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Estimation of Origin-Destination Matrices Based on Markov Chains

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Abstract

The method of estimating origin-destination matrices of correspondence using observational data on traffic flows based on the Markov chain theory is considered in this paper. The method is based on the transportation network, which is associated with the graph of the corresponding Markov chain and on the canonical form of the graph proposed. The classification of observation models on flows in a transportation network is provided. The properties of the method proposed are investigated on several simple networks. The recommendations for the practical application of the method proposed in real transportation networks are given.

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Keywords: transport modelling, origin-destination matrices, Markov chains

1. Introduction

Mathematical equilibrium models of traffic flows are widely used to support decision-making process in the management of transport systems of large cities, agglomerations. The development of these models can be divided into two stages. The first stage is to estimate origin-destination (OD) matrices based on the initial data. At the second stage the origin-destination matrices obtained are distributed to the transportation network. Elements of an OD matrix area total number of users moving from one point of the transportation network to another. The problem

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of obtaining origin-destination flows is that they are not explicitly observable, and we have to find them indirectly. Different methods that depend on the initial data are commonly used to estimate OD flows. And all methods can be divided into 2 classes. The first class includes gravity and entropy models described by Vasil'eva *et al.* (1981) and by Wilson (1967). The gravity model is used by analogy with the famous law of gravitation; the entropy model is based on a maximum entropy method. A significant disadvantage of these methods is that they are described by a common function whose parameters are determined based on the mobility of the population, i.e. the indirect information about movements of citizens.

The second class of methods involves the estimation of OD flows using the observations on the traffic flow. Vardi (1996) defined this class of methods as "network tomography". A variety of such methods has been already developed. Tebaldi and West (1998), Li (2005) considered the Bayesian approach to the problem of OD matrix estimation in their papers. Hazelton (2001) conducted a comprehensive study of the problem and identified its fundamental issues. All the methods above assume that a so-called assignment matrix exists, i.e. the matrix that defines the connection between origin-destination flows and their paths, and between paths and links in which traffic flows are observed. Another approach to the problem of network tomography is based on a Markov representation of transport flows (not a Monte Carlo Markov chain (MCMC) method). In the papers of Crisostomi *et al.* (2010) and Morimura *et al.* (2013) approaches to the description of origin-destination trips using Markov chains were presented. Li (2009) considered a method to estimate OD matrices of the public transport using Markov approach. Khabarov *et al.* (2012) used Markovian approach to assess the OD matrix by the measurements on borders of transport zones. In this paper, we consider the approach to the estimation of OD matrices using observations on traffic flows on network nodes based on a Markov representation of transport OD flows.

In the section 2.1, the model of a transportation network is considered, and the classification of observation models on flows in a transportation network for the problems of estimation OD flows is provided. In the section 2.2, the possibility of presenting a transportation network as a Markov chain is considered, and a method for obtaining OD flows based on their Markov properties is provided. In the section 2.3, the properties of the method proposed are studied. Section 2.4 discusses practical aspects of the application of the method proposed. Section 3 is the conclusion.

2. Estimation of OD matrices

2.1. Observation models

Let us present a transportation network as a directed weighted graph **G** (**V**, **E**), where **V** is the set of vertices, and **E** is the set of edges. The transportation network **G** (**V**, **E**) is associated with a physical transportation network. The vertices are the transportation network nodes (junctions, interchanges, origins and destinations of trips), and the edges are network links, i.e. some connections between nodes. Some microscopic objects, such as cars, move through the transportation network and form a traffic flow. There are some physical and technological limitations in observing the transportation network. These restrictions give rise to 5 basic observation models in the graph **G**:

1. *The observation model at the network node*. A total number of micro-objects located in the node $v \in \mathbf{V}$ are observed during the time interval t: $n_v(t)$.
2. *The observation model on the network link*. A total number of micro-objects on the link $e \in \mathbf{E}$ of the transportation network **G** is observed during the time interval t: $n_e(t)$.
3. *The observation model at the network turns*. For such an observation model it is necessary to introduce the concept of the dual graph **L** (**G**) described by Harary (1969). Vertices of the dual graph **L** (**G**) are associated with the edges of the graph **G**. Consequently, edges of the dual graph **L** (**G**) connect vertices that correspond to adjacent edges of the graph **G** (see. Fig. 1). A total number of micro-objects on the link $e_l \in \mathbf{L}(\mathbf{E})$ of the dual **L** (**G**) is observed during the time interval t: $n_{el}(t)$. It can be interpreted as the flow intensity of some turn.
4. *The observation model at the network route*. The route is the k-th degree turn. The k-th degree turn, where $k > 1$, means the number of micro-objects passing through a chain of k adjacent links of **G**. A total number of micro-objects on the link $q \in \mathbf{L}^k(\mathbf{G}) = \mathbf{L}(\mathbf{L}^{k-1}(\mathbf{G}))$ (see. Fig. 1) is observed during the time interval t: $n_q(t)$. It may be interpreted as the intensity of the k-th degree turns.

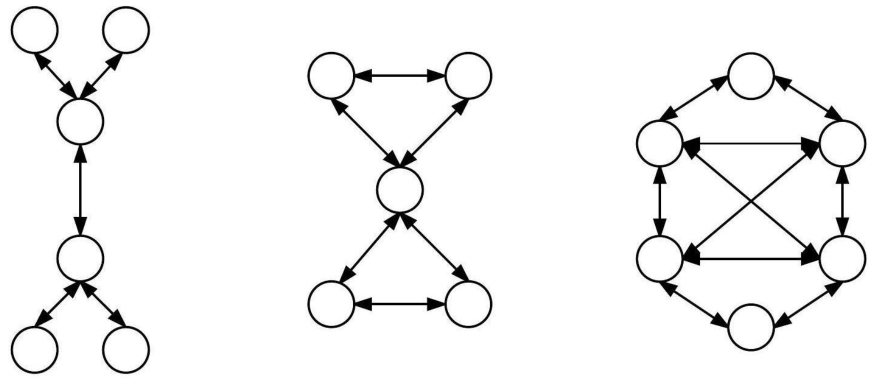

Fig. 1. Transformation of transportation networks.

5. *The model of origin-destination observations*. A complete graph **F** (**G**) is built based on the graph **G**. A total number of micro-objects on the link $q_f \in$**F** (**G**), moved from the node i to the node j during the time interval t: $\rho_{ij}(t)$. It may be interpreted as an intensity of OD flow.

It is important to note that the graphs **L** (**G**), $\mathbf{L}^k$ (**G**) and **F** (**G**) can also be considered as transportation networks. This allows us to consider models 2–4 in the same way in terms of estimating OD flows.

The initial data to the OD flow estimation problem is a certain sample, obtained using one or the other observation model, as well as some prior information on movements of the population. Thus the following problem is considered. It is necessary to estimate a size of the OD flow between nodes of the transportation network using sample data obtained according to some observation model. In the observation models 2–4 measurements are carried out on links of some transportation networks, so a variety of statistical methods of the OD matrix estimation can be applied, for example, Vardi (1996) or Tebaldi and West (1998) or Hazelton (2001), etc.

In the observation models considered the question of the physical implementation of the observation raises. In this regard, it is important to note that the observation model 1–3 can be implemented using a single observer. All other models require a network-distributed observation model, which is quite difficult physically to put into practice.

In this paper the method for observation models 2–4 using Markov properties of transport OD flows is considered.

2.2. OD matrix estimation method based on Markov chains

The transportation network **G**(**V**, **E**) can be associated with a transition graph of an a periodic Markov chain with discrete time and the transition probability matrix **P** that is described in papers by Khabarov *et al.* (2012), Khabarov and Tesselkin (2016).

$$P = \left\{ \hat{p}_{ij} \right\}, 0 \le \hat{p}_{ij} \le 1 \ \ and \ \forall i, j \sum_j \hat{p}_{ij} = 1 . \tag{1}$$

In this case, the Markov chain can be interpreted as follows. A micro-object of the transportation network, for instance, a car, located in the node i of the network (the state i), moves to the node j of the network at the time t with the probability p_{ij}. This model is applicable for the observation models 2–4.

