proposed [5–7] as a generalization of the traditional fuzzy sets introduced by Zadeh [2], have the property to incorporate the uncertainty of the information. The IFSs offer a new possibility to represent imperfect knowledge and, therefore, to describe in a more adequate way many real problems. Such problems appear when we face with human opinions involving two or more answers of the type: "Yes", "Not", "I do not know", "I am not sure", etc.

Many applications have highlighted the superiority of intuitionist fuzzy logic compared to traditional fuzzy logic; in the following we mention some of them. In [8] a new approach for fuzzy inference in intuitionistic fuzzy system applied to monitoring a non-linear dynamic plant were described. In [9] an optimized method to reduce the points number to be used in order to identify a person using fuzzy fingerprints is described. The design of an intuitionistic fuzzy logic controller for heater fans on the basis of intuitionistic fuzzy systems were presented in [10]. In [11] an intuitionistic fuzzy logic controller to determine washing time for a washing machine is developed. Because the conventional traffic controller is unable to solve a set of problems (for instance, the congestion at the intersection) in [12] an intelligent transport system based of intuitionistic fuzzy logic is proposed. In [13] a method to construct type-1 intuitionistic fuzzy inference that is able to handle more uncertainty than type-1 fuzzy inference system and performs faster than a type-2 fuzzy inference system were described. The results show that the intuitionistic fuzzy inference system performs better than other methods. Many works have been dedicated to improving the architectures and theory used in the construction of control systems. For instance, in [14] a new general procedure is proposed to construct the membership and non-membership functions of the fuzzy reliability using time-dependent intuitionistic fuzzy sets and in [15] an approach for graphically representing intuitionistic fuzzy sets for their use in Mamdani fuzzy inference systems were proposed. The importance of intuitionistic fuzzy sets is explained, also, in [16] was proved that any type-2 fuzzy set $\hat{A}$ can be represented by intuitionistic fuzzy set A_g^*.

Some questions appear. What happens if the knowledge base changes with time, and non-contradictory information becomes into doubtful or contradictory information, or any combination of these three situations? What inference system can be used? Mediative fuzzy logic, presented for first time in Montiel et al. [17], is a novel approach which is able to deal with kind of inconsistent information providing a common sense solution when contradiction exists; this is a mediate solution. Mediative fuzzy logic can be reduced to intuitionistic and traditional fuzzy logic, depending on how the membership functions (agreement or non-agreement) are established.

As is specified in [18], there is a lot of applications where information is inconsistent. In economics, for estimating the gross

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domestic product it is possible to use different variables and for estimating the exportation rates it is necessary to use a combination of different variables. In forecasting prediction, uncertainty always appears, because to obtain a reliable prediction it is necessary to have a number of decisions, each one based on a different group.

In medicine, information from experiments can be somewhat inconsistent because patients being might respond different to some experimental medication. The contradictory results from multiple clinical trials are attributed either to methodological deficiencies in the design of one of the trials or to small sample sizes [19].

After the introduction of mediative fuzzy logic, a number of works have demonstrated the benefits of its use [18,20–22]. In [21] mediative fuzzy logic were used with promising results for solving nonlinear optimization problems avoiding the problem of cycling, in [18] the results of mediative fuzzy logic were compared with those given by intuitionistic fuzzy logic and traditional fuzzy logic and the conclusion is: mediative fuzzy logic reflects contradictory knowledge at the system output and it can gives a softener transition when we have hesitation and contradiction fuzzy sets. In [22] mediative fuzzy logic is used to construct an intelligent method for controlling population size in evolutionary algorithms. Experimental results gave a significantly reduction in time and an improvement in precision.

In this paper we propose a reasoning system based on mediative fuzzy logic applied in a diagnosis medical problem. Further, this paper is organized as follows. In the second section we present the basic concepts concerning the mediative fuzzy logic; all these notions will be used in the next sections. In Section 3 we modify the standard Mamdani fuzzy logic controller in order to be used in mediative fuzzy logic. The Section 4 is devoted to application of the proposed system to heart disease diagnosis. The conclusions are discussed in Section 5.

2. Mediative fuzzy logic

A traditional fuzzy set in X is given as a set $A = \{(x, \mu_A(x))/x \in X\}$ [2]. In [5] is defined the notion of intuitionistic fuzzy set as follows:

Definition 1. An intuitionistic fuzzy set A in X is defined as

$$A = \{(x, \mu_A(x), \nu_A(x))/x \in X\} \tag{1}$$

where $\mu_A, \nu_A : X \to [0, 1]$ satisfy the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1 \quad \forall\, x \in X. \tag{2}$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and non-membership of x to A, respectively. Obviously, a fuzzy set A corresponds to the following intuitionistic fuzzy set $A = \{(x, \mu_A(x), 1 - \mu_A(x)), x \in X\}$. For each intuitionistic fuzzy set A in X,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{3}$$

is called the intuitionistic fuzzy index of x in A; it is a hesitancy degree of x to A [5–7] and satisfies the inequality

$$0 \le \pi_A(x) \le 1 \quad \forall\, x \in X. \tag{4}$$

Therefore, if we want to describe an intuitionistic fuzzy set we must use any two functions from the triplet: (membership function, non-membership function, intuitionistic fuzzy index).

Fuzzy inference in intuitionistic case implies working with the membership function μ and the non-membership function ν. Hence, the output of an intuitionistic fuzzy system can be calculated as follows [23]:

$$\text{IFS} = (1 - \pi)^*\text{FS}_\mu + \pi^*\text{FS}_\nu \tag{5}$$

where FS_μ is the traditional output of a fuzzy system using the membership function μ, and FS_ν is the output of the fuzzy system using the non-membership function ν. For $\pi = 0$ in the last relation, the IFS is reduced to the output of a traditional fuzzy system.

A contradiction fuzzy set C in X is given by

$$\zeta_C(x) = \min(\mu_C(x), \nu_C(x)). \tag{6}$$

In this case, the functions μ_C and ν_C are named agreement membership function and agreement non-membership function, respectively; as is specified in [18], these names are more adequate for contradictory fuzzy sets. In order to compute the system's output, Montiel et al. [18] proposed the following expressions:

$$\text{MFS} = \left(1 - \pi - \frac{\zeta}{2}\right)^*\text{FS}_\mu + \left(\pi + \frac{\zeta}{2}\right)^*\text{FS}_\nu \tag{7}$$

$$\text{MFS} = \min\left(((1 - \pi)^*\text{FS}_\mu + \pi^*\text{FS}_\nu),\ 1 - \frac{\zeta}{2}\right) \tag{8}$$

$$\text{MFS} = ((1 - \pi)^*\text{FS}_\mu + \pi^*\text{FS}_\nu)^*\left(1 - \frac{\zeta}{2}\right). \tag{9}$$

In the case when the contradictory index ζ is equal to zero, the system's output is reduced to an intuitionistic fuzzy output.

3. Knowledge representation

The knowledge base of a rule-based system may contain imprecisions which appear in the description of the rules given by the expert. In order to represent imprecise values, the majority of applications work with intuitionistic fuzzy numbers which was proposed by Burillo et al. [24], but various definitions were proposed in the literature.

Definition 2 (*[25]*). An intuitionistic fuzzy subset

$A = \{(x, \mu_A(x), \nu_A(x))|x \in \mathbb{R}\}$

of the real line is called intuitionistic fuzzy number if:

(a) A is if-normal: there exists $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_0) = 0$.
(b) A is if-convex: its membership function is fuzzy convex $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \le min(\mu_A(x_1),\ \mu_A(x_2))\ \ \forall\, x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$ and its non-membership function is fuzzy concave $\nu_A(\lambda x_1 + (1 - \lambda) x_2) \ge max(\nu_A(x_1), \nu_A(x_2))\ \forall\, x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$.
(c) μ_A and ν_A are continuous by parts.

