

# The Compensation for Hysteresis of Silicon Piezoresistive Pressure Sensor

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**Abstract**—The aim of this paper is to examine compensating for the hysteresis error of silicon pressure sensor in order to improve the sensor accuracy. The object of investigation is large-range diffused silicon piezoresistive pressure sensors in the industrial field, based on MEMS technology. Due to the complex hysteresis characteristic of the sensor and difficulties in compensation, there are currently no published precedents in relevant studies. The author has analyzed the causation and impacting factors of the hysteresis characteristic and demonstrated, through experiment, that the silicon pressure sensor does satisfy the necessary and sufficient conditions of the general Preisach model. Through utilizing the Preisach model of the sensor and compensating for the hysteresis error using the compensation algorithm on inverse general Preisach model, the experiment has demonstrated that the hysteresis error decreases significantly after compensation, hence enhancing the precision of the sensors.

**Index Terms**—General Preisach model, hysteresis characteristic, silicon pressure sensor.

## I. INTRODUCTION

**H**YSTERESIS characteristics are found extensively in mechanical systems such as sensors as an unusual form of nonsmooth nonlinearity with multivalued mapping. In the sensor field, hysteresis characteristics are the inconsistencies of static characteristics between ascending and descending input variables, within the full scale [1]. Of the three indicators in the judgment of sensor performances, nonlinearity errors, hysteresis errors and repeatability errors, hysteresis error alone contributes to nearly 30% of all intrinsic errors, making it the key factor in sensor precision. Up until now, existing publications relevant to the compensation of silicon pressure sensor have all focused upon nonlinearity errors and in that aspect already produced systematic and comprehensive results [2]–[4]. Moreover, repeatability errors are examples of random error whose compensation is extremely difficult to realize, hence the significance of the investigation on the causation and compensation of hysteresis errors.

The sensor investigated is composed of foundation support, silicon piezoresistive pressure-sensitive chip, a stainless steel

casing, silicon oil, and a thin stainless steel diaphragm. Being an unusual example of mechanic-microelectronics-hydraulic system, the hysteresis characteristic of the silicon pressure sensor is impacted upon by the mechanic, microelectronics and hydraulic, for example, the mechanical property of the stainless steel diaphragm, the compressibility of the silicon oil, the mechanical property of the silicon chip, the silicon piezoresistive effect, the gap and friction between parts, consequently, the pattern of the hysteresis is extremely complex. Regarding silicon pressure sensors, studies into hysteresis of the unusual structure-composed of mechanic, microelectronics and hydraulic systems carries great theoretical importance.

However, up until now, there has been little academic attention paid to the hysteresis characteristic of silicon pressure sensors, on which no relevant publication can be located. Existing studies on hysteresis generally focuses upon the material field, exemplified by studies on ferromagnet and piezoelectric ceramic [5], [6]. In these areas, scholars have, after comprehensive research, proposed multiple hysteresis models and compensatory methods. There is the mechanical electronic equivalence model [7] based on physics, the Preisach model [8], the Bounc-wen model [9] and the Dahl model [10] based on phenomenology. The mechanical electronic equivalence model based on the law of conservation of energy describes the hysteresis using the differential equations easily, has a simple structure but suffers from lower precision and deficient descriptions. The Bounc-wen model with few parameters can be easily designed; its precision is dependant upon the initial status and could only describe the hysteresis deficiently. The Dahl model which describes the hysteresis using form of force can be understood easily, but in consequence of its numerous parameters the design proves to be rather difficult. The Preisach model based on phenomenology is not limited by the specific physical nature; its hysteresis operator could describe almost any hysteresis, including mechanical, hydraulic and microelectronic hysteresis. Moreover, the Preisach model is relatively simple to understand and capable of describing the hysteresis characteristic. However, on the downside it does demand a large quantity of experiments in order to establish the model. In consideration of the specific structure and complex hysteresis characteristic of the sensor, the author has adopted the Preisach mode, in the study, due to its capability to thoroughly describe any given hysteresis.

