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Highlights

- Mutual fund management is conceptualized as a three-stage production process.
- Individual processes are considered as operating under different risk profiles.
- Risk type is modelled through conditions imposed on the linking variables.
- A general multiplier based network DEA model is developed.
- Overall fund management efficiency is decomposed into stage level efficiencies.
- Proposed linkage performance metric is found to improve discriminatory power of performance.

A new network DEA model for mutual fund performance appraisal: An application to U.S. equity mutual funds

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Abstract

Mutual fund is a popular investment vehicle for investors. Investors usually judge fund manager performance relative to target benchmarks. Fund managers, on the other hand, are interested in knowing how/why they perform well or poorly relative to their peers in different aspects of fund management as well. To acquire more insights about this issue and design a comprehensive performance measure, fund management function is conceptualised as a three-stage production process. To assess overall and stage-level performance, a network data envelopment analysis model is developed. The stage-level processes are deemed to operate under two different environmental conditions - levels of risk exposure. Operation under different levels of risk exposure is modelled through conditions imposed on the intermediate measures. A new index proposed to assess linkage performance is demonstrated empirically to improve discriminatory power of performance. Further applications of the proposed model are discussed.

JEL Classification: G10; G11; G20; C67

Keywords: Mutual fund management; Network data envelopment analysis; Performance appraisal; Efficiency decomposition

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1. Introduction

U.S. mutual fund (MF) industry is the largest in the world with nearly $16 trillion in assets at year-end 2015. Fifty-two per cent of these MF assets are equity funds with bond funds (21%), money market funds (18%) and hybrid funds (9%) making up the rest (Investment Company Institute, 2016). Equity, bond and hybrid MFs are typical long-term investments whereas money market funds provide short-term yields. Interest in MFs is generally widespread across households, business and institutional investors. MF is an attractive financial instrument for households because MFs are managed by financial experts and owning shares in a MF is a cost effective way of diversification. In the U.S., approximately eighty-nine per cent of MF assets are held by households. However, as large number of companies offers a wide choice, MF selection is not an easy task. Rating agencies such as Morningstar give guidance to this end by providing star ratings based on risk-adjusted return. Nevertheless, it is in the best interest of investors, whether big or small, to have a general overview of fund performance in addition to risk-adjusted return. Fund managers, on the other hand, will be interested in knowing why/how they perform well or poorly overall as well as in different aspects of fund management compared to their peers. Our aim is to investigate this issue through a novel multi-stage data envelopment analysis model.

Two traditional measures of financial entity performance are the Sharpe and Treynor ratios. Both are two-dimensional ratio measures with the association between return and risk being the foundation for their derivation. However, when many performance measures are involved, two-dimensional ratio measure of performance upgrades to an analysis in a multi-dimensional framework. Data envelopment analysis (DEA) is a non-parametric mathematical programming technique that can assess performance in a multi-dimensional output-input framework. Performance appraisal using DEA is consistent with the concept of output-input ratio based efficiency measurement of production processes. In DEA, it is not required to specify a functional form for the technology transformation function. This is considered as an advantage over other frontier-based performance appraisal methodologies such as stochastic frontier analysis where specification of functional form for the frontier is mandatory. MF performance evaluation using parametric frontier estimation methods is rare. One such attempt is in Babalos, Mamatzakis and Matousek (2015).

Application of DEA for MF performance appraisal can be traced back to the late 1990s (Murthi et al, 1997; McMullen and Strong, 1998). A major reason advanced for using DEA in this context is its mathematical ability to handle multiple inputs and multiple outputs. Early DEA studies of MF performance appraisal treat fund operation as a black box. Recent studies of MF performance appraisal using DEA look inside the black box to capture internal structures of the overall fund management process. Basso and Funari (2016) provide a comprehensive list of DEA studies that evaluate MFs and other managed funds such as pension and hedge funds dating up to 2014. Out of the sixty-one DEA approaches of MF performance listed therein, only one study adopts a network structure, viz Premachandra et al. (2012). Galagedera et al. (2016) assess MF performance by extending Premachandra et al. (2012)'s two-stage network structure to accommodate independent output at the first stage.

In this paper, we propose a new three-stage network model in multiplier DEA setting for MF performance appraisal. In multi-stage processes, the measures that link consecutive stages are referred to as "intermediate measures". Intermediate measures represent resources deemed generated and consumed within the production process, and therefore they may be considered as internal resources (Chen and Yan, 2011; An et al., 2016). Premachandra et al. (2012) and Galagedera et al. (2016) in their two-stage MF performance appraisal modelling framework, consider *operational management* and *portfolio management* as the two consecutive processes with net asset value (NAV) as an intermediate variable. Further, they consider NAV, fund size, standard deviation of returns and expense ratio as the inputs of the second stage (portfolio management process). When MF management process is conceptualised as a multi-stage production process, we argue that fund size and NAV may serve well as performance measures when fund size and NAV are modelled as input and output of a sub-process rather than as two inputs of a sub-process. We label the new management process that we propose to accommodate this scenario as *resource management* process. Another advantage of introducing resource management process as a separate stage is that we may then consider the portfolio management process that follows the resource management process as generating returns primarily with risk as input. In short, we conceptualise overall MF management process as a serially linked three-stage process comprising of operational management, resource management and portfolio management processes.

Moreover, guided by the practices adopted in the MF industry, we argue that the individual stage-level processes of our proposed three-stage process may operate under different levels of risk exposure (environmental conditions). Accordingly, we sort the three processes into two groups such that operational management and resource management together forms one group and Portfolio management another. Clearly, in the case of MFs, portfolio management process is a high risk undertaking compared to operational and resource management processes. The operational and resource management activities, on the other hand, are relatively low risk undertakings. Therefore, we group the operational and resource management processes together and refer to them as an allied process. We model the difference in the risk exposure of the allied process and the portfolio management process through restrictions imposed on the weights associated with intermediate measures. When two consecutive individual stage-level processes operate under similar levels of risk exposure, we assume that there is no internal resource imbalance (IRI) and when they operate under different levels of risk exposure there is potential for IRI. We construct a new index to measure IRI of the proposed three-stage MF management process. Overall, we (i) formulate a general DEA model to assess MF performance by conceptualising overall fund management process as a serially linked three-stage process under which the first two stages operate as an allied process and (ii) propose an index that measures IRI which is of practical value - both are new additions to MF performance appraisal literature.

An advantage of our modelling approach is that not only we can assess overall fund management performance but we can also assess performance from three different standpoints; operational management, resource management and portfolio management. This offers valuable information to MF managers as they will be able to judge their performance relative to their peers from different aspects of management. In our empirical investigation of 298 U.S. equity MFs, we demonstrate that level of IRI may be used to discriminate funds with similar ranking in terms of performance. Through our modelling framework, we could ascertain whether overall inefficiency results from inefficiency of individual stage-level process, internal resource imbalance or both and therefore provide more insights and new information on MF performance and on MF management practice. Because we develop a general model, our model may be easily recast to appraise performance of other types of financial entities and may not be limited to applications in the finance sector. Our models are readily applicable in situations where the activities of two consecutive stages of a three-stage process are carried out in-house and the activities of the other stage are outsourced. The in-house and outsourced activities referred here aligns with the concept of internal and external entities described in Pournader et al. (2017) with reference to practices in supply chain management.

The rest of the paper is organized as follows. In section 2, we discuss MF performance appraisal using DEA briefly, and introduce our proposed network structure. We develop a new network DEA model in section 3. In section 4, we describe the data and present the input, intermediate and output measures used in the empirical investigation. The results are discussed in section 5 followed by robustness check of the results in section 6. Section 7 concludes the paper with some remarks.

2. Mutual fund performance appraisal using DEA

2.1 Background

The studies that use conventional DEA models to evaluate MF performance generally consider MF management process as a single-stage production process with multiple inputs and multiple outputs (See e.g., Basso and Funari, 2001; Choi and Murthi, 2001; Tarim and Karan, 2001; Galagedera and Silvapulle, 2002). When a production process is modelled as a single-stage process, we are blinded as to what happens within the production process at large. Where this matters, network DEA models add value over the information obtained via conventional DEA models. Network DEA models incorporate the internal structure of the production process into performance analysis. Because of this versatility, network DEA is becoming increasingly popular in performance appraisal and a variety of network structures is presented in the literature. The network structure depends on the production process. For a detailed description of DEA model development of systems with network structures see Kao (2017). A popular application area of network DEA is supply chain management (Mirhedayatian et al., 2014; Pournader, et al., 2017). In the case of MF performance appraisal, as far as we are aware, only two studies use network DEA approach (Premachandra et al., 2012; Galagedera et al., 2016). The network structure that they apply is a two-stage process. Both these studies highlight that stage-level inefficiency may vary across MFs and their contribution towards overall inefficiency provide useful managerial information.

2.2 Proposed network structure

Studies highlight that MF performance may be associated with many factors including fund size, returns, variability in the returns, cost, fees, redemption and net asset value. Adverse macroeconomic conditions can make the task of MF management even more difficult. Furthermore, MF specific micro-level information is not available freely. Hence, MF performance appraisal is not a straightforward exercise. Subject to these limitations, we endeavour to apprise MF performance by considering a large set of measures deemed important in MF performance appraisal. We draw a parallel between MF performance appraisal and bank performance appraisal in variable section. In bank performance appraisal, whether to treat deposits as an input or as an output is a dilemma. One way that this issue is addressed in the literature is to conceptualise bank operation as a two-stage production process. A similar dilemma arises in MF performance appraisal as fund size and net asset value may be considered as inputs or as outputs. Using both fund size and net asset value together as inputs or as outputs however is not prudent as both proxy scale of operation in some ways and are generally highly positively correlated. We propose an alternative. We argue that net asset value may be considered as total funds transformed through a management process different from operational management and portfolio management. This is the foundation for augmenting the two-stage process proposed in Premachandra et al. (2012) for MF performance appraisal to a three-stage process.[1]

Figure 1 depicts the empirical framework of our proposed three-stage MF management process. We select the input, intermediate and output measures shown in Figure 1 guided by previous DEA

[1] We are mindful here that the required number of individual stage-specific measures increases as the number of stages included in a multi-stage production process increases. Therefore, given that it not easy to obtain fund specific information of a large number of MFs, application of multi-stage network DEA models for MF performance appraisal can be a challenging task. When the relevant stage-specific measures are not available for performance appraisal in a multi-stage framework, it is better to conceptualise MF management process as a single-stage process.

studies of MF performance appraisal (Premachandra, et al. 2012; Malhotra, Martin and Russel, 2007; Wilkens and Zhu, 2005; Choi and Murthi, 2001; Murthi, Choi and Desai, 1997).

