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# Reliability evaluation of the Eurocode model for fatigue assessment of steel bridges



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### ABSTRACT

In European countries, the design of bridges is conducted following the specifications in the Eurocodes. For verification against fatigue of steel bridges, a simplified model is suggested based on a single vehicle load model together with  $\lambda$  factors to estimate a representative stress range. Since the release of the Eurocodes the accuracy of this format has been discussed and questioned. In the current paper, a probabilistic model for fatigue assessment is suggested estimating the load effect from bridge weigh-in-motion (BWIM) measurements. The probabilistic model has been used to estimate the reliability reached with the existing verification format for road bridges. The result shows a large scatter depending foremost on the bridge geometry and the traffic volume. A tentative calibration of the verification format has been performed and new functions for two of the  $\lambda$  factors have been derived. With these new functions a significant improvement in the consistency of the reliability level has been achieved. The study demonstrates the need for a more extensive calibration of the Eurocode model and indicates the parameters to focus on.

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#### 1. Introduction

Designing a steel bridge against fatigue damage requires estimations of the load effect and the resistance at critical details. The load effect needs to be estimated using a structural model and depends on the traffic intensity and the type of vehicles travelling over the bridge. In reality, a bridge is typically loaded by a range of different vehicles with varying axle configurations and gross weights which is not feasible to consider during the design phase. In practice, the design is performed using predefined simplified load models described in regulations.

The Eurocode is the governing regulation for design of bridges in Europe. In part EN 1993-2 [1], a verification format considering fatigue is defined as

$$\gamma_{\rm Ff} \Delta \sigma_{\rm E2} \le \frac{\Delta \sigma_{\rm C}}{\gamma_{\rm Mf}} \tag{1}$$

where  $\Delta \sigma_{E2}$  is a damage equivalent stress range and  $\Delta \sigma_{C}$  is the fatigue strength, both valid for 2 million load cycles. The partial safety factors  $\gamma_{Ff}$  and  $\gamma_{Mf}$  represent the model uncertainties associated with the load model and the fatigue strength, respectively. The

fatigue strength is a characteristic value based on fatigue tests under constant amplitude stress conditions. The damage equivalent stress range is determined as

$$\Delta \sigma_{\rm E2} = \lambda \Phi_2 \Delta \sigma_p \tag{2}$$

where  $\lambda$  is a damage equivalent factor,  $\Phi_2$  a damage equivalent impact factor, and  $\Delta \sigma_p$  the stress range for the load p. The stress range is defined as the difference between the maximum stress and the minimum stress caused by the load model evaluated using influence areas [1]. For road bridges, the load to be considered is the fatigue load model 3 (FLM3).

The model based on damage equivalent factors was suggested already in [2] for railway bridges. The calibration considering traffic loads on road bridges is described in [3,4], and [5]. The characteristics of the FLM3 and the associated damage equivalent factors were calibrated using traffic measurements from Auxerre on the motorway A6 between Paris and Lyon in France [5]. In the background documents, statistical considerations regarding vehicle interaction were considered using stochastic queueing theory. The validation of the model was, however, performed in a deterministic manner against the traffic measurements from Auxerre.

The accuracy of the verification model (1) and the associated FLM3 has been subject to discussions in, e.g., [6,7], and [8]. The model is suspected to render misleading results for regions with traffic

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Nomenclature		
AADHT	Average annual daily heavy traffic	
BMN	Bimodal normal distribution	
BWIM	Bridge weigh-in-motion	
CoV	Coefficient of variation	
FORM	First order reliability method	
FLM3	Fatigue load model 3 in the Eurocode EN 1991-2	
GW	Gross weight	
SD	Standard deviation	
Lc	Critical length of the influence length or area	
Nobs	Number of vehicles heavier than 100 kN	
$P_{\mathrm{f}}$	Probability of failure	
Q <sub>m1</sub>	The average gross weight (kN) of the lorries in the	
	slow lane	
$\Delta \sigma$	Stress range	
$\Delta \sigma_{C}$	Fatigue strength at 2 million cycles	
$\Delta\sigma_{ m E2}$	Damage equivalent stress range	
$\Delta \sigma_p$	Maximum stress range for load p	
$\Phi(\mathbf{x})$	Standardized normal distribution function of <i>x</i>	
$\Phi_2$	Damage equivalent impact factor	
β	Reliability index	
γ	Partial factor	
λ	Damage equivalent factor	
Stochastic variables are listed in Table 2.		

intensities different from the reference location in Auxerre. And the model itself is claimed to be too simplistic to capture all possible cases of bridge geometry and traffic scenarios. In Sweden doubts about its validity have been raised mainly because now, fatigue more often becomes decisive in the design of new bridges than before when the superseded Swedish regulations were governing [9].