## II. FACTORS IN HYSTERESIS CHARACTERISTIC OF SILICON PRESSURE SENSOR

The silicon pressure sensor in the research is intended mainly for application in the harsh environment, including the automobile, aerospace and chemical industry. Utilizing isolated metal

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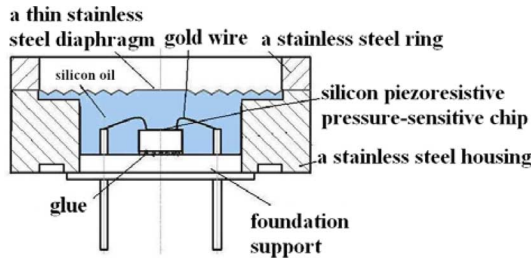


Fig. 1. Schematic diagram of silicon pressure sensor.

package, the schematic diagram of the sensor structure is as shown in Fig. 1.

Silicon piezoresistive pressure-sensitive chip is made with a micromachined silicon diaphragm with piezoresistive strain gauges diffused into it, fused to a glass backplate. The silicon chip is bonded to foundation support through gold wires to the soldering pins. The hermetic cavity is composed of the thin stainless steel diaphragm, the stainless steel casing and foundation ceramic support, which is filled with silicon oil.

Under operation, the pressure acts on the thin stainless steel diaphragm and is transferred through the silicon oil onto the silicon chip. The change of the resistor induced by piezoresistive effect breaks the balance of Wheatstone Bridge Circuit, which results in voltage signal approximately proportional to the pressure. From the perspective of energy transfer, the energy of the pressure signal is transmitted from the diaphragm through silicon oil to silicon chip and it is inevitable that some delay and energy loss will occur during this process, which is the primary cause of the sensor hysteresis.

The mechanical property of the stainless steel diaphragm, the compressibility of the silicon oil, the mechanical property of the silicon chip, the silicon piezoresistive effect, the gap and friction between parts are the immediate cause of the hysteresis. Furthermore, the level of expertise in the manufacturing process, for example, the purification of silicon oil, electrostatic bonding of the silicon chip and cementing the chip, also have deep impacts upon the hysteresis characteristic. . . Thus, it can be seen that, the hysteresis characteristics are affected by a range of various factors including the sensor structure and manufacturing quality, hence the complexity of the hysteresis characteristic patterns.

### III. EXPERIMENT OF NECESSARY AND SUFFICIENT CONDITIONS FOR THE SILICON PRESSURE SENSOR

The classical Preisach model based on magnetic hysteresis, proposed by German physicist Preisach in 1935 [8], was the first nonlinear modeling technique used to describe hysteresis behavior; subsequently, it was frequently applied to simulate the hysteresis behavior of various materials. The wiping-out and congruency properties constitute necessary and sufficient conditions for the classical Preisach model. American scholar I. D. Mayergoyz of the University of Maryland proposed the general Preisach model in 1988, expanding the congruency property of the classical Preisach model to vertical chords congruency property, which resulted in a broader area of applicability of the hysteresis model.

The wiping-out and vertical chords congruency properties constitute necessary and sufficient conditions for the general

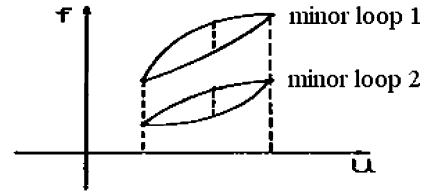


Fig. 2. Vertical chords congruency property.