First stage (operational management process – labelled stage A) is where funds are raised and therefore fund size is considered as stage A output. Stage A inputs are marketing and distribution fees and management fees. The second stage (resource management process – labelled stage B) is where the funds raised in stage A are secured for investment in the third stage. We consider net asset value (NAV) as stage B output. Disbursements such as transaction costs and costs incurred for recordkeeping, custodial services, taxes, legal expenses and accounting and auditing fees are not accounted for in stage A. Therefore, we consider management expense ratio as stage B input in addition to fund size and turnover ratio. The third stage (portfolio management process - labelled stage C) involves management of assets to generate returns with risk taken. Hence, we consider total risk, systematic risk, downside risk and NAV as inputs and return as output of stage C. We discussed earlier that operational and resource management processes operate under similar environmental conditions. In other words, the operational and resource management processes are subject to similar levels of risk exposure. Because of this commonality, we consider these two processes as an allied process. We show the alliance between stage A and stage B in Figure 1 by enclosing them in a dash-lined rectangle.

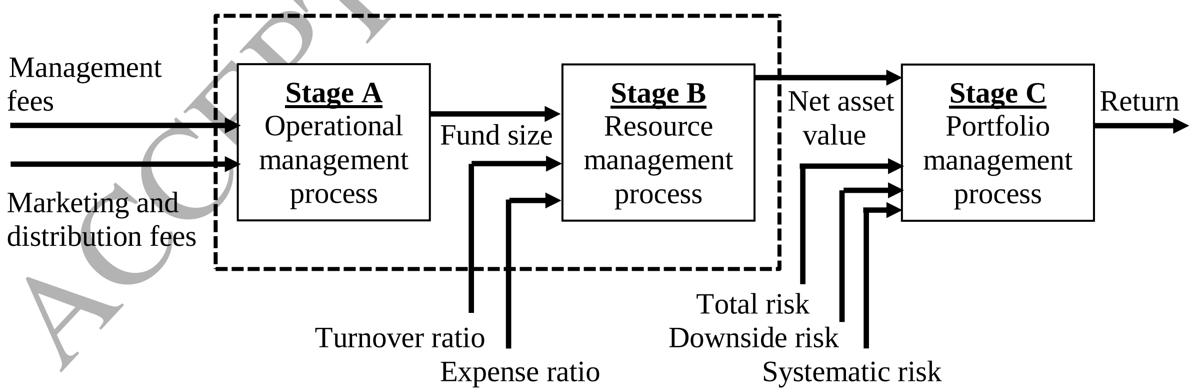

Fig 1. Empirical framework of three-stage MF management process

3. Model development

3.1 Three-stage production process with alliance between the first two stages

A general serially linked three-stage process with alliance between the first two stages is illustrated in Figure 2. The dash-lined rectangle of Figure 2 signifies the alliance between stage A and stage B. In this section, we derive DEA models for performance appraisal in the general three-stage network structure depicted in Figure 2. The network structure that we propose for our empirical study, depicted in Figure 1, is a special case of the general network structure in Figure 2 where stage A and stage B processes have no additional output; Y^A and Y^B. Therefore, when adopting the models derived in this section for performance appraisal in our empirical study, we set $Y_j^A = \{\}$ and $Y_j^B = \{\}$. Throughout this section, we assume there are *n* homogeneous MFs generically referred to as decision making units (DMUs) in the DEA terminology.

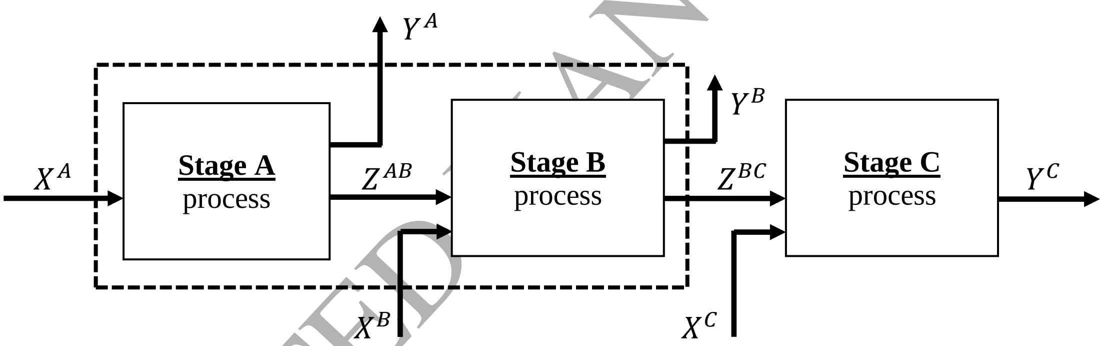

Fig 2. Three-stage process with alliance between the first two stages.

Let i_A = the number of independent inputs at stage A, $X_j^A = \left(x_{1j}^A, x_{2j}^A, \dots, x_{i_A j}^A\right)$ denote the inputs of DMU$_j$ observed at stage A, i_B = the number of independent inputs at stage B, $X_j^B = \left(x_{1j}^B, x_{2j}^B, \dots, x_{i_B j}^B\right)$ denote the inputs of DMU$_j$ observed at stage B, i_C = the number of independent inputs at stage C, $X_j^C = \left(x_{1j}^C, x_{2j}^C, \dots, x_{i_C j}^C\right)$ denote the inputs of DMU$_j$ observed at stage C, d_{AB} = the number of intermediate measures linking stage A and stage B, $Z_j^{AB} = \left(z_{1j}^{AB}, z_{2j}^{AB}, \dots, z_{d_{AB} j}^{AB}\right)$ denote the observed levels of the intermediate measures linking stage A and stage B of DMU$_j$, d_{BC} = the number of intermediate measures linking stage B to stage C, $Z_j^{BC} =$

$\left(z_{1j}^{BC}, z_{2j}^{BC}, \ldots, z_{d_{BC}j}^{BC}\right)$ denote the observed levels of the intermediate measures linking stage B and stage C of DMU$_j$, r_A = the number of independent outputs at stage A, $Y_j^A = \left(y_{1j}^A, y_{2j}^A, \ldots, y_{r_A j}^A\right)$ denote the observed outputs of DMU$_j$ at stage A, r_B = the number of independent outputs at stage B, $Y_j^B = \left(y_{1j}^B, y_{2j}^B, \ldots, y_{r_B j}^B\right)$ denote the observed outputs of DMU$_j$ at stage B, r_C = the number of independent outputs at stage C, and $Y_j^C = \left(y_{1j}^C, y_{2j}^C, \ldots, y_{r_C j}^C\right)$ denote the observed outputs of DMU$_j$ at stage C.

Generally, MF performance metrics are positive with the exception of average return. In our case, average return is an output of the final stage and to ensure that average return is positive we add a positive constant to the returns. Such transformation of output does not affect the optimal solutions of Banker et al. (1984) input-oriented variable returns to scale model, known as the BCC model (Pastor, 1996). Therefore, analogous to the studies that used DEA to appraise MF performance with average return as output, we employ input-oriented models under the variable returns to scale assumption. Furthermore, this is consistent with our empirical situation as in practice; MF managers have more control over the inputs (fees, expenses and risk) than with output (return).

We begin by modelling each stage as an independent disjoint process (no link with any of the other processes). To compute the efficiency of DMU$_0$ at stages A, B and C (denoted by θ_0^A, θ_0^B and θ_0^C) we adopt the input-oriented BCC model in multiplier form. Formally, the model that computes θ_0^A can be expressed as

$$\theta_0^{A*} = Max \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AAB} z_{d0}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{r0}^A + \zeta^A}{\sum_{i=1}^{i_A} v_i^A x_{i0}^A} \quad (1)$$

subject to $$\sum_{d=1}^{d_{AB}} \eta_d^{AAB} z_{dj}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{rj}^A + \zeta^A \leq \sum_{i=1}^{i_A} v_i^A x_{ij}^A, \qquad j = 1,2, \ldots, n,$$

All $\eta_d^{AAB}, u_r^A, v_i^A \geq \epsilon$; ζ_0^A is unrestricted.

where n is the number of DMUs. The intermediate measures play a dual role in model formulation. To make this distinction clear, we denote the weight assigned to z_{d0}^{AB} as η_d^{AAB} when as z_{d0}^{AB} is viewed as an output of stage A and when z_{d0}^{AB} is viewed as an input of stage B we denote the weight assigned to it as η_d^{BAB}. The intermediate measures z_{d0}^{BC}, $d = 1,2, \ldots, d_{BC}$ link stage B and stage C. Therefore, when z_{d0}^{BC} is viewed as an output of stage B, the weight assigned to it is denoted as η_d^{BBC} and when z_{d0}^{BC} is viewed as an input of stage C the weight assigned to it is denoted as η_d^{CBC}. In model (1), the decision variables $\eta_d^{AAB}, u_r^A, v_i^A$ (referred to as multipliers) are constrained to positive values ($\geq \epsilon$, where ϵ is a small positive number).[2] Similarly, the model that computes θ_0^B may be written as

$$\theta_0^{B*} = Max \frac{\sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{r0}^B + \zeta^B}{\sum_{d=1}^{d_{AB}} \eta_d^{BAB} z_{d0}^{AB} + \sum_{i=1}^{i_B} v_i^B x_{i0}^B} \tag{2}$$

subject to

$$\sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{dj}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{rj}^B + \zeta_0^B \leq \sum_{d=1}^{d_{AB}} \eta_d^{BAB} z_{dj}^{AB} + \sum_{i=1}^{i_B} v_i^B x_{ij}^B, \quad j = 1,2, \dots, n,$$

$$\text{All } \eta_d^{BBC}, \eta_d^{BAB}, u_r^B, v_i^B \geq \epsilon; \ \zeta^B \text{ is unrestricted,}$$

and the model that computes θ_0^C may be written as

$$\theta_0^{C*} = Max \frac{\sum_{r=1}^{r_C} u_r^C y_{r0}^C + \zeta^C}{\sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C} \tag{3}$$

subject to $$\sum_{r=1}^{r_C} u_r^C y_{rj}^C + \zeta_0^C \leq \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{dj}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{ij}^C, \quad j = 1,2, \dots, n,$$

$$\text{All } \eta_d^{CBC}, u_r^C, v_i^C \geq \epsilon; \ \zeta_0^C \text{ is unrestricted.}$$

