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A Case Study of Bank Queueing Model

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Abstract:- This paper deals with the Queueing theory and the analysis of queueing system by using probability curves. Starting with the basis of the distributions and important concepts of queueing theory, probability curves of Gamma distribution are used to analyse the banking service.

Keywords:- Arrival rate, Service rate, Poission process, Probability distribution, Gamma function, Gamma distributions, Probability graph, Random variable.

I. INTRODUCTION

Important concepts

Discrete Random variables [4]

If ξ is an experiment having a sample space S and X is a function that assigns a real number X(e) to every outcome e Є S, then X(e) is called Random variable. And when it is associated with counting then the Random variable is termed as Discrete Random variable.

Continuous random variable

If ξ is an experiment having a sample space S and X is a function that associates with one or more intervals, then X is called continuous.

Probability Distribution of a Random variable

The family of probability distributions that has special importance in probability is the following.
If X is a discrete random variable, we associate a number $P_x(x_i) = P_x(X = x_i)$ with each outcome x_i in R_x for I = 1, 2 …, n ... where the numbers $P_x(x_i)$ satisfy the following:

$P_x(x_i) > 0$ for all i.

$\sum_{i=1}^{\infty} P_x(x_i) = 1$

The function P_x is called the probability law of random variable and the pairs $[x_i, P_x(x_i)]$ is called the probability distribution of X.

If X is continuous random variable, then for δ > 0,

$$P_X(X = x) = \lim_{\delta \to 0}[F_x(x+\delta) - F_x(x)] = 0$$

We define probability density function $f_x(x)$ as $f_x(x) = \frac{d}{dx} F_X(x)$

And it follows that $F_X(x) = \int_{-\infty}^{x} f_x(x).$

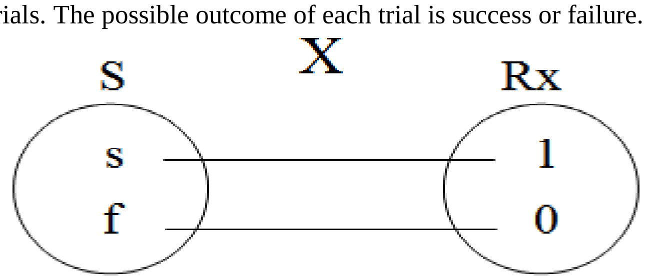
$f_x(x) \geq 0$ for all x Є R_X

$\int f(x)\,dx = 1$

Then $P\{e \in S: a <= X(e) \leq b\} = P_X(a \leq X \leq b) = \int_a^b f(x)\,dx$

Characteristics of some discrete distributions:

The stem of the probability distributions is Bernolli trials. There are experiments such as each experiment consists of n trials. The possible outcome of each trial is success or failure.



Bernoulli's Distribution:

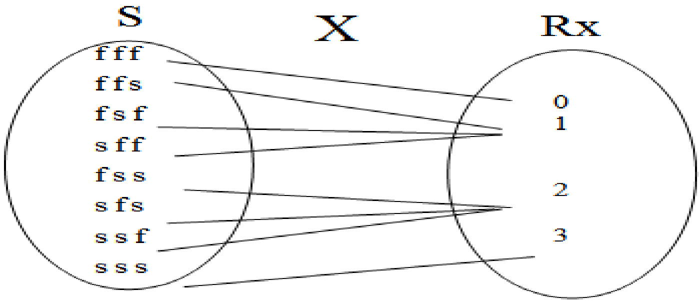
Perform an experiment ξ_I (i^{th}) whose sample space is $S_I = \{S, F\}$. Now the random variable X_i defined as $X_I(s)$ is mapped to 1 and $X_I(f)$ is mapped to 0 of Real numbers. The range space R_x is subset of real numbers is [0, 1]. The probability law defined in this Bernoulli's distribution with respective to the random variable is

$P_i(r_i) = p$ when $r_i = 1$, I = 1, 2 ……n.
$= 1-p$ when $r_i = 0$, I = 1, 2,……n.

The random process of Bernoulli's trials was the origin for some of the discrete distributions. The binomial, the geometric, the pascal, and the negative binomial are based on the proceedings of Bernoulli's trials.

