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Soils and Foundations 57 (2017) 488–495

www.elsevier.com/locate/sandf

Bearing capacity of foundations on soft clays with granular column and trench

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Received 3 August 2016; received in revised form 17 December 2016; accepted 4 February 2017
Available online 18 May 2017

Abstract

The bearing capacity of foundations on soft clays under undrained conditions has been computed with inclusion of (i) a single vertical granular trench below a strip footing and (ii) a granular column placed below a circular footing. A lower bound plane strain and axisymmetric limit analysis, in conjunction with finite elements and an optimization procedure, has been used. The efficiency factor (ξ) has been determined by varying B_t/B_f; where (i) B_t = diameter of the column (width of the trench) and (ii) B_f = diameter of the circular footing (width of the strip footing). The effect of (i) the depth (D) of the column (trench) and (ii) the angle of internal friction (ϕ) of the column (trench) material has been explored for a wide range of $c_u/(\gamma B_f)$; c_u and γ imply undrained cohesion and the unit weight of the clay mass, respectively. Factor ξ increased quite significantly with increases in B_t/B_f and D/B_f. Factor ξ improved further with (i) increases in ϕ and (ii) decreases in $c_u/(\gamma B_f)$.

Keywords: Bearing capacity; Clays, Finite elements; Foundations; Limit analysis; Optimization; Plasticity; Stone columns

1. Introduction

A number of investigations, that examined the improvement in the bearing capacity of foundations by the insertion of granular trenches and columns, have been reported in literature. The studies are based on (i) analytical approaches (Madhav and Vitkar, 1978; Bouassida and Hadhri, 1995; Bouassida et al., 1995), (ii) elasto-plastic finite element analyses (Schweiger and Pande, 1986; Mitchell, 1985), and (iii) numerical lower and upper bound finite element limit analyses (Bouassida et al., 2015).

Series of small-scale model experiments (Hamed, 1986; Nazir and Azzam, 2010; Bouassida and Porbaha, 2004) and full-scale field tests (Mitchell, 1981, 1985; Stuedlein and Holtz, 2012) have been carried out by a few researchers. These different model and field tests revealed that the bearing capacity of foundations can be increased quite significantly with an increase in the depth of the granular trench.

In the present paper, the aim is to determine the bearing capacity of both strip and circular footings, placed on soft to medium soft clays reinforced with a single granular trench and column, respectively, by using the lower bound limit analysis with finite elements and the optimization principle.

2. Problem statement

A strip footing with width B_f and a circular footing with diameter B_f are placed over a soft clay deposit with $\phi = 0$.

Peer review under responsibility of The Japanese Geotechnical Society.
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http://dx.doi.org/10.1016/j.sandf.2017.05.013
0038-0806/

The strip footing is provided with a vertical granular trench of width B_t and the circular footing is reinforced with a vertical column of diameter B_t. The depth of the column (trench) is D. The undrained shear strength of the clayey deposit is c_u and the internal friction angle of the granular trench/column material is ϕ. The clayey deposit and granular soil media are assumed to follow the Tresca and Mohr-Coulomb yield criteria, respectively. The internal friction angle of ϕ of the clay is taken as being equal to 0. The associated flow rule is assumed to be applicable for granular material as well as for clay. It is to determine the magnitude of P_u for different values of B_t/B_f, D/B_f, $c_u/(\gamma B_f)$, and ϕ. The interface between the footing and the underlying soil mass is assumed to be perfectly rough. The unit weights of both clay and granular materials are chosen to be the same. Computations for a number of cases, however, have also been exclusively carried out by varying the ratio of the unit weights of these two different materials.

3. Problem domain and boundary conditions and finite element mesh

A rectangular domain KLST, shown in Fig. 1(a), has been considered for solving the problem. The entire problem domain remains symmetric about the vertical axis MN passing through the centre of the footing. Accordingly, zone MNST has been chosen for the analysis. The horizontal distance (L_r) from the right edge of the footing to the vertical boundary, ST, is varied from $5B_f$ to $15B_f$ for different values of ϕ. Depending upon D/B_f and ϕ, the vertical extent (L_d) of the domain is chosen to be between $6B_f$ and $15B_f$. The values for L_r and L_d are selected in a manner such that (i) the yielded elements do not approach any of the chosen domain boundaries (ST and NS) and (ii) increments in the size of the domain do not affect the magnitude of the collapse load.

The stress boundary conditions, applicable along different boundaries of the domain, are illustrated in Fig. 1(a).