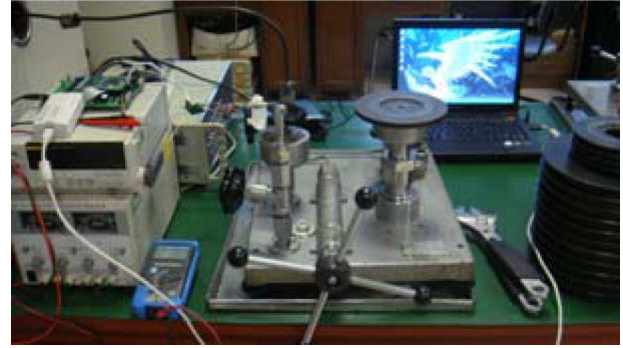


Fig. 3. Experimental system.

Preisach model; therefore, the author has designed the experiment to prove that the silicon pressure sensor does satisfy the necessary and sufficient conditions of the general Preisach model.

- 1) The wiping-out property: when a new local maximum or minimum exceeds the existing record in history, the previous value will be replaced and cease to affect the system in the future.
- 2) Vertical chords congruency property: all minor loops resulting from back and forth input variations between the same two consecutive extrema have equal vertical chords for the same input values. Fig. 2 shows vertical chords congruency property.

The object of the experiment is the silicon piezoresistive pressure sensor with isolated metal package. Its input pressure range is 0–40 Mpa. The output signal voltage is related to the input pressure and the range is 0–50 mV. Experimental instruments include piston pressure gauge, constant-current power and digital multimeter. The sensor is supplied by the +5 V constant-current power. The input pressure is from the piston pressure gauge. The output signal voltage is measured by the digital multimeter. The experiment system is as illustrated in Fig. 3. The experiment is conducted under a room temperature of 26 °C and 56% relative humidity.

#### A. Experiment of the Wiping-Out Property

To investigate the wiping-out property of the silicon pressure sensor, experiments are performed using the input pressure shown in Fig. 4(a) and (b). The corresponding output voltages are compared in Fig. 4(c).

As seen in Fig. 4(c) the outputs overlaps during the 60–90 s timeframe coincide, indicating that the previous input extrema of 20 and 10 Mpa was no longer impacting on the output once the input reached the new maximum of 40 Mpa.