The Binomial Distribution: The random variable X which is defined as "the number of successes in n Binomial trials" has Binomial distribution probability. Defined as

$P(r) = nc_r\, p^r (1-p)^{n-r}$ when n = 0, 1, 2, 3…, n.
$= 0$ otherwise.

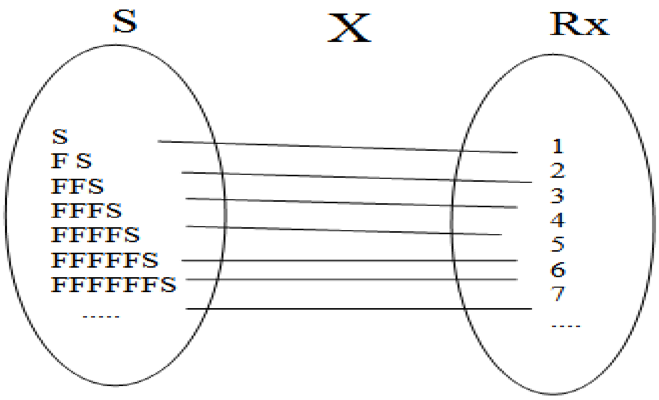


X is defined as no. of successes in n Bernoulli's trials.

Geometric distribution:

The geometric distribution is also dependant on the Bernoulli's trials. The difference between the Binomial Bernoulli's trials and Geometric Bernoulli's trials is the number of trials in Binomial is fixed and in that of geometric the number of trials is not fixed. The random variable is defined as "the number of trials required to get first success". The probability distribution of random variable X is

$P(r) = p\, q^{r-1}$ when r = 1, 2, 3……
$= 0$ otherwise.



The Pascal distribution:

The Pascal distribution also has its origin in Bernoulli's trials. In this random variable X described as the trial on which the r th success occurs, where r is an integer. The probability of X is defined as

$P(r) = n\text{-}1c_{r-1}\, p^r (1-p)^{n-r}$ when n = r, r+1, r+2 ….
$= 0$ otherwise.

The Poission distribution:

This distribution is used in real world phenomenon. We are not doing any artificial experiments in which we get the results either success or failure. We consider only time oriented occurrences (arrivals). As in the previous distributions we do not make n trials but instead we take number of arrivals that occur in the particular time interval [0, t].

This distribution is discrete distribution and developed in two ways. The first development involves the Poission process. The second development involves that the Poission distribution to be a limiting case of the Binomial distribution.

Development of Poission process: In this process the random variable has practical applicability. In defining Poission process, we initially consider a collection of arbitrary time oriented occurrences also called "arrivals or births". The random variable X_t is the "number of arrivals that occur in the particular time interval [0, t]". The range space R_{Xt} = {0, 1, 2, 3 ….}. In developing the Poission probability distribution of random variable X_t, it is necessary to take some assumptions. They are

1. The number of arrivals during the non overlapping time intervals is independent random variables.

2. We make assumption that there exists a positive quantity λ such that for any small interval, Δt the following postulates are satisfied.

✓ The probability that exactly one arrival will occur in an interval of width Δt is approximately $\lambda\Delta t$. The approximation is in the sense that the probability is
$\lambda\Delta t. + o_1(\Delta t)$ where the function $[o_1(\Delta t)/\Delta t] \to 0$ as $\Delta t \to 0$.

✓ The probability that exactly zero arrival will occur in the interval is approximately
$1-\lambda\Delta t$. Again this is in the sense that it is equal to $1-\lambda\Delta t+o_2(\Delta t)$ and $[o_2(\Delta t)/\Delta t] \to 0$
As $\Delta t \to 0$.

✓ The probability that two or more arrivals occur in the interval is equal to a quantity
$O_3(\Delta t)$, where $[o_3(\Delta t)/\Delta t] \to 0$ as $\Delta t \to 0$.