We consider the problem of estimating transition probabilities based on observations on the Markov chain at the time $t = \{0,1,\ldots,T\}$ using the sample statistics

$$n_{ij} = \sum_{t=0}^{T} n_{ij}(t), \quad (2)$$

where n_{ij} (t) is a number of chain transitions from the state i to the state j at the time point t. n_i is a total number of chain transitions from the state i over the time T.

$$n_i = \sum_{j=1}^{m_i} n_{ij} . \quad (3)$$

A chain transition from the state i to state j is determined by a binomial distribution with the probability p_{ij}. According to Lee *et al.* (1970) estimates p_{ij} can be obtained using a maximum likelihood methods

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i}, \; i = 1,...,m; j = 1,\ldots,m_i . \quad (4)$$

Let us consider the problem of OD matrix estimation, so as for the selected observation the transition probability matrix of the Markov chain is estimated according to (4). Further, it is desirable to transform the transportation network transport the graph **G** (**V**, **E**) to a canonical form. In this regard, we divide the set of nodes **V** into three subsets: the elements of the first subset **S**⊆**V** are called origins; the second subset **M**⊆**V** consists of intermediate or internal nodes; the third one **D**⊆**V** contains so-called destinations. Every trip in the transportation network starts at some origin s∈**S**, passes through various internal nodes and ends at some destination d∈**D**. Moreover, the sets **S**, **M** and **D** do not intersect.

Let us introduce the origin-destination matrix ρ, as a fundamental feature of a transportation network. The element of the matrix ρ_{ij} is the number of micro-objects, moving from the i-th node of **S** to the j-th vertex of **D**.

The OD matrix ρ (**G**) is calculated using the diagonal matrix φ whose elements φ_{ii} are equal to the total number of micro-objects (the total flow) leaving the i-th origin, and using the matrix of indirect transition probabilities **B** between the elements of the sets **S** and **D**.

$$\rho = \varphi \cdot B . \quad (5)$$

Since the total flow leaving all origins is equal to the total flow entering all destinations, thus, we can similarly consider the matrix of the flow entering destinations, ψ.

$$\rho = B \cdot \psi . \quad (6)$$

Thus, the equations (5) and (6) make it possible to obtain the OD matrix, using only the prior data on the volume of traffic flows in origins and destinations and the matrix of transition probabilities of the corresponding Markov chain.

Further, some features of the Markov chain that describes the transportation network **G** (**V**, **E**) with size m.

- The state i belongs to the minimal set of states, if ∀j one can get from the state i to the state j, then the state of i can be reached from the state i in one or more steps.
- If we exclude all minimal sets M_{min} from the set of states of the Markov chain **V**, the minimal set of **U**\ $\mathbf{M}_{min}$ will be called the "first level" set corresponding to the minimal set. Similarly, we can define the "second level" set, etc.

Let us divide the matrix of transition probabilities into blocks. The first block of the matrix corresponds to the sub graph whose nodes form a minimal set, the second block corresponds to a "first level" set, etc. Further, according to Kemeny and Snell (1960), such a representation will be called a *canonical decomposition* of a transportation network.

Based on features of the transportation network **G** (**V**, **E**), the matrix **P** is divided into three blocks. It should also be noted that the set **D** consists entirely of absorbing states, and it is impossible to get from any state of the set **S** to another state of this set. The sets **S** and **D** are connected to each other only indirectly. Taking into account these facts, the transition probability matrix of the corresponding Markov chain can be written in a block canonical form:

$$P = \begin{pmatrix} I & 0 & 0 \\ R_{MD} & P_M & 0 \\ 0 & R_{SM} & 0 \end{pmatrix}. \tag{7}$$

Where $\mathbf{P}_M$ is a matrix of transition probabilities inside the set **M**. $\mathbf{R}_{SM}$, $\mathbf{R}_{MD}$ are matrices of transition probabilities from the states of the sets **S** and **M** to the states of **M** and **D**, respectively.