In our empirical investigation, we conceptualise that the first two stages operate as an allied process. We model this alliance by imposing the condition that the corresponding intermediate measures are valued the same regardless of their role, either as output at stage A or as input at stage B. This is a commonly used assumption in DEA studies of two-stage processes (Liang et al., 2006; Kao and Hwang, 2008; Liang et al., 2008; Chen et al., 2009; Aviles-Sacato et al., 2015). Specifically, we assume that the multipliers associated with the intermediate measures linking stage A and stage B are the same so that in Models (1) and (2) $\eta_d^{AAB} = \eta_d^{BAB} = \eta_d^{AB}$; $d = 1,2, \dots, d_{AB}$. This condition ensures that the implied value of stage A output associated with the intermediate resources Z_0^{AB} is equal to the implied value of the stage B input associated with the same set of intermediate resources, i.e. $\sum_{d=1}^{d_{AB}} \eta_d^{AAB} z_{d0}^{AB} = \sum_{d=1}^{d_{AB}} \eta_d^{BAB} z_{d0}^{AB}$. We interpret this condition as stage A and stage B operating with no intermediate resource imbalance (IRI). We define the level of IRI between stage A and stage B as

$$I_0^{A-B} = \frac{\sum_{d=1}^{d_{AB}} \eta_d^{BAB} z_{d0}^{AB}}{\sum_{d=1}^{d_{AB}} \eta_d^{AAB} z_{d0}^{AB}} \tag{4}$$

In our case, the alliance assumption on the multipliers $(\eta_d^{AAB} = \eta_d^{BAB} = \eta_d^{AB})$ implies $I_0^{A-B} = 1$.

We express efficiency of the allied processes, θ_0^{AB} as a weighted average of stage A and stage B efficiencies, θ_0^A and θ_0^B such that $\theta_0^{AB} = w_A \theta_0^A + w_B \theta_0^B$ and $w_A + w_B = 1$ where w_A and w_B are user-

[2] In our empirical analysis, we use $\epsilon = 10^{-5}$.

specified weights. Chen et al. (2009) suggest that the relative 'size' of the inputs of a stage may reflect the importance of that stage. They use the ratio of the implied value of the inputs (resources) of a stage to the implied value of the inputs of all stages as the weight for the efficiency of that stage. We do the same because this is a reasonable assumption given that the model we apply is input-oriented. Accordingly, w_A and w_B can be defined as $w_A = \frac{\sum_{i=1}^{i_A} v_i^A x_{i0}^A}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B}$ and $w_B = \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_B} v_i^B x_{i0}^B}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B}$ and the aggregate efficiency of the allied process, θ_0^{AB} can be written as

$$\theta_0^{AB} = \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{r0}^A + \zeta^A + \sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{r0}^B + \zeta^B}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B}. \tag{5}$$

Similarly, we express overall efficiency of the three-stage process of DMU_0, θ_0^{ABC} as a weighted average of the allied process efficiency, θ_0^{AB}, and stage C efficiency, θ_0^C, such that $\theta_0^{ABC} = w_{AB}\theta_0^{AB} + w_C\theta_0^C$ and $w_{AB} + w_C = 1$ where w_{AB} and w_C are user-specified weights. Following the same line of argument used in specifying w_A and w_B, we write w_{AB} and w_C as

$$w_{AB} = \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C} \tag{6}$$

and

$$w_C = \frac{\sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C}. \tag{7}$$

Then θ_0^{ABC} can be written as

$$\theta_0^{ABC} = \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{r0}^A + \zeta^A + \sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{r0}^B + \zeta^B + \sum_{r=1}^{r_C} u_r^C y_{r0}^C + \zeta^C}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C}. \tag{8}$$

In our empirical setup, we assume that the allied and stage C processes operate under different environmental conditions in respect of risk. This situation is somewhat analogous to a two-stage process where each process is undertaken by a different firm. In that case, it is not just to enforce the condition that both firms assign the same value to a resource when that resource plays a dual role-output in one case and input in the other. Such dilemmas arise in network representation of supply chains involving subcontractors. For example, suppose the firm that operates the second stage is a subcontractor of the firm that operates the first stage. In that case, the subcontractor will be under no obligation to accede to a model-implied *conditional* valuation scheme unfavourable to it when appraising its performance. Following a similar line of reasoning for processes that operates under different levels of risk exposure, we allow some extent of flexibility in the choice of multipliers associated with the intermediate measures that link the allied and stage C processes. Specifically, we do not impose the restriction that the multipliers of the intermediate measures linking the allied process with stage C process have the same value. The constraints that we impose on the multipliers associated with the intermediate measures Z^{BC} are $\eta_d^{BBC} \geq \eta_d^{CBC}$ for $d = 1,2,\ldots,d_{BC}$.[3] These conditions ensure that the sum of the implied value of the intermediate measures as output of the allied process is greater than or equal to the sum of the implied value of the same set of intermediate measures as input at stage C, i.e. $\sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{dj}^{BC} \geq \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{dj}^{BC}$ for all *j*=1,2,…,*n*. Hence, there is a possibility of imbalance in the implied value of intermediate resources z_d^{BC}. In this case, the level of IRI between the allied process and stage C is measured by

$$I_0^{AB-C} = \frac{\sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{dj}^{BC}}{\sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{dj}^{BC}}. \tag{9}$$

The assumption on the multipliers guarantees that $0 < I_0^{AB-C} \leq 1$.

3.2 Assessing overall performance

Following the arguments presented in the previous subsection, the model we use to determine the overall efficiency of the three-stage process depicted in Figure 1, θ_0^{ABC} can be formulated as

$$\theta_0^{ABC*} = Max \frac{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{r0}^A + \zeta^A + \sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{r0}^B + \zeta^B + \sum_{r=1}^{r_C} u_r^C y_{r0}^C + \zeta^C}{\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} v_i^A x_{i0}^A + \sum_{i=1}^{i_B} v_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{i0}^C} \tag{10}$$

subject to

$$\sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{dj}^{AB} + \sum_{r=1}^{r_A} u_r^A y_{rj}^A + \zeta^A \leq \sum_{i=1}^{i_A} v_i^A x_{ij}^A, \qquad j = 1,2,\ldots,n,$$

$$\sum_{d=1}^{d_{BC}} \eta_d^{BBC} z_{dj}^{BC} + \sum_{r=1}^{r_B} u_r^B y_{rj}^B + \zeta^B \leq \sum_{d=1}^{d_{AB}} \eta_d^{AB} z_{dj}^{AB} + \sum_{i=1}^{i_B} v_i^B x_{ij}^B, \qquad j = 1,2,\ldots,n,$$

$$\sum_{r=1}^{r_C} u_r^C y_{rj}^C + \zeta^C \leq \sum_{d=1}^{d_{BC}} \eta_d^{CBC} z_{dj}^{BC} + \sum_{i=1}^{i_C} v_i^C x_{ij}^C, \qquad j = 1,2,\ldots,n,$$

$$\eta_d^{BBC} \geq \eta_d^{CBC}, \qquad d = 1,2,\ldots,d_{BC},$$

[3] When assessing performance with a metric defined as the ratio of composite output to composite input, it is desirable to have high composite output and low composite input. In stage C, $z_d^{BC}: d = 1,2,\ldots,d_{BC}$ are input measures and therefore when assessing stage C performance, it is desirable to value them low than high suggesting that η_d^{CBC} may not exceed η_d^{BBC} for $= 1,2,\ldots,d_{BC}$ is a realistic scenario.

All $\eta_d^{AB}, \eta_d^{BBC}, \eta_d^{CBC}, u_r^A, v_i^A, u_r^B, v_i^B, u_r^C, v_i^C \geq \epsilon$; ζ^A, ζ^B and ζ^C are unrestricted.

The constraints of model (10) are the constraints of models (1), (2) and (3) with $\eta_d^{AAB} = \eta_d^{BAB} = \eta_d^{AB}$; $d = 1,2,\ldots,d_{AB}$ plus a set of constraints on the multipliers of the intermediate measures linking the allied process with stage C process. The overall process of DMU_0 is called "efficient" if and only if $\theta_0^{ABC*} = 1$.

