A Cardinality-Constrained Robust Approach for the Ambulance Location and Dispatching Problem

Vittorio Nicoletta, Ettore Lanzarone, Valérie Bélanger and Angel Ruiz

Abstract Emergency Medical Services (EMS) systems aim to provide immediate care in case of emergency. A careful planning is a major prerequisite for the success of an EMS system, in particular to reduce the response time. Unfortunately, the demand for emergency services is highly variable and uncertainty should not be neglected while planning the activities. Several optimization models have been proposed in the literature to deal with EMS planning-related problems, e.g. the Ambulance Location and Dispatching Problem (ALDP). However, most of the models are deterministic and neglect demand uncertainty. In this paper, we formulate and validate a robust counterpart of the ALDP to deal with demand uncertainty, exploiting the cardinality-constrained approach. Numerical experiments inspired by a real case show promising results and prove the practical applicability of the approach.

Keywords Emergency medical services · Demand uncertainty · Robust optimization · Cardinality-constrained approach

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P. Cappanera et al. (eds.), *Health Care Systems Engineering*, Springer Proceedings in Mathematics & Statistics 210, https://doi.org/10.1007/978-3-319-66146-9_9

1 The Ambulance Location and Dispatching Problem

Emergency Medical Services (EMS) systems consist of the clinical activities and the ambulance transportation in response to an emergency call. EMS plays an important role in modern health care systems, as an adequate response to distress calls may have a crucial impact on patients' health conditions. In particular, one of the main issues is to reduce, or to keep under a given threshold, the time between the distress call and the arrival of the ambulance to the emergency site. The body of literature devoted to EMS design and management is huge; we refer the interested reader to [2] and [6], which report classifications of EMS problems, formulations, and solving approaches. Unsurprisingly, both reviews identify uncertainty on the demand and availability of ambulances upon a call arrival as open and important issues to address in future research.

This paper deals with the Ambulance Location and Dispatching Problem (ALDP), which aims to choose at the same time the location for the available ambulances and a dispatch policy, which decides which ambulance should answer any arriving call. Although most of location problems assume that any call is answered by the closest available ambulance, this policy is not always optimal [3, 11]. Thus, we consider the formulation proposed in [4, 5], where the dispatch policy takes the form of a list of ambulances for each demand zone, in which the ambulances are sorted according to their priority to answer the calls in the zone.

To deal with stochastic demands, we introduce a robust counterpart of the deterministic ALDP, based on the cardinality-constrained approach [8]. Such an approach has been recognized to be very effective to cope with uncertainty in health care problems [1] and has been successfully applied to other health care facilities [9], but it has never been applied to EMS. In fact, although several works deal with uncertainty in the context of ambulance location [7, 12], none of them has previously used this approach.

The paper is organized as follows. The ALDP formulation and the proposed robust counterpart are presented in Sect. 2. Computational experiments are detailed in Sect. 3, and the results in Sect. 4. Finally, conclusions are drawn in Sect. 5.

2 Robust Problem Formulation

2.1 Sets, Parameters and Decision Variables

The ALDP is defined on a graph $G = (V, E)$. V is given by $I \cup J$, where $I = \{v_1, \dots, v_n\}$ and $J = \{v_{n+1}, \dots, v_{n+m}\}$ represent all demand zones and potential waiting positions, respectively. $E = \{(v_i, v_j) : v_i, v_j \in V\}$ is the set of the edges connecting the nodes in the graph. A demand zone $v_i \in I$ is characterized by the coordinates of its centroid and a demand d_i, which is considered to be uncertain in our framework. A potential waiting position $v_j \in J$ is defined as a demand zone where p_j

vehicles ($p_j \geq 1$) can be located and from where they move to join the different calls. A travel time t_{ij} is associated to each edge $(v_i, v_j) \in E$.

Moreover, K denotes the set of all available vehicles, and Z_i the set of vehicles in the dispatch list of $v_i \in I$. Finally, we define the capacity W_i as the maximum number of demands an ambulance i can serve in a given period, and the busy fraction q_i as the time fraction an ambulance i is expected to be busy (and therefore unavailable to answer calls).

The ALDP considers two types of decisions: location decisions to select a waiting position for each vehicle, and dispatch decisions.