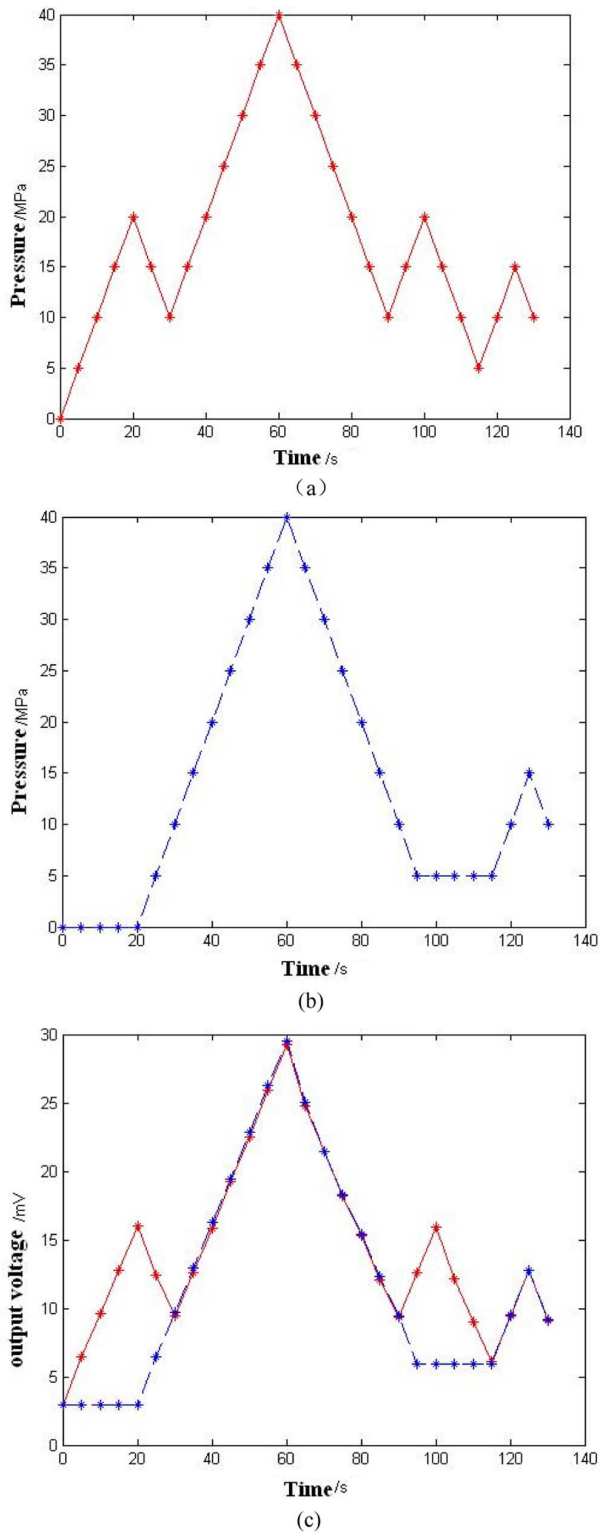


Fig. 4. Experiment of the wiping-out property. (a) Input pressure 1. (b) Input pressure 2. (c) Output voltages.

Likewise, it can be seen that the outputs overlapped during the 115–130 s timeframe coincide, indicating that the previous input extrema of 10 and 20 Mpa no longer had any influence over the output once the input reached the new minimum of 5 Mpa. In other words, the previous extrema were wiped out. Therefore the wiping-out property is active in the silicon pressure sensor.