The parameter λ is sometimes called mean arrival rate or mean occurrence rate. And Poisson developed and summarized the following

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \text{ when } n = 0, 1, 2, 3\ldots.$$
$$= 0 \text{ otherwise.}$$

The development of Poisson distribution from the Binomial:
The Binomial distribution is

$$P(r) = nc_r\, p^r (1-p)^{n-r} \text{ when } n = 0, 1, 2, 3\ldots, n.$$
$$= 0 \text{ otherwise.}$$

If we let $np = c$ so that $p = \frac{c}{n}$ and $1-p = 1-\frac{c}{n} = \frac{n-c}{n}$ and if we then replace terms involving p with corresponding terms involving c we obtain

$$P(r) = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}\left[\frac{c}{n}\right]^r\left[\frac{n-c}{n}\right]^{n-r}$$
$$= \frac{c^r}{r!}\left[(1)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r-1}{n}\right)\right]\left(1-\frac{c}{n}\right)^n\left(1-\frac{c}{n}\right)^{-r} \text{------------(1)}$$

In letting $n \to \infty$ and $p \to 0$ in such a way that $np = c$ remains fixed, the terms $(1-\frac{1}{n})$, $(1-\frac{2}{n})$, $\ldots(1-\frac{r-1}{n})$ all approach to 1, as does $(1-\frac{c}{n})^{-r}$. we know $(1-\frac{c}{n})^n \to e^c$ as $n \to \infty$.

Thus limiting form of equation (1) is $P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ which is Poisson distribution.

Some continuous distributions:

The uniform distribution: A random variable X is said to follow continuous uniform distribution in an interval [a, b] if its density function is constant over the entire range of X. I.e. $f(x) = K$, $a \le x \le b$.

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\to \int_a^b K\,dx = 1$$
$$\to K[b-a] = 1$$
$$\to K = \frac{1}{b-a}$$

Therefore $f(x) = \frac{1}{b-a}$ $a \le x \le b$.

Then $p(c \le x \le d) = \int_c^d \frac{dx}{b-a} = \frac{d-c}{b-a}$.

The exponential distribution: In this random variable defined as "the time between the occurrences". It has the density function $f(x) = \lambda e^{-\lambda x}$ when $x \ge 0$.

0 otherwise, where λ is real positive constant.

Gamma distribution: Gamma density function is $f(t) = \frac{\lambda}{\Gamma(r)}(\lambda t)^{r-1}e^{-\lambda r}$, $t > 0$

$= 0$ otherwise.

The parameters are $r > 0$ and $\lambda > 0$.
The parameter r is called shape parameter,
And the parameter λ is called scale parameter.

The Weibull distribution: The important usage of Weibull distribution is that it gives a fairly accurate approximation to the probability law of many random variables. Time to failure in electrical and mechanical components is one of the applications of this distribution. The probability density function of Weibull distribution is

$$f(x) = \frac{\beta}{\delta}\left[\frac{x-\gamma}{\delta}\right]^{\beta-1}\exp\left[-\left(\frac{x-\gamma}{\delta}\right)^{\beta}\right],\ x \ge \gamma,$$
$$= 0, \text{ otherwise.}$$

Introduction

Constructive time expended constitutes a small percentage of the total time spent by us on various activities. Severe traffic congestions and bottlenecks eat away a major chunk of time while travelling. Due to a boom in accounts and growing population, a visit to the bank or post office results in a lot of time wastage as a huge number of customers are waiting to be serviced. Super markets are usually overcrowded which leads to a delay in making day to day purchases. In general, customers get irate when there is delay in getting their respective works completed. On the other hand, employees providing service to customers also dread this situation as it may lead to loss of business. This situation is a direct result of lack of organized service providing mechanism. A detailed study of the queuing systems is a rational way of providing solution to this problem. Such a study helps us understand the size of the queue, behaviour of the customers in the queue, system capacity, arrival process, service availability, service process in the system. This analysis helps in providing valuable inputs to the management to take remedial measures.

A queue is a waiting line. Queueing theory is mathematical theory of waiting lines. The customers arriving at a queue may be calls, messages, persons, machines, tasks etc. we identify the unit demanding service, whether it is human or otherwise, as a customer. The unit providing service is known as server. For example (1) vehicles requiring service wait for their turn in a service center. (2) Patients arrive at a hospital for treatment. (3) Shoppers are face with long billing queues in super markets. (4) Passengers exhaust a lot of time from the time they enter the airport starting with baggage, security checks and boarding.