The latter are made according to the list Z_i of each demand zone i, choosing the first available ambulance starting from the beginning of the list. However, for life-threatening calls, the nearest available vehicle is sent to the scene of the incident. If no vehicle is available in the dispatch list of a zone, the nearest available vehicle is sent. Finally, if no vehicle is available at all, the call is placed on a waiting queue or redirected to another service; in the latter case, we consider an arbitrary value T for the response time.

The following assumptions are included (see [4, 5]):

1. Location and dispatch decisions are taken for a given planning horizon.
2. The number of vehicles available in the planning horizon is fixed and known.
3. All vehicles have the same workload capacity ($W_i = W \ \forall i$).
4. All vehicles have the same busy fraction ($q_i = q \ \forall i$); although this assumption needs to be empirically confirmed, it is broadly used in EMS design problems.
5. All of the dispatch lists have the same cardinality, without loss of generality.

Two groups of decision variables are defined to adequately consider location and dispatch decisions. They are summarized in Table 1, together with problem sets and parameters.

2.2 Objective Function and Constraints

A brief description of the deterministic model is given in the following; details can be found in [4] and [5].

The ALDP aims at minimizing the overall expected response time. Based on the dispatch lists, the expected response time for a demand zone is given by three contributions: *i)* the sum of the response times of the vehicles in the dispatch list, weighted by their probability to answer the call; *ii)* the time corresponding to the vehicles available to respond to emergency calls, but not in the dispatch list; *iii)* the time of the calls placed in queue or referred to another service because no vehicle is available. Only the first contribution is considered when formulating the ALDP; the others are calculated afterwards when the solution is executed, based on the decisions and the system characteristics.

Table 1 ALDP sets, parameters and decision variables

Sets	
I	demand zones
J	potential waiting positions for vehicles
K	available vehicles
Z_i	dispatch list of zone i (all with the same cardinality $\|Z\|$)
Parameters	
d_i	demand of zone i
t_{ji}	travel time from zone j to zone i
p_j	maximum number of vehicles in j
W	capacity, i.e. maximum number of demands a vehicle can serve in the time
	horizon (same for all vehicles)
q	busy fraction (same for all vehicles)
Decision variables	
w_i^{zk}	equal to 1 if vehicle k is in position z of the dispatch list of zone v_i,
	0 otherwise
y_{ij}^{zk}	equal to 1 if vehicle k, located in zone v_j, is in position z of the dispatch list
	of v_i, 0 otherwise

The formulation is completed by three sets of constraints. The first set ensures that each vehicle is located at a waiting position, guaranteeing that the maximum number of vehicles p_j is respected. The second set ensures that the demands assigned to a vehicle respect its capacity, considering for each vehicle the busy fraction and the presence in one or more dispatch lists. Finally, the third set imposes that a vehicle cannot occupy more than one position in the dispatch list of each zone, and that exactly one vehicle is at each position of each dispatch list.

2.3 Cardinality-Constrained Robust Formulation

To model the uncertain demands, we apply the cardinality-constrained approach to the parts of the model in [4, 5] where parameters d_i appear. First, we convert the objective function into a set of constraints by adding a new variable η_k. Thus, the new objective function is:

$$\min \sum_{k \in K} \eta_k \tag{1}$$

with the additional constraints:

$$\sum_{i \in I} \sum_{z \in Z} \sum_{j \in J} (1-q) q^{z-1} d_i t_{ji} y_{ij}^{zk} \leq \eta_k \quad \forall k \tag{2}$$

Then, we consider the uncertain demands as independent random variables $\widetilde{d}_i$ ($i \in I$). According to [8], each of them is characterized by a nominal value $\bar{d}_i$ and a maximum variation $\hat{d}_i$, i.e. $\widetilde{d}_i \in \left[\bar{d}_i - \hat{d}_i, \bar{d}_i + \hat{d}_i\right]$.

Thus, demands can be expressed as $\widetilde{d}_i = \bar{d}_i + \alpha_i \, \hat{d}_i$, where each $\alpha_i \in [-1, 1]$ represents the deviation of the demand of zone i from its nominal value $\bar{d}_i$, standardized by the half-length of the uncertainty interval $\hat{d}_i$. For example, $\alpha_i = 0$ corresponds to $\widetilde{d}_i = \bar{d}_i$, $\alpha_i = 1$ to $\widetilde{d}_i = \bar{d}_i + \hat{d}_i$, and $\alpha_i = -1$ to $\widetilde{d}_i = \bar{d}_i - \hat{d}_i$.