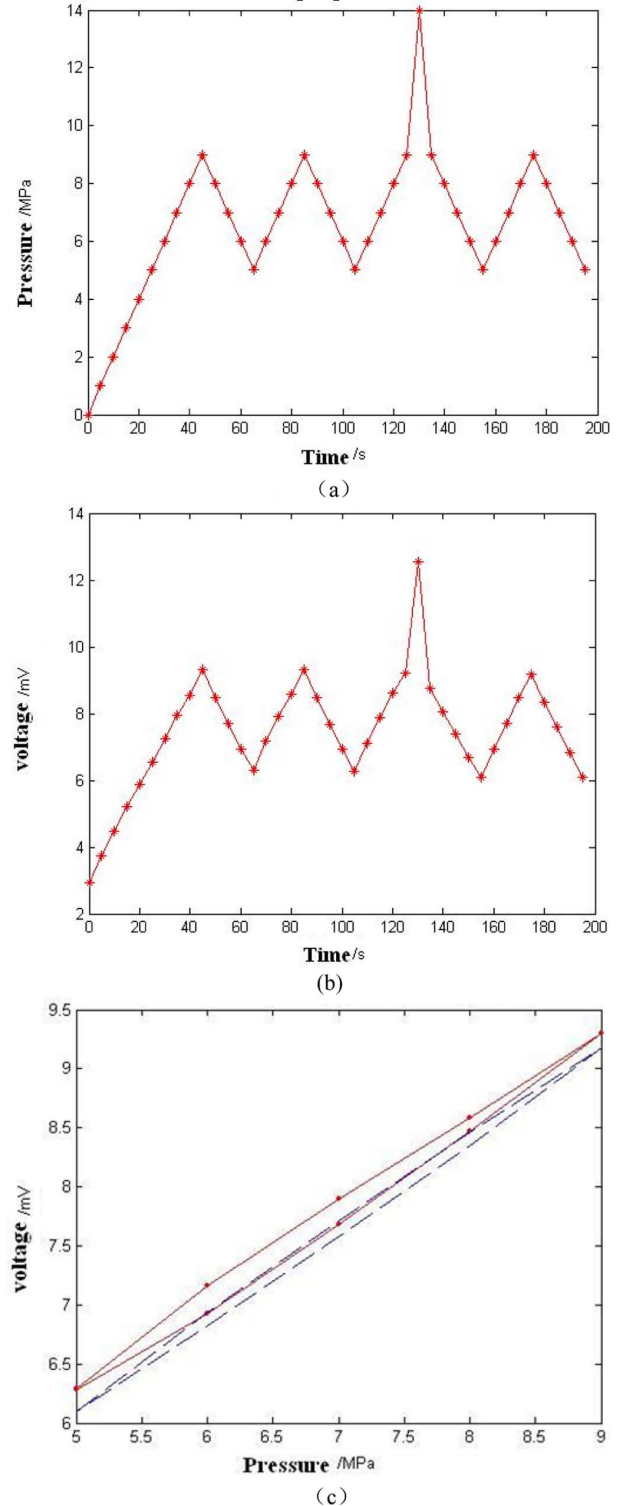


Fig. 5. Experiment of the vertical chords congruency property. (a) Input pressure. (b) Output voltage. (c) The minor loops.

### B. Experiment of the Vertical Chords Congruency Property

To investigate the vertical chords congruency property of the silicon pressure sensor, the input pressure is as shown in Fig. 5(a). The corresponding output voltage is given in Fig. 5(b). As Fig. 5(c) demonstrates, the output voltage of same reversal input pressure values (5, 9, 5 Mpa) has, at different time intervals, produced two minor loops. (The minor loop

formed between 65–105 s is represented by solid line, while the minor loop formed between 155–195 s is represented by broken line).

However, the difference is subtle. Hence, the vertical chords congruency property does not completely satisfy the silicon pressure sensor here. This has, to certain extents, affected the accuracy of the compensation of the hysteresis of the sensor using the Preisach model. However, it will be seen later that the accuracy of the Preisach model is still within acceptable range.

#### IV. THE ESTABLISHMENT OF THE GENERAL PREISACH MODEL FOR THE SILICON PRESSURE SENSOR

To describe the Preisach model, an infinite set of hysteresis operators  $\gamma_{\alpha\beta}$  are considered, and they are represented by rectangular loops and defined as follows:  $\gamma_{\alpha\beta} = 1$  if  $p(t) > \alpha$  and  $\gamma_{\alpha\beta} = 0$  if  $p(t) < \beta$ . Along with the above set of operators, we have used weight function  $\mu(\alpha, \beta)$ , the Preisach model is then given by

$$x(t) = \iint_{s^+} \mu(\alpha, \beta) \cdot \gamma_{\alpha\beta}[p(t)] d\alpha d\beta \quad (1)$$

where  $x(t)$  is the output voltage,  $\mu(\alpha, \beta)$  is a weight function,  $\gamma_{\alpha\beta}$  is the hysteresis operator having an output of 1 or 0,  $\alpha$  and  $\beta$  correspond to “up” and “down” switching values of the input pressure  $p(t)$ , respectively.  $S^+$  is the area where  $\gamma_{\alpha\beta}$  is 1.

Mayergoyz has also proposed the numerical expression of the general Preisach model for application [11]. When input pressure decreases from  $\alpha'$  to  $\beta'$ , the change of the output voltage is given by

$$x'(\alpha', \beta') = x_{\alpha'} - x_{\alpha'\beta'} \quad (2)$$

where  $x_{\alpha'}$  is the output voltage when input pressure increases from 0 to  $\alpha'$ ,  $x_{\alpha'\beta'}$  is the output voltage when input pressure decreases from  $\alpha'$  to  $\beta'$ . Function  $x'(\alpha', \beta')$  is defined as a Preisach function and represents the output voltage also when input pressure increases from  $\alpha'$  to  $\beta'$ . A pressure loading process consists of the continuous varying pressure. For successive  $n$  process ( $n \geq 2$ ), the output voltage, using the numerical expression of the general Preisach model, can be written as the following.