For an easy implementation, majority applications work with trapezoidal intuitionistic fuzzy numbers.

Definition 3. An intuitionistic fuzzy number denoted as $A = \{(x, \mu_A(x), \nu_A(x))|x \in \mathbb{R}\}$ is given by

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \frac{x - a_4}{a_3 - a_4} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{for } a_4 < x \end{cases}$$

$$\nu_A(x) = \begin{cases} 1 & \text{if } x < b_1 \\ \frac{x - b_2}{b_1 - b_2} & \text{if } b_1 \le x \le b_2 \\ 0 & \text{if } b_2 \le x \le b_3 \\ \frac{x - b_3}{b_4 - b_3} & \text{if } b_3 \le x \le b_4 \\ 1 & \text{if } b_4 < x \end{cases}$$

where $b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4 \in \mathbb{R}$ and $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$.

$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$.

The rule

if X is A then Y is B

is represented by its conditional possibility distribution $\pi_{Y/X}$ defined as follows [26,27]

$$\pi_{Y/X}(v, u) = \mu_A(u) \to \mu_B(v), \quad \forall\, u \in U, \quad \forall\, v \in V$$

where $\to$ is an implication operator, μ_A and μ_B are the membership functions of the fuzzy sets A and B, respectively, U is the domain of X and V is the domain of Y.

Definition 4. A fuzzy implication is a function $I: [0, 1]^2 \to [0, 1]$ satisfying the following conditions, for all values $x, y, z \in [0, 1]$:

I1: If $x \le z$ then $I(x, y) \ge I(z, y)$.
I2: If $y \le z$ then $I(x, y) \le I(x, z)$.
I3: $I(0, y) = 1$ (falsity implies anything).
I4: $I(x, 1) = 1$ (anything implies tautology).
I5: I(1,0) = 0 (Booleanity).

The following properties for fuzzy implication are required by different authors and they have their significance in various applications:

I6: $I(1, x) = x$ (tautology cannot justify anything) for all $x \in [0, 1]$.
I7: $I(x, I(y, z)) = I(y, I(x, z))$ (exchange principle) for all $x, y, z \in [0, 1]$.
I8: $x \le y$ if and only if $I(x, y) = 1$ (implication defines ordering) for all $x, y \in [0, 1]$.
I9: $I(x, 0) = N(x)$ for all $x \in [0, 1]$ is a strong negation.
I10: $I(x, y) \ge y$ for all $x, y \in [0, 1]$.
I11: $I(x, x) = 1$ (identity principle) for all $x \in [0, 1]$.
I12: $I(x, y) = I(N(y), N(x))$ for all $x, y \in [0, 1]$ and a strong negation N.
I13: I is a continuous function.

Czogala and Leski [28] analyzing a set of eight implications (Kleene-Dienes, Reichenbach, Lukasiewicz, Godel, Rescher-Gaines, Goguen, Zadeh, Fodor) concluded that the Lukasiewicz implication

$$I_L(x, y) = \min(1 - x + y, 1)$$

satisfies all set of previous properties and, therefore, it is one of the most important implications.

4. Proposed mediative fuzzy system

In medicine we often find inconsistent information that can come from different human experts that do not fully agree all the time. In this case traditional and intuitionistic fuzzy logics are not effective while mediative fuzzy logic can be successfully used [29]. So is mentioned in first chapter, applications based on mediative fuzzy logic have shown its superiority to other fuzzy logics (traditional or intuitive). Starting from Mamdani fuzzy logic controller and from Fuzzy expert system from [30], in this section we describe a mediative fuzzy logic controller which can be used for heart disease diagnosis. For this we extend the system from [30] replacing fuzzy logic with mediative fuzzy logic, via intuitionistic fuzzy logic. In the experiment from [18] the best results were obtained using Eq. (7) in order to compute the system's output; that is why we will use the same equation.

4.1. Input and output variables

First step to designing an expert system is determination of input and output variables. We will work with linguistic variables. A linguistic variable is defined by a quin-tuple $(x, T(x), U, G, M)$ where: x is the name of variable, $T(x)$ is the set of terms (set of linguistic values of x), G is a syntactic rule for generating the name of the terms and M is a semantic rule for associating each term with its meaning.

We will use the same variables as in [30]; a value of a variable is a linguistic value or an integer number. A linguistic value is represented as an intuitionistic fuzzy number defined by its membership function and its non-membership function. For fuzzy numbers we will specify only the intervals with non-zero values.

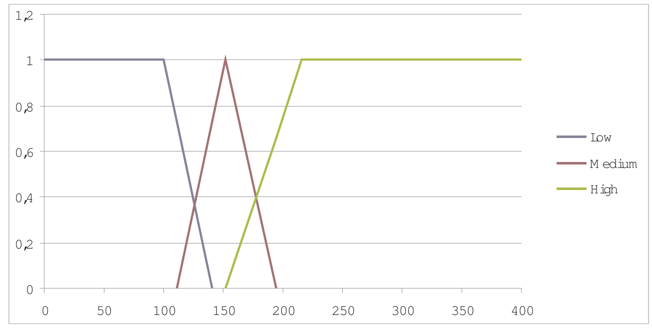

Fig. 1. The membership function of the input variable blood pressure.

Input variables are:

1. Chest pain. This variable has numerical values: 1 = typical angina, 2 = atypical angina, 3 = non-angina pain and 4 = asymptomatic.

2. Blood pressure. This variable has 4 values: Low, Medium, High and Very High, defined as intuitionistic fuzzy sets. Low and Very high sets are represented by trapezoidal fuzzy numbers while Medium and High sets are triangular fuzzy numbers. The membership functions (μ) and the non-membership functions (ν) corresponding to these values are listed below (Figs. 1 and 2):

$$\mu_{\text{Low}}(x) = \begin{cases} 1 & \text{if } x \le 111 \\ \frac{134 - x}{23} & \text{if } 111 \le x \le 134 \end{cases}$$

$$\nu_{\text{Low}}(x) = \begin{cases} \frac{x - 123}{21} & \text{if } 123 \le x \le 144 \\ 1 & \text{if } 144 < x \end{cases}$$

$$\mu_{\text{Medium}}(x) = \begin{cases} \frac{x - 127}{12} & \text{if } 127 \le x \le 139 \\ 1 & \text{if } x = 139 \\ \frac{153 - x}{14} & \text{if } 139 \le x \le 153 \end{cases}$$

$$\nu_{\text{Medium}}(x) = \begin{cases} 1 & \text{if } x < 115 \\ \frac{139 - x}{24} & \text{if } 115 \le x \le 139 \\ 0 & \text{if } x = 139 \\ \frac{x - 139}{31} & \text{if } 139 \le x \le 170 \\ 1 & \text{if } 170 < x \end{cases}$$

$$\mu_{\text{High}}(x) = \begin{cases} \frac{x - 142}{15} & \text{if } 142 \le x \le 157 \\ 1 & \text{if } x = 157 \\ \frac{172 - x}{15} & \text{if } 157 \le x \le 172 \end{cases}$$

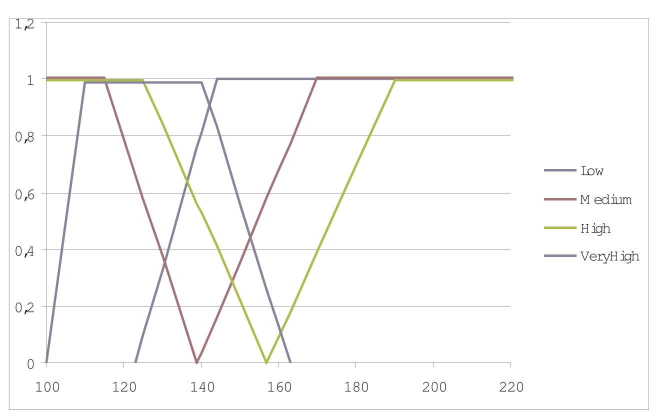

Fig. 2. The non-membership function of the input variable blood pressure.

