



Contents lists available at ScienceDirect

# Expert Systems With Applications

journal homepage: [www.elsevier.com/locate/eswa](http://www.elsevier.com/locate/eswa)

## Project resources scheduling and leveling using Multi-Attribute Decision Models: Models implementation and case study



Ch. Markou, G.K. Koulinas, A.P. Vavatsikos\*

Production and Management Engineering Dept., Democritus University of Thrace, Vas. Sofias 12, Xanthi 67100, Greece

### ARTICLE INFO

#### Article history:

Received 19 September 2016

Revised 16 December 2016

Accepted 25 January 2017

Available online 2 February 2017

#### Keywords:

Project management

Project scheduling

Resource leveling

Multi-Attribute Decision Models

### ABSTRACT

Project scheduling is one of the most vital processes in Project Management. It is a widely discussed topic in academic and practical circles due to its importance and complexity. Manpower, machines, materials and equipment are used for the execution of project activities, but these mostly have limited availability, which can constrain project scheduling procedures. Project resources might exceed or fall short of the resource demand in a project's time horizon. These considerations present issues to project managers who must try to properly allocate among these demands in order to achieve a near optimal utilization during a project's lifetime. Resource leveling is among the greatest challenges faced by project managers as the success of a project largely depends on it. This is because peaks and valleys in the resource usage histogram are responsible for cost overruns due to the necessary recruitment, dismissal and training of the personnel. Moreover, issues may arise regarding the efficient management of available resources given that large peaks correspond to fluctuations in resource allocation during a project's life cycle or construction period. To address these issues, resource leveling provides procedures and frameworks that ensure the efficient management of resources to obtain smooth resource usage profiles. These procedures attempt to identify activities that should be delayed to resolve resource over-allocations under time and cost constraints. Given the existence of a variety of available rules that could be followed by project managers to prioritize activities, the paper at hand examines the implementation of five Multi-Attribute Decision Making models and how they perform in the scheduling of a solar park construction project. Namely these models are the Weighted Sum Method, Analytic Hierarchy Process, PROMETHEE, TOPSIS, Ordered Weighted Average (OWA) and Hybrid Weighted Average (HWA). Finally, the derived results are discussed in comparison with those obtained by the standard resource leveling procedures of MS-Project.

© 2017 Elsevier Ltd. All rights reserved.

### 1. Introduction

The efficient management of available resources is one of the greatest and most complex problems that project managers (PMs) have to overcome. The Resource Leveling Problem (RLP) is a classic resource management problem faced by practitioners, managers and researchers. Resource leveling aims to minimize peaks and valleys in the resource histogram without increasing the project duration beyond the original critical path duration (Harris, 1990; Shtub, Bard, & Globerson, 2005). However, the problem can also come to bear in cases of limited resources, which often lead to extensions of the initial project duration (Hiyassat, 2001; Neumann & Zimmermann, 2000). During the past six decades, several different approaches have been developed for solving the RLP. Exact algorithms have been proposed in the literature, including integer and

dynamic programming techniques (Bandelloni, Tucci, & Rinaldi, 1994; Neumann & Zimmermann, 2000). These approaches are suitable for small-sized networks due to the so-called "combinatorial explosion" phenomenon. Several heuristic procedures have also been developed to overcome the RLP, most of them are based on shifting heuristics or priority rule methods (Burgess & Killebrew, 1962; Neumann & Zimmermann, 2000). In addition, metaheuristic approaches such as genetic algorithms (Kyriklidis & Dounias, 2016; Kyriklidis, Vassiliadis, Kirytopoulos, & Dounias, 2014; Leu, Yang, & Huang, 2000; Li & Demeulemeester, 2016; Ponz-Tienda, Yepes, Pellicer, & Moreno-Flores, 2013) and simulated annealing algorithms (Anagnostopoulos & Koulinas, 2010; Son & Skibniewski, 1999) have attempted to find an optimum solution to this problem. Recently, hyperheuristic algorithms proposed to treat the RLP and resource allocation problems (Anagnostopoulos & Koulinas, 2010; Koulinas & Anagnostopoulos, 2011; Koulinas, Kotsikas, & Anagnostopoulos, 2014) have offered some promising results. The basic idea of these approaches is to create a resource profile based on the early start

\* Corresponding author.

E-mail addresses: [charalamposmarkou@outlook.com](mailto:charalamposmarkou@outlook.com) (Ch. Markou), [gkoulina@pme.duth.gr](mailto:gkoulina@pme.duth.gr) (G.K. Koulinas), [avavatsi@pme.duth.gr](mailto:avavatsi@pme.duth.gr) (A.P. Vavatsikos).

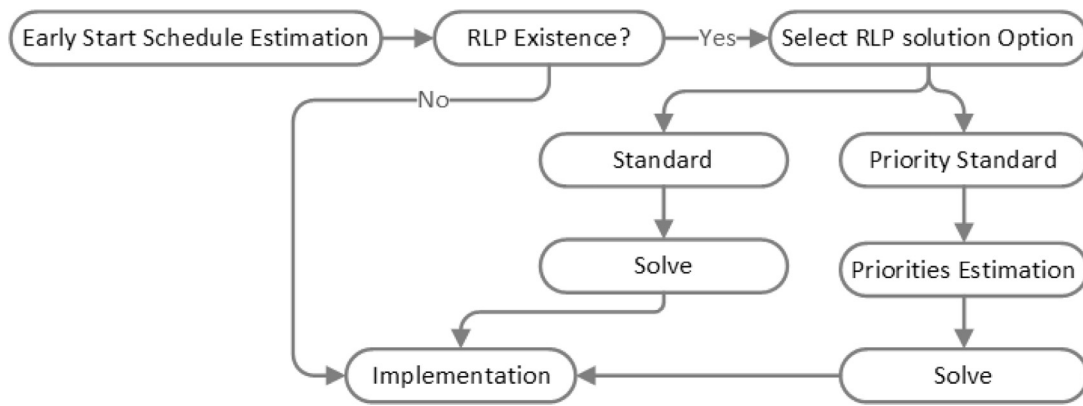


Fig. 1. RLP solution options.

schedule calculated from the Critical Path Method (CPM), and then shift noncritical activities according to fixed heuristic rules.

This paper describes the application of Multi-Attribute Decision Making (MADM) methods to define the activity priorities to treat resource leveling under constraints when priority rule methods are implemented. The proposed framework aims to optimize resource usage without exceeding a pre-determined resource limit. This goal is accomplished by allowing PMs' participation to the priorities determination phase of. A