A linear programming equivalent of model (10) may be obtained through Charnes-Cooper transformation (Charnes and Cooper, 1962) as

$$\theta_0^{ABC*} = Max \begin{pmatrix} \sum_{d=1}^{d_{AB}} \pi_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} \mu_r^A y_{r0}^A + \xi^A + \sum_{d=1}^{d_{BC}} \pi_d^{BBC} z_{d0}^{BC} \\ + \sum_{r=1}^{r_B} \mu_r^B y_{r0}^B + \xi^B + \sum_{r=1}^{r_C} \mu_r^C y_{r0}^C + \xi^C \end{pmatrix} \tag{11}$$

subject to

$$\sum_{d=1}^{d_{AB}} \pi_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \omega_i^A x_{i0}^A + \sum_{i=1}^{i_B} \omega_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \pi_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} \omega_i^C x_{i0}^C = 1,$$

$$\sum_{d=1}^{d_{AB}} \pi_d^{AB} z_{dj}^{AB} + \sum_{r=1}^{r_A} \mu_r^A y_{rj}^A + \xi^A \leq \sum_{i=1}^{i_A} \omega_i^A x_{ij}^A, \qquad j = 1,2,\ldots,n,$$

$$\sum_{d=1}^{d_{BC}} \pi_d^{BBC} z_{dj}^{BC} + \sum_{r=1}^{r_B} \mu_r^B y_{rj}^B + \xi^B \leq \sum_{d=1}^{d_{AB}} \pi_d^{AB} z_{dj}^{AB} + \sum_{i=1}^{i_B} \omega_i^B x_{ij}^B, \qquad j = 1,2,\ldots,n,$$

$$\sum_{r=1}^{r_C} \mu_r^C y_{rj}^C + \xi^C \leq \sum_{d=1}^{d_{BC}} \pi_d^{CBC} z_{dj}^{BC} + \sum_{i=1}^{i_C} \omega_i^C x_{ij}^C, \qquad j = 1,2,\ldots,n,$$

$$\pi_d^{BBC} \geq \pi_d^{CBC}, \qquad d = 1,2,\ldots,d_{BC},$$

All $\mu_r^A, \omega_i^A, \mu_r^B, \omega_i^B, \mu_r^C, \omega_i^C, \pi_d^{AB}, \pi_d^{BBC}, \pi_{d0}^{CBC} \geq \epsilon$; ξ^A, ξ^B and ξ^C are unrestricted.

The optimal efficiency of the three-stage process of DMU_0 may be computed from the optimal values of the decision variables of model (11) as[4]

$$\theta_0^{ABC*} = \begin{matrix} \sum_{d=1}^{d_{AB}} \pi_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \mu_r^{A*} y_{r0}^A + \xi^{A*} + \sum_{d=1}^{d_{BC}} \pi_d^{BBC*} z_{d0}^{BC} \\ + \sum_{r=1}^{r_B} \mu_r^{B*} y_{r0}^B + \xi^{B*} + \sum_{r=1}^{r_C} \mu_r^{C*} y_{r0}^C + \xi^{C*} \end{matrix} \tag{12}$$

3.3 Assessing stage-level performance

Step-I: Computing efficiency of overall process components

In our modelling framework, overall efficiency score is a weighted average of the efficiency scores of the allied and stage C processes. Hence, after solving model (11), we may compute the efficiency scores of the allied process and stage C process by substituting the optimal values of the decision variables obtained in the solution to model (11) in (5) and thereafter in $\theta_0^{ABC} = w_{AB}\theta_0^{AB} + w_C\theta_0^C$.

[4] The decision variables with * superscript indicate that they are the optimal values obtained in the corresponding model.

However, as the optimal values of the decision variables of model (11) may not be unique, it is plausible that the decomposed overall efficiency scores are not unique. Kao and Hwang (2008) suggest a way forward: computing component efficiency scores by giving priority to one of the processes that makes up the overall process (in our case, allied process and stage C process) and determining its efficiency score first while maintaining the optimal efficiency of the overall process fixed. Suppose the allied process is given priority over the stage C process. Then, the efficiency of the allied process, θ_0^{AB} may be computed first while maintaining the optimal overall efficiency computed in (11) fixed at θ_0^{ABC*} using

$$\theta_0^{AB_1*} = Max \frac{\sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} \varrho_r^A y_{r0}^A + \varsigma^A + \sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} \varrho_r^B y_{r0}^B + \varsigma^B}{\sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \gamma_i^A x_{i0}^A + \sum_{i=1}^{i_B} \gamma_i^B x_{i0}^B} \quad (13)$$

subject to

$$\sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{dj}^{AB} + \sum_{r=1}^{r_A} \varrho_r^A y_{rj}^A + \varsigma^A \le \sum_{i=1}^{i_A} \gamma_i^A x_{ij}^A, \qquad j = 1,2,\dots,n,$$

$$\sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{dj}^{BC} + \sum_{r=1}^{r_B} \varrho_r^B y_{rj}^B + \varsigma^B \le \sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{dj}^{AB} + \sum_{i=1}^{i_B} \gamma_i^B x_{ij}^B, \qquad j = 1,2,\dots,n,$$

$$\sum_{r=1}^{r_C} \varrho_r^C y_{rj}^C + \varsigma^C \le \sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{dj}^{BC} + \sum_{i=1}^{i_C} \gamma_i^C x_{ij}^C, \qquad j = 1,2,\dots,n,$$

$$\tau_d^{BBC} \ge \tau_d^{CBC}, \qquad d = 1,2,\dots,d_{BC},$$

$$\sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} \varrho_r^A y_{r0}^A + \varsigma^A + \sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} \varrho_r^B y_{r0}^B + \varsigma^B + \sum_{r=1}^{r_C} \varrho_r^C y_{r0}^C + \varsigma^C = \theta_0^{ABC*} \left(\sum_{d=1}^{d_{AB}} \tau_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \gamma_i^A x_{i0}^A + \sum_{i=1}^{i_B} \gamma_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} \gamma_i^C x_{i0}^C \right),$$

All $\varrho_r^A, \gamma_i^A, \varrho_r^B, \gamma_i^B, \varrho_r^C, \gamma_i^C, \tau_d^{AB}, \tau_d^{BBC}, \tau_d^{CBC} \ge \epsilon$; ς^A, ς^B and ς^C are unrestricted.

The efficiency of the allied process may be obtained as

$$\theta_0^{AB_1*} = \frac{\sum_{d=1}^{d_{AB}} \tau_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \varrho_r^{A*} y_{r0}^A + \varsigma^{A*} + \sum_{d=1}^{d_{BC}} \tau_d^{BBC*} z_{d0}^{BC} + \sum_{r=1}^{r_B} \varrho_r^{B*} y_{r0}^B + \varsigma^{B*}}{\sum_{d=1}^{d_{AB}} \tau_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_A} \gamma_i^{A*} x_{i0}^A + \sum_{i=1}^{i_B} \gamma_i^{B*} x_{i0}^B} \quad (14)$$

Normally, stage C process efficiency, $\theta_0^{C_2}$ is obtained thereafter using $\theta_0^{ABC*} = w_{AB}^* \theta_0^{AB_1*} + w_C^* \theta_0^{C_2}$ where w_{AB}^* and w_C^* are the optimal weights for the allied process and stage C process are obtained by substituting the optimal decision variable values of the solution to model (13) in (6) and (7).[5]

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[5] The superscript 1 and 2 attached to θ_0^{AB} and θ_0^C indicate which of the two components of the overall process is considered relatively more important over the other in overall efficiency decomposition. For example, superscript 1 of $\theta_0^{AB_1}$ indicates that the allied process is considered relatively more important than stage C process. Similarly, if stage C process is preferred to the allied process, we may first maximise $\theta_0^{C_1} = \frac{\sum_{r=1}^{r_C} \varrho_r^C y_{r0}^C + \varsigma^C}{\sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} \gamma_i^C x_{i0}^C}$ subject to the same set of constraints given in model (13). Suppose the efficiency of stage C process computed this way is $\theta_0^{C_1*}$ and the corresponding efficiency of the allied process is $\theta_0^{AB_2}$. Then,

Nonetheless, we do not compute stage C efficiency this way, because there is potential for imbalance in the intermediate resources linking the allied and stage C processes. Therefore, not only the stage level efficiencies, the IRI computed with optimal multipliers obtained from model (13) also may not be unique. In view of this, we compute stage C efficiency after controlling for IRI by obtaining a unique value for I_0^{AB-C}. Computations of a unique value for I_0^{AB-C} and stage-level efficiencies are described next.

Step-II: Controlling for intermediate resource imbalance

To obtain a unique value for I_0^{AB-C}, we maximise $\left(\sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{d0}^{BC} - \sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{d0}^{BC}\right)$ while maintaining the efficiency of the three-stage process (overall efficiency) computed in model (11) fixed at θ_0^{ABC*} and the allied process efficiency computed in model (13) at $\theta_0^{AB_1*}$. Maximising $\left(\sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{d0}^{BC} - \sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{d0}^{BC}\right)$ may be interpreted as allowing $\sum_{d=1}^{d_{BC}} \tau_d^{BBC} z_{d0}^{BC}$ (implied value of an output of the allied process) to attain a high value and $\sum_{d=1}^{d_{BC}} \tau_d^{CBC} z_{d0}^{BC}$ (implied value of an input of stage C process) to attain a low value. This is in line with giving some sort of flexibility to the allied and stage C processes in their pursuit to show performance in a manner that is favourable to them and therefore is consistent with the assumption that allied process and stage C process have different risk profiles and may be considered as operating under different environmental conditions.

The linear programming model that we solve here is

$$Max \left(\sum_{d=1}^{d_{BC}} \psi_d^{BBC} z_{d0}^{BC} - \sum_{d=1}^{d_{BC}} \psi_d^{CBC} z_{d0}^{BC}\right) \tag{15}$$

subject to

$$\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{dj}^{AB} + \sum_{r=1}^{r_A} \phi_r^A y_{rj}^A + \delta^A \le \sum_{i=1}^{i_A} \varphi_i^A x_{ij}^A, \quad j = 1,2,\dots,n,$$

$$\sum_{d=1}^{d_{BC}} \psi_d^{BBC} z_{dj}^{BC} + \sum_{r=1}^{r_B} \phi_r^B y_{rj}^B + \delta^B \le \sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{dj}^{AB} + \sum_{i=1}^{i_B} \varphi_i^B x_{ij}^B, \quad j = 1,2,\dots,n,$$

$$\sum_{r=1}^{r_C} \phi_r^C y_{rj}^C + \delta^C \le \sum_{d=1}^{d_{BC}} \psi_d^{CBC} z_{dj}^{BC} + \sum_{i=1}^{i_C} \varphi_i^C x_{ij}^C, \quad j = 1,2,\dots,n,$$

$$\psi_d^{BBC} \ge \psi_d^{CBC}, \quad d = 1,2,\dots,d_{BC},$$

$$\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^A y_{r0}^A + \delta^A + \sum_{d=1}^{d_{BC}} \psi_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^B y_{r0}^B + \delta^B + \sum_{r=1}^{r_C} \phi_r^C y_{r0}^C + \delta^C =$$
$$\theta_0^{ABC*}\left(\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \varphi_i^A x_{i0}^A + \sum_{i=1}^{i_B} \varphi_i^B x_{i0}^B + \sum_{d=1}^{d_{BC}} \psi_d^{CBC} z_{d0}^{BC} + \sum_{i=1}^{i_C} \varphi_i^C x_{i0}^C\right),$$

if $\theta_0^{AB_1*} = \theta_0^{AB_2}$ and $\theta_0^{C_1*} = \theta_0^{C_2}$ we may conclude that efficiency decomposition is unique. Models that test whether the efficiency decomposition is unique are available in Liang, Cook and Zhu (2008).

$$\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^A y_{r0}^A + \delta^A + \sum_{d=1}^{d_{BC}} \psi_d^{BBC} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^B y_{r0}^B + \delta^B =$$
$$\theta_0^{AB_1*}\left(\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \varphi_i^A x_{i0}^A + \sum_{i=1}^{i_B} \varphi_i^B x_{i0}^B\right),$$

All $\phi_r^A, \varphi_i^A, \phi_r^B, \varphi_i^B, \phi_r^C, \varphi_i^C, \psi_d^{AB}, \psi_d^{BBC}, \psi_d^{CBC} \geq \epsilon$; δ^A, δ^B and δ^C are unrestricted.