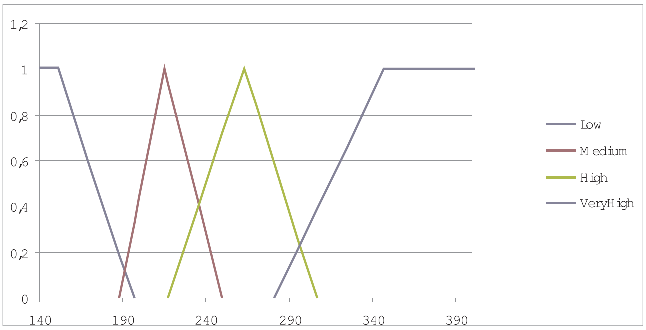

Fig. 3. The membership function of the input variable cholesterol.

$$\nu_{\text{High}}(x) = \begin{cases} 1 & \text{if } x < 125 \\ \frac{157-x}{32} & \text{if } 125 \le x \le 157 \\ 0 & \text{if } x = 157 \\ \frac{x-157}{33} & \text{if } 157 \le x \le 190 \\ 1 & \text{if } 190 < x \end{cases}$$

$$\mu_{\text{Very High}}(x) = \begin{cases} \frac{x-154}{17} & \text{if } 154 \le x \le 171 \\ 1 & \text{if } 171 \le x \end{cases}$$

$$\nu_{\text{Very High}}(x) = \begin{cases} 1 & \text{if } x < 140 \\ \frac{163-x}{23} & \text{if } 140 \le x \le 163 \end{cases}$$

3. Cholesterol. For this input variable one uses the value of low density lipoprotein cholesterol. For to represent this value we use the intuitionistic fuzzy numbers given by the next functions (Figs. 3 and 4):

$$\mu_{\text{Low}}(x) = \begin{cases} 1 & \text{if } x \le 151 \\ \frac{197-x}{46} & \text{if } 151 \le x \le 197 \end{cases}$$

$$\nu_{\text{Low}}(x) = \begin{cases} \frac{x-174}{36} & \text{if } 174 \le x \le 210 \\ 1 & \text{if } 210 < x \end{cases}$$

$$\mu_{\text{Medium}}(x) = \begin{cases} \frac{x-188}{27} & \text{if } 188 \le x \le 215 \\ 1 & \text{if } x = 215 \\ \frac{250-x}{35} & \text{if } 215 \le x \le 250 \end{cases}$$

$$\nu_{\text{Medium}}(x) = \begin{cases} 1 & \text{if } x < 170 \\ \frac{215-x}{45} & \text{if } 170 \le x \le 215 \\ 0 & \text{if } x = 215 \\ \frac{x-215}{55} & \text{if } 215 \le x \le 270 \\ 1 & \text{if } 270 < x \end{cases}$$

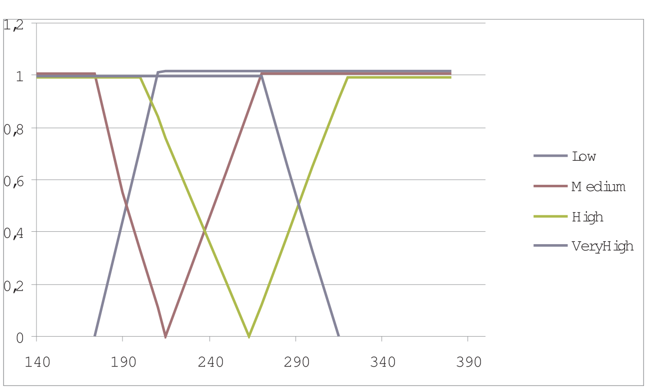

Fig. 4. The non-membership function of the input variable cholesterol.

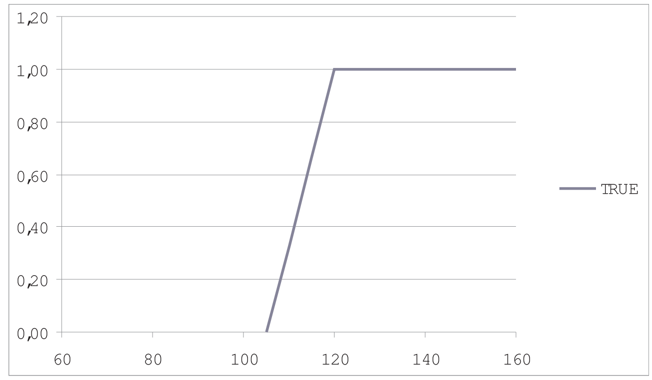

Fig. 5. The membership function of the input variable blood sugar.

$$\mu_{\text{High}}(x) = \begin{cases} \frac{x-217}{46} & \text{if } 217 \le x \le 263 \\ 1 & \text{if } x = 263 \\ \frac{307-x}{44} & \text{if } 263 \le x \le 307 \end{cases}$$

$$\nu_{\text{High}}(x) = \begin{cases} 1 & \text{if } x < 200 \\ \frac{263-x}{63} & \text{if } 200 \le x \le 263 \\ 0 & \text{if } x = 263 \\ \frac{x-263}{57} & \text{if } 263 \le x \le 320 \\ 1 & \text{if } 320 < x \end{cases}$$

$$\mu_{\text{Very High}}(x) = \begin{cases} \frac{x-281}{66} & \text{if } 281 \le x \le 347 \\ 1 & \text{if } 347 \le x \end{cases}$$

$$\nu_{\text{Very High}}(x) = \begin{cases} 1 & \text{if } x < 270 \\ \frac{315-x}{45} & \text{if } 270 \le x \le 315 \\ 0 & \text{if } 315 \le x \end{cases}$$

4. Blood sugar (diabetes). This input field has just one linguistic value: True, represented as (Figs. 5 and 6)

$$\mu_{\text{True}}(x) = \begin{cases} \frac{x-105}{15} & \text{if } 105 \le x \le 120 \\ 1 & \text{if } 120 \le x \end{cases}$$

$$\nu_{\text{True}}(x) = \begin{cases} 1 & \text{if } x < 80 \\ \frac{112-x}{32} & \text{if } 80 \le x \le 112 \\ 0 & \text{if } 112 \le x \le 160 \end{cases}$$

5. Resting electrocardiography (ECG). This variable has 3 linguistic values: Normal, ST-T Abnormal, Hypertrophy. The intuitionistic fuzzy numbers corresponding to these values are given by the following functions (Figs. 7 and 8):

Fig. 6. The non-membership function of the input variable blood sugar.

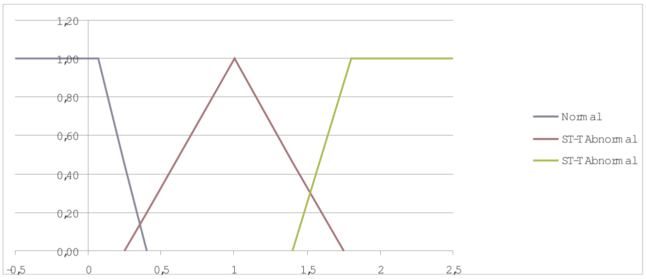

Fig. 7. The membership function of the input variable ECG.