When the last variable of the input pressure is increasing, the output voltage is

$$x(t) = \sum_{k=1}^{x-1} [x'(\alpha_k, \beta_{k-1}) - x'(\alpha_k, \beta_k)] + x'(p(t), \beta_{x-1}). \quad (3)$$

When the last variable of the input pressure is decreasing, the output voltage is

$$x(t) = \sum_{k=1}^{x-1} [x'(\alpha_k, \beta_{k-1}) - x'(\alpha_k, \beta_k)] + [x'(\alpha_x, \beta_{x-1}) - x'(\alpha_x, p(t))]. \quad (4)$$

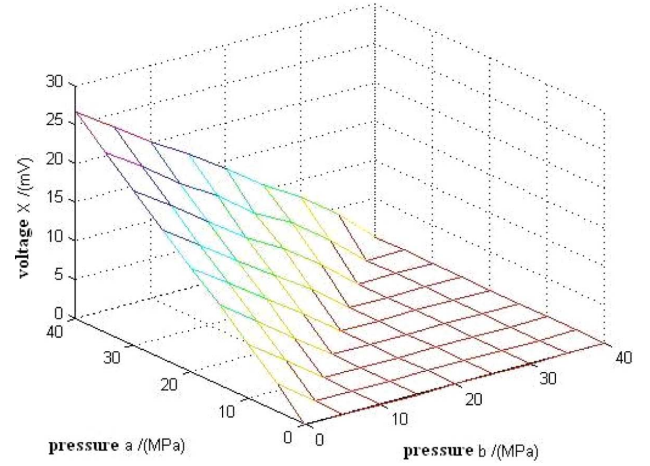


Fig. 6. The plot of the Preisach function.

To acquire the output voltage using the Preisach model, the Preisach function needs to be worked out first. The experiment to acquire the Preisach function is designed in the following. Considering the number of samples and complexity of the experiment, the range of 0–40 Mpa is divided into eight sections on average. First, the input pressure was increased from 0 to 5 Mpa, then decreased to 0 Mpa, from which the Preisach function  $x(5, 0)$  was calculated. The output voltage was recorded after each change in pressure. Second, The input pressure is then increased to 10 Mpa, the output voltage was recorded, then decreased to 5 Mpa, the output voltage was again recorded and the Preisach function  $x(10, 5)$  calculated, then decreased to 0 Mpa, the output voltage was again recorded and the Preisach function  $x(10, 0)$  calculated; and so on until the Preisach function  $x(40, 0)$  is acquired; Finally, the experimental data of the Preisach function  $x'(\alpha', \beta')$  are acquired and drawn as in Fig. 6.

#### V. THE ESTABLISHMENT OF THE INVERSE GENERAL PREISACH MODEL

From the experimental data of the Preisach function  $x'(\alpha', \beta')$ , Preisach function describing the hysteresis of the sensor can be acquired by regression analysis. If the array of the input pressure extrema in history can be known, the present output voltage can be calculated according to the Preisach function. However, the inverse function of the Preisach function is needed in order to acquire the present input pressure without hysteresis error in the signal processing.

The Preisach function  $x'(\alpha', \beta')$  monotonically increases with respect to  $\alpha'$  and  $\beta'$  for input pressure [12]. For a fixed  $\beta'$ , the inverse of  $x'(\alpha, \beta)$  is defined as  $\alpha' = X_{\alpha}^{-1}[x'(\alpha', \beta'), \beta']$  and for a fixed  $\alpha'$ , the inverse of  $x'(\alpha, \beta)$  is defined as  $\beta' = X_{\beta}^{-1}[\beta', x'(\alpha', \beta')]$ . Hence, the inverse Preisach model can be derived from (3) and (4) as the following [13], shown in (5) at the bottom of the next page.

From the inverse Preisach model, it can be seen that if the initial input pressure and array of the output voltage extrema in history was known, the present input pressure can be calculated using formula (5).