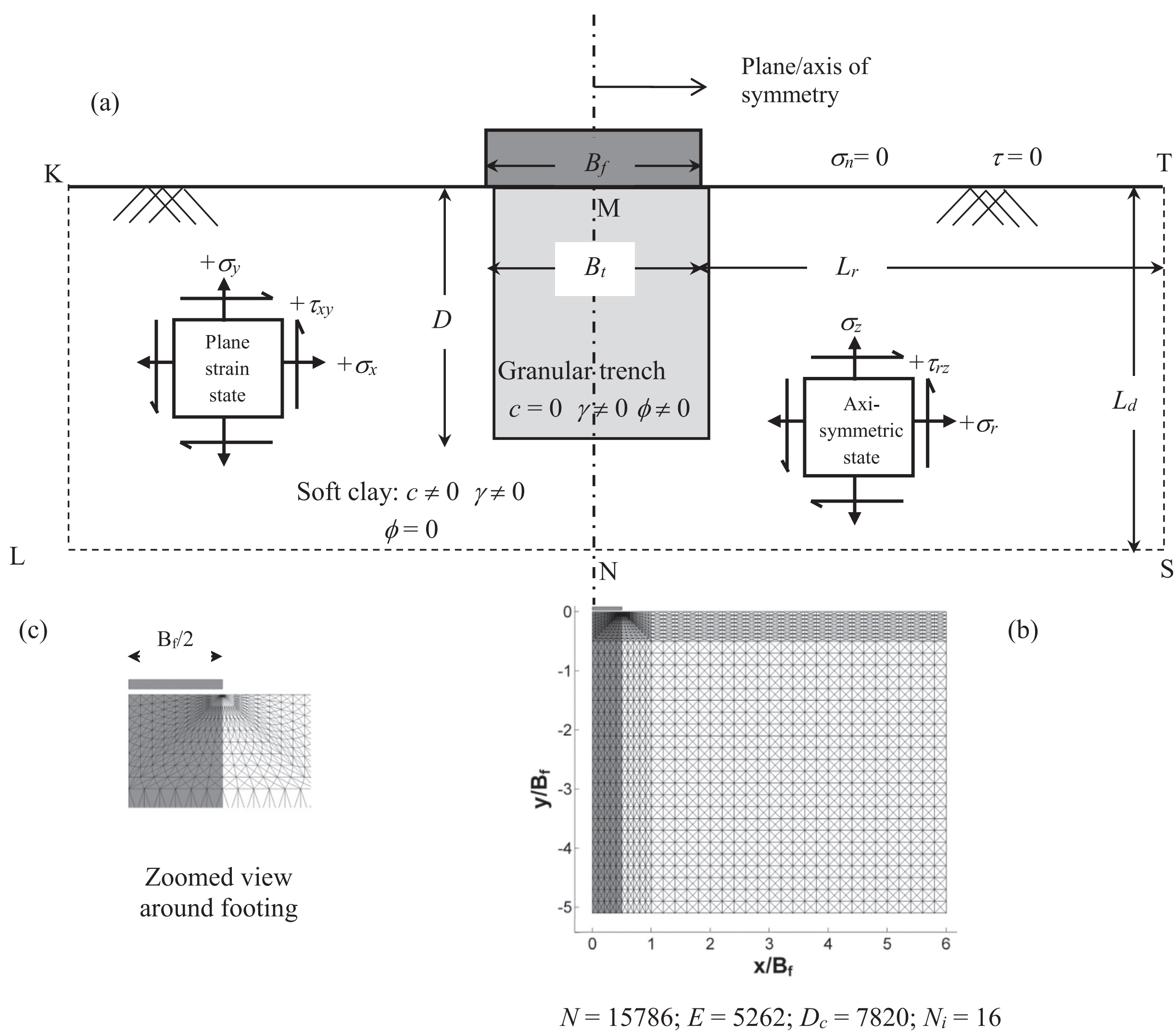

Fig. 1. (a) Schematic diagram of the problem, (b) typical finite element mesh for a strip/circular footing with a granular trench, (c) zoomed view of mesh around footing.

Apart from the imposition of the inequality constraint to avoid the yielding of the elements, no implicit stress boundary conditions need to be specified along boundaries NS or ST. Since vertical line MN represents the axis (plane) of symmetry, the shear stress along MN becomes equal to zero. A shear slip is permitted along the interface of the footing and surrounding the soil mass. Along the footing-soil interface, the following stress boundary conditions are imposed:

$$\text{For a strip footing}: |\tau_{xy}| \leqslant (c\ \cot\ \phi - \sigma_y)\tan\ \delta \quad (1a)$$

$$\text{For a circular footing}: |\tau_{rz}| \leqslant (c\ \cot\ \phi - \sigma_z)\tan\ \delta \quad (1b)$$

The problem domain was discretized into a number of three-node triangular elements in such a way that the sizes of all the elements decreased continuously towards the edge of the footing. Typical finite element meshes, for strip and circular footings with a granular trench (column), having B_t/B_f equal to 1 and $D = L_d$, are shown in Fig. 1(b); N, E, D_c, and N_i imply the total number of nodes, elements, stress discontinuities, and nodes along the soil-footing interface, respectively.