Demands $\widetilde{d}_i$ appear in the new constraint (2) and in the workload capacity limit for each vehicle (see [4, 5]) which are rewritten in the cardinality-constrained robust framework as:

$$\sum_{i \in I} \sum_{z \in Z} \sum_{j \in J} (1-q) q^{z-1} \bar{d}_i t_{ji} y_{ij}^{zk} + \sum_{i \in I} \sum_{z \in Z} \sum_{j \in J} (1-q) q^{z-1} (\alpha_i \hat{d}_i) t_{ji} y_{ij}^{zk} \leq \eta_k \quad \forall k \tag{3}$$

$$\sum_{z \in Z} \sum_{i \in i} (1-q) q^{z-1} \bar{d}_i w_i^{zk} + \sum_{z \in Z} \sum_{i \in i} (1-q) q^{z-1} (\alpha_i \hat{d}_i) w_i^{zk} \leq W \quad \forall k \tag{4}$$

Satisfying constraints (3) and (4) for all possible demand realizations, i.e. for all combinations of $\{\alpha_i, \ i \in I\}$, would lead to a too conservative (and unrealistic) solution. In fact, it is very unlikely that all of the demand coefficients assume their worst (highest) values simultaneously (i.e. $\alpha_i = 1 \ \forall i \in I$). Thus, in the cardinality-constrained approach, we limit the number of zones that ask for the highest demand, in each constraint, by means of the robustness parameters $\{\varGamma_k, \ k \in K\}$. Indeed, the robust cardinality-constrained solution guarantees that the solution remains feasible if up to $\varGamma_k$ parameters α_i go to the maximum value equal to 1 in each constraint, while the others remain at the nominal value equal to 0.

In the following, we consider the same value $\varGamma$ for all vehicles (i.e. $\varGamma_k = \varGamma \ \forall k$); however, the robust counterpart we derive can be easily extended to the case in which the robustness parameters $\{\varGamma_k, \ k \in K\}$ vary from vehicle to vehicle. Parameter $\varGamma$ controls the level of robustness of the solution and can be set equal to $\{0, 1, \ldots, |I|\}$ (we consider only integer values). Fixing $\varGamma = 0$ guarantees feasibility only if all of the random variables assume their nominal value (deterministic solution), whereas setting $\varGamma = |I|$ means no restrictions (most conservative solution).

We underline that, in our formulation, the optimal values α_i can be different from constraint to constraint. This simply increases the level of robustness of the solution and has to be accounted while analyzing the impact of $\varGamma$.

Briefly, to derive the robust counterpart, (3) and (4) are rewritten as:

$$\sum_{i\in I}\sum_{z\in Z}\sum_{j\in J}(1-q)q^{z-1}\bar{d}_i t_{ji} y_{ij}^{zk} + \beta_k \le \eta_k \qquad \forall k \tag{5}$$

$$\sum_{z\in Z}\sum_{i\in i}(1-q)q^{z-1}\bar{d}_i w_i^{zk} + \gamma_k \le W \qquad \forall k \tag{6}$$

where β_k and γ_k are the optima of the two following knapsack problems (generated for each k):

$$\begin{aligned} \beta_k = \max & \sum_{i\in I}\sum_{z\in Z}\sum_{j\in J}(1-q)q^{z-1}(\alpha_i \hat{d}_i) t_{ji} y_{ij}^{zk} \\ & \sum_{i\in I}\alpha_i \le \Gamma \\ & \alpha_i \in [0,1] \quad \forall i \in I \end{aligned} \tag{7}$$

$$\begin{aligned} \gamma_k = \max & \sum_{z\in Z}\sum_{i\in i}(1-q)q^{z-1}(\alpha_i \hat{d}_i) w_i^{zk} \\ & \sum_{i\in I}\alpha_i \le \Gamma \\ & \alpha_i \in [0,1] \quad \forall i \in I \end{aligned} \tag{8}$$

Applying the Strong Duality Theorem [8], we obtain their dual problems, which can be substituted in (5) and (6) to obtain the following robust formulation:

$$\min \sum_{k\in K}\eta_k \tag{9}$$

s.t.