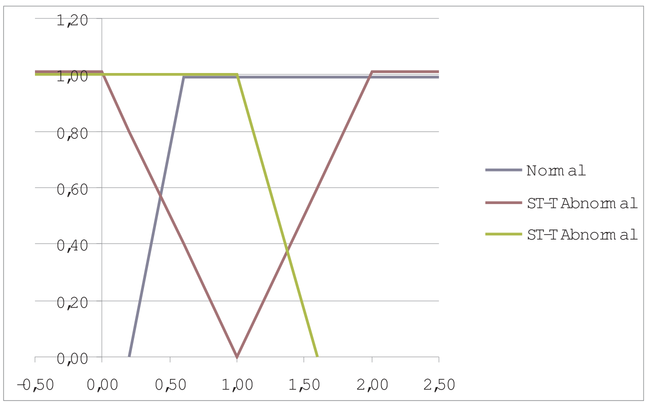

Fig. 8. The non-membership function of the input variable ECG.

$$\mu_{\text{Normal}}(x) = \begin{cases} 1 & \text{if } -05 \leq x \leq 0.07 \\ \frac{0.4-x}{0.33} & \text{if } 0.07 \leq x \leq 0.4 \end{cases}$$

$$\nu_{\text{Normal}}(x) = \begin{cases} 0 & \text{if } -0.5 \leq x \leq 0.2 \\ \frac{x-0.2}{0.4} & \text{if } 0.2 \leq x \leq 0.6 \\ 1 & \text{if } 0.6 < x \end{cases}$$

$$\mu_{\text{ST-TAbnormal}}(x) = \begin{cases} \frac{x-0.25}{0.75} & \text{if } 0.25 \leq x \leq 1 \\ 1 & \text{if } x = 1 \\ \frac{1.75-x}{0.75} & \text{if } 1 \leq x \leq 1.75 \end{cases}$$

$$\nu_{\text{ST-TAbnormal}}(x) = \begin{cases} 1 & \text{if } x < 0 \\ 1-x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x = 1 \\ x-1 & \text{if } 1 \leq x \leq 2 \\ 1 & \text{if } 2 < x \end{cases}$$

$$\mu_{\text{Hypertrophy}}(x) = \begin{cases} \frac{x-1.4}{0.4} & \text{if } 1.4 \leq x \leq 1.8 \\ 1 & \text{if } 1.8 \leq x \leq 2.5 \end{cases}$$

$$\nu_{\text{Hypertrophy}}(x) = \begin{cases} 1 & \text{if } x < 1 \\ \frac{1.6-x}{0.6} & \text{if } 1 \leq x \leq 1.6 \\ 0 & \text{if } 1.6 \leq x \leq 2.5 \end{cases}$$

6. Maximum heart rate. The value of this input variable expresses the maximum rate of man in 24 hours and has the values: Low, Medium, High (Figs. 9 and 10):

$$\mu_{\text{Low}}(x) = \begin{cases} 1 & \text{if } x \leq 100 \\ \frac{141-x}{41} & \text{if } 100 \leq x \leq 141 \end{cases}$$

$$\nu_{\text{Low}}(x) = \begin{cases} \frac{x-120}{40} & \text{if } 120 \leq x \leq 160 \\ 1 & \text{if } 160 < x \end{cases}$$

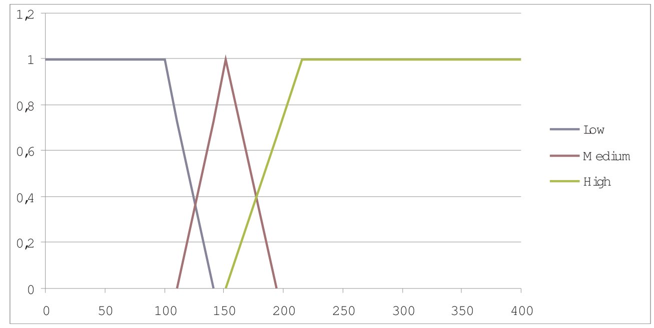

Fig. 9. The membership function of the input variable maximum heart rate.

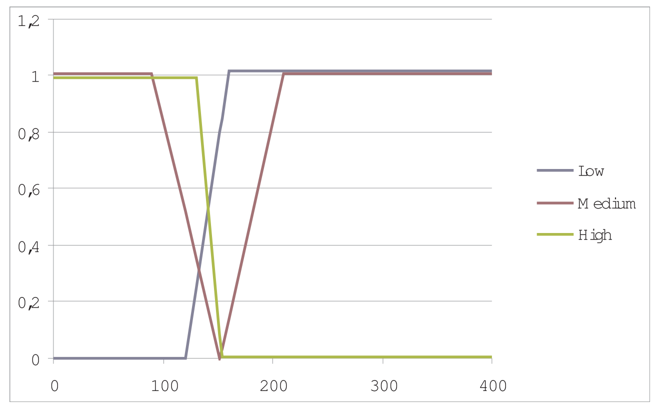

Fig. 10. The non-membership function of the input variable maximum heart rate.

$$\mu_{\text{Medium}}(x) = \begin{cases} \frac{x-111}{41} & \text{if } 111 \leq x \leq 152 \\ 1 & \text{if } x = 152 \\ \frac{194-x}{42} & \text{if } 152 \leq x \leq 194 \end{cases}$$

$$\nu_{\text{Medium}}(x) = \begin{cases} 1 & \text{if } x < 90 \\ \frac{152-x}{62} & \text{if } 90 \leq x \leq 152 \\ 0 & \text{if } x = 152 \\ \frac{x-152}{58} & \text{if } 152 \leq x \leq 210 \\ 1 & \text{if } 210 < x \end{cases}$$

$$\mu_{\text{High}}(x) = \begin{cases} \frac{x-152}{64} & \text{if } 152 \leq x \leq 216 \\ 1 & \text{if } 216 \leq x \end{cases}$$

$$\nu_{\text{High}}(x) = \begin{cases} 1 & \text{if } x < 130 \\ \frac{154-x}{24} & \text{if } 130 \leq x \leq 154 \end{cases}$$

7. Exercise. This input variable has 2 values: if doctor determines exercise test for patient, value is 1 and, otherwise, value is 0.

8. Old peak. This input variable means ST depression induced by exercise relative to rest and it has 3 values: Low, Risk, Terrible (Figs. 11 and 12):

$$\mu_{\text{Low}}(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } 1 \leq x \leq 2 \end{cases}$$

$$\nu_{\text{Low}}(x) = \begin{cases} \frac{x-1.5}{1.5} & \text{if } 1.5 \leq x \leq 3 \\ 1 & \text{if } 3 < x \end{cases}$$

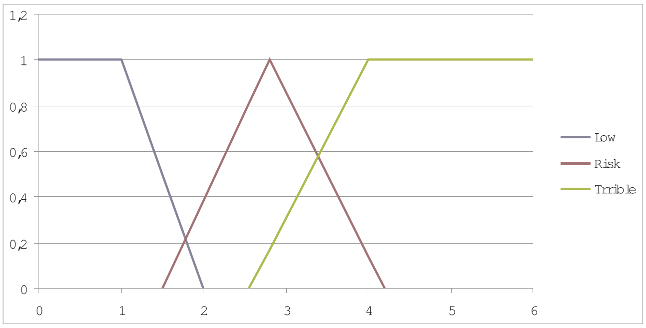

Fig. 11. The membership function of the input variable old peak.