4. Analysis

The methodology used in this paper has been followed from (i) Sloan (1988), for solving a plane strain problem, and (ii) Kumar and Khatri (2011), for dealing with an axisymmetric problem. The nodal stresses, (i) σ_x, σ_y, and τ_{xy}, for a strip anchor and (ii) σ_r, σ_z, σ_θ, and τ_{rz}, for a circular footing, become the basic unknown stress variables; σ_θ is the circumferential normal stress for an axisymmetric problem.

Statically admissible stress discontinuities are permitted along all the common edges shared by any two adjacent elements. Along the interface of a column (trench) material and surrounding the clay, the values for normal and shear stresses need to always be continuous. The Mohr-Coulomb failure criterion for a plane strain problem can be written in following form:

$$F = (\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 - [2c\ \cos\ \phi - (\sigma_x + \sigma_y)\sin\ \phi]^2 = 0 \quad (2)$$

In the case of the Tresca failure criterion, the value for ϕ becomes equal to 0. For an axisymmetric problem, σ_x, σ_y, and τ_{xy} are simply replaced with σ_r, σ_z, and τ_{rz}, respectively. For an axisymmetric problem, there is an additional stress variable, σ_θ, apart from σ_r, σ_z, and τ_{rz}. The value for σ_θ, following the Harr-von Karman hypothesis (Haar and von Karman, 1909), is kept closer to minor principal stress σ_3 (Kumar and Khatri, 2011)

$$\sigma_{\theta j} \geqslant \sigma_{rj} \quad (3a)$$

$$\sigma_{\theta j} \geqslant \sigma_{zj} \quad (3b)$$

$$\sigma_{\theta j} \leqslant \sigma_{3fj} \quad (3c)$$

where σ_{3f} is the minor principal stress at failure and j implies the node number. The expression for σ_{3f} is given below.

$$\sigma_{3f} = \frac{\sigma_z + \sigma_r}{2} + \{-\frac{\sigma_z + \sigma_r}{2} + c\ \cot\ \phi\}\sin\ \phi \quad (4)$$

For an axisymmetric problem, in addition to the inequality constraints given by Eq. (2), the constraints given by Eqs. 3 and 4 also need to be imposed. The original Mohr-Coulomb yield function was linearized by following the methodology proposed by Bottero et al. (1980). The yield function (Eq. (2)) becomes a circle in the X-Y plane which is approximated by a regular polygon of sides p inscribed to the parent yield circle; $X = \sigma_x - \sigma_y$ and $Y = 2\tau_{xy}$. The nonlinear inequality condition can thus be replaced by p numbers of linear inequality constraints as given herein:

$$A_k\sigma_{xj} + B_k\sigma_{yj} + c_k\tau_{xyj} \leqslant D \quad k = 1, 2, \ldots, p \quad (5)$$

where, $A_k = \cos\left(\frac{2\pi k}{p}\right) + \sin\phi\cos\left(\frac{\pi}{p}\right)$; $B_k = \sin\phi\cos(\frac{\pi}{p}) - \cos\left(\frac{2\pi k}{p}\right)$; $C_k = 2\sin\left(\frac{2\pi k}{p}\right)$; $D = 2c\cos\phi\cos\left(\frac{\pi}{p}\right)$; c and ϕ refer to the cohesion and the internal friction angle of the soil mass, respectively. In case of an axisymmetric problem, the inequality constraints (5) are imposed by replacing σ_x, σ_y, and τ_{xy} with σ_r, σ_z, and τ_{rz}, respectively, in the following form:

$$A_k\sigma_{rj} + B_k\sigma_{zj} + c_k\tau_{rzj} \leqslant D \quad k = 1, 2, \ldots\ldots, p \quad (6)$$

$$P_u = \int_{Footing\text{-}soil\ \ interface} (-\sigma_n dA) \quad (7)$$

where σ_n is the normal stress acting over the element of area dA on the surface of the footing.