In this paper, a reliability-based model for fatigue assessment is proposed which allows a consideration of the inherent uncertainties of the resistance and the load effect. The resistance is modeled using linear damage accumulation and a bilinear relation between the fatigue endurance and the stress ranges. The load effect is estimated from real traffic considering vehicle characteristics determined by bridge-weigh-in-motion (BWIM). The model has been used in a calibration of the Eurocode verification format. The calibration was performed as described in the flowchart shown in Fig. 1 which is a



Fig. 1. Calibration procedure modified after [10].

modification of the general strategy outlined in [10]. The details of the figure will be explained in the subsequent sections.

In comparison to previous calibrations of the Eurocode model [5,7], the current calibration is performed considering a complete probabilistic formulation of the verification. Similar calibrations have been performed for wind turbines in, e.g., [11] and [12]. For bridges, early contributions on reliability-based calibrations can be found in, e.g., [13,14], and [15]. The model proposed in the current paper allows a more general description of the stochastic variables and especially for the consideration of the traffic loads. A calibration of the Eurocode verification format is presented based on BWIM measurements performed from year 2005 to 2009 at various locations in Sweden. The database consists of a total of 872 090 registered heavy vehicles and can be seen as a representation of the average traffic in Sweden. The contribution of the paper is the stochastic model for fatigue assessment considering BWIM measurements, and a calibrated deterministic model with a significantly improved consistency of the reliability level for different bridge geometries and traffic volumes. The proposed stochastic model can be used for calibration of the Eurocode model against national or regional specific traffic conditions determined by BWIM.

The paper has the following outline. The Eurocode verification format is briefly described in Section 2. In Section 3, the probabilistic model is elaborated and the procedure for calibration is explained. The database of vehicles from BWIM measurements is presented in Section 4. The results of the probabilistic calculations and the calibration are presented in Section 5.

#### 2. The Eurocode verification format

For bridges in steel, the Eurocode EN 1993-2 prescribes the verification to be performed using Eq. (1). The load effect is represented by  $\Delta\sigma_{\rm E2}$ , an equivalent stress range which for road bridges should be calculated using Eq. (2) with a stress range determined by structural analysis using the fatigue load model 3 (FLM3). The single vehicle load model consists of four axles with equal weights of 120 kN, see Fig. 2. The impact factor due to dynamics  $\Phi_2$  may be taken as equal to unity as it is already included in FLM3 [1].

The damage equivalent stress range should be related to 2 million cycles irrespective of the bridge component considered or the traffic situation. This is obtained by the damage equivalent factor  $\lambda$  which in turn is divided into four separate factors as

$$\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \quad \text{but} \quad \lambda \le \lambda_{\text{max}} \tag{3}$$

where  $\lambda_1$  depends on the length of the critical influence line or area,  $\lambda_2$  depends on the traffic volume,  $\lambda_3$  considers the design life of the bridge, and  $\lambda_4$  is a factor for traffic on other lanes. The product of the factors should be limited to  $\lambda_{max}$  taking account of the fatigue limit. The current study has shown that  $\lambda_1$  and  $\lambda_2$  are decisive for the reliability level. According to EN 1993-2, the former should be determined as



Fig. 2. Fatigue load model 3 (FLM3).

for midspan sections where  $L_c$  is the critical length of the influence line in meter as defined in EN 1993-2. For bending moment at midspan sections,  $L_c$  should be taken as the span length. For bending moment at support sections,  $L_c$  should be determined as the mean value of the two adjacent spans. For support sections, the  $\lambda_1$  factor should be determined as

$$\lambda_1 = \begin{cases} 2.0 - 0.3 \frac{L_c - 10}{20}, & 10 \le L_c \le 30\\ 1.7 + 0.5 \frac{L_c - 30}{50}, & 30 < L_c \le 80 \end{cases}$$
(5)

The factor was according to Sedlacek et al. [5], determined so that the damage caused by the FLM3 and 2 million cycles was equal to the accumulated damage caused by the measured traffic from Auxerre. It is also pointed out that other types of traffic would lead to other  $\lambda_1$  values.