To acquire the inverse function, the data of the Preisach function  $x'(\alpha, \beta)$  can be translated into the data of the inverse function  $\alpha' = X_\alpha^{-1}[x'(\alpha', \beta'), \beta']$  and  $\beta' = X_\beta^{-1}[\beta', x'(\alpha', \beta')]$ , respectively, from which the inverse function can be derived through regression analysis.

## VI. THE EXPERIMENT OF THE COMPENSATION FOR THE HYSTERESIS ERROR OF THE SENSOR

To investigate the compensation for the hysteresis error of the silicon pressure sensor using the inverse Preisach model, the input pressure designed is as shown in Fig. 7(a). The corresponding output voltages are given in Fig. 7(b).

To compare with the algorithm of the compensation for hysteresis error, the output voltages were calculated using the algorithm of the compensation for nonlinearity error. The algorithm of the compensation for nonlinearity error is given by

$$p(t) = -0.00267329 \times u(t)^2 + 1.61455u(t) - 5.05939 \quad (6)$$

where  $p(t)$  is the calculated pressure,  $u(t)$  is the output voltage.

The output data are calculated using the algorithm of the compensation for hysteresis error and the algorithm of the compensation for nonlinearity error, respectively, and the comparison of error between the algorithms is shown in Fig. 7(c). (The solid line expresses the error of the hysteresis compensated algorithm and the broken line expresses the error of the nonlinearity compensated algorithm in Fig. 7(c).) The comparison of the errors using different algorithms is shown in Table I. Because the hysteresis error of the sensor is related to the pressure loading process, the average value and RMS value of errors are used to estimate the algorithm. Compared with the algorithm of the compensation for nonlinearity error, the algorithm of the compensation for hysteresis error reduces the average value of errors by 51.1% and reduces the RMS value of errors by 84%, as seen in Table I.

From the experimental result, it can be seen that the pressure error compensated by hysteresis algorithm is obviously smaller than the error compensated by the nonlinearity algorithm. Because the inverse Preisach model recorded the information of the hysteresis error and the nonlinearity error, the algorithm of the inverse Preisach model can compensate for the hysteresis error and the nonlinearity error. However, the algorithm of nonlinearity compensation has only recorded the information of the nonlinearity error, so the algorithm of nonlinearity compensation can not compensate for the hysteresis error. Thus,