The linear optimization problem is finally defined by the following form:

$$\text{Maximize the objective function}: -\{g\}^T\{\sigma\} \quad (8a)$$

$$\text{Subjected to (i) equality constraints}: \{A_{eq}\}\{\sigma\} = \{b_{eq}\} \quad (8b)$$

$$\text{(ii) inequality constraints}: \{A_{ineq}\}\{\sigma\} \leqslant \{b_{ineq}\} \quad (8c)$$

5. Definition of efficiency factor (ξ)

The efficiency factor is defined as the ratio of the bearing capacity (p_{um}) of the foundation, with an inclusion of the vertical granular column (trench), to one (p_u) without any column (trench).

$$\text{Efficiency factor } \xi = \frac{p_{um}}{p_u} \quad (9)$$

6. Results and comparisons

The results have been obtained for different values of (i) B_t/B_f, namely, 0.2, 0.4, 0.6, 0.8, 1.0, and 2.0 and (ii) $c_u/(\gamma B_f)$, namely, 0.10, 0.25, 0.50, 0.75, and 1.0. The depth of the column (trench) was varied from B_f to $10B_f$, and two different values were used for the internal friction angle (ϕ) of the column (trench) material, namely, 40° and 45°. The variations in ξ are shown in Figs. 2(a,b,c) and 2(d,e,f) for (i) a rough strip footing with a granular trench and