The scale factor  $\boldsymbol{\lambda}_2$  considers the traffic volume and should be determined as

$$\lambda_2 = \frac{Q_{\rm m1}}{Q_0} \left(\frac{N_{\rm obs}}{N_0}\right)^{1/5} \tag{6}$$

A decisive parameter in Eq. (6) is the number of heavy vehicles,  $N_{obs}$ , with a gross weight more than 100 kN estimated per year and per slow lane. The other parameters are the average gross weight (kN) of the lorries in the slow lane and the reference values  $Q_0 = 480$  kN and  $N_0 = 0.5 \times 10^6$ . In Sweden, a traffic category is determined based on the *AADHT*, the average annual daily heavy traffic, which in turn gives a value for  $N_{obs}$ . This relation is shown in Table 1. It should be noted that *AADHT* counts the number of vehicles heavier than 35 kN while  $N_{obs}$  counts the number of vehicles heavier than 100 kN.

#### 3. Probabilistic model

Starting with the Palmgren–Miner rule for linear damage accumulation, a limit state equation can be defined as [17]

$$g(\mathbf{x}, n) = \delta - \sum_{i} \frac{n_{i}}{N_{\text{R}i}} = \delta - \sum_{i} \frac{n_{i} \,\Delta \sigma_{i}^{m_{i}}}{K_{i}} \tag{7}$$

where  $g(\mathbf{x}, n)$  depends on the stochastic variables in the vector  $\mathbf{x}$  and the number of cycles n,  $\delta$  represents the resistance expressed as the accumulated damage at failure, and the summation is the accumulated damage of the load. The variables K and m describe the fatigue endurance as linear in log-log scale, and  $\Delta \sigma$  is the stress range caused by the load. A state of failure is defined for  $g \le 0$  and the probability of failure is defined as  $P_f = P(g \le 0)$  [10]. The reliability index  $\beta$  is related to the probability of failure as [10]

$$\beta = -\Phi^{-1} \left( P_{\rm f} \right) \tag{8}$$

where  $\Phi^{-1}()$  is the inverse of the standardized normal distribution function.

 Table 1

 Number of heavy vehicles expected per year and per slow lane as suggested in EN 1991-2 (Table 4.5) [16] based on traffic data specified in the Swedish National Annex.

Traffic categor	у	N <sub>obs</sub>
1	$6000 < AADHT \le 24000$	$2,0 \times 10^{6}$
2	$1500 < AADHT \leq 6000$	$0,5 \times 10^{6}$
3	$600 < AADHT \le 1500$	0, 125 × 10 <sup>6</sup>
4	$AADHT \le 600$	0,05 × 10 <sup>6</sup>

To enable a consideration of the model uncertainties, the limit state Eq. (7) is extended to

$$g(\mathbf{x}, n) = \delta - \frac{1}{K_1} \sum_{i} n_i (C_{\rm S} \Delta \sigma)_i^{m_1} - \frac{1}{K_2} \sum_{j} n_j (C_{\rm S} \Delta \sigma)_j^{m_2}$$
(9)

which is formulated for a bilinear fatigue endurance where *i* counts the stress ranges along the upper part of the *S*–*N* curve and *j* along the lower part, see Fig. 7.1 in EN 1993-1-9 [18]. The stochastic variable  $C_{\rm S}$  represents the model uncertainty of the stress ranges and can be split into

$$C_{\rm S} = C_{\rm G} C_{\rm SCF} C_{\rm D} C_{BW} \tag{10}$$

which represent the uncertainty of the global model for stress analysis, the stress concentration factor, the influence of dynamics, and the uncertainty of the vehicle characterization, respectively.