After solving model (15), we may compute the optimal adverse level of IRI between the allied and stage C processes of DMU_0 as $I_0^{AB-C*} = \frac{\sum_{d=1}^{d_{BC}} \psi_d^{CBC*} z_{d0}^{BC}}{\sum_{d=1}^{d_{BC}} \psi_d^{BBC*} z_{d0}^{BC}}$ where ψ_d^{CBC*} and ψ_d^{BBC*} are the optimal values of the corresponding decision variables. Then the optimal adverse level of IRI of the overall fund management process of DMU_0, I_0^*, can be expressed as

$$I_0^* = I_0^{A-B*} \times I_0^{AB-C*} \quad (16)$$

We refer to I_0^* as IRI index of DMU_0. By assumption we have $I_0^{A-B*} = 1$ (from (4)). Therefore, $I_0^* = I_0^{AB-C*}$. For a given DMU_0, $I_0^* = 1$ indicates no imbalance in the use of intermediate resources in the overall fund management process which we consider as a desired outcome for DMU_0. Further, since we restrict ψ_d^{BBC} and ψ_d^{CBC} for all $d = 1,2, \ldots d_{BC}$ to be strictly positive ($\geq \epsilon$) and $\psi_d^{BBC*} \geq \psi_d^{CBC*}, d = 1,2, \ldots, d_{BC}$, we have that $0 < I_0^* \leq 1$. Therefore, the smaller the value of I_0^* is the higher the level of IRI between the allied and stage C processes or generally in the overall management process. In section 5.3 we show that I_0^* may be used as an additional criterion to discriminate performance.

Step-III: Computing stage level efficiency

The individual stage-wide efficiency scores of DMU_0 are obtained after computing its optimal allied process efficiency score while maintaining its optimal overall efficiency score and under the worst case scenario of IRI. We compute the efficiency scores of stage A, stage B and stage C using the optimal decision variable values obtained from model (15) as

$$\theta_0^{A*} = \frac{\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^{A*} y_{r0}^A + \delta^{A*}}{\sum_{i=1}^{i_A} \varphi_i^{A*} x_{i0}^A}, \quad (17)$$

$$\theta_0^{B*} = \frac{\sum_{d=1}^{d_{BC}} \psi_d^{BBC*} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^{B*} y_{r0}^B + \delta^{B*}}{\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_B} \varphi_i^{B*} x_{i0}^B} \quad (18)$$

and

$$\theta_0^{C*} = \frac{\sum_{r=1}^{r_C} \phi_r^{C*} y_{r0}^{C} + \delta^{C*}}{\sum_{d=1}^{d_{BC}} \psi_d^{CBC*} z_{d0}^{BC} + \sum_{i=1}^{i_C} \varphi_i^{C*} x_{i0}^{C}}. \tag{19}$$

The following assertions are stated with respect to our serially linked three-stage production process with alliance between the first two stages.

Lemma 3.3.1: The allied process of DMU_0 is efficient if and only if its individual processes are efficient. The *proof* is given in the Appendix.

Theorem 3.3.2: The overall process of DMU_0 is efficient if and only if its individual stages are efficient. The *proof* is given in the Appendix.

4. Application to U.S. equity mutual funds

We focus on U.S. equity MFs. Our sample is obtained from the 2015 fund profile in the Morningstar Direct database. Initially we collected data on a large number of funds and later reduced to 298 funds according to our sample selection criteria. We require that funds have inception dates prior to 1 January 2006, have been in active trading and survived up to 31 December 2015. Hence, our sample is free from survivorship and age-bias. Further, we restrict the sample to large funds – exceeding US$ 1 billion in size – and therefore the sample may be considered as comprising of healthy funds. We require all funds to have non-zero values for all measures in all sampled years. Table 1 lists the measures used in the analysis and gives a brief description of their calculation. Our sample does not include funds that do not report an overall Morningstar rating at the end of the sample period, 2015. Morningstar Direct report funds under different share classes and where such representation duplicate funds, we avoid double counting.

Table 1
Input, intermediate and output measures used in DEA models.

Stage A	
Input measures	
Management fees (x_1^A) :	Fees paid to investment advisors expressed as a percentage.
Marketing and distribution fees (x_2^A) ("12b-1" fees):	Cost of marketing and selling fund shares expressed as a percentage.
Intermediate measure that links stage A and stage B	
Fund size (z^{AB}):	Market value of portfolio in base currency.
Stage B	
Input measures	
Net expense ratio (x_1^B) :	Annual fee expressed as a percentage to cover expenses such as administrative fees, operating costs and all other asset-based costs incurred by the fund.
Turnover ratio (x_2^B):	Percentage of holdings replaced.
Intermediate measure that links stage B and stage C	
Net Asset Value (z^{BC}):	Total value of portfolio less liabilities in base currency.
Stage C	
Input measures	
Total risk (x_1^C):	Standard deviation of weekly return.
Systematic risk (x_2^C):	CAPM beta computed using weekly return.
Downside risk (x_3^C):	Downside standard deviation of weekly return.
Output measure	
Annual return (y^C):	Expressed as a percentage.

The sample comprise of 246 domestic equity MFs (82.6%) and 52 international equity MFs (17.4%). Domestic equity MFs have at least 70 per cent of total assets invested in U.S. stock markets and international equity MFs have at least 40 per cent of their exposure in overseas equity markets. In our sample, 14 (4.7%), 61 (20.5%) and 137 (46%) MFs are rated 5-star, 4-star and 3-star respectively and 73 (24.5%) and 13 (4.3%) funds 2-star and 1-star respectively by Morningstar. This is an indication that the MFs in our sample are not biased towards a specific risk-adjusted return performance level.

5. Results and discussion

5.1 Overview of overall, allied process and stage level performance

Table 2 gives summary statistics of relative efficiency scores of domestic and international MFs separately and together. Panel A of Table 2 reveals that the average overall management efficiency score of all MFs is 0.676 with coefficient of variation (CV) 0.170. The overall performances of domestic and international MFs also have similar characteristics. The average overall management efficiency score of domestic MFs is 0.678 (CV = 0.171) and of international MFs is 0.666 (CV = 0.165). To test whether the difference in performance between domestic and international MFs are statistically significant, we adopt a non-parametric test of equality of the median efficiency scores of two groups proposed in Banker et al. (2010).[6] According to this test, the difference in the median efficiency scores of domestic and international funds is not statistically significant at the 5 per cent level in all aspects of management investigated in the study. The results are reported in the last row of Table 2. Ours is a cross-sectional study. The difference in relative performance of domestic and international equity MFs may depend on the observation period.

Table 2
Summary of relative efficiency scores of domestic, international and all funds.

	Operational management relative efficiency	Resource management relative efficiency	Portfolio management relative efficiency	Allied process relative efficiency	Overall management relative efficiency
Panel A: All funds (n=298)					
Average	0.267	0.481	0.741	0.297	0.676
Std. deviation	0.148	0.215	0.187	0.170	0.115
Minimum	0.004	0.001	0.000	0.065	0.277
Median	0.241	0.482	0.778	0.247	0.677
Maximum	1.000	1.000	1.000	1.000	1.000
No of efficient funds	4 (1.34%)	3 (1.01%)	7 (2.35%)	2 (0.67%)	1 (0.34%)
Panel B: Domestic funds (n=246)					
Average	0.262	0.487	0.745	0.293	0.678
Std. deviation	0.145	0.211	0.188	0.170	0.116
Minimum	0.004	0.046	0.000	0.065	0.277
Median	0.241	0.488	0.781	0.246	0.680
Maximum	1.000	1.000	1.000	1.000	1.000
No of efficient	4 (1.63%)	2 (0.81%)	5 (2.03%)	2 (0.81%)	1 (0.41%)

[6] This test is based on order statistics. The test statistic is $\hat{Z} = (\widehat{p_1} - \widehat{p_2}) \Big/ \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}$ where $\widehat{p_1} = \frac{n_1}{N_1}$, $\widehat{p_2} = \frac{n_2}{N_2}$, $\hat{p} = (N_1\widehat{p_1} + N_2\widehat{p_2})/(N_1 + N_2)$, N_1 and N_2 are the sample sizes of group 1 and group 2 and n_1 and n_2 are the number of observations in group 1 and group 2 that are lower than the median observation in the full sample.

funds					
Panel C: International funds (n=52)					
Average	0.291	0.456	0.725	0.314	0.666
Std. deviation	0.163	0.232	0.181	0.167	0.110
Minimum	0.005	0.001	0.004	0.154	0.436
Median	0.244	0.399	0.752	0.250	0.660
Maximum	0.998	1.000	1.000	0.995	0.992
No of efficient funds	0 (0.00%)	1 (1.92%)	2 (3.85%)	0 (0.00%)	0 (0.00%)
Test of the difference in the median relative efficiency scores of domestic and international funds					
Z-statistic*	0.809	-1.221	-1.221	1.076	-0.916

*None of the differences are statistically significant at the 5% level. The statistical test used here is described in footnote 5.