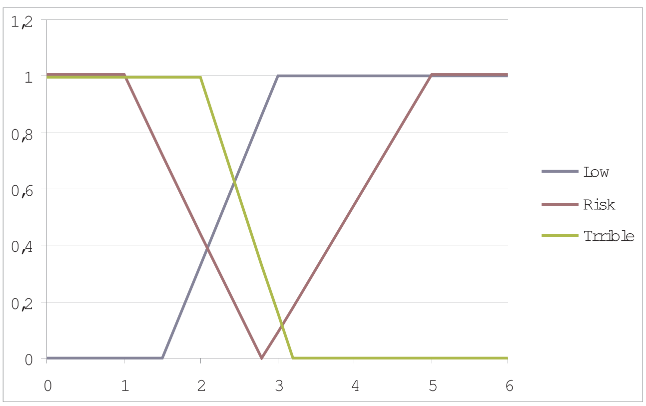

Fig. 12. The non-membership function of the input variable old peak.

$$\mu_{\text{Risk}}(x) = \begin{cases} \frac{x-1.5}{1.3} & \text{if } 1.5 \le x \le 2.8 \\ 1 & \text{if } x = 2.8 \\ \frac{4.2-x}{1.4} & \text{if } 2.8 \le x \le 4.2 \\ 0 & \text{for } a_4 < x \end{cases}$$

$$\nu_{\text{Risk}}(x) = \begin{cases} 1 & \text{if } x < 1 \\ \frac{2.8-x}{1.8} & \text{if } 1 \le x \le 2.8 \\ 0 & \text{if } x = 2.8 \\ \frac{x-2.8}{2.2} & \text{if } 2.8 \le x \le 5 \\ 1 & \text{if } 5 < x \end{cases}$$

$$\mu_{\text{Terrible}}(x) = \begin{cases} \frac{x-2.55}{1.45} & \text{if } 2.55 \le x \le 4 \\ 1 & \text{if } 4 \le x \end{cases}$$

$$\nu_{\text{Terrible}}(x) = \begin{cases} 1 & \text{if } x < 2 \\ \frac{3.2-x}{1.2} & \text{if } 2 \le x \le 3.2 \end{cases}$$

9. Thalium scan. This input variable has 3 numerical values: 3, 6 and 7 named Normal, Fixed Defect and Reversible Defect, respectively.

10. Sex. This variable has 2 values: value 0 means that patient is male and value 1 means that patient is female.

11. Age. This input fields has 4 linguistic values: Young, Mild, Old and Very Old, represented as intuitionistic fuzzy numbers given by the following functions (Figs. 13 and 14):

$$\mu_{\text{Young}}(x) = \begin{cases} 1 & \text{if } x \le 29 \\ \frac{38-x}{9} & \text{if } 29 \le x \le 38 \end{cases}$$

$$\nu_{\text{Young}}(x) = \begin{cases} \frac{x-35}{10} & \text{if } 35 \le x \le 45 \\ 1 & \text{if } 45 < x \end{cases}$$

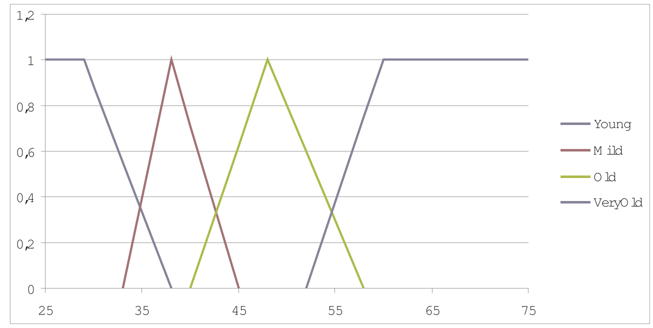

Fig. 13. The membership function of the input variable age.

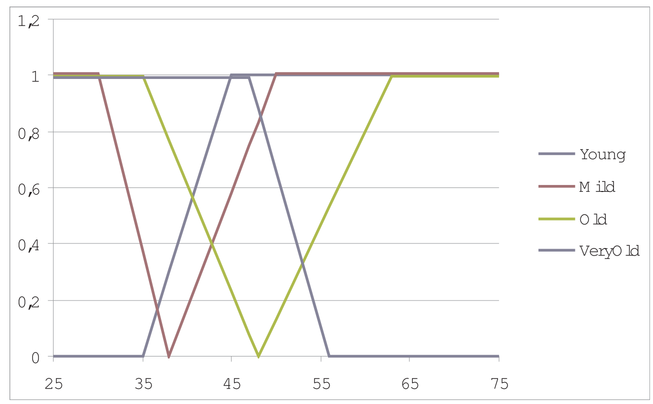

Fig. 14. The non-membership function of the input variable age.

$$\mu_{\text{Mild}}(x) = \begin{cases} \frac{x-33}{5} & \text{if } 33 \le x \le 38 \\ 1 & \text{if } x = 38 \\ \frac{45-x}{7} & \text{if } 38 \le x \le 45 \end{cases}$$

$$\nu_{\text{Mild}}(x) = \begin{cases} 1 & \text{if } x < 30 \\ \frac{38-x}{8} & \text{if } 30 \le x \le 38 \\ 0 & \text{if } x = 38 \\ \frac{x-38}{12} & \text{if } 38 \le x \le 50 \\ 1 & \text{if } 50 < x \end{cases}$$

$$\mu_{\text{Old}}(x) = \begin{cases} \frac{x-40}{8} & \text{if } 40 \le x \le 48 \\ 1 & \text{if } x = 48 \\ \frac{58-x}{10} & \text{if } 48 \le x \le 58 \end{cases}$$

$$\nu_{\text{Old}}(x) = \begin{cases} 1 & \text{if } x < 35 \\ \frac{48-x}{13} & \text{if } 35 \le x \le 48 \\ 0 & \text{if } x = 48 \\ \frac{x-48}{15} & \text{if } 48 \le x \le 63 \\ 1 & \text{if } 63 < x \end{cases}$$

$$\mu_{\text{Very Old}}(x) = \begin{cases} \frac{x-52}{8} & \text{if } 52 \le x \le 60 \\ 1 & \text{if } 60 \le x \end{cases}$$

$$\nu_{\text{Very Old}}(x) = \begin{cases} 1 & \text{if } x < 47 \\ \frac{56-x}{9} & \text{if } 47 \le x \le 56 \end{cases}$$

Output variable. The system has a single output variable that refers to presence of heart disease in the patient. It is an integer value from 0 to 4; these values are referred as Healthy, Sick1, Sick2, Sick3 and Sick4. By increasing of an integer value, heart disease risk increases for patient. The linguistic values for output variable are given by the following functions that define intuitionistic fuzzy numbers (Figs. 15 and 16):

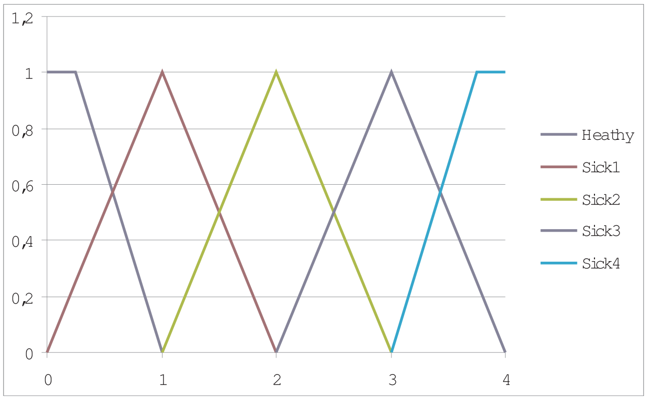

Fig. 15. The membership function of the output variable.