We note that all states of the sets **M** and **S** are transient. Let **B** is a matrix of indirect transition probabilities and **T** is a set of transient states. The indirect transition probability b_{ij} from the transient state i to the absorbing state j can be obtained as follows:

$$b_{ij} = p_{ij} + \sum_{k \in T} p_{ik} b_{kj} \tag{8}$$

or in a matrix form:

$$B = R + P_T B . \tag{9}$$

Consequently:

$$B = \left(I - P_T\right)^{-1} R . \tag{10}$$

It is known from the book by Kemeny and Snell (1960) that the matrix $(\mathbf{I} - \mathbf{P}_T)^{-1}$ is a fundamental matrix of the Markov chain.

It is necessary to obtain the matrix of indirect transition probabilities between the sets **S** and **D**, the matrix $\mathbf{B}_{SD} = \{b_{ij}\}$, to estimate the final OD matrix. As it is known that it is impossible to get directly from the state i of **S** to the state j of **D**, therefore:

$$B_{SD} = B_{SM} B_{MD} . \tag{11}$$

Let us return to a canonical form of the transition matrix (7) of the transportation network. According to (10), indirect transition probabilities of the sets **M** and **D** form the matrix $\mathbf{B}_{MD}$:

$$B_{MD} = \left(I - P_M\right)^{-1} R_{MD} . \tag{12}$$

Since the set **S** consists of transient states we can consider states of **M** as absorbing states corresponding to states of the set **S**. As direct transitions between states of **S** do not exist, then we have:

$$B_{SM} = \left(I - 0\right)^{-1} R_{SM} = R_{SM} . \tag{13}$$

According to (11), (12) and (13), the matrix of indirect transition probabilities $\mathbf{B}_{SD}$ is equal to:

$$B_{SD} = R_{SM}\left(I - P_M\right)^{-1} R_{MD}\,. \tag{14}$$

It is only remained to substitute the equation (14) into the equation (5) to get the final origin-destination matrix.

2.3. Investigation of the method

The method proposed has been investigated in several simple transportation networks depending on the size of the network and the violation of the Markov property. The study was conducted as follows. At the first stage the true OD matrix was distributed onto the network in the software package PTV Vision Visum using the equilibrium distribution. Simulated traffic intensities on nodes, links and turns of the network were taken as observations on traffic flows. The "observations" obtained allowed us to form a dual graph of the transportation network, which the method was applied to. The software implementation of the method has been written in C# language. Figure 2 shows examples of transportation networks, on which the study was conducted. In the figures blue polygons indicate the so-called "transportation districts", that are starting and ending points of every trip, and blue points and black sections form internal nodes and links of the transportation network.

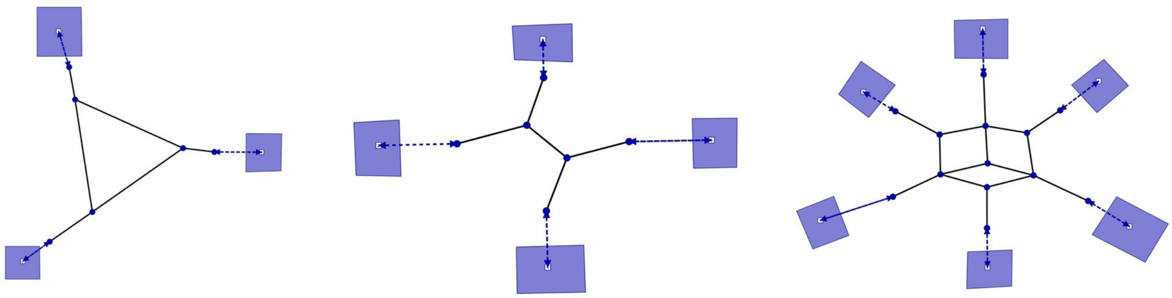

Fig. 2. Transportation networks 1, 2 and 3.

