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A NORTON APPROACH TO DISTRIBUTION NETWORK MODELING FOR HARMONIC STUDIES

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Abstract: This paper presents a Norton approach for modeling distribution networks where the system configuration is not fully known. Traditionally, harmonic studies use complex distribution networks modeled by harmonic current sources for specific frequencies. Although this approach has been proved to be adequate for some studies, this may not happen for other applications. When changing the operating condition of the supply-side system, the harmonic currents injected by the distribution network might change. This information is in this paper used to estimate a Norton model of the load-side distribution network. The estimated models can be used to analyze, for example, the effect of harmonic filters under different supply system configurations or operating conditions. The method of estimating the Norton models is illustrated on a test system, simulated on the well-known simulation program EMTDC. The performance of the estimated models is, for different configurations of the supply system, compared to the performance of the traditionally used "constant current" approach.

keywords: Harmonics, Power Quality, Nonlinear loads, Filters

I. INTRODUCTION

The use of power electronics-based devices in power systems has increased steadily over the last decades. Electronics devices are nonlinear and thus they create distorted currents even when supplied with purely sinusoidal voltage. These distorted currents cause voltage and current distortion throughout the system which can result in additional heating in power system equipment, unmotivated switching of breakers, blowing of fuses, and interference with communication systems [1]. These electronics devices are also more sensitive to power quality variations than equipment applied in the past. Microprocessor based controls applied in these electronics devices are sensitive to many types of disturbances which can result in nuisance tripping, misoperation or actual device failures. This is becoming a major concern for the industry.

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As the number and ratings of power electronic devices connected to the power systems increase, the harmonic currents injected into the power system and the resulting voltage distortion can become a major problem for power quality. Furthermore, the increasing emphasis on overall power system efficiency is causing a continued growth in the application of shunt capacitors for power factor correction. This is occurring both within customer facilities as well as on the grid companies power system. Capacitors change the system frequency characteristics which can result in resonances that magnify specific harmonic voltages and transient disturbances. Thus, harmonic filters are needed at some locations in the power system to minimize the harmonic distortion.

There are a number of techniques presently being used for power system harmonic analysis [2]. These techniques vary in terms of data requirements, modeling complexity, problem formulation and solution algorithms.

The most commonly used method in commercial software is the admittance matrix method [2]. In these software, e.g. [3], the harmonic sources are usually modeled as current sources for each harmonic.

In harmonic iteration methods [2, 4, 5] the harmonic producing device is modeled as a supply voltage-dependent current source. Programs based on this method requires more knowledge of the harmonic producing equipment than those based on the admittance matrix method.

Other methods that take into account the voltage dependent nature of the nonlinear devices are based on Newton type algorithms [2, 6]. These methods normally requires that the device equations are available in a closed form or in a form where the derivatives can be easily calculated.

Besides the frequency-domain methods mentioned above, techniques have also been developed for harmonic analysis in the time-domain. The simplest approach is to run a time simulation until a steady state is reached. Electromagnetic transient programs such as EMTDC [8] can be used as such a tool.

Assume now that you want to do a harmonic study in a system where there is a distribution system connected, and that the system configuration of this distribution system is not fully known.

One problem with harmonic iteration methods, methods based on Newton type algorithms and time-domain simulations is that they normally require a complete knowledge of the system being modeled. Programs based on the admittance matrix *method* [2, 3] normally do not require this detailed knowledge. These programs allow for specifying a harmonic current source based on measurements. By measuring the harmonic current or voltage at the bus where the modeled network is connected, the network can be included in the analysis as a harmonic current or voltage source. However, modeling a distribution network as a current or voltage source may not be accurate enough if the operating conditions of the supply system deviate much from that under which the harmonic current or voltage spectra was determined [7]. As distribution systems generally consists of several shunt connected impedances, changing the operating condition of the supply system might change the harmonic currents injected at the measurement bus. Thus, there is a need for more general models of large networks. A first approach towards this would be to model the network as a Thévenin or Norton model. These models are more accurate for a wider range of operating conditions than the "constant current" or voltage source models.

The purpose of this paper is to show how a Norton model of a distribution system can be estimated from harmonic voltage and current measurements.

II. INFLUENCE OF CAPACITOR SWITCHING ON NETWORK HARMONIC GENERATION

To illustrate the behavior of load-side network harmonic current generation under different operating conditions of the supply-side network, harmonic current measurements at a substation transformer in Stockholm is described before and after switching of substation capacitor banks [7]. The system under study is shown schematically in figure 1.