Queueing theory studies arrival process in to the system, waiting time in the queue, waiting time in the system and service process. And in general we observe the following type of behavior with the customer in the queue. They are

Balking of Queue: Some customers decide not to join the queue due to their observation related to the long length of queue, in sufficient waiting space. This is called Balking.

Reneging of Queue: This is the about impatient customers. Customers after being in queue for some time, few customers become impatient and may leave the queue. This phenomenon is called as Reneging of Queue.

Jockeying of Queue: Jockeying is a phenomenon in which the customers move from one queue to another queue with hope that they will receive quicker service in the new position.

History In Telephone system we provide communication paths between pairs of customers on demand. The permanent communication path between two telephone sets would be expensive and impossible. So to build a communication path between a pair of customers, the telephone sets are provided a common pool, which is used by telephone set whenever required and returns back to pool after completing the call. So automatically calls experience delays when the server is busy. To reduce the delay we have to provide sufficient equipment. To study how much equipment must be provided to reduce the delay we have to analyse queue at the pool. In 1908 Copenhagen Telephone Company requested Agner K.Erlang to work on the holding times in a telephone switch. Erlang's task can be formulated as follows. What fraction of the incoming calls is lost because of the busy line at the telephone exchange? First we should know the inter arrival and service time distributions. After collecting data, Erlang verified that the Poisson process arrivals and exponentially distributed service were appropriate mathematical assumptions. He had found steady state probability that an arriving call is lost and the steady state probability that an arriving customer has to wait. Assuming that arrival rate is λ, service rate is μ and $\rho=\frac{\lambda}{\mu}$ he derived formulae for loss and deley.

(1) The probability that an arriving call is lost (which is known as Erlang B-formula or loss formula).

$$P_n = \frac{\frac{\rho}{n!}}{\sum \frac{\rho^k}{k!}} = B(n,\rho)$$

(2) The probability that an arriving has to wait (which is known as Erlang C-formula or deley formula).

$$P_n = \frac{n}{n-\rho(1-B(n,\rho))} B(n,\rho)$$

Erlang's paper "On the rational determination of number of circuits" deals with the calculation of the optimum number of channels so as to reduce the probability of loss in the system.

Whole theory started with a congestion problem in tele-traffic. The application of queueing theory scattered many areas. It include not only tele-communications but also traffic control, hospitals, military, call-centers, supermarkets, computer science, engineering, management science and many other areas.

Important concepts in Queueing theory

Little law

One of the feet of queueing theory is the formula Little law. This is

$$N = \lambda T$$

This formula applies to any system in equilibrium (steady state).

Where λ is the arrival rate

T is the average time a customer spends in the system

N is the average number of customers in the system

Little law can be applied to the queue itself.

I.e. $N_q = \lambda T_q$

Where λ is the arrival rate
T_q the average time a customer spends in the queue
N_q is the average number of customers in the queue

II. CLASSIFICATION OF QUEUING SYSTEMS

Input process

If the occurrence of arrivals and the offer of service strictly follow some schedule, a queue can be avoided. In practice this is not possible for all systems. Therefore the best way to describe the input process is by using random variables which we can define as "Number of arrivals during the time interval" or "The time interval between successive arrivals".

If the arrivals are in group or bulk, then we take size of the group as random variable. In most of the queueing models our aim is to find relevant probability distribution for number of customers in the queue or the number of customers in the system which is followed by assumed random variable.

Service Process

Random variables are used to describe the service process which we can define as "service time" or "no of servers" when necessary. Sometimes service also be in bulk. For instance, the passengers boarding a vehicle and students understanding the lesson etc..

Number of servers

Queueing system may have Single server like hair styling salon or multiple servers like hospitals. For a multiple server system, the service may be with series of servers or with c number of parallel servers. In a bank withdrawing money is the example of former one where each customer must take a token and then move to the cashier counter. Railway reservation office with c independent single channels is the example of parallel server system which can serve customers simultaneously.