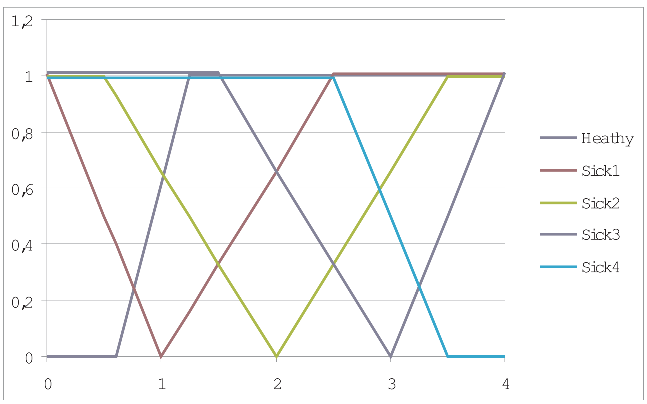

Fig. 16. The non-membership function of the output variable.

$$\mu_{\text{Healthy}}(x) = \begin{cases} 1 & \text{if } x \le 0.25 \\ \frac{1-x}{0.75} & \text{if } 0.25 \le x \le 1 \end{cases}$$

$$\nu_{\text{Healthy}}(x) = \begin{cases} \frac{x-0.6}{0.65} & \text{if } 0.6 \le x \le 1.25 \\ 1 & \text{if } 1.25 < x \end{cases}$$

$$\mu_{\text{Sick1}}(x) = \begin{cases} x & \text{if } x \le 1 \\ 1 & \text{if } x = 1 \\ 2-x & \text{if } 1 \le x \le 2 \end{cases}$$

$$\nu_{\text{Sick1}}(x) = \begin{cases} 1-x & \text{if } x \le 1 \\ 0 & \text{if } x = 1 \\ \frac{x-1}{1.5} & \text{if } 1 \le x \le 2.5 \\ 1 & \text{if } 2.5 < x \end{cases}$$

$$\mu_{\text{Sick2}}(x) = \begin{cases} x-1 & \text{if } 1 \le x \le 2 \\ 1 & \text{if } x = 2 \\ 3-x & \text{if } 2 \le x \le 3 \end{cases}$$

$$\nu_{\text{Sick2}}(x) = \begin{cases} 1 & \text{if } x < 0.5 \\ \frac{2-x}{1.5} & \text{if } 0.5 \le x \le 2 \\ 0 & \text{if } x = 2 \\ \frac{x-2}{1.5} & \text{if } 2 \le x \le 3.5 \\ 1 & \text{if } 3.5 < x \end{cases}$$

$$\mu_{\text{Sick3}}(x) = \begin{cases} x-2 & \text{if } 2 \le x \le 3 \\ 1 & \text{if } x = 3 \\ 4-x & \text{if } 3 \le x \le 4 \end{cases}$$

$$\nu_{\text{Sick3}}(x) = \begin{cases} 1 & \text{if } x < 1.5 \\ \frac{3-x}{1.5} & \text{if } 1.5 \le x \le 3 \\ 0 & \text{if } x = 3 \\ x-3 & \text{if } 3 \le x \le 4 \end{cases}$$

$$\mu_{\text{Sick4}}(x) = \begin{cases} \frac{x-3}{0.75} & \text{if } 3 \le x \le 3.75 \\ 1 & \text{if } 3.75 \le x \le 4 \end{cases}$$

$$\nu_{\text{Sick4}}(x) = \begin{cases} 1 & \text{if } x < 2.5 \\ 3.5-x & \text{if } 2.5 \le x \le 3.5 \end{cases}$$

4.2. Fuzzy rule base

The main part of a fuzzy inference system is rule base. Our system uses the same rule base as in [30], that contains 44 rules and 11 input variables. Every rule is of the type one input–one output. The rules are shown below.

R_1. If Chest Pain is Typical Angina then Result is Healthy
R_2. If Chest Pain is Atypical Angina then Result is Sick1
R_3. If Chest Pain is Non-angina then Result is Sick2
R_4. If Chest Pain is Asymptomatic then Result is Sick3
R_5. If Chest Pain is Asymptomatic then Result is Sick4
R_6. If Sex is Female then Result is Sick1
R_7. If Sex is Male then Result is Sick2
R_8. If Blood Pressure is Low then Result is Healthy
R_9. If Blood Pressure is Medium then Result is Sick1
R_{10}. If Blood Pressure is High then Result is Sick2
R_{11}. If Blood Pressure is High then Result is Sick3
R_{12}. If Blood Pressure is Very High then Result is Sick4
R_{13}. If Cholesterol is Low then Result is Healthy
R_{14}. If Cholesterol is Medium then Result is Sick1
R_{15}. If Cholesterol is High then Result is Sick2
R_{16}. If Cholesterol is High then Result is Sick3
R_{17}. If Cholesterol is Very High then Result is Sick4
R_{18}. If Blood Sugar is True then Result is Sick2
R_{19}. If EKG is Normal then Result is Healthy
R_{20}. If EKG is Normal then Result is Sick1
R_{21}. If EKG is ST-T Abnormal then Result is Sick2
R_{22}. If EKG is Hypertrophy then Result is Sick3
R_{23}. If EKG is Hypertrophy then Result is Sick4
R_{24}. If Max Heart Rate is Low then Result is Healthy
R_{25}. If Max Heart Rate is Medium then Result is Sick1
R_{26}. If Max Heart Rate is Medium then Result is Sick2
R_{27}. If Max Heart Rate is High then Result is Sick3
R_{28}. If Max Heart Rate is High then Result is Sick4
R_{29}. If Exercise is True then Result is Sick2
R_{30}. If Old Peak is Low then Result is Healthy
R_{31}. If Old Peak is Low then Result is Sick1
R_{32}. If Old Peak is Terrible then Result is Sick2
R_{33}. If Old Peak is Terrible then Result is Sick3
R_{34}. If Old Peak is Risk then Result is Sick4
R_{35}. If Thallium is Normal then Result is Healthy
R_{36}. If Thallium is Normal then Result is Sick1
R_{37}. If Thallium is Fixed Defect then Result is Sick2
R_{38}. If Thallium is Reversible Defect then Result is Sick3
R_{39}. If Thallium is Reversible Defect then Result is Sick4
R_{40}. If Age is Young then Result is Healthy
R_{41}. If Age is Mild then Result is Sick1
R_{42}. If Age is Old then Result is Sick2
R_{43}. If Age is Old then Result is Sick3
R_{44}. If Age is Very Old then Result is Sick4

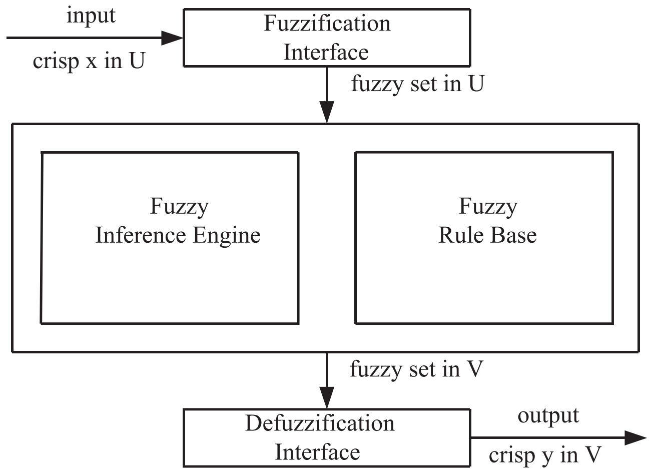

Fig. 17. Fuzzy logic controller.