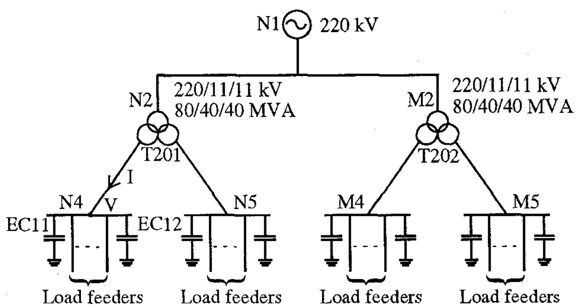

Fig. 1. Schematic one-line diagram of a distribution system

The substation is at Bus N1 fed by a 220 kV transmission network. The substations total load is then divided between two, equally rated, three-winding transformers. On each of the four low voltage windings, together with the load feeders, two capacitor banks for power factor correction are connected.

In this study, measurement of harmonic voltage and current levels were performed during 75 minutes on one of these low voltage windings, with different combinations of capacitor banks EC11 and EC12 connected as shown in figure 1. After subtracting the currents flowing in the capacitor banks, the 5^{th} and 7^{th} harmonic load current in percent of the fundamental is shown in figure 2. In the figure it can be seen that for different combinations of capacitor banks connected, the harmonic current injected by the load-side network change.

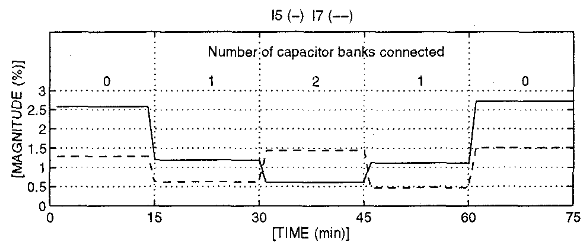

Fig. 2. Measured harmonic load currents in percent of fundamental

This study shows that modeling the load-side network as a "constant current" source for each harmonic may not give accurate results. The modeled load-side harmonic current can be either over- or underestimated, depending on the situation. The study also shows that by changing the operating condition in the supply network, some information of the studied load-side network could be obtained.

III. FREQUENCY-DOMAIN NORTON CIRCUT

The results from the previous chapter suggest that information obtained from harmonic current and voltage measurements with different operating conditions of the supply system could be used to estimate a model capable of representing the network under a wider range of operating conditions than a simple current or voltage source.

To estimate a Norton model as given in figure 3, measurements of harmonic current, I_h, and voltage, V_h, with two different operating conditions of the supply system has to be performed. The change in supply system operating condition can for example be obtained by switching a shunt capacitor, disconnecting a parallel transformer or some other changes that give a significant change in the supply system harmonic impedance. It is however required that voltage and current can be measured, or estimated, on both sides of the device that are being switched.

Through the circuit in figure 3 it can be seen that by changing the operating conditions of the supply system, then harmonic voltage V_h, and harmonic currents I_h and $I_{ZN,h}$ will be changed.

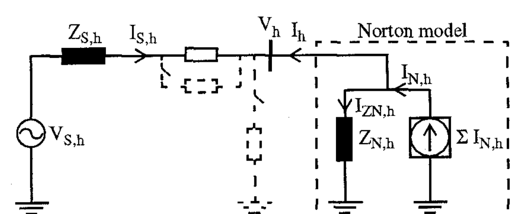

Fig. 3. Norton model and Thévenin equivalent of the load-side network and the supply system

In figure 3:

$V_{S,h}$ - Supply system harmonic voltage
$Z_{S,h}$ - Supply system harmonic impedance
$I_{S,h}$ - Supply system harmonic current
V_h - Harmonic voltage at load-side network
I_h - Harmonic current injected into supply system
$I_{N,h}$ - Harmonic current generated by load-side network
$I_{ZN,h}$ - Harmonic current through Norton impedance
$Z_{N,h}$ - Harmonic Norton impedance

The harmonic current, $I_{N,h}$, generated by the load-side network finds an electrical path which consists of a parallel combination of $Z_{N,h}$ and the total impedance of the supply system. As $Z_{N,h}$ normally is greater than this impedance, the majority of $I_{N,h}$ flows towards the supply. However, as shown in the previous chapter, modifying the operating conditions of the supply system forces the currents flowing in both the supply system impedance and $Z_{N,h}$ to change.