System capacity

Sometimes there is finite waiting space for the customers who enter the queueing system. This type of queueing systems are referred as finite queueing system.

Queue discipline

This is the rule followed by server in accepting the customers to give service. The rules are

- FCFS (First come first served).
- LCFS (Last come first served).
- Random selection (RS).
- Priority will be given to some customers.
- General discipline (GD).

Kendall's notation

Notation for describing all characteristics above of a queueing model was first suggested by David G Kendall in 1953.

The notation is with alphabet separated by slashes.

- A/B/X/Y/Z

Where A indicates the distribution of inter arrival times
B denotes the distribution of the service times
X is the capacity of the system
Y denotes number of sources
Z refers to the service discipline

Examples of queueing systems that can be defined with this convention are M/M/1
M/D/n
G/G/n

Where M stands for Markov
D stands for deterministic
G stands for general

Definition- state dependent service: The situation in which service depends on the number of customers waiting is referred to as state dependent service.

A survey was conducted in relation to the operations of Andhra bank, JNTUH, Hyderabad. The bank provides service to various types of customers on daily basis. The mean deviation of arrival rate of customers in each week is different. In the first week the bank has 6 customers/minute. For the second week it was 4.662 customers/minute. For third and fourth weeks 3.666 customers/minute, and 3 customers/minute respectively.

Since the service is state dependent service, the time taken by employee to complete one service on average is different in different weeks. By observations and data from the management of the Bank the time to serve one customer on average is listed below:

For first week ----- 2 minutes/customer

For second week ------ 2.5 minutes/customer
For third week ------ 2.5 minutes/customer
For fourth week ----- 3 minutes/customer

In this paper we consider the application of Gamma distribution which involves the Gamma function. Gamma function: The Gamma function was first introduced by Leonhard Euler. It is the solution of following interpolation problem. "Find a smooth curve that connects the points (x, y) given by y = (x-1)!" It is easy to interpolate factorial function to non negative values. However, the Gamma function is the solution to provide a formula that describes the resulting curve. It is defined by the use of tools integrals from calculus.

Gamma function is defined for all complex numbers except for non positive integers as

$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt$ with positive real part.

$\Gamma(z) = (z-1)!$ when z is a positive integer.

Gamma distribution: Gamma distribution is a general type of statistical distribution with two parameters r, λ and density function $f(t) = \frac{\lambda}{\Gamma(r)} (\lambda t)^{r-1} e^{-\lambda r}$, $t > 0$

= 0 otherwise.

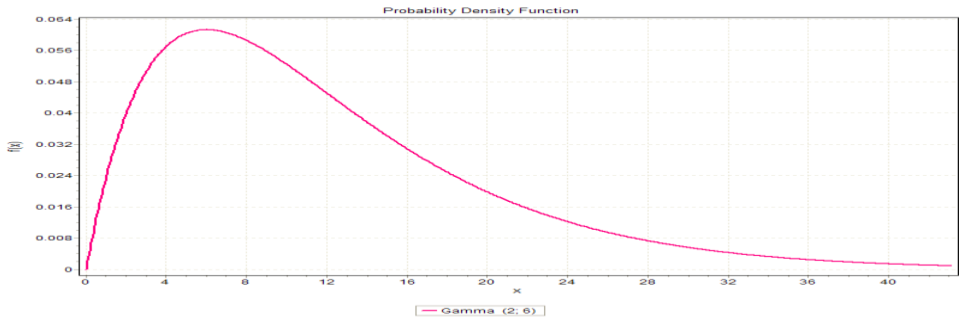
The parameter r > 0 is called shape parameter.