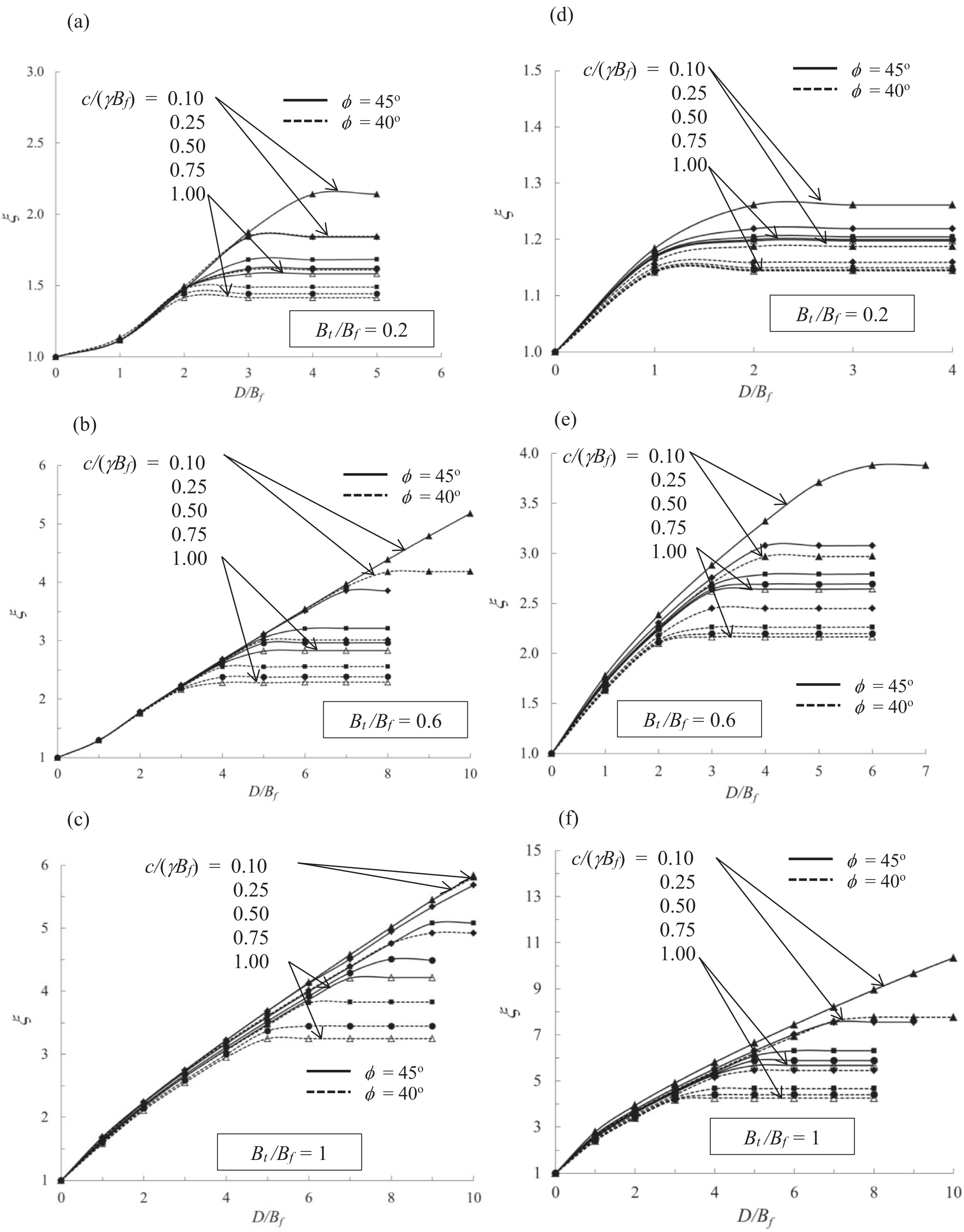

Fig. 2. The variation in efficiency factor (ξ) with D/B_f for: strip footing with (a) $B_t/B_f = 0.2$, (b) $B_t/B_f = 0.6$, (c) $B_t/B_f = 1$, and circular footing with (d) $B_t/B_f = 0.2$, (e) $B_t/B_f = 0.6$, (f) $B_t/B_f = 1$.

(ii) a circular rough footing with a granular column, respectively. The non-dimensional bearing capacity factor ($N_c = \frac{P_u}{A_f c}$) for the strip and the circular footings was found to be 5.09 and 6.05, respectively; $A_f = \pi B_f^2/4$ for the circular footing and $A_f = (B_f \times 1)$ for the strip footing. These values for N_c were found to match well with the corresponding values of 5.14 and 6.05 for the strip and the circular footings on the basis of the slip line method (Martin, 2005). By knowing the values for ξ and N_c, the magnitude of P_u can be simply determined using $P_u = N_c A_f \xi$.

The value for ξ increases continuously with an increase in D/B_f up to a certain value for $(D/B_f)_{\text{opt}}$, after which the magnitude of ξ becomes almost constant. The magnitude of ξ becomes greater for larger values of B_t/B_f and ϕ. On the other hand, the magnitude of ξ increases continuously with a decrease in $c_u/(\gamma B_f)$ by keeping the values of B_t/B_f and ϕ constant. In other words, employing the granular column (trench) leads to a considerable increase in ξ especially for very small values of $c_u/(\gamma B_f)$. The optimal values for D/B_f and the corresponding ξ for the strip and circular foundations are presented in Tables 1 and 2, respectively. It can be noted that the optimal values for D/B_f and the corresponding ξ increase continuously with (i) a decrease in $c_u/(\gamma B_f)$ and (ii) an increase in B_t/B_f. The optimum magnitude of D/B_f and the corresponding ξ value for a circular foundation placed on clayey soil with $c_u/(\gamma B_f) = 0.25$, with a granular column of $B_t/B_f = 0.6$, are (i) 4.0 and 3.08 for $\phi = 45°$ and (ii) 3.0 and 2.45 for $\phi = 40°$. On the other hand, the optimum magnitude of D/B_f and the corresponding ξ value for a strip foundation placed on clayey stratum with $c_u/(\gamma B_f) = 0.25$, with a granular column of $B_t/B_f = 0.6$, are (i) 7.0 and 3.85 for $\phi = 45°$ and (ii) 5.0 and 3.01 for $\phi = 40°$. Note that with the same values for $c_u/(\gamma B_f)$, B_t/B_f, and ϕ, as compared to a circular footing, the improvement in the bearing capacity becomes greater for a strip footing.

For (i) $\gamma_{sand}/\gamma_{clay} = 1.0$ and 1.5, (ii) $B_t/B_f = 2$, and (iii) $c_u/\gamma_{clay}B_f = 1.0$, 1.5, 2.0, and 2.5, the values of ξ for different magnitudes of D/B_f are presented in Table 3 for both strip and circular footings. The maximum difference in ξ by

Table 1
Maximum efficiency factor and corresponding optimal values of D/B_f for a strip footing reinforced with a granular trench.

B_t/B_f	$c_u/\gamma B_f$	$\phi = 45°$		$\phi = 40°$	
		$(D/B_f)_{\text{opt}}$	ξ	$(D/B_f)_{\text{opt}}$	ξ
0.2	0.10	4	2.14	3	1.85
	0.50	3	1.68	2	1.48
	1.00	3	1.58	2	1.42
0.6	0.10	>10	5.60	8	4.18
	0.50	7	3.21	4	2.55
	1.00	6	2.83	4	2.28
1.0	0.10	>10	11.13	>10	7.86
	0.50	9	5.08	6	3.82
	1.00	7	4.21	5	3.25

Table 2
Maximum efficiency factor and corresponding optimal values of D/B_f for a circular footing reinforced with a granular column.