The quality of the matrices estimated was calculated using following criteria described in Bera and Krishna Rao (2011):

- Relative error (RE)

$$RE = \sqrt{\frac{1}{2}\cdot\sum_{i,j}\left(\frac{\tilde{\rho}_{ij}-\rho_{ij}}{\rho_{ij}}\right)^2}\,; \tag{15}$$

- Total Demand Deviation (TDD)

$$TDD = \frac{\left|\sum_{i,j}\tilde{\rho}_{ij} - \sum_{i,j}\rho_{ij}\right|}{\sum_{i,j}\rho_{ij}}\,; \tag{16}$$

- Mean absolute error (MAE)

$$MAE = \frac{\sum_{i,j} \left| \tilde{\rho}_{ij} - \rho_{ij} \right|}{N}; \tag{17}$$

- Root Mean Square Error (RMSE)

$$RMSE = \frac{\sqrt{\frac{1}{N} \cdot \sum_{i,j} \left(\tilde{\rho}_{ij} - \rho_{ij} \right)^2}}{\frac{1}{N} \cdot \sum_{i,j} \rho_{ij}}, \tag{18}$$

where $\tilde{\rho}$ is the true OD matrix and N is the number of origin-destination pairs. The TDD gives the quality of the estimated ODM. The RMSE error quantifies the total error of the estimate. The mean error indicates the existence of consistent under-or-over-prediction in the estimate. Smaller values of these measures will indicate the higher quality of the OD matrix estimated.

Table 1. Results of the investigation.

Criteria	Transportation network 1	Transportation network 2	Transportation network 3
Relative error (RE)	0	1,196E-16	8,783E-16
Total Demand Deviation (TDD)	0	0	1,739E-16
Mean absolute error (MAE)	0	5,684E-14	4,926E-13
Root Mean Square Error (RMSE)	0	4,192E-17	3,858E-16

Table 1 shows results of the study carried out with assumption that the Markov, or memory less, property is not violated, i.e. the transition probability in the current node does not depend on transition probabilities in previous nodes. Values of elements of the true OD matrix are in the range from 300 to 1100 vehicles per hour.

Thus, whatever numbers of nodes and links are, if the Markov assumption is not violated, the method will estimate OD flows correctly.

Table 2 shows results of the study carried out on the transportation network 2, given that the Markov property is violated, i.e. states of the Markov chain are dependent. The violation of the Markov property is divided into 2 categories: "small" violation, when the dependence exists, however, deviations in the probabilities do not exceed 15%, and "strong" violation the true transition probability from the current state depends entirely on the transition probability in the previous state.

Table 2. Results of the investigation (violation of the Markov assumption).

Criteria	No violation	"Small" violation	"Strong" violation
Relative error (RE)	1,196E-16	0,846	44,390
Total Demand Deviation (TDD)	0	1,185E-16	4,743E-16
Mean absolute error (MAE)	5,684E-14	193,212	850,892
Root Mean Square Error (RMSE)	4,192E-17	0,061	0,301

According to the study we can conclude that the method proposed with the "strong" violation of the Markov property estimates the OD matrix incorrectly, and it cannot be used. However, the "small" violation of the Markov property affects the method, but, nevertheless, it allows to get results close to true values.

The key drawback of the method proposed is its dependence on the Markov assumption of transport OD flows. This assumption works better in weakly-connected networks, but in strongly-connected networks this assumption can lead to big errors. The dependence on this assumption can be reduced by the usage of a "higher level" Markov chain and, consequently, a "higher level" observation model, for example, the observation model 4 with $k \geq 2$, but it will lead to additional difficulties in obtaining observations.

Known in the literature methods of estimating OD matrices using observations described in papers by Vardi (1996), Tebaldi and West (1998), Hazelton (2001), etc. are based on two equations:

$$y = B \cdot z$$
$$z = A \cdot x$$

Where y is a vector of all OD flows, z is a vector of all possible routes between origin-destination nodes; x is a vector of observations on links. The matrices **A** and **B**, the so-called assignment matrices, relate routes and links, and OD flows and routes, respectively. Thus, unlike our method, we need to form the matrices **A**, **B** additionally to use these methods that complicates the estimation procedure.