4.3. Reasoning system

In order to infer imprecise conclusions from a set of imprecise premises, Zadeh gave a theory of approximate reasoning [27]; this theory is based on fuzzy logic inference processes and an important part of this reasoning is represented by Fuzzy Logic Control (FLC), that consists of four parts: Fuzzification Interface, Fuzzy Rule-Base, Fuzzy Inference Engine and Defuzzification Interface (Fig. 17).

Because the inputs and the outputs of a FLC system are fuzzy sets, we have to fuzzify the crisp inputs and to defuzzify the fuzzy outputs.

4.3.1. Fuzzification and firing level

A fuzzification operator transforms crisp data into intuitionistic fuzzy sets. For instance, $x_0 \in U$ is fuzzified into $\bar{x}_0$ according to the relations:

$$\mu_{\bar{x}_0}(x) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$\nu_{\bar{x}_0}(x) = \begin{cases} 0 & \text{if } x = x_0 \\ 1 & \text{otherwise} \end{cases} \tag{11}$$

The μ-firing level and the ν-firing level of an intuitionistic fuzzy set A and a crisp value x_0 as input are $\mu_A(x_0)$ and $\nu_A(x_0)$, respectively.

4.3.2. Inferred conclusion

Let R : *if X is A then Y is C* a rule from the rule base. Because we work with rules with one input, the firing level of a rule is the firing level of the fuzzy set from antecedent. Using the μ-firing level l_μ and the ν-firing level l_ν corresponding to rule R, we compute the intuitionistic fuzzy index $\pi = 1 - l_\mu - l_\nu$, the contradiction fuzzy index $\zeta = min(l_\mu, l_\nu)$ and the rule-firing level $l = (1 - \pi - \frac{\zeta}{2})l_\mu + (\pi + \frac{\zeta}{2})l_\nu$. The rule is represented by Lukasiewicz implication and the conclusion is inferred using Mamdani's model.

We get two conclusions:

- the μ-conclusion C^μ, which is the traditional output of the rule computed using the membership function and the firing level l_μ given by

  $\mu_{C^\mu}(v) = I_L(l_\mu, \mu_C(v)), \forall\, v \in V.$

- the ν-conclusion C^ν, which is the output of the rule computed using the non-membership function and the firing level l_ν given by

  $\mu_{C^\nu}(v) = I_L(l_\nu, \mu_C(v)), \forall\, v \in V.$

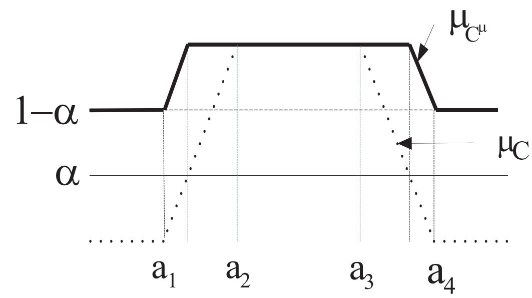

Fig. 18. μ-conclusion obtained with Lukasiewicz implication and firing level α.

If the conclusion of the rule is the trapezoidal intuitionistic fuzzy number $(b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ then the conclusion inferred are given by the following expressions [31]:

$$\mu_{C^\mu}(x) = \begin{cases} 1 - l_\mu & \text{if } x \le a_1 \\ \frac{x - a_1}{a_2 - a_1} + 1 - l_\mu & \text{if } x \in [a_1, l_\mu(a_2 - a_1) + a_1] \\ 1 & \text{if } x \in [l_\mu(a_2 - a_1) + a_1, l_\mu(a_3 - a_4) + a_4] \\ \frac{x - a_4}{a_3 - a_4} + 1 - l_\mu & \text{if } x \in [l_\mu(a_3 - a_4) + a_4, a_4] \\ 1 - l_\mu & \text{if } a_4 \le x \end{cases}$$

$$\mu_{C^\nu}(x) = \begin{cases} 1 & \text{if } x \le l_\nu(b_1 - b_2) + b_2 \\ \frac{x - b_2}{b_1 - b_2} + 1 - l_\nu & \text{if } x \in [l_\nu(b_1 - b_2) + b_2, b_2] \\ 1 - l_\nu & \text{if } x \in [b_2, b_3] \\ \frac{x - b_3}{b_4 - b_3} + 1 - l_\nu & \text{if } x \in [b_3, l_\nu(b_4 - b_3) + b_3] \\ 1 & \text{if } x \ge l_\nu(b_4 - b_3) + b_3 \end{cases}$$

The relationship between the conclusion of the rule and the conclusions inferred is illustrated graphically in Figs. 18 and 19 .

4.3.3. Defuzzification and output result

The conclusions C^μ and C^ν are defuzzified into y^μ and y^ν using the Middle of Maxima and Middle of Minima technique, respectively; this is defined as the main off all values of the universe of discourse having maximal (minimal, respectively) membership grades. The defuzzified output corresponding to conclusion C is computed as

$$y = \left(1 - \pi - \frac{\zeta}{2}\right)y^\mu + \left(\pi + \frac{\zeta}{2}\right)y^\nu.$$

The crisp output is computed by discrete Center of Gravity method: if the number of fired rules is N then the final control action is

$$y_0 = \frac{\sum_{j=1}^{N} y_j l_j}{\sum_{j=1}^{N} l_j}$$

where l_j is the firing level and y_j is the crisp output of the ith rule.

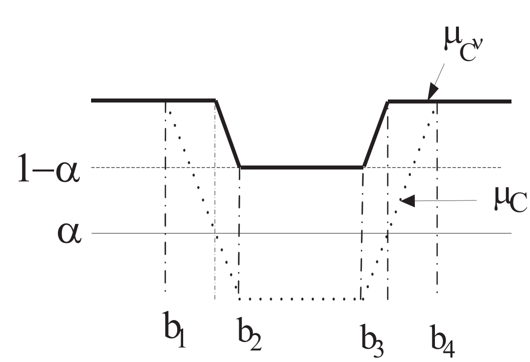

Fig. 19. ν-conclusion obtained with Lukasiewicz implication and firing level α.

Table 1
Crisp outputs of firing rules.

Rule	Firing level	Crisp output
R_4	1.0	3.0
R_7	1.0	2.0
R_8	0.19	0.4625
R_{16}	0.62	2.96
R_{18}	1.0	2.0
R_{19}	1.0	0.125
R_{27}	0.0156	2.38
R_{28}	0.0156	2.38
R_{29}	1.0	2.0
R_{30}	0.36	0.415
R_{31}	0.36	1.1
R_{38}	1.0	3.0
R_{39}	1.0	3.5
R_{44}	1.0	3.5

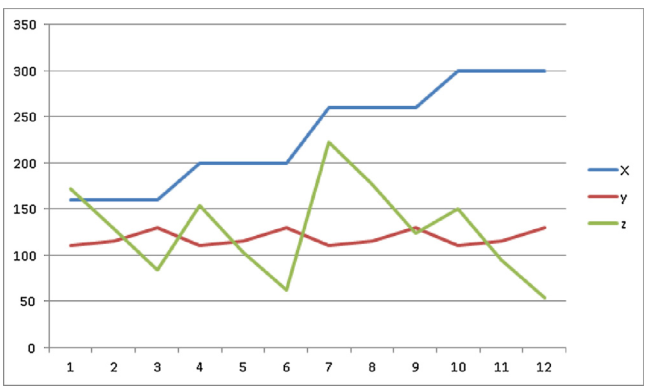

Fig. 20. Graphical representation for cholesterol (x), blood sugar (y) and output system (z).