B_t/B_f	$c_u/\gamma B_f$	$\phi = 45°$		$\phi = 40°$	
		$(D/B_f)_{\text{opt}}$	ξ	$(D/B_f)_{\text{opt}}$	ξ
0.2	0.10	2	1.26	2	1.19
	0.50	2	1.20	1	1.15
	1.00	2	1.19	1	1.14
0.6	0.10	6	3.88	4	2.96
	0.50	4	2.79	3	2.26
	1.00	4	2.64	3	2.16
1.0	0.10	>10	11.18	8	7.77
	0.50	6	6.31	4	4.67
	1.00	5	5.67	4	4.26

Table 3
A comparison between efficiency factors for $\gamma_{sand}/\gamma_{clay} = 1$ and $\gamma_{sand}/\gamma_{clay} = 1.5$.

Footing type	B_t/B_f	$\gamma_{sand}/\gamma_{clay}$	D/B_f	Efficiency factor (ξ)	
				$c/\gamma_{clay}B_f = 1.0$	$c/\gamma_{clay}B_f = 2.0$
Strip	1	1.0	4	2.89	2.83
		1.5		2.60	2.70
		1.0	Full	3.25	2.94
		1.5		3.17	2.90
	2	1.0	4	3.81	3.39
		1.5		3.30	3.34
		1.0	Full	4.16	3.40
		1.5		4.21	3.53
Circular	1	1.0	4	4.27	4.07
		1.5		4.22	4.04
		1.0	Full	4.27	4.07
		1.5		4.22	4.04
	2	1.0	4	7.89	6.22
		1.5		8.36	6.88
		1.0	Full	7.89	6.21
		1.5		8.38	6.89

varying $\gamma_{sand}/\gamma_{clay}$ from 1.0 to 1.5 is found to be around 13.4%. This implies that for different $\gamma_{sand}/\gamma_{clay}$ values, the results do not differ as significantly as the results obtained when assuming $\gamma_{sand}/\gamma_{clay} = 1.0$. This is because the shear strength of the soil mass plays a major role in determining the magnitude of the failure load. Therefore, all the results were finalized by assuming $\gamma_{sand}/\gamma_{clay} = 1$. It was noted that for a greater value of $c_u/\gamma_{clay}B_f$, with larger B_t/B_f and D/B_f, an increase in the magnitude of $\gamma_{sand}/\gamma_{clay}$ causes a marginal increase in the efficiency factor. On the other hand, a nominal decrease in the value of ξ was noted with an increase in $\gamma_{sand}/\gamma_{clay}$ with smaller B_t/B_f and D/B_f. For $c_u/\gamma_{clay}B_f = 1.5$, an increase in the value of $\gamma_{sand}/\gamma_{clay}$ was found to cause an increase in the magnitude of ξ. In contrast, for $c_u/\gamma_{clay}B_f = 1.0$, an increase in the value of $\gamma_{sand}/\gamma_{clay}$ generally leads to a decrease in ξ. For $c_u/\gamma_{clay}B_f = 1.5$, an increase in the magnitude of $\gamma_{sand}/\gamma_{clay}$ was found to cause an increase in ξ. This reverse trend occurs due to the fact that for very soft clays, the increased weight of the column (trench) itself exerts an additional load on the underlying soft clay stratum and, as a result, the efficiency factor decreases with an increase in $\gamma_{sand}/\gamma_{clay}$. This observation implies that for very soft clays, it would be advantageous to use such a granular material for filling the column (trench) for which the shear strength is good, but at the same time light in weight. An admixture of gravel, sand, fly ash, and lime could perhaps be one suitable option; note that fly ash is light in weight which may be beneficial for reducing the overall density of the compacted admixture. However, more research will be needed to explore the suitability of the trench/column material for very soft clays.

7. Comparisons of the results

The efficiency factors obtained from the present numerical analysis were compared with different numerical, analytical, and experimental results reported in literature for both strip and circular footings with the usage of a granular column (trench).

For a smooth strip footing laid on soft clays reinforced with a granular trench, a comparison between the present results and the upper bound solution of Madhav and Vitkar (1978) has been presented in Fig. 3 with two different values for $c_u/(\gamma B_f)$, namely, 0.1 and 1.0 and for $\phi = 40°$ and 45°. Note that the results from both analyses for the smooth footing match well. In most of the cases, the present analysis provides lower values for the efficiency factors compared to those of the solution by Madhav and Vitkar