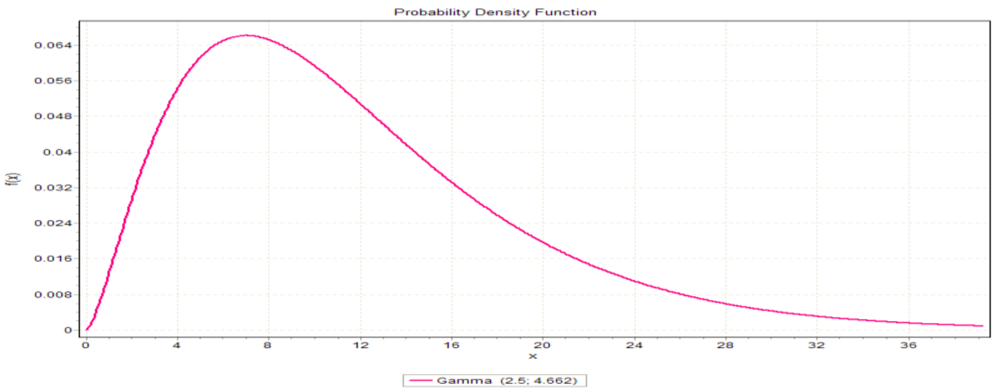
And the parameter λ>0 is called scale parameter.

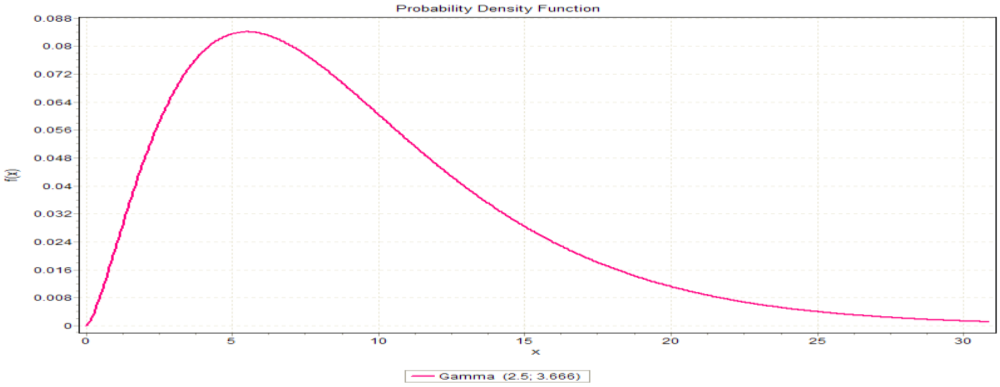
Since arrivals to the bank are time oriented occurrences, they follow Poisson probability distribution. And as arrival times in the Poisson process have Gamma distribution, we can use gamma probability distribution to the arrivals of Andhra Bank. Considering the "time to complete the service" as random variable, we calculate the probability of completing the service in time t for different weeks.

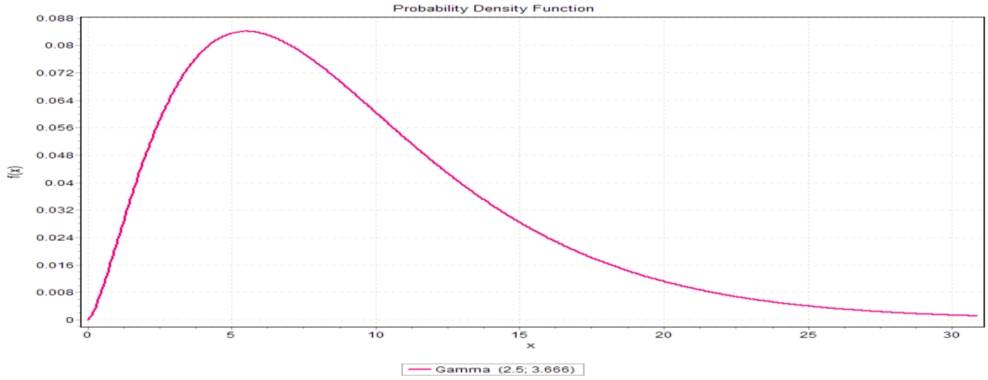
Calculating the probabilities by using probability density function of gamma distribution