The stress ranges  $\Delta\sigma$  and the associated number of cycles *n* in Eq. (9) can be determined by cycle counting for the response of each vehicle type using, e.g., the rainflow method [19]. By performing the stress analysis for each vehicle with deterministic geometric properties, the stress ranges can be scaled by a stochastic variable for the gross weight. However, this entails a need to consider a sufficient number of vehicle types to reach an accurate representation of the traffic response. To consider each type of vehicle, the limit state equation is extended once more to

$$g(\mathbf{x}, n) = \delta - \sum_{k} \left( \frac{1}{K_1} \sum_{i} n_i \left( C_{\mathsf{S}} C_{\mathsf{GW}, k} \Delta \sigma \right)_i^{m_1} + \frac{1}{K_2} \sum_{j} n_j \left( C_{\mathsf{S}} C_{\mathsf{GW}, k} \Delta \sigma \right)_j^{m_2} \right)$$
(11)

where  $C_{\text{GW},k}$  is the stochastic variable for the gross weight of vehicle k and the summation is performed for all vehicles  $k = 1 \dots N_k$ .

#### 3.1. Uncertainties

The stochastic variables considered are listed in Table 2 together with their statistical distributions and parameters. A short review of the background for the stated values is given below.

The variables in the probabilistic model were in principal assigned distributions and parameters as suggested in the JCSS Probabilistic Model Code [20]. The stochastic variable for the accumulated damage  $\delta$  defines the limit when the fatigue endurance is exhausted but also reflects the uncertainty of the Palmgren–Miner rule. It was modeled with a lognormal distribution, a mean value of unity and a coefficient of variation (CoV) of 0.3 [20,21]. The model uncertainty for the stress calculation was considered by  $C_{\rm G}$ . Statistical properties

Table 2
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Stochastic variables. N  $\sim$  Normal, LN  $\sim$  Lognormal, BMN  $\sim$  Bimodal normal, DET  $\sim$  Deterministic.

Variable	Distribution	Mean	CoV
δ	LN	1	0.3
C <sub>G</sub>	LN	1	0.1
C <sub>SCF</sub>	-	-	-
CD	N	1.1	0.1
C <sub>BW</sub>	Ν	1	0.02
C <sub>GW</sub>	BMN	$\mu_{GW}$	$V_{\rm GW}$
<i>K</i> <sub>1</sub>	LN	$\mu_{K1}$	0.49
K <sub>2</sub>	LN	$\mu_{K2}$	$V_{\rm K2}$
$\Delta \sigma_{\rm L}$	LN	$\mu_{ m L}$	$V_{\rm L}$
$m_1$	DET	3	-
<i>m</i> <sub>2</sub>	DET	5	-
$\Delta \sigma$	DET		

are suggested in [20] for different calculation models. A distribution pertaining to global nominal stress analysis was used in the current study with a lognormal ditribution, a mean value of unity and a CoV of 0.1. No stress concentration factor was considered why  $C_{\rm SCF}$  was omitted.

The influence of dynamics was modeled by  $C_D$  assigned a normal distribution with a mean value of 1.1 and a CoV of 0.1 as suggested in [22]. The distribution is based on a literature study presented in [23].

The gross weights of the vehicles were taken from BWIM measurements of which the accuracy was considered by the variable  $C_{BW}$ . A distribution was fitted by a comparison of the gross weight determined by the BWIM system and a portable scale used by the Police with an assumed high accuracy. Results for 16 vehicles presented in [24] were used to fit a normal distribution rendering a mean value of unity and a CoV of 0.02. This low CoV indicates an exceptional accuracy in comparison to WIM systems in general [25]. However, the purpose of the BWIM measurements was to attain accurate vehicle data and well-adapted bridges were instrumented accordingly. More information about the BWIM system can be found in Section 4. The gross weight of each vehicle type was considered by the variable  $C_{GW}$  described in Section 4.

The fatigue endurance is in the Eurocode described by bilinear S-N curves based on fatigue tests. It is described in the limit state equation by the variables K and m. The mean value of  $K_1$  depends on the geometry of the studied detail and can be estimated from the characteristic strength stated in the Eurocode as

$$\mu_{\ln K} = \ln K_{\rm C} + k \,\sigma_{\ln K} \tag{12}$$

where  $K_C$  is related to the fatigue strength as  $K_C = 2 \cdot 10^6 \Delta \sigma_C^m$ , and k is a tolerance interval factor in this case set to k = 2 as suggested in [11]. The standard deviation for *lnK* can be calculated as

$$\sigma_{\ln K} = \sqrt{\ln\left(V_{\rm K}^2 + 1\right)} \tag{13}$$

where  $V_{\rm K}$  is the CoV for  $K_1$  stated as 0.49 in Table 2 based on a standard deviation for  $log_{10}N$  equal to 0.2 as suggested in [26]. The lower part of the *S*–*N* curve described by  $K_2$  is typically assumed directly correlated to  $K_1$  as

$$K_2 = \left(\frac{K_1^5}{\left(5 \cdot 10^6\right)^2}\right)^{1/3} \tag{14}$$

which agrees with the Eurocode description of the *S*–*N* curve for  $m_1 = 3$  and  $m_2 = 5$ .