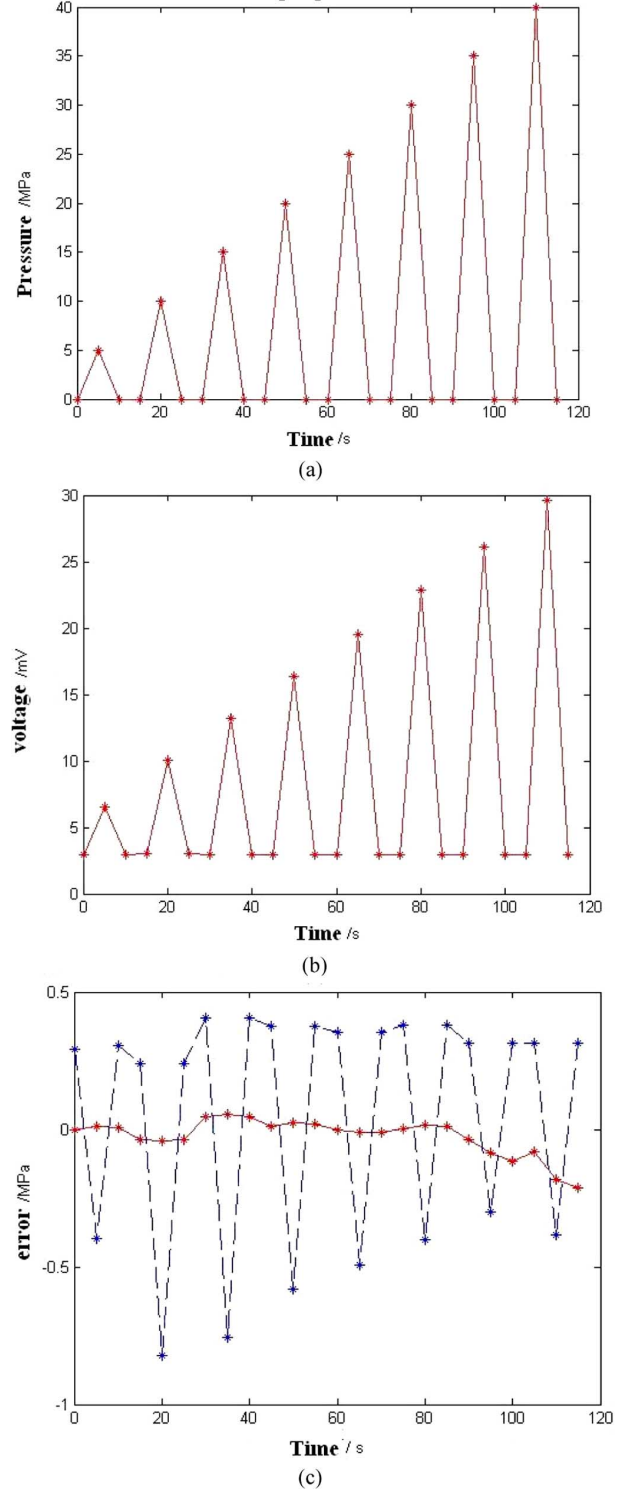


Fig. 7. The experiment of the compensation for the hysteresis error of the sensor. (a) Input pressure, (b) output voltage, (c) the error comparison.

$$p(t) = \begin{cases} X_\alpha^{-1} \left\{ x(t) - \sum_{k=1}^{x-1} [x'(\alpha'_k, \beta'_{k-1}) - x'(\alpha'_k, \beta'_k)], \beta'_x \right\}, & (p(t) \geq 0) \\ X_\beta^{-1} \left\{ \alpha'_x, \sum_{k=1}^{x-1} [x'(\alpha'_k, \beta'_{k-1}) - x'(\alpha'_k, \beta'_k)] + x'(\alpha'_x, \beta'_{x-1}) - x(t) \right\}, & (p(t) < 0) \end{cases} \quad (5)$$

TABLE I

comparison of the errors		Average value of errors	RMS value of errors
hysteresis algorithm	compensated	-0.0247	0.0675
nonlinearity compensated algorithm		0.0505	0.4252

the pressure error compensated by hysteresis algorithm is obviously smaller than the error compensated by the nonlinearity algorithm. Therefore, it is effective to compensate for the hysteresis error of the silicon pressure sensor on the inverse Preisach model.

## VII. CONCLUSION

From the causes and influencing factors of the sensor hysteresis, one can clearly see the complexity in the patterns and rules processed by the hysteresis characteristic of the silicon pressure sensor, a special mechanic, microelectronics and hydraulic system. The author has proposed the usage of general Preisach model of the sensor hysteresis in combination of the inverse general Preisach model which compensate for occurring hysteresis errors. The experiment regarding the necessary and sufficient conditions and the experiment of compensation for the hysteresis errors have been designed and performed, the results are as described in the following paragraph.

- 1) The experiment of necessary and sufficient conditions demonstrates that the silicon pressure sensor satisfy conditions for the wiping-out property to be effective, however, the presence of vertical chords congruency property may, to some extent, affect the accuracy of the compensation for the hysteresis of the sensor.
- 2) The experiment of the compensation for the hysteresis error shows that the hysteresis error has decreases significantly through the usage of compensation based on the algorithm of the Preisach model, in comparison to compensation for nonlinearity.

In summary, the adoption of the general Preisach model has been effective for the hysteresis of silicon pressure sensors and

compensation for hysteresis errors. As evident in the improvements made to the sensor precision and reduction of hysteresis error during compensation.

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