Now we discuss the association between the rankings of funds based on their relative efficiency scores. Table 3 reports the Spearman's rank correlation. The strongest association is observed in operational and allied process management efficiency scores with rank correlation coefficient at 0.844. The other counterpart of the allied process is resource management. The association between the rankings of resource management and allied process management efficiency scores however is not strong. The rank correlation coefficient in this case is 0.165. Interestingly, the association between the rankings based on the stage level efficiency scores is not statistically significant pair wise.[7] We advance this finding as empirical justification for the three stages that we propose to represent the overall fund management process.

Out of the three management processes that make up the overall fund management process, it is the portfolio management performance that has the strongest association with overall performance with rank correlation coefficient at 0.702. This is uncovered in spite of giving priority to the allied process over the portfolio management process in overall efficiency decomposition. We check robustness of the results to change in priority from allied process to portfolio management process in overall efficiency decomposition in section 6.1.

[7] Supportive evidence of this is found in the Wilcoxon signed rank sum test for matched pairs.

Table 3
Spearman's rank correlation of relative efficiency scores.

	Operational management	Resource management	Portfolio management	Allied process management	Overall management
Operational management	1				
Resource management	0.067	1			
Portfolio management	0.011	0.021	1		
Allied process management	0.844*	0.165*	0.000	1	
Overall management	0.352*	0.243*	0.702*	0.438*	1

* indicates statistically significant at the 1% level.

5.2 Performance at the individual fund level

Here we limit the discussion to the 20 best performing MFs in each management process.

Overall performance

The 20 best performers in overall management are listed in Table 4. Out of the 298 MFs considered in the analysis, only one MF is efficient overall. This may be due to the augmented structure of the network representation (Figure 1). Previous studies that adopt network representation of production processes reveal that increased structure may add discriminatory power (Färe and Whittaker, 1995; Galagedera, et al., 2016). This MF, as anticipated (Theorem 3.3.2), is also efficient in all other aspects of management modelled in the analysis and is rated 4-star by Morningstar. Out of the top twelve overall performers, nine are ranked very poorly in portfolio management and ten are ranked eleven or better in allied process management. This is reversed in the case of the other eight MFs listed in Table 4. In fact, out of the eight MFs listed in the bottom rows of Table 4, seven are ranked within the top ten in portfolio management (see Table 6) whereas many of them are ranked poorly in allied process management. These results reveal that generally, good overall performance may not suggest good allied process performance or good portfolio management performance. This brings us to the question; which of the three management processes may influence overall performance the most.

Allied process performance

Table 5 reports the efficiency scores of the top 20 performers in allied process management. In this case, two funds are efficient and as expected (Lemma 3.3.1) they are efficient in operational management and resource management as well. Further, eleven of the twenty funds listed in Table 4 are also listed in Table 5 suggesting positive association in the allied and overall management process performance, particularly at the high end. We find that portfolio management performance of the MFs listed in Table 5 is generally poor. This is not surprising because we give priority to the allied process over the portfolio management process in overall efficiency decomposition. When we do the opposite, we find a similar result; there is no positive association between portfolio management performance and allied process performance. We advance lack of positive association between allied process performance and portfolio management performance uncovered here as empirical justification (value added) for conceptualising overall fund management process as a production process comprising of multiple stages.

Operational management performance

We find that four funds are operational management efficient. Moreover, fifteen out of the 20 operational management performers are also among the top twenty allied process performers. This observation and the rank correlation between operational management and allied process management efficiency scores at 0.844 (see Table 3) suggest that the association between them is positive and strong. This is important information to MF managers. Given the earlier finding that there is a strong positive association between allied process performance and overall performance, especially at the high end, a positive step towards achieving excellence in overall performance is to manage the operational management process efficiently.

Resource management performance

Only three funds are resource management efficient. Consistent with the rank correlation coefficients reported in Table 3, there is no evidence to suggest that resource management performance may be associated with operational management and portfolio management performance. Because resource management performance is positively associated with overall performance, (see Table 3), MF managers should not take resource management process lightly in their pursuit for excellence in overall management.

Portfolio management performance

Table 6 lists the top 20 portfolio management performers. Here we find that seven MFs are portfolio management efficient. Six of them have 4-star Morningstar rating. Half the MFs listed in Table 6 are also listed in Table 4 suggesting that portfolio management performance and overall management performance may have a strong positive association in the case of high end portfolio management performers. In Table 4, we find that high overall performance may not imply high portfolio management performance. Hence, the empirical evidence suggests that while good portfolio management performance may suggest good overall performance, good overall performance may not suggest good portfolio management performance. We find further that the top performing portfolio management MFs are ranked poorly in other aspects of management namely operational management and resource management. This is important information to MF managers because without overall efficiency decomposition they would be blinded as to what management processes or which aspects of fund management may influence their overall performance. It is often debated whether good fund performance is due to management skill or luck (Fama and French, 2010). MF performance in that context is assessed in terms of abnormal returns after controlling for costs such as fees and expenses and focussing primarily on the management of the portfolio. Our coverage of MF performance appraisal is much broader and therefore we contribute to this debate from a wider perspective.

Table 4
Overall management: top 20 performers.

Name of the mutual fund	Operational management		Resource management		Portfolio management		Allied process management		Overall management		Intermediate resource imbalance		Morningstar
	RES	Rank	RES	Rank	RES	Rank	RES	Rank	RES	Rank	I_0^*	Rank	Rating
SsgA S&P 500 Index N	1	2.5	1	2	1	4	1	1.5	1	1	0.00	295.5	4
JHFunds2 Capital App 1	1	2.5	0.288	241	0.213	286	0.995	5.5	0.995	2.5	0.03	269.5	4
Jhancock Blue Chip Growth 1	1	2.5	0.387	198	0.011	289	0.996	3.5	0.995	2.5	0.04	264.5	5
Jhancock Equity-Income 1	0.999	5	0.150	276	0.006	291	0.996	3.5	0.994	4	0.06	254	3
JHFunds2 International Value 1	0.998	6	0.107	283	0.004	293	0.995	5.5	0.992	5	0.05	259.5	3
JHFunds2 Mid Cap Stock 1	0.996	7	0.207	270	0.007	290	0.990	7	0.989	6	0.02	274.5	3
MM S&P 500® Index R4	0.481	16	0.967	7	0.032	287	0.966	8	0.966	7	0.01	285	3
American Funds Growth Fund of Amer A	1	2.5	1	2	0.000	297	1	1.5	0.930	8.5	0.08	192	3
Principal Large Cap S&P 500 Index R2	0.334	31	0.290	239	0.992	10	0.332	47	0.930	8.5	1.00	4	3
State Farm S&P 500 Index A Legacy	0.246	119	0.920	10	0.013	288	0.919	9	0.919	10.5	0.01	285	3
AXA Aggressive Allc A	0.477	17	0.402	187	0.950	16	0.472	27	0.919	10.5	1.00	4	1
JNL/Mellon Cap S&P 500 Index A	0.010	295	0.901	12	0.002	294	0.900	11	0.895	12	0.00	295.5	4
EQ/Equity 500 Index IA	0.198	268	0.457	160	0.997	9	0.455	28	0.877	13	0.01	285	4
Invesco Diversified Dividend A	0.294	44	0.402	186	1	4	0.402	33	0.865	14	0.01	285	4
Oppenheimer International Small-Mid Co A	0.258	77	0.399	188	1	4	0.263	98	0.861	15.5	0.10	161.5	4
First Eagle Overseas A	0.294	45	0.305	231	1	4	0.295	67	0.861	15.5	0.08	192	4
ClearBridge Large Cap Growth A	0.237	188	0.503	140	1	4	0.241	198	0.859	17.5	0.50	26.5	4
Diamond Hill Small Cap A	0.234	224	0.577	102	1	4	0.239	228	0.859	17.5	0.50	26.5	4
Pear Tree Polaris Foreign Value Ord	0.005	297	0.862	16	0.004	292	0.860	13	0.858	19.5	0.00	295.5	4
Fidelity Advisor® Intl Sm Cap Opps A	0.230	254	0.613	82	0.998	8	0.234	255	0.858	19.5	0.50	26.5	2
Average	0.515	95.2	0.537	141.6	0.511	148.7	0.678	52.5	0.921	10.5	0.20	199.1	

Notes: RES = relative efficiency score. When ranking MFs (n=298) based on performance, we sort MFs according to their relative efficiency scores (RES) in descending order and assign rank 1 to the MF with the highest relative efficiency score and rank 298 to the MF with the lowest relative efficiency score. MFs with the same relative efficiency score are assigned their average rank.

Table 5
Allied process management: top 20 performers.