A cut-off limit for fatigue damage has been considered also directly correlated to  $K_1$ . It can be calculated from  $K_2$  as

$$\Delta\sigma_{\rm L} = \left(\frac{K_2}{10^8}\right)^{1/5} \tag{15}$$

which gives a cut-off limit at 10<sup>8</sup> cycles as specified in the Eurocode.

The stress ranges below the cut-off limit  $\Delta\sigma_L$  are not considered to contribute to any fatigue damage. The model for the fatigue endurance is shown in Fig. 3 valid for a characteristic fatigue strength of  $\Delta\sigma_C = 80$  MPa.

#### 3.2. Target reliability

Fatigue failure is considered as an ultimate limit state as it can lead to collapse of a structure. However, a fatigue failure of a structural detail or a local member does not necessarily lead to high



**Fig. 3.** Mean value and dispersion for the fatigue strength  $\Delta \sigma_{C} = 80$  MPa. The shaded area shows the mean value plus/minus two standard deviations (SD).

consequences as loss of human life. This is captured in [11] by defining an acceptable probability of failure considering fatigue as

$$P_{\rm f,fat} = \frac{P_{\rm f}}{P(f|fat)} \tag{16}$$

where  $P_{\rm f}$  is the annual probability of failure for the ultimate limit state and P(f|fat) is the probability of a complete collapse given that a fatigue failure has occurred. It suggests that a higher probability of failure can be accepted considering fatigue in comparison to the general ultimate limit state. A target value of  $\beta_1 = 4.3$  for a reference period of one year is suggested for fatigue critical details with large consequences of failure [11]. It corresponds to safety class (consequence class) 2 in [27] which is the basis for the safety levels adopted in Sweden. With a reference period of 100 years, the corresponding target reliability becomes  $\beta_{100} = 3.1$  which was adopted in the current study. This target agrees with the value suggested for *not inspectable* details in ISO 13822 [28]. It is, furthermore, close to the value of  $\beta = 3$  suggested in [3].

#### 4. Bridge weigh-in-motion in Sweden

The Swedish Transport Administration (Trafikverket, former Vägverket) regularly performs traffic measurements at varying locations in the road network. The measurements are performed on bridges instrumented with 8 to 16 strain gauges transforming the bridge into a scale. After a calibration of the system with known vehicles, continuous measurements are performed during free flowing traffic. The technique is called bridge weigh-in-motion (BWIM) and can be performed without the notice of the road-users [25]. Measurements performed from year 2005 to 2009 using the commercial system SiWIM were used in this study. During those years, 152 individual measurements were performed at locations spread over the country, rendering a total of 872 090 registered vehicles. Only vehicles with a gross weight of 35 kN or more are included in the result, out of which 92 % has a gross weight above 100 kN. The relation between the number of axles and the gross weight of the vehicles is shown in Fig. 4 as a box plot. The boxes for each group enclose 50% of the vehicles. The whiskers has the length of 1.5 times the length of the box. The grey crosses are the outliers. The level 640 kN indicated in the figure is the maximum allowed gross weight in Sweden. About 0.4 % of the registered vehicles exceeds this level.

A classification of the registered vehicles is automatically performed by the BWIM system based on gross weight, number of axles, and axle distances. Out of the 872 090 registered vehicles, 804 957 were sorted in 30 classes with 2 to 8 axles. In the current calibration, one vehicle type from each class was determined by selecting the



**Fig. 4.** The relation between the number of axles and gross weight (GW) for the registered vehicles.

vehicle with the median length. This vehicle determined the axle distances and the distribution of the axle loads for the vehicle type but the gross weight was assigned a distribution representing the whole vehicle class. This means that 30 vehicle types were considered each with individual geometrical definitions and different gross weight distributions. The three most frequent vehicle classes were number 40, 113 and 94 shown in Fig. 5. Together, these classes constitute about 40% of the vehicle passages. Descriptions of all 30 vehicle types can be found in [29].