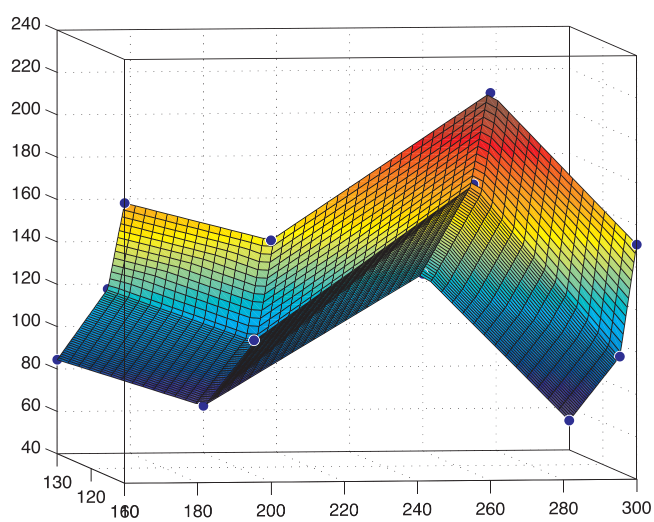

Fig. 21. Output surface generated by cholesterol and blood sugar.

5. System testing

We have tested the designed system with the same input values as in [30]:

Chest Pain = 4 (Asymptomatic),
Blood Pressure = 117 (Low),
Cholesterol = 230 (High),
Blood Sugar = 130 (True),
ECG = 0 (Normal),

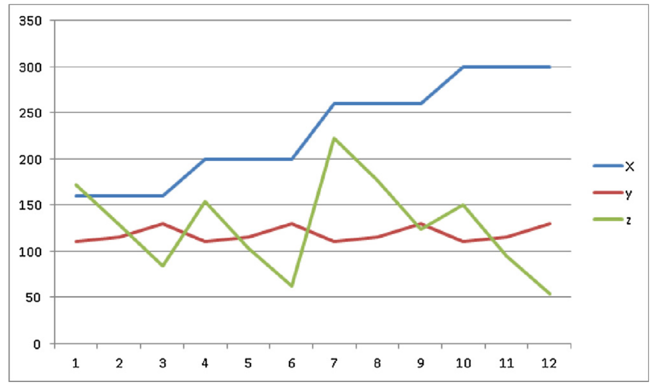

Fig. 22. Graphical representation for blood pressure (x), blood sugar (y) and output system (z).

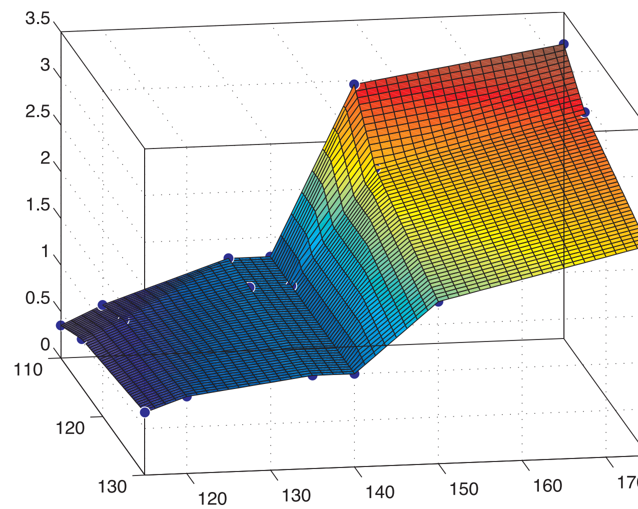

Fig. 23. Output surface generated by blood pressure and blood sugar.

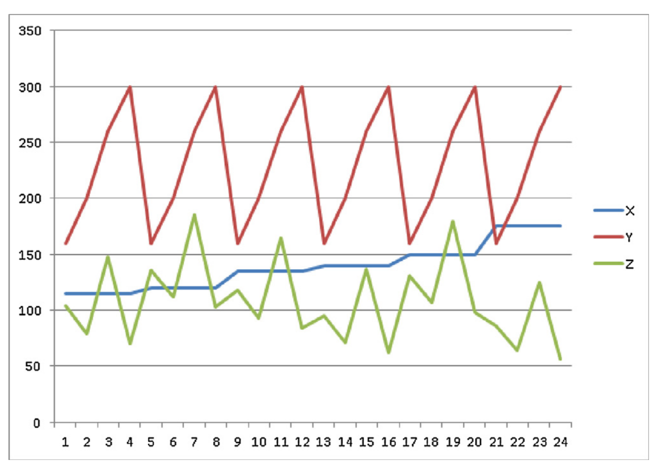

Fig. 24. Graphical representation for blood pressure (x), cholesterol (y) and output system (z).

Maximum Heart Rate = 160 (High),
Exercise = 1 (True),
Old Peak = 1.4 (Low),
Thallium = 7 (Reversible),
Sex = 1 (Male) and
Age = 60 (Very Old).

The fired rules with their firing levels and crisp outputs are

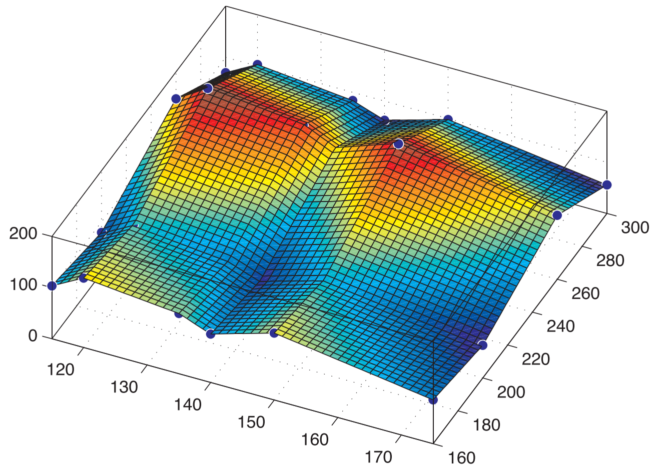

Fig. 25. Output surface generated by blood pressure and cholesterol.

presented in Table 1.

The relations between output variable and some input variables are presented in Figs. 20–25.

The output of the reasoning system is $y = 2.27$ which correspond to Sick2, which the same as in [30]. The crisp output from [30] is $y = 2$, the difference appears because the logic used in our system works with inconsistent information and models more adequate problems from real world; in other words, the proposed system is more complex and gives a better approximation.

6. Conclusion

Uncertainty appears in different forms and. affects decision making. Nowadays, there are mathematical models to handle the uncertainty. But if we work with a knowledge base that changes with time, and with non-contradictory information that becomes doubtful or contradictory, or with any combination of these three situations then we need to use mediative fuzzy logic which is able to process inconsistent information.

So is mentioned in first chapter, applications based on mediative fuzzy logic (see papers [17–22]) have shown its superiority to other fuzzy logics (traditional or intuitive). For this reason, in this paper, we extends and improves the system from [30] based on fuzzy logic by working with intuitionistic fuzzy sets to represent the input and output variables and with mediative fuzzy logic for reasoning. Superiority of our system is given by the possibility to handle contradictory and doubtful information. As is mentioned in [22] the fact of having the possibility of complementing the knowledge with non-agreement functions give us the possibility of implementing a more realistic fuzzy inference system.

In future papers we intend to improve this system by

- tuning the membership functions and rules used in inference system; for instance, if-then rules can be obtained from training patterns
- using a procedure to generate inference rules so that each has a degree of certainty
- improving the reasoning system by using the degree of certainty for determining the inferred conclusion
- test with other equations to compute the mediate output; for instance (8) and (9).

Acknowledgements

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