Since the methods described above are based on routes, they will provide exactly the same result as our method on the transportation network 1. It can be described by the fact that every network route has links belonging to only one route. In other cases, the accuracy of methods varies.

2.4. Practical application

As mentioned above the applicability of the method proposed depends on the violation of the Markov assumption of OD flows. In strongly-connected networks, for example, a network of the individual transport in a large city, the method proposed does not provide reliable results, but in weakly-connected networks, such as a network of roads between cities and villages, results are expected to be very accurate. The method proposed was applied to the base line freight transportation network of Novosibirsk city (see Figure 3) during the development of Novosibirsk complex transport model. General characteristics of the transport model, as well as the main characteristics of the base line freight transportation network are presented in Table 3.

Table 3. Novosibirsk transport model characteristics.

Objects	Total number	Total number in freight transportation network
Nodes	3481	91
Links	9702	208
Zones	397	49
Nodes with observations	61	42

During the development of the transport model a flow survey was carried out at key nodes of the transportation network of Novosibirsk. The data was collected at corresponding nodes during morning and evening peak hours using video cameras. The data was processed according to the observation model 3. The most informative nodes were selected for the survey. For this purpose the results proposed in the paper by Khabarov *et al.* (2015) were used. Finally, 42 key nodes of the baseline network were observed. Such a number of nodes were chosen according to possibilities available. In general, the choice of necessary number of observations requires an additional study. The OD matrix of the freight transportation network was obtained based on the data collected using our method. The matrix has been expertly tested and used in the transport model.

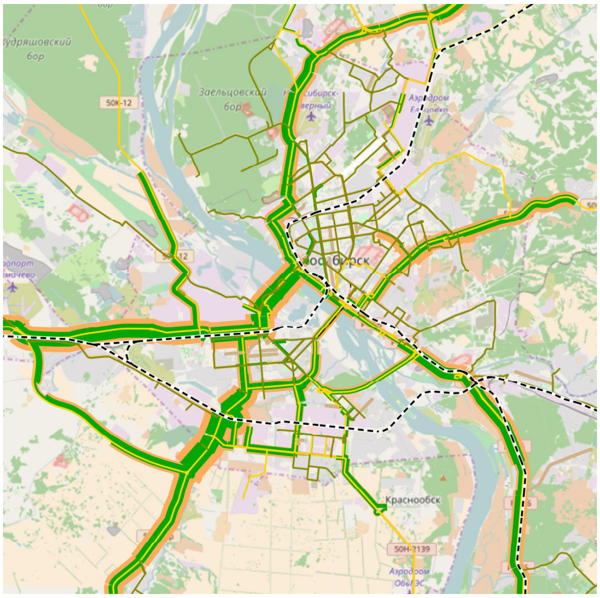

Fig. 3. Novosibirsk freight transportation network.

3. Conclusions

In this paper the OD matrix estimation method based on the representation of the network as a Markov chain was proposed. Models of observation on traffic flows on the network were considered. The OD matrix estimation method is based on a canonical decomposition of a transition matrix of a Markov chain. If observations obtained according to the observation models 2 – 4 are available, it is possible to apply the method proposed to estimate origin-destination flows and, which is connected to the estimation of indirect transition probabilities of the chain using its fundamental matrix.

The method proposed makes it possible to calculate OD flows based on Markov transition probabilities which greatly simplifies the estimation problem. If Markov assumption about OD flows is not violated, the method helps in obtaining accurate results on any network. The main drawback of the method is Markov assumption about OD flows that may be incorrect, especially in strongly-connected networks.

The further study is connected with the effectiveness of the method proposed in cases of a Markov chain with missing data, as well as the usage of Bayesian approach for the estimation problem in the context of the violation of Markov assumption of origin-destination flows.

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