Assuming no change in operating conditions in the modeled load-side distribution network between the two measurements, it is seen from figure 3 that for each harmonic, the measured currents $I_{h,1}$ and $I_{h,2}$ can be expressed as:

$$I_{h,1} = I_{N,h} - I_{ZN,h,1} \quad (1)$$

$$I_{h,2} = I_{N,h} - I_{ZN,h,2} \quad (2)$$

where all quantities are complex.

The current through the harmonic Norton impedance, $I_{ZN,h}$, can before and after change in supply system operating condition be calculated as:

$$I_{ZN,h,1} = \frac{V_{h,1}}{Z_{N,h}} \quad ; \quad I_{ZN,h,2} = \frac{V_{h,2}}{Z_{N,h}} \quad (3)$$

Using equation (3) in equation (1) and (2) gives:

$$I_{h,1} = I_{N,h} - \frac{V_{h,1}}{Z_{N,h}} \quad ; \quad I_{h,2} = I_{N,h} - \frac{V_{h,2}}{Z_{N,h}} \quad (4)$$

Subtracting $I_{h,1}$ from $I_{h,2}$ gives:

$$I_{h,2} - I_{h,1} = \frac{V_{h,1} - V_{h,2}}{Z_{N,h}} \quad (5)$$

Solving for $Z_{N,h}$ in equation (5) then gives the Norton impedance for each harmonic as:

$$Z_{N,h} = \frac{(V_{h,1} - V_{h,2})}{(I_{h,2} - I_{h,1})} \quad (6)$$

The harmonic Norton current source can then be calculated as:

$$I_{N,h} = I_{h,1} + \frac{V_{h,1}}{Z_{N,h}} \quad (7)$$

Notice that, since equations (6) and (7) are complex, it is important to have correct measurements of not only the harmonic voltage and current magnitudes but also the phase angles. It is also important that voltage and current measurements are referred to some common bus phase angle that does not change with system condition. In figure 3 this is the fundamental voltage at the bus labeled $V_{S,h}$.

A. Estimating the supply system Thévenin equivalent

Since it is usually very difficult to measure phase angles with respect to some remote bus, the phase shift between the measured voltages and the remote bus have to be estimated.

Figure 4 a) and b) show the supply system before and after the change in impedance in figure 3.

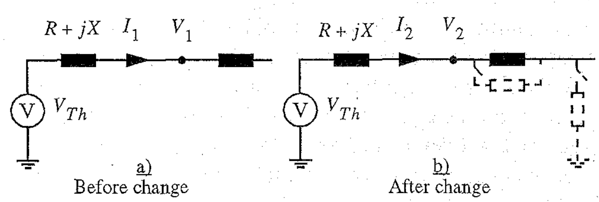

Fig. 4. Supply system before and after change in impedance

If it is assumed that the Thévenin voltage phase angle is zero, the following equations can be formulated from figure 4:

$$V_1 e^{j\theta_{V1}} = V_{Th} - (R + jX) I_1 e^{j\theta_{I1}} \quad (8)$$

$$V_2 e^{j\theta_{V2}} = V_{Th} - (R + jX) I_2 e^{j\theta_{I2}} \quad (9)$$

If the phase shift between the current and voltage measurements are denoted ϕ_1 and ϕ_2, equations (8) and (9) can be rewritten as:

$$V_1 = V_{Th}\cos\theta_{V_1} - RI_1\cos\phi_1 + XI_1\sin\phi_1 \quad (10)$$

$$0 = -V_{Th}\sin\theta_{V_1} - RI_1\sin\phi_1 - XI_1\cos\phi_1 \quad (11)$$

$$V_2 = V_{Th}\cos\theta_{V_2} - RI_2\cos\phi_2 + XI_2\sin\phi_2 \quad (12)$$

$$0 = -V_{Th}\sin\theta_{V_2} - RI_2\sin\phi_2 - XI_2\cos\phi_2 \quad (13)$$

In equations (10)-(13) V_1, V_2, ϕ_1 and ϕ_2 are known and V_{Th}, R, X, θ_{V1} and θ_{V2} unknown, i.e. five unknown variables and only four equations. To solve the above system of equations, a fifth equation is needed. The fifth equation can be formulated if it is assumed that the X over R ratio, ***X/R***, is roughly known from system short circuit information, i.e.

$$X = X/R \cdot R \quad (14)$$

By solving the above system of non-linear equations the phase shift between the remote bus and the voltage and current measurements used in equations (1) to (7) can be found.