The distributions for the stochastic variables  $C_{GW,k}$  were estimated based on the measured gross weights for each vehicle type  $k = 1 \dots N_k$  using the maximum likelihood method. In general, a good fit was reached using a bimodal normal distribution expressed as

$$F(x, \boldsymbol{\theta}) = \theta_5 \,\Phi(x, \theta_1, \theta_3) + (1 - \theta_5)\Phi(x, \theta_2, \theta_4) \tag{17}$$

where  $\Phi(x, \mu, \sigma)$  is the distribution function for a normal distribution with a mean value of  $\mu$  and a standard deviation of  $\sigma$ . An example of a fitted distribution is shown in Fig. 6 for one of the 30 vehicle types. A complete presentation of all vehicle types, fitted distributions, and parameters can be found in [29].



**Fig. 5.** Examples of the three most frequent vehicle types with axle distances and axle load distributions. A total of 30 vehicle types were considered in the evaluation.



**Fig. 6.** The distribution of the gross weight for the vehicles assigned to class 94 together with the fitted bimodal normal (BMN) distribution. The distribution has the parameters  $\theta_1 = 395.3$ ,  $\theta_2 = 580.2$ ,  $\theta_3 = 96.87$ ,  $\theta_4 = 37.68$ , and  $\theta_5 = 0.5869$ .

#### 5. Results

The calibration of the verification model (1) was performed as described by Fig. 1 against the BWIM data from Sweden. The basic input specified as the first box consists of bridge specific input. To enable analysis of several bridge cases the bridge models in this study were simplified to two-dimensional beam structures with constant cross-sections. This enabled an efficient calculation of the vehicle response using an analytical routine based on the displacement method. Constant bending stiffness *EI* along the beams also makes the section-forces independent on cross-section properties.

#### 5.1. Deterministic fatigue assessment

Nine different bridge cases schematically shown in Fig. 7 were studied. The cases covered beams with 1, 2, 3, 4 and 6 spans with different lengths. The span dimensions were collected from real bridges but were simplified as two-dimensional beams. A summary of the span lengths is given in Table 3. From the bridge geometry a value for  $\lambda_1$  was calculated following the specifications in EN 1993-2.

A deterministic calculation following the Eurocode needs, besides the bridge geometry, the number of heavy vehicles expected per year and per slow lane  $N_{obs}$ . In Sweden, this is obtained through the *AADHT* for the road as specified in Table 1. Another input value is the



Fig. 7. Schematic figures of the nine bridge cases.

Table 3	
Span lengths for the nine bridge cases. See Fig. 7.	

Case	Number of spans	Span lengths/m
(a)	1	10
(b)	1	42
(c)	2	19 + 19
(d)	2	62 + 65
(e)	3	12 + 18 + 12
(f)	3	58 + 74 + 58
(g)	4	21 + 30 + 29 + 16
(h)	4	64 + 79 + 79 + 64
(i)	6	38 + 44 + 44 + 44 + 44 + 38

average gross weight of the lorries in the slow lane  $Q_{m1}$ . In Sweden, this value is specified to  $Q_{m1} = 410$  kN for normal conditions. From these two values  $\lambda_2$  was calculated following the specifications in EN 1993-2. The design life was set to 100 years which gave  $\lambda_3 = 1$  and the influence from other lanes was assumed negligible giving  $\lambda_4 = 1$ . The final  $\lambda$  values according to Eq. (3) for one midspan section in each bridge case is shown in Fig. 8 for different  $N_{obs}$ .

It is evident in Fig. 8 that the number of heavy vehicles has a significant influence on the  $\lambda$  value, which in turn will influence the damage equivalent stress range. For the cases when the  $\lambda$  value exceeded  $\lambda_{max}$ , the subsequent calculations were based on the latter. This was generally the case when  $N_{obs}$  was larger than  $0.5 \times 10^6$ .