$$\sum_{i\in I}\sum_{z\in Z}\sum_{j\in J}(1-q)q^{z-1}\bar{d}_i t_{ji} y_{ij}^{zk} + \Gamma a_k^{of} + \sum_{i\in I} b_{ki}^{of} \le \eta_k \qquad \forall k \tag{10}$$

$$\sum_{z\in Z}\sum_{i\in i}(1-q)q^{z-1}\bar{d}_i w_i^{zk} + \Gamma a_k^{con} + \sum_{i\in I} b_{ki}^{con} \le W \qquad \forall k \tag{11}$$

$$a_k^{of} + b_{ki}^{of} \ge (1-q)q^{z-1}\hat{d}_i t_{ji} y_{ij}^{zk} \qquad \forall k, i, z, j \tag{12}$$

$$a_k^{con} + b_{ki}^{con} \ge (1-q)q^{z-1}\hat{d}_i w_i^{zk} \qquad \forall k, i, z \tag{13}$$

$$\sum_{z\in Z} w_i^{zk} \le 1 \qquad \forall k, i \tag{14}$$

$$\sum_{k\in K} w_i^{zk} = 1 \qquad \forall z, i \tag{15}$$

$$w_i^{zk} = \sum_{j \in J} y_{ij}^{zk} \qquad \forall z, i, k \qquad (16)$$

$$w_i^{zk} \in \{0, 1\} \qquad \forall z, i, k \qquad (17)$$

$$y_{ij}^{zk} \in \{0, 1\} \qquad \forall z, i, k, j \qquad (18)$$

$$\eta_k, a_k^{of}, a_k^{con} \geq 0 \qquad \forall k \qquad (19)$$

$$b_{ki}^{of}, b_{ki}^{con} \geq 0 \qquad \forall k, i \qquad (20)$$

Constraints (14)–(18) are the same as in the deterministic model, while (10)–(13) and (19)–(20) are those modified or added in the robust counterpart. The new variables $a_k^{of}, b_{ki}^{of}, a_k^{con}, b_{ki}^{con}$ are the dual of those appearing in the two knapsack problems.

3 Computational Experiments

Numerical tests have been run considering a set of instances based on the case of Montréal, QC, Canada. Instances (see [10]) have been generated using public annual reports published by *Urgences-santé* (2006) and *Statistics Canada* (2011). They include 30 demand zones, which represent the central part of the city of Montréal, whose nominal demands $\bar{d}_i$ range from 41 to 496. Moreover, vehicle capacity W has been set equal to 1500, and busy fraction q to 0.5.

The deterministic model has been solved considering the nominal demand $\bar{d}_i$ for each zone $i \in I$. Then, for the robust model, we have set the maximum variation $\hat{d}_i$ equal to 0.25 $\bar{d}_i$ for each demand zone $i \in I$.

We analyze the impact of Γ in terms of feasibility and price to pay for the improved feasibility (price of robustness). For this purpose, we consider 11 values of Γ, ranging from $\Gamma = 0$ (the deterministic model) to $\Gamma = |I|$ (the case in which each demand takes its maximum value) as follows:

$$\Gamma = \frac{k}{10}|I|, k \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

Being $|I| = 30$, the considered values of Γ are integer.

3.1 Execution and Solutions Evaluation

Three types of demand scenarios have been generated to evaluate the behavior of the obtained solutions, based on the values $\bar{d}_i$ and $\hat{d}_i$ in each zone i:

- *normal scenarios*: each d_i follows a Normal distribution centered in $\bar{d}_i$, and $\hat{d}_i$ represents about the 95% quantile, i.e. $d_i \sim N\left(\mu = \bar{d}_i, \sigma = \frac{\bar{d}_i + \hat{d}_i}{2}\right)$.
- *uniform scenarios*: each d_i is equal to $\bar{d}_i + \alpha_i \hat{d}_i$, where each α_i follows a uniform distribution in the interval $[-1, 1]$.
- *worst case scenarios*: each d_i is equal to $\bar{d}_i + \alpha_i \hat{d}_i$, where each α_i follows a uniform distribution in the interval $[0, 1]$. Unlike the previous cases, these demands are not centered in their respective $\bar{d}_i$. Therefore, these scenarios refer to a situation in which the demands have been underestimated, which represent an interesting case for health care managers.