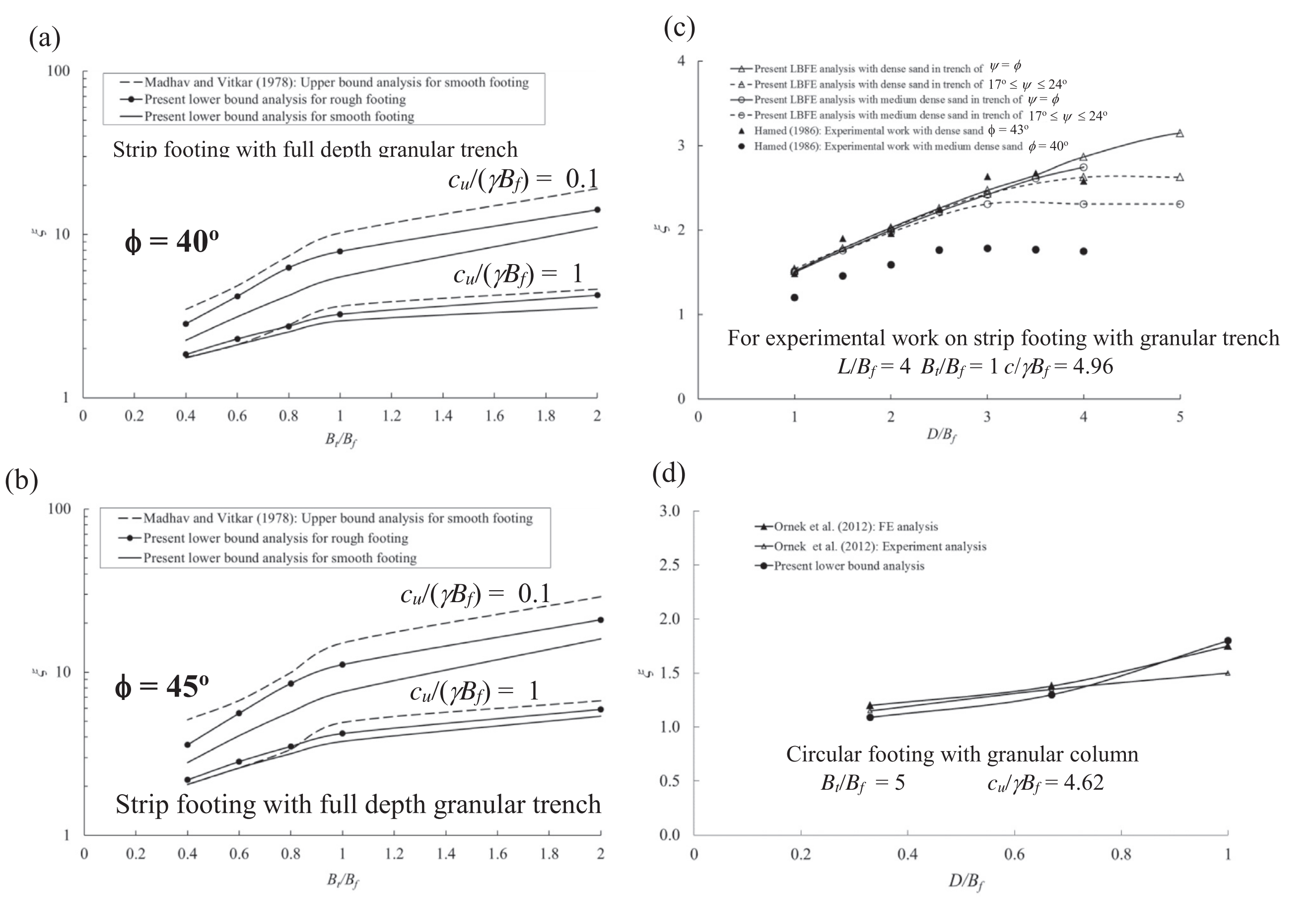

Fig. 3. A comparison of the present analysis with the results of Madhav and Vitkar (1978) for (a) $\phi = 40°$ and (b) $\phi = 45°$, (c) experimental data of Hamed (1986) for a strip footing reinforced with the trench, and (d) experimental and numerical data of Ornek et al. (2012) for a circular footing reinforced with a granular column.

Table 4
A comparison between present numerical results and that reported by Bouassida et al. (2015) for a strip footing ($B_f = 3$ m) with a deep granular trench for $c_u/(\gamma B_f) = 0.39$ and = 30°.

B_t/B_f	$Q/B_f\, c_u$		
	FELA	FELA 3D	Present work
1/3	$7.3^{*}(7.45)^{**}$	7.4	7.3

*Numerical lower bound finite element analysis using nonlinear optimization by Kuhn-Tucker optimality up to one decimal place.
**Numerical upper bound finite element analysis up to one decimal place.