When the  $\lambda$  factor had been determined, the bending moments at the midspan and support sections were calculated by the analytical structural analysis. Based on Eqs. (1) and (2), a design variable *w* could then be calculated as

$$w = \gamma_{\rm Ff} \gamma_{\rm Mf} \lambda \Phi_2 \frac{\max[M] - \min[M]}{\Delta \sigma_{\rm C}}$$
(18)

where *M* is the bending moment dependent on the position of the vehicle. The design variable is determined to fulfill the deterministic verification format in the Eurocode. The partial safety factors were set to  $\gamma_{\text{Ff}} = 1$  and  $\gamma_{\text{Mf}} = 1.35$  and the impact factor was set to  $\Phi_2 = 1$  as suggested in the Eurocode. The fatigue strength was set to  $\Delta\sigma_{\text{C}} = 80$  MPa. It should, however, be noted that this value has no noticeable influence on the estimated reliability in the following step. The probabilistic fatigue strength is connected to the deterministic value through the assumed statistical distribution and the characteristic value, see Eq. (12). A change of detail category will also change the mean value but the CoV remains the same. Thereby, only the dispersion of the fatigue strength is relevant for the reliability evaluation.



Fig. 8. Calculated  $\lambda$  values for one midspan section in each bridge case.

#### 5.2. Probabilistic fatigue assessment

Based on the design variable w determined in the deterministic assessment, a reliability-based assessment using the model described in Section 3 was performed. Structural analyses were performed for each bridge case and for each vehicle type rendering stress histories for the decisive sections. Stress range spectra were calculated by rainflow analysis using the toolbox WAFO in Matlab [30]. The total number of vehicle passages considered was determined as

$$N_{\rm vs} = 100.365.25 \frac{AADHT}{2}$$
(19)

for 100 years, 365.25 days per year, and divided by two due to two traffic directions assumed (or two lanes). The total number of vehicle passages was then divided between the frequency of the 30 vehicle types determined from the BWIM measurements. The limit state Eq. (11) was then used to calculate the probability of failure by crude Monte Carlo simulations. Initially, samples of one million simulations were generated. For cases where the reliability index approached 4.2 or higher, the simulation was remade with 10 million values to reach at least ten values within the failure region. The nonlinearity of the limit state equation, e.g., due to the fatigue cut-off limit, made approximative methods as the first order reliability method (FORM) unreliable.

The reliability indices estimated for the different bridge cases and different *AADHT*'s are shown in Fig. 9 for the decisive midspan sections, and in Fig. 10 for the support sections. For the midspan sections, the reliability indices varies between 1.6 and 4.9. A trend of decreasing reliability with increasing critical length  $L_c$  is evident in Fig. 9. For the support sections, the reliability indices vary between 0.86 to 3.8. An opposite trend is indicated for these sections, with an increasing reliability with increasing critical length, see Fig. 10. Both figures show a significant scatter caused mainly by the critical length and the *AADHT*. Considering a target reliability of 3.1, the result indicates both conservative and nonconservative estimates.

#### 5.3. Calibration

Considering the flowchart in Fig. 1, the comparison against the target reliability shows a significant deviation and large scatter. This implies a need to calibrate the deterministic verification format. Previous studies where the load effect has been determined by in-situ



**Fig. 9.** Estimated reliability indices at midspan sections for different critical lengths of the influence line  $L_c$  and different *AADHT*.



**Fig. 10.** Estimated reliability indices at support sections for different critical lengths of the influence line  $L_c$  and different *AADHT*.

measurements have shown that the product  $\gamma_{Ff}\gamma_{Mf} = 1.35$  gives an acceptable agreement with a target reliability of 3.1, see e.g. [17]. Hence, the calibration in this study was focused on the  $\lambda$  factors for the load effect estimation.

An iterative approach was implemented to determine a  $\lambda$  factor that yielded a reliability index equal to the target  $\beta = 3.1$ . The reliabilities for all 112 result sections were estimated for  $\lambda$  factors in the range 0.5 to 2 times the original  $\lambda$  factor according to the Eurocode. The  $\lambda$  value giving a target reliability of 3.1 was determined by linear interpolation over the range of  $\lambda$  values and for every case. For the new set of  $\lambda$  factors, statistical curve fitting using regression analysis was performed first to derive a function for  $\lambda_2$  minimizing the scatter with respect to *AADHT*, and then for  $\lambda_1$  with respect to the critical length. For  $\lambda_1$ , different functions were required for midspan and support sections. For midspan sections, a function for  $\lambda_1$  was derived to

$$\lambda_1^{\text{midspan}} = \begin{cases} 0.9922 - \frac{L_c}{147.7}, & L_c \le 18.62\\ 0.6566 + \frac{L_c}{76.82} - \frac{L_c^2}{10540}, & L_c > 18.62 \end{cases}$$
(20)

where  $L_c$  is the critical length of the influence line in meter as defined in EN 1993-2. For support sections the corresponding expression was derived to

$$\lambda_1^{\text{support}} = 1.154 - \frac{L_c}{91.61} + \frac{L_c^2}{7788}$$
(21)

Both functions are shown in Fig. 11.