Name of the fund	Operational management		Resource management		Portfolio management		Allied process management		Overall management		Intermediate resource imbalance		Morningstar
	RES	Rank	RES	Rank	RES	Rank	RES	Rank	RES	Rank	I_0^*	Rank	Rating
SSgA S&P 500 Index N	1	2.5	1	2	1	4	1	1.5	1	1	0.00	295.5	4
American Funds Growth Fund of Amer A	1	2.5	1	2	0.000	297	1	1.5	0.930	8.5	0.08	192	3
JHancock Equity-Income 1	0.999	5	0.150	276	0.006	291	0.996	3.5	0.994	4	0.06	254	3
JHancock Blue Chip Growth 1	1	2.5	0.387	198	0.011	289	0.996	3.5	0.995	2.5	0.04	264.5	5
JHFunds2 Capital App 1	1	2.5	0.288	241	0.213	286	0.995	5.5	0.995	2.5	0.03	269.5	4
JHFunds2 International Value 1	0.998	6	0.107	283	0.004	293	0.995	5.5	0.992	5	0.05	259.5	3
JHFunds2 Mid Cap Stock 1	0.996	7	0.207	270	0.007	290	0.990	7	0.989	6	0.02	274.5	3
MM S&P 500® Index R4	0.481	16	0.967	7	0.032	287	0.966	8	0.966	7	0.01	285	3
State Farm S&P 500 Index A Legacy	0.246	119	0.920	10	0.013	288	0.919	9	0.919	10.5	0.01	285	3
American Funds Invmt Co of Amer A	0.657	11	0.996	4	0.000	298	0.902	10	0.854	22	0.01	285	2
JNL/Mellon Cap S&P 500 Index A	0.010	295	0.901	12	0.002	294	0.900	11	0.895	12	0.00	295.5	4
American Funds Washington Mutual A	0.683	9	0.996	5	0.000	296	0.867	12	0.840	24	0.01	285	4
Pear Tree Polaris Foreign Value Ord	0.005	297	0.862	16	0.004	292	0.860	13	0.858	19.5	0.00	295.5	4
American Funds Fundamental Invs A	0.634	12	0.824	21	0.849	70	0.770	14	0.790	40	0.01	285	3
American Funds Capital World Gr&Inc A	0.674	10	0.815	22	0.766	163	0.742	15	0.763	52	0.07	225.5	3
American Funds New Perspective A	0.579	13	0.751	31	0.824	98	0.665	16	0.757	55	0.02	274.5	4
American Funds Europacific Growth A	0.886	8	0.304	232	0.816	102	0.652	17	0.749	64	0.05	259.5	2
Lord Abbett Alpha Strategy A	0.443	19	0.629	74	0.814	106	0.628	18	0.791	39	0.07	225.5	3
ClearBridge Aggressive Growth A	0.012	294	0.593	90	0.001	295	0.593	19	0.588	236	0.00	295.5	3
Gabelli Equity Income AAA	0.201	265	0.586	93	0.864	50	0.585	20	0.747	67	0.01	285	3
Average	0.625	69.8	0.664	94.5	0.311	219.5	0.851	10.5	0.871	33.9	0.03	269.6	

Notes: RES = relative efficiency score. When ranking MFs (n=298) based on performance, we sort MFs according to their relative efficiency scores (RES) in descending order and assign rank 1 to the MF with the highest relative efficiency score and rank 298 to the MF with the lowest relative efficiency score. MFs with the same relative efficiency score are assigned their average rank.

Table 6
Portfolio management: top 20 performers.

Name of the fund	Operational management		Resource management		Portfolio management		Allied process management		Overall management		Intermediate resource imbalance		Morningstar
	RES	Rank	RES	Rank	RES	Rank	RES	Rank	RES	Rank	I_0^*	Rank	Rating
ClearBridge Large Cap Growth A	0.237	188	0.503	140	1	4	0.241	198	0.859	17.5	0.50	26.5	4
Invesco Diversified Dividend A	0.294	44	0.402	186	1	4	0.402	33	0.865	14	0.01	285	4
SSgA S&P 500 Index N	1	2.5	1	2	1	4	1	1.5	1	1	0.00	295.5	4
Oppenheimer International Small-Mid Co A	0.258	77	0.399	188	1	4	0.263	98	0.861	15.5	0.10	161.5	4
Principal SAM Strategic Growth A	0.243	136	0.586	94	1	4	0.247	146	0.856	21	0.25	87	2
Diamond Hill Small Cap A	0.234	224	0.577	102	1	4	0.239	228	0.859	17.5	0.50	26.5	4
First Eagle Overseas A	0.294	45	0.305	231	1	4	0.295	67	0.861	15.5	0.08	192	4
Fidelity Advisor® Intl Sm Cap Opps A	0.230	254	0.613	82	0.998	8	0.234	255	0.858	19.5	0.50	26.5	2
EQ/Equity 500 Index IA	0.198	268	0.457	160	0.997	9	0.455	28	0.877	13	0.01	285	4
Principal Large Cap S&P 500 Index R2	0.334	31	0.290	239	0.992	10	0.332	47	0.930	8.5	1.00	4	3
Goldman Sachs US Eq Div and Premium A	0.235	217	0.507	136	0.982	11	0.238	230	0.840	23	0.33	54	3
Calvert Equity A	0.238	176	0.581	99	0.969	12	0.242	180	0.831	26	0.17	133.5	3
AB Large Cap Growth A	0.241	148	0.368	207	0.959	13	0.244	171	0.819	30	0.09	176.5	4
First Eagle US Value A	0.238	179	0.505	138	0.958	14	0.242	191	0.824	28	0.50	26.5	2
Franklin Growth A	0.008	296	0.566	108	0.952	15	0.565	21	0.823	29	0.00	295.5	3
AXA Aggressive Allc A	0.477	17	0.402	187	0.950	16	0.472	27	0.919	10.5	1.00	4	1
Principal Capital Appreciation A	0.244	130	0.486	148	0.947	17	0.248	139	0.815	32	0.50	26.5	3
EQ/Common Stock Index Portfolio IA	0.260	75	0.476	151	0.945	18	0.475	26	0.815	31	0.00	295.5	3
AB Wealth Appreciation Strategy A	0.235	215	0.760	29	0.936	19	0.240	210	0.806	33	0.50	26.5	2
Eaton Vance Tx-Mgd Growth 1.1 A	0.238	177	0.249	259	0.924	20	0.248	134	0.797	34	0.33	54	3
Average	0.287	145.0	0.502	144.3	0.975	10.5	0.346	121.5	0.856	21.0	0.32	124.1	

Notes: RES = relative efficiency score. When ranking MFs (n=298) based on performance, we sort MFs according to their relative efficiency scores in descending order and assign rank 1 to the MF with the highest relative efficiency score and rank 298 to the MF with the lowest relative efficiency score. MFs with the same relative efficiency score are assigned their average rank.

5.3 Intermediate resource imbalance

Our measure of intermediate resource imbalance I^* varies between 0 and 1 (inclusive) with $I^* = 1$ revealing no IRI. We interpret $I^* = 1$ as efficient use of internal resources. The results reveal that I^* based fund rankings are not associated with the rankings based on the performance in any of the three aspects of fund management considered in the analysis. Therefore I^* may be considered as an index that provides additional information on managerial performance.

Table 7, lists the MFs with $I^* = 1$. There are seven such MFs. None of these MFs are efficient in any of the management aspects investigated in this study. Only two of them namely, Principal Large Cap S&P 500 Index R2 and AXA Aggressive Allc A belong to the top 20 category in overall performance. The practical value of I^* is that we may use I^* to discriminate funds ranked equal at any level of performance. For example, Table 5 reveals that two MFs are allied process management efficient. These two MFs have the same efficiency score of unity and hence the same rank. However, their I^* is different. We rank the efficient MF with the highest I^*, American Funds Growth Fund of Amer A with I^* = 0.08 above the other MF which is SSgA S&P 500 Index N because higher the I^* the better the efficiency in utilisation of internal resources.[8]

5.4 Association with Morningstar rating

Morningstar rates MFs based on weighted average of 3-year, 5-year and 10-year risk-adjusted return relative to funds that belongs to specific fund categories; the best performers receive five-star rating and the worst receive one-star rating according to a pre-specified distribution of funds across the rating levels (Morningstar, 2009). In our case, we assess fund performance in the cross-section relative to a mix of different fund categories and by imposing a network structure on the overall fund management process. Therefore, Morningstar ratings and performance assessed in our models are not readily comparable. However, we find weak evidence of compatibility between DEA-based rankings and Morningstar ratings. Morningstar rates eleven of the top 20 overall performers (Table 4), five of

[8] It is possible that two or more funds with the same rank whether efficient or otherwise may also have the same I^*. In that case it is not possible to discriminate funds based on I^* alone. For example, Table 6 reveals that ClearBridge Large Cap Growth A and Diamond Hill Small Cap A are portfolio management efficient and both these funds have the same I^*.

the top 20 operational management performers, nine of the top 20 resource management performers and eight of the top 20 portfolio management performers (Table 6) 4-star or higher.

6. Robustness check

6.1 Change in priority in overall efficiency decomposition

When decomposing the overall efficiency score using model (11), we give priority to the allied process over the portfolio management process. All the results discussed thus far are those obtained under this assumption. Here, we compare the results when the portfolio management process is given priority over the allied process and vice versa. Figure 3 presents the graphs of the rankings of MFs based on their stage level performance obtained under the two priority schemes. Panel (a) of Figure 3 reveal that change in priority has no significant impact on the rankings of MFs that perform relatively well and relatively poorly. It is the funds that are in the middle band that are affected due to priority change. The ranking based on operational management performance of 154 (51.7%) MFs improve when priority is changed from the allied process to portfolio management process. In many of these 154 funds the improvement is substantial. In approximately 47 per cent of them the rank improves by at least 25. On the other hand, Panel (c) of Figure 3 reveals that change in priority does not affect portfolio management performance-based rankings. According to Panel (b) of Figure 3, the ranking of MFs based on resource management performance get affected the most due to priority change.

Moreover, when the allied process is given priority in overall efficiency decomposition, the number of funds with $I^* = 1$ is 7 and when preference is given to the portfolio management process over the allied process, the number increases to 131 suggesting loss of discriminatory power of IRI. Therefore, given that the rankings based on portfolio management efficiency scores are not affected much by change in priority (see Panel (c) of Figure 3) and the high correlation between portfolio and overall management processes, giving priority to the allied process over the portfolio management process in overall efficiency decomposition is the better option.

Table 7
Funds with no intermediate resource imbalance.