From equations (6) and (7) it is seen that all information needed for the calculations of the harmonic Norton model can be found in the two measurements of harmonic voltage and current. No information about the supply system harmonic impedance or the modeled load-side network is needed. This makes the estimation simple and the calculations fast.

In [10] continues harmonic measurements, at a customer feeding point, and equation (6) was used to estimate the customers contribution to the power system harmonic disturbance.

IV. ILLUSTRATION OF MODEL PERFORMANCE

To illustrate the method of estimating the Norton model and to show the performance of the estimated models, the method was tested on a simple power system shown in figure 5. This system was simulated on the Electromagnetic Transients Simulation Program (EMTDC) [8]. For the supply system, the fundamental voltage phase shift between the remote bus 1 and bus 3 are first estimated using the method described in the previous chapter. For the load-side distribution network indicated in figure 5, the Norton models are then estimated. After that the performance of the estimated models are investigated for different configurations of the supply system. In this case tuning of the two capacitor banks at bus 3 into filters are chosen as examples. For each example the performance is compared to results received by the commonly used "constant current" approach.

A. Test System

The test system, shown in figure 5, consists of a 40 kV network, which at bus 3 is feeding two distribution networks. The short circuit MVA at bus 3 is S_k=122,11 MVA, and the X over R ratio, X/R=6,22. At bus 3 there are two capacitor banks for power factor correction connected. Switching of these capacitor banks will provide the two different operating conditions, needed for the estimation of the Norton models. The harmonic loads at buses 8 and 9 injects 5^{th}, 7^{th}, 11^{th}, 13^{th}, 17^{th} and 19^{th} harmonic currents into the system.

To provide realistic knowledge of the supply system, needed for the estimation of the Thévenin equivalent, it is assumed that the short circuit MVA is known to be approximately 125 MVA and the X over R ratio, $X/R \approx 5$

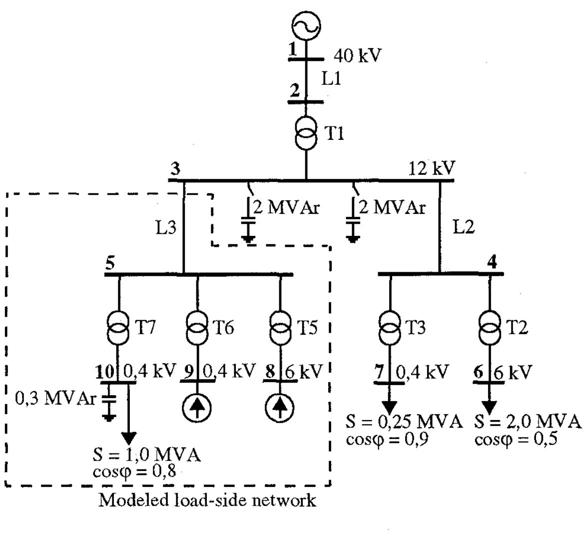

Fig. 5. Simulated test system

B. Model Estimation

In the simulated EMTDC-model it is possible to "measure" the voltage at bus 3 and the current flowing in line L3 and transformer T1, with different combinations of capacitor banks connected. In this case measurements of harmonic voltages and currents with one and two capacitor banks connected at bus 3 are used to estimate the Thévenin equivalent of the supply system and the Norton models of the distribution network.

The two sets of voltage and current measurements are supplied to a modeling program written in Matlab [9]. In the program, the Thévenin equivalent of the supply system is first estimated using equations (10)-(14). After this, the Norton models of the load-side network are calculated for each harmonic, using equations (6) and (7). Table 1 shows the initial guess, the final solution and the exact values for the supply system Thévenin equivalent. The voltage spectra at bus 3, and current spectra in line L3 for the two measurements are shown in figure 6. Table 2 shows the estimated Norton currents and figure 7 the estimated Norton impedance magnitudes and phase angles.