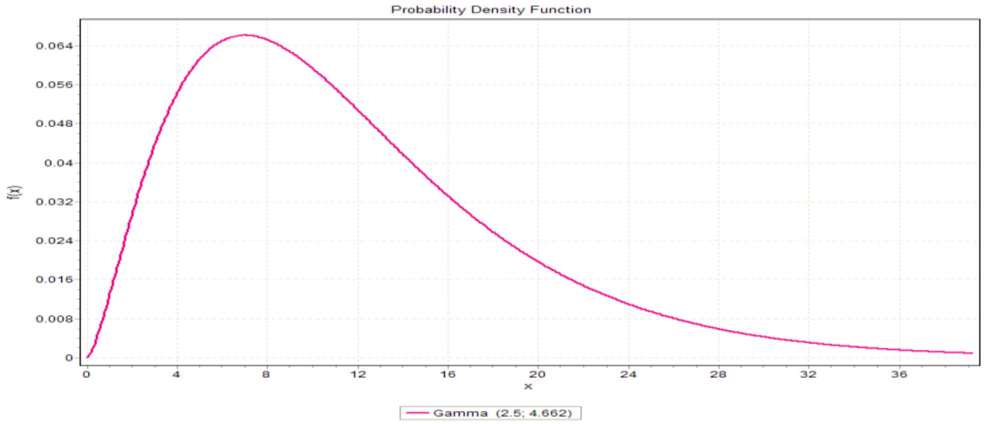
Graph of pdf:

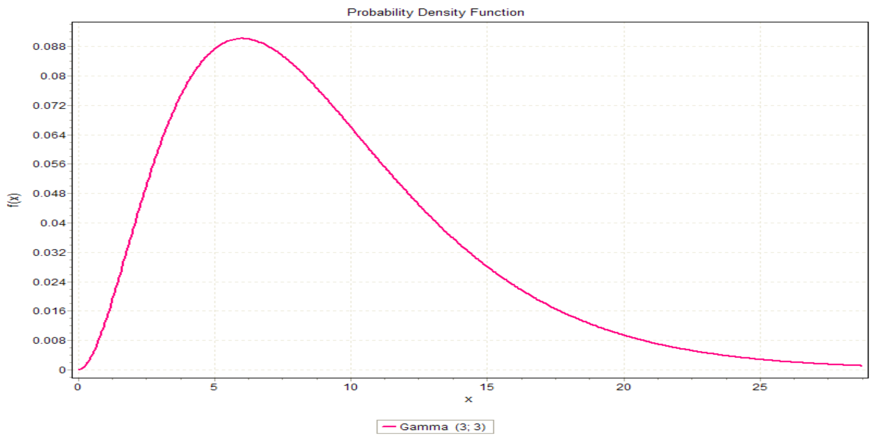












Let us take time t = 5 minutes, then time taken to complete the service within 5 minutes for different weeks is tabulated.

time t = 5 min.	1st week	2nd week	3rd week	4th week
r	2	2.5	2.5	3
λ	6	4.662	3.666	3
probability of completing service in t min	0.060361	0.054344	0.074055	0.087442

Though the arrival rate is more in first week, due to the high service rate the probability of completing the service in 5 minutes is 0.060361. Due to the little bit decrease in service rate, the probability in second week is 0.054344. With all most all equal service rate as second week and with less arrival rate, in third week the probability is 0.074055. In the last week of month though service rate is less, the probability of getting serviced in less time is 0.087442. So a customer enters the bank will completes his work quickly in 4th week among all weeks. And he takes more time in 2nd week in comparative with other weeks.

III. CONCLUSION

This paper studies the Queueing theory and by the use of probability curves the analysis of the queueing system. From the work surveyed above we have found arrival and service rates at the bank are different in different weeks. The conclusions are developed from the application of Gamma probability distribution. The arrival rate is higher in week 1. However, the higher arrival rate is offset by higher service productivity. Therefore, the probability of completing the service in 5 minutes in week 1 is 0.060361. The efficiency factor is not constant. The service productivity diminishes and as a result the probability in the second week drops to 0.054344. In the third week the rate of arrival decreases. Consequently, maintaining the same service rate as in the second week, the probability goes up to 0.074055. The service rate is less in the last week of the month as the arrival rates decrease considerably, the probability of completing the service with in less time peaks as compared to the remaining weeks.i.e0.087442.

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