**Fig. 11.** Functions for  $\lambda_1$  based on statistical curve fittings.



**Fig. 12.** Function for  $\lambda_2$  based on a statistical curve fitting.

The influence of the traffic volume is considered with the factor  $\lambda_2$  in the Eurocode model. A new function based on the AADHT was derived to

$$\lambda_2 = 9.854 \, AADHT^{0.02769} - 10.46 \tag{22}$$

The function is shown in Fig. 12.

Using the derived functions for  $\lambda_1$  and  $\lambda_2$ , while keeping the other variables in the verification format (1) unchanged, gave the reliability indices shown in Fig. 13. It is evident that the consistency of the reliability level is greatly improved in comparison to Figs. 9 and 10. The calibration was performed to reach a mean value of all result points as close as possible to the target reliability of  $\beta = 3.1$ . A total of 112 result points was included in the evaluation. The reliability indices range from 3.0 to 3.3 for the midspan sections and from 2.4 to 3.4 for the support sections. The reason for the larger scatter of the support sections is that the critical length  $L_c$ , provides a less accurate estimate of the stress ranges close to the supports.

The presented calibration was performed adhering to the Eurocode model basing the derived expressions for  $\lambda_1$  and  $\lambda_2$  on already existing variables. A new definition of the critical length for support sections might be motivated in a more extensive calibration of the model.

#### 6. Discussion

The probabilistic analyses were based on vehicles classified by BWIM measurements in Sweden during the years 2005 to 2009. The population of vehicles is assumed representative for heavy traffic



Fig. 13. Reliability indices for the calibrated verification format considering 112 result sections.

in Sweden in general. For a specific bridge along a specific road, however, the distribution may very well be different. Also for other countries the distribution of heavy vehicles may be different.

The results presented in Section 5 are valid for single vehicle passages. If two vehicles are passing the bridge simultaneously, e.g., during an overtaking, the interaction between the vehicles might influence the response at a critical detail. This is considered by the factor  $\lambda_4$  in the Eurocode, which in this study was set to unity. A theoretical consideration of vehicle interaction requires a threedimensional structural model of the bridge and a statistical model for the probability of occurrence. The issue is touched upon in [4] where a model based on queue theory is suggested. The result is strongly dependent on the shape of the influence surface and becomes unique for each studied case. To reduce the uncertainties related to the influence of vehicle interaction, in-situ measurements on bridges are suggested as a continuation of the current study.

Another possible interaction is when closely spaced vehicles are passing a bridge in the same lane. This can frequently occur during rush-hour traffic. Results presented in [3] show, however, that the highest fatigue damage occurs when the distance between lorries corresponds to free traffic. Also this kind of interaction is a topic for further studies by in-situ measurements.

#### 7. Conclusions

With the purpose to validate the Eurocode model for fatigue assessment of road bridges in steel, a probabilistic model has been suggested. It is based on vehicle loads determined by BWIM measurements. The model has been used to estimate the reliability achieved using the existing verification format by analyses of some simple bridge models. By an iterative process, the Eurocode verification format has been calibrated to render a more consistent reliability level. Considering the results, the following conclusions can be drawn:

- The estimated reliability indices for the existing verification format in the Eurocode EN 1993-2 show a large scatter for the investigated cases. The values range from  $\beta = 0.86$  to  $\beta = 4.9$ . In general, higher reliability indices were reached for midspan sections in comparison to support sections.
- The scatter in the reliability indices is caused mainly by differences in the critical length of the influence lines and different values for N<sub>obs</sub>, the number of heavy vehicles per slow lane per year.
- A tentative calibration of the verification format has been performed by deriving new functions for the factors λ<sub>1</sub> and λ<sub>2</sub> by statistical curve fittings.
- The calibration against a target reliability of  $\beta = 3.1$ , for a reference period of 100 years, rendered a significant improvement in the consistency of the reliability level, see Fig. 13.

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