TABLE 1 Supply system Thévenin equivalent

	Initial guess	Final solution	Exact
V_{Th} (kV)	12,0000	12,0132	12,0007
R (Ω)	0,2259	0,2333	0,1872
X (Ω)	1,1295	1,1666	1,1644
θ_{V1} (°)	0,0	-1,4761	-1,4825
θ_{V2} (°)	0,0	-1,6782	-1,6477
θ_{V2}-θ_{V1} (°)	0,0	-0,2021	-0,1652
S_k (MVA)	125,0000	121,3075	122,1100
X/R	5,0	5,0	6,22

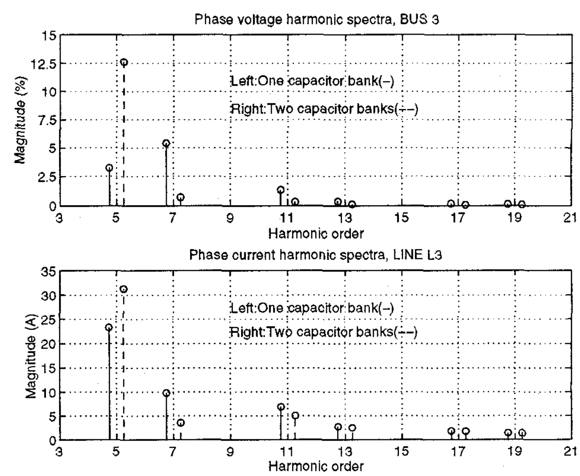

Fig. 6. Phase current harmonic spectra

TABLE 2 Estimated Norton model currents

Harmonic	Magn (A)	Phase (°)
5	20,4103	104,8096
7	5,5011	169,7832
11	4,4775	49,0331
13	2,3932	70,8301
17	1,8002	-15,8229
19	1,3645	-17,3578

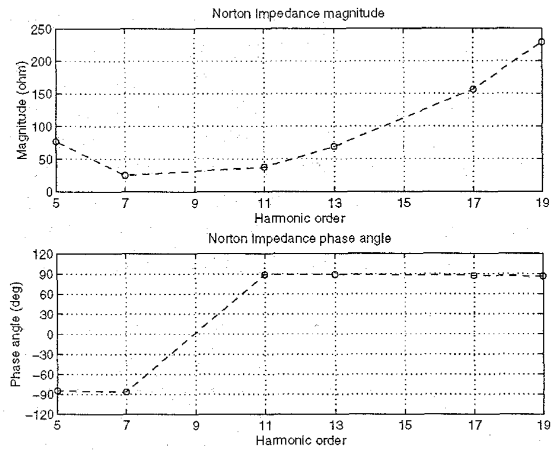

Fig. 7. Estimated Norton impedance

Notice that, for the estimation of the Norton models, only the difference, θ_{V2}-θ_{V1}, in table 1 is of interest. If the ***X/R*** ratio was assumed to be 8, this difference would be -0,1316 degrees. Close to th exact ***X/R*** ratio the error in the difference, θ_{V2}-θ_{V1}, is approximately 0.0014 degrees per percent error in the estimation of the ***X/R*** ratio.

C. Model Performance

After the Norton models for the network are calculated, the performance is investigated.

When performing a harmonic study on a system like the one shown in figure 5 it is of interest to investigate harmonic voltages and currents for different configurations of the supply system. In this study tuning the two capacitor banks at bus 3 into single-tuned filters will be used as examples.

To be able to calculate harmonic voltages and currents in the system, a simple harmonic load flow program, based on the bus admittance matrix method [3], is written in Matlab. In such a load flow program, the bus admittance matrix $\mathbf{Y_{bus}}$ is built at each frequency of interest and network equation (15), which relates the injected current vector **I** to the bus voltage vector **V**, is solved for each frequency.

$$I_{bus}(\omega) = Y_{bus}(\omega) \cdot V_{bus}(\omega) \qquad (15)$$

The written Matlab program is thus based on similar principles to those in [3], but the possibility of including the estimated Norton models is included.

In the test system it is then possible to calculate the voltages at each bus (1-4, 6, 7), and the branch currents, using both the proposed Norton model and the commonly used approach, where the load-side network is modeled as a fundamental voltage dependent current source, $I_h = kV_{FND}$, for each harmonic. The voltages received from the harmonic load flow calculations with these two models can then be compared with those received by EMTDC.

Looking at the voltage spectra in figure 6 it is seen that with one capacitor bank connected there is a parallel resonance close to the seventh harmonic. When connecting the second capacitor bank this resonance is shifted towards the fifth harmonic.

Since the lowest harmonic present is also the dominant harmonic with both capacitor banks connected, a first approach to mitigate the harmonic distortion in the supply system could be to tune one of the capacitor banks into a fifth harmonic filter.

After tuning one of the capacitor banks at bus 3 into a fifth harmonic filter, the filter is included in the harmonic load flow calculations for both the Norton and the "constant current" approach. Figure 8 shows the calculated harmonic voltage at bus 3 and figure 10, the harmonic current spectra injected at bus 3.