(1978), except for $B_t/B_f < 0.8$ with $c_u/(\gamma B_f) = 1$ where both analyses provide almost the same results (Table 4).

For the strip footing placed on soft clay reinforced with a granular trench, a comparison between the present results and the experimental data provided by Hamed (1986), based on small-scale model tests, has been presented in Fig. 3. In this comparison, the variation in the efficiency factor with changes in D/B_f has been illustrated for $c_u/(\gamma B_f) \approx 4.96$ and $B_f/B_t = 1$. The trench is comprised of sand with two different relative densities: (i) dense sand with $\phi = 43°$ and (ii) medium dense sand with $\phi = 40°$. The value of the dilatancy angle (ψ) was obtained by Bolton's empirical expression (Bolton, 1986, 1987). Although the lower bound limit analysis is strictly applicable to materials that follow an associated flow rule, one can approximately discover the magnitude of the collapse load (Sloan, 2013) by using reduced shear strength parameters c^* and ϕ^*, instead of c and ϕ, based on the following expressions established by Davis (1968):

$$\tan\ \phi^* = \eta\ \tan\ \phi;\ c^* = \eta c \text{ and } \eta = \frac{\cos\psi\cos\phi}{1-\sin\psi\sin\phi} \qquad (10)$$

It can be noted from Fig. 3(a) that the present results match reasonably well with the experimental data reported by Hamed (1986). The values for the efficiency factor from the model tests for a trench with dense sand compare very closely with the present results. On the other hand, the efficiency factor for a trench with medium dense sand has been found to be approximately 20% greater than that of the present solution. It should be mentioned that in the model tests, the length to width ratio of the footing was kept equal to 4, whereas the present analysis is strictly meant for a strip footing. The size effect of the footing was also not explored while making the comparison. It has already been noted that the efficiency factor for a strip footing becomes significantly greater than that for a circular footing.

For a circular footing stabilized with a granular column, the results from the present analysis have been compared with the experimental and numerical works performed by Ornek et al. (2012) for $B_t/B_f = 5$ with $c_u/(\gamma B_f) = 4.62$. A comparison of these results is presented in Fig. 3(b) with $\phi = 43°$ and $\psi = 13°$. The maximum difference between the two solutions has been found to be 10% for $D/B_f = 0.33$. However, the values for the efficiency factors from the experiments have been found to be a little lower for $D/B_f = 1.0$.

For (i) a strip foundation reinforced with a granular trench and (ii) a circular foundation reinforced with a granular column, the results from the present analysis have also been compared with the expression proposed by Stuedlein and Holtz (2013) on the basis of the test results given in Mitchell (1981) to discover the ultimate bearing capacity of a single aggregate pier. A corresponding comparison of the results is shown in Table 5 with $\phi = 45°$ for a granular column/trench and $c_u/(\gamma B_f) = 2.5$ for in-situ clay. The values for the efficiency factors from the present analysis are found to be smaller than the values given by Stuedlein and Holtz (2013) and Mitchell (1981). The difference between the two series of results increases with (i) an increase in B_t/B_f for a strip footing reinforced with a granular trench and (ii) a decrease in B_t/B_f for a circular footing with a single granular column.

8. Conclusions

The improvement in the bearing capacity of strip and circular foundations on soft clays with the application of a single vertical granular column (trench) below the centre of the footing has been evaluated by using the lower bound finite element limit analysis. The efficiency factor has been found to increase continuously with an increase in the diameter (width) of the column (trench). The internal friction angle (ϕ) of the column (trench) material has a significant impact on the improvement in the bearing capacity. The efficiency factor (ξ) increases continuously with an increase in the depth (D) of the column (trench) up to a certain depth before attaining a certain maximum value. The optimal values for D/B_f and the corresponding ξ increase continuously with (i) a decrease in $c_u/(\gamma B_f)$ and (ii) an

Table 5
A comparison of the efficiency factor (ξ) between the present analysis and the empirical expression of Stuedlein and Holtz (2013) following the tests' results of Mitchell (1981).

Type of footing	B_t/B_f	Efficiency factor (ξ)	
		Present analysis	Stuedlein and Holtz (2013) using test results of Mitchell (1981)
Strip footing with a granular trench	0.2	1.523	1.646
	0.6	2.180	2.939
Circular footing with a granular column	0.2	1.193	1.984
	0.6	2.554	3.455

increase in B_t/B_f. With the same values for $c_u/(\gamma B_f)$, B_t/B_f and ϕ, the improvement in the bearing capacity becomes significantly greater for a strip footing than for a circular footing.

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