For each alternative, 100 Monte Carlo samples have been drawn from each demand distribution, thus obtaining as many execution scenarios of each type.

To evaluate the feasibility of a solution in a scenario, we compute the maximum value of the workload W_{max} as:

$$W_{max} = \max_{k \in K} \sum_{z \in Z} \sum_{i \in i} (1 - q) q^{z-1} d_i w_i^{zk} \tag{21}$$

where d_i denotes here the demand in the scenario. A solution is considered to be *unfeasible* if the associated W_{max} is greater than the workload capacity W used to solve the problem. Similarly, to evaluate the *price of robustness* in a scenario, we first compute the value of the objective function OF as:

$$OF = \sum_{i \in I} \sum_{z \in Z} \sum_{k \in K} \sum_{j \in J} (1 - q) q^{z-1} d_i t_{ji} y_{ij}^{zk} \tag{22}$$

where d_i denotes, once again, the demand in the scenario. Then, we compute the price of the robust solution (i.e. its additional cost when executed) as the difference Δ_{OF} between the robust OF and the corresponding deterministic OF when $\Gamma = 0$:

$$\Delta_{OF} = OF_{robust} - OF_{deterministic} \tag{23}$$

4 Results

All instances have been solved to optimality within 1 hour, although the computational times increase from 108 up to 2514 seconds as the value of Γ increases.

Results are provided in Fig. 1. Solutions are always feasible for $\Gamma \geq 0.3|I|$ in the normal and uniform scenarios, while more than the 75% of the solutions are feasible for $\Gamma \geq 0.5|I|$ in the worst case scenario. As for the price of robustness, we observe that Δ_{OF} values are always below 6 s; thus, considering that OF values are around