From the figures it can be seen that tuning one capacitor bank into a fifth harmonic filter creates a parallel resonance close to the eleventh harmonic, and for this harmonic it is seen from that the Norton approach significantly improve the results.

When installing a harmonic filter it is hoped that after installation, the waveform of the voltage (and current) will have very little dominant harmonic component and other harmonic components will not increase. As seen in figure 8, the fifth harmonic voltage is decreased. However, as mentioned before, it is also seen that a parallel resonance is created close to the eleventh harmonic. This result suggests that by tuning the other capacitor bank into a seventh harmonic filter, the harmonic distortion in the system could be reduced.

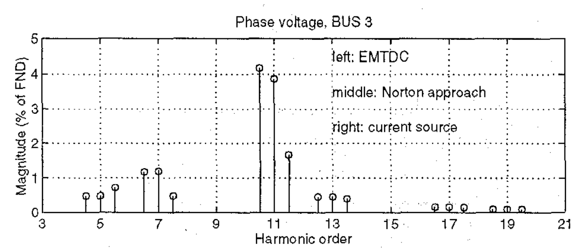

Fig. 8. Voltage spectra with one capacitor bank tuned to a fifth harmonic filter

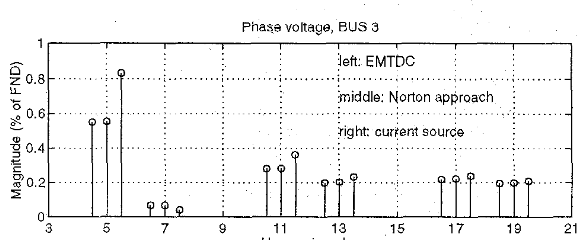

Fig. 9. Voltage spectra with one capacitor bank tuned to a fifth harmonic filter and the other to a seventh harmonic filter

The calculated voltage and current spectra for this case are shown in figures 9 and 10 respectively. From the figures it can be seen that the parallel resonance for the eleventh harmonic is reduced. It can also be seen that compared to the results received by the "constant current" approach, the Norton approach give more accurate results, particularly for lower harmonic orders.

Comparing figures 6 and 10 it can be seen that since the fundamental voltage do not change much when tuning the capacitor banks into filters, the harmonic current injected into the supply system no not change much for the "constant current" approach. For the Norton approach the injected currents change when the harmonic impedance of the supply system change. This explains the improved performance of the proposed Norton approach.

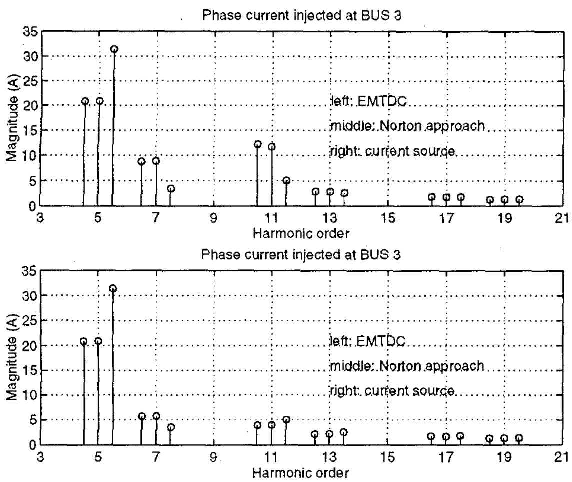

Fig. 10. Current spectra injected into supply system with fifth and seventh harmonic filter

V. CONCLUSIONS

The subject of this paper has been modeling of large distribution networks where the system configuration is not fully known. In chapter II it was shown that changing the operating condition of the supply system might change the harmonic currents injected by the distribution network. In this paper this result was used to estimate a harmonic Norton model of the distribution network. It has been shown that by measuring, or estimating, the voltage and current on both sides of the device being switched, when changing the operating condition of the supply system, the phase shift between the measurements and the system remote bus can be estimated. To estimate the Norton models for each frequency of interest, all information needed can then be found form the two measurements of harmonic voltage and injected currents. The Norton model estimation method was applied to a test-system, simulated on the well-known simulation program EMTDC, and the performance was, for different configurations of the supply system, compared to the results received by the commonly used "constant current" source model. The results received from the numerical example show that the Norton approach gives a better estimation of the behavior of the system for a wider range of operating conditions than the "constant current" source.

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VII. BIOGRAPHIES

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