Name of the fund	Operational management		Resource management		Portfolio management		Allied process management		Overall management		Intermediate resource imbalance		Morningstar
	RES	Rank	RES	Rank	RES	Rank	RES	Rank	RES	Rank	I_0^*	Rank	Rating
Principal Large Cap S&P 500 Index R2	0.334	31	0.290	239	0.992	10	0.332	47	0.930	8.5	1	4	3
AXA Aggressive Allc A	0.477	17	0.402	187	0.950	16	0.472	27	0.919	10.5	1	4	1
Invesco Equally-Wtd S&P 500 B	0.455	18	0.241	263	0.872	41	0.440	31	0.839	25	1	4	3
Principal MidCap S&P 400 Index R2	0.295	42	0.658	59	0.813	107	0.304	57	0.766	51	1	4	3
Nationwide Mid Cap Market Index A	0.241	145	0.673	49	0.808	111	0.247	151	0.738	80	1	4	3
Principal SmallCap S&P 600 Index R2	0.295	43	0.663	56	0.741	183	0.304	58	0.700	127	1	4	4
Oppenheimer Global A	0.277	60	0.394	192	0.590	269	0.314	54	0.588	237	1	4	3
Average	0.339	50.9	0.474	149.3	0.823	105.3	0.345	60.7	0.783	77.0	1	4	

Notes: RES = relative efficiency score

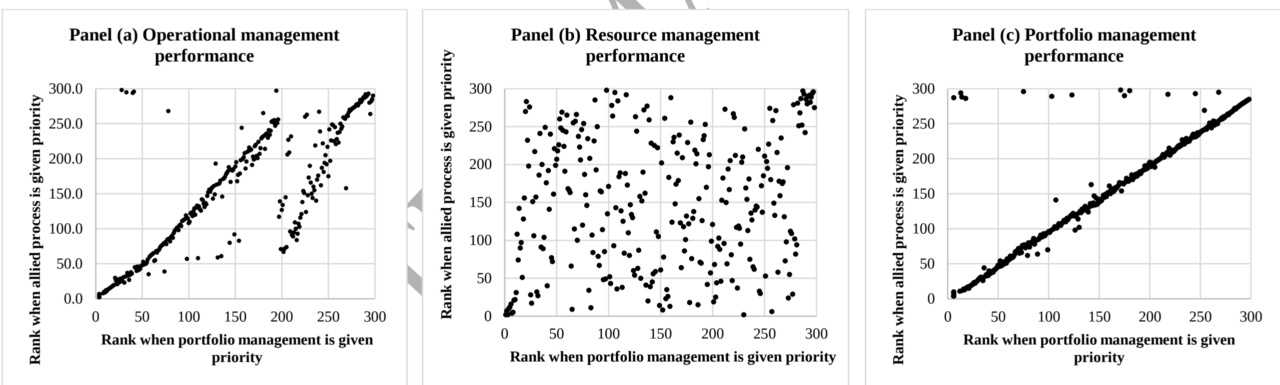

Figure 3. Rankings based on stage-wide performance assessed under the two priority schemes in overall efficiency decomposition.

6.2 Relaxing the worst case scenario condition on intermediate resource imbalance

The stage level efficiency scores we discussed thus far are computed using the optimal decision variable values of model (13). The objective function of model (13) is maximising IRI. We do that to obtain a unique value for IRI. Here we discuss the implication of relaxing the worst case scenario of IRI on stage level performance. When we compute stage level efficiency scores using the optimal decision variable values of model (11), the metric that gets affected the most is IRI. The number of funds with $I^* = 1$ is now 190 (63.8%) compared to 7 when IRI is maximised. An implication of this, in addition to I^* not being unique is that IRI diminishes discriminatory power substantially.

7. Concluding remarks

Studies highlight that comprehensive appraisal of MF performance have to go beyond simple risk-return ratio measures and assessing MF performance in a multi-dimensional framework by conceptualising the overall fund management process as a black box operation may be inadequate. We propose a new network DEA model to appraise MF performance in a multi-dimensional framework. We conceptualise MF management process as a serially linked three-stage process comprising of operational management, resource management and portfolio management processes. When formulating DEA models to determine overall and stage-level efficiency scores, we incorporate the condition that operational and resource management processes are low risk undertakings compared to portfolio management. The differences in the level of risk exposure of stage-level processes are modelled through conditions imposed on the multipliers associated with intermediate measures. In so doing we develop an index to measure internal resource imbalance. Through our performance appraisal procedure, MF performance may be assessed (i) from three different aspects of fund management and (ii) in the model implied use of intermediate resources.

In the MF performance appraisal literature, the focus is predominantly on the fund portfolio with main considerations being risk, returns and cost. Our performance appraisal approach is beneficial to MF management because findings are based on models that accommodate multiple measures of performance and in a comprehensive network representation of the overall fund management process.

The models proposed here may not be limited to application on MFs. Another application would be appraisal of superannuation fund performance. Superannuation funds are established mainly with funds contributed by employers on behalf of their employees. A common practise of industry specific superannuation funds is outsourcing portfolio management. The network representation that we propose can readily accommodate this feature; two processes are undertaken in-house and the other outsourced. Another contribution that we make is development of an index that measures efficiency in the use (implied in the model) of internal resources that links multiple stages. In our production process network representation, we consider two types of linkages and derive a composite measure to determine internal resource use performance in the overall production process. This approach can be easily extended to assess internal resource use performance in different types of production processes. Through our empirical application, we show that the index that measures internal resource imbalance may improve discriminatory power of performance assessed in network DEA models where internal resource imbalance is allowed.

Appendix

Proof of Lemma 3.3.1:

Suppose all stages of the allied process of DMU_0 is efficient $\left(\theta_0^{A*} = \theta_0^{B*} = 1\right)$. Then, $\theta_0^{AB*} = 1$ as θ_0^{AB} is a convex combination of θ_0^{A} and θ_0^{B}. Suppose the allied process of DMU_0 is efficient $\left(\theta_0^{AB_1*} = 1\right)$. From the first constraint of model (15) we have that $\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^{A*} y_{r0}^{A} + \delta^{A} - \sum_{i=1}^{i_A} \varphi_i^{A*} x_{i0}^{A} \leq 0$. Then, from the last constraint of model (15), we have $\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^{A*} y_{r0}^{A} + \delta^{A*} - \sum_{i=1}^{i_A} \varphi_i^{A*} x_{i0}^{A} = \sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_B} \varphi_i^{B*} x_{i0}^{B} - \left(\sum_{d=1}^{d_{BC}} \psi_d^{BBC*} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^{B*} y_{r0}^{B} + \delta^{B*}\right) \leq 0$ and from the second constraint of model (15) we have that $\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_B} \varphi_i^{B*} x_{i0}^{B} - \left(\sum_{d=1}^{d_{BC}} \psi_d^{BBC*} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^{B*} y_{r0}^{B} + \delta^{B*}\right) \geq 0$. Both these conditions are satisfied only when θ_0^{A*} and θ_0^{B*} defined in (17) and (18) equal unity.□

Proof of Theorem 3.3.2:

Suppose that all stage level performance of DMU_0 is efficient $\left(\theta_0^{A*} = \theta_0^{B*} = \theta_0^{C*} = 1\right)$. Then, from Lemma 2.3.1, we have that $\theta_0^{AB*} = 1$. Hence $\theta_0^{ABC*} = 1$ as θ_0^{ABC} is a convex combination of θ_0^{AB} and θ_0^{C}. Suppose DMU_0 is overall efficient $\left(\theta_0^{ABC*} = 1\right)$. Then, from the second last constraint of model (15), we have $\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{r=1}^{r_A} \phi_r^{A*} y_{r0}^{A} + \delta^{A*} + \sum_{d=1}^{d_{BC}} \psi_d^{BBC*} z_{d0}^{BC} + \sum_{r=1}^{r_B} \phi_r^{B*} y_{r0}^{B} + \delta^{B*} = \sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_A} \varphi_i^{A*} x_{i0}^{A} + \sum_{i=1}^{i_B} \varphi_i^{B*} x_{i0}^{B} + \sum_{d=1}^{d_{BC}} \psi_d^{CBC*} z_{d0}^{BC} + \sum_{i=1}^{i_C} \varphi_i^{C*} x_{i0}^{C} - \sum_{r=1}^{r_C} \phi_r^{C*} y_{r0}^{C} - \delta^{C*}$. The LHS components here are the same as the LHS components of the last constraint of model (15). By substitution, we have that $\left(\sum_{d=1}^{d_{AB}} \psi_d^{AB*} z_{d0}^{AB} + \sum_{i=1}^{i_A} \varphi_i^{A*} x_{i0}^{A} + \sum_{i=1}^{i_B} \varphi_i^{B*} x_{i0}^{B}\right)\left(1 - \theta_0^{AB_1*}\right) = \left(\sum_{r=1}^{r_C} \phi_r^{C*} y_{r0}^{C} + \delta^{C*} - \sum_{d=1}^{d_{BC}} \psi_d^{CBC*} z_{d0}^{BC} - \sum_{i=1}^{i_C} \varphi_i^{C*} x_{i0}^{C}\right)$. From (19), we have that $\left(\sum_{r=1}^{r_C} \phi_r^{C*} y_{r0}^{C} + \delta^{C*} - \sum_{d=1}^{d_{BC}} \psi_d^{CBC*} z_{d0}^{BC} - \sum_{i=1}^{i_C} \varphi_i^{C*} x_{i0}^{C}\right) \leq 0$. Hence it follows that $\left(\sum_{d=1}^{d_{AB}} \psi_d^{AB} z_{d0}^{AB} + \sum_{i=1}^{i_A} \varphi_i^{A} x_{i0}^{A} + \sum_{i=1}^{i_B} \varphi_i^{B} x_{i0}^{B}\right)\left(1 - \theta_0^{AB_1*}\right) \leq 0$. This condition is satisfied only when $\theta_0^{AB_1*} = 1$ because we assume that all observed values and the multipliers are positive and $\theta_0^{AB_1*} \leq 1$. Now we have $\theta_0^{ABC*} = 1$ and

${\theta_0^{AB_1}}^* = 1$. Then from the last two constraints of model (15) we have that $\left(\sum_{r=1}^{r_C} \phi_r^{C*}\, y_{r0}^C + \delta^{C*} - \sum_{d=1}^{d_{BC}} \psi_d^{CBC*}\, z_{d0}^{BC} - \sum_{i=1}^{i_C} \varphi_i^{C*}\, x_{i0}^C\right) = 0$ leading to $\theta_0^{C*} = 1$. From Lemma 3.3.1 we have that $\theta_0^{A*} = \theta_0^{B*} = 1$.□

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