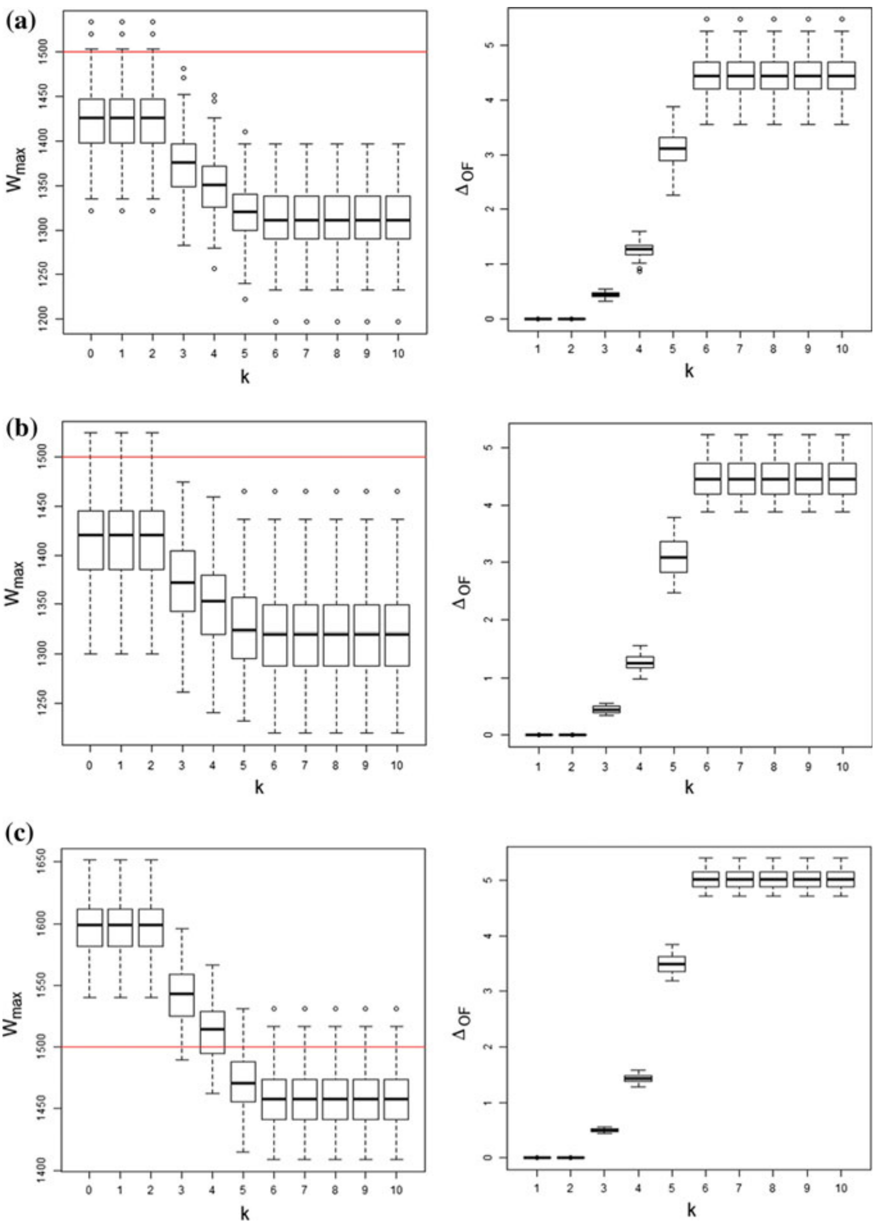

Fig. 1 Feasibility W_{max} (left column) and price of robustness Δ_{OF} (right column) for normal scenario (**a**), uniform scenario (**b**), and worst case scenario (**c**).

1500 s, they can be considered negligible. In other words, we can conclude that the price of robustness is highly affordable in order to guarantee feasible solutions.

5 Conclusions

In this paper, we propose and validate a robust counterpart of the ALDP in [4, 5] based on the cardinality-constrained approach. Results from the application to a realistic test case show that demand variations with respect to the expected values impair the feasibility of the deterministic solution, while its robust counterpart performs better for proper values of parameter Γ. In particular, in the considered test case, values of Γ between the 30 and the 50% of the demand zones (meaning that the $30-50\%$ of the demand zones assume the worst case value) allow the solution to remain feasible when tested against several demand scenarios. At the same, the observed increase of the objective function is negligible.

We may conclude that including the robustness in the ALDP problem is promising, at least in the tested case, because of the capability to increase the feasibility of the solutions while keeping limited the price to pay in terms of increased objective function value.

References

1. Addis, B., Carello, G., Grosso, A., Lanzarone, E., Mattia, S., Tànfani, E.: Handling uncertainty in health care management using the cardinality-constrained approach: advantages and remarks. Oper. Res. Health Care **4**, 1–4 (2015)
2. Aringhieri, R., Bruni, M.E., Khodaparasti, S., van Essen, J.T.: Emergency medical services and beyond: addressing new challenges through a wide literature review. Comp. Oper. Res. **78**, 349–368 (2017)
3. Bandara, D., Mayorga, M.E., McLay, L.A.: Priority dispatching strategies for EMS systems. J. Oper. Res. Soc. **65**, 572–587 (2014)
4. Bélanger, V., Lanzarone, E., Ruiz, A., Soriano, P.: The ambulance relocation and dispatching problem. Technical report CIRRELT 2015–59 (2015) https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2015-59.pdf
5. Bélanger, V., Lanzarone, E., Ruiz, A., Soriano, P.: The ambulance location and dispatching problem. Paper under second review (2017)
6. Bélanger, V., Ruiz, A., Soriano, P.: Recent Advances in Emergency Medical Services Management. Technical report CIRRELT 2015–28 (2015) https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2015-28.pdf
7. Beraldi, P., Bruni, M.E.: A probabilistic model applied to emergency service vehicle location. Eur. J. Oper. Res. **196**, 323–331 (2009)
8. Bertsimas, D., Sim, M.: The price of robustness. Oper. Res. **52**, 35–53 (2004)
9. Carello, G., Lanzarone, E.: A cardinality-constrained robust model for the assignment problem in home care services. Eur. J. Oper. Res. **236**, 748–762 (2014)
10. Kergosien, Y., Bélanger, V., Soriano, P., Gendreau, M., Ruiz, A.: A generic and flexible simulation-based analysis tool for EMS management. Int. J. Prod. Res. **53**, 7299–7316 (2015)

11. Schmid, V.: Solving the dynamic ambulance relocation problem and dispatching problem using approximate dynamic programming. Eur. J. Oper. Res. **219**, 611–621 (2012)
12. Zhang, Z., Jiang, H.: A robust counterpart approach to the bi-objective medical service design problem. Appl. Math. Model. **38**, 1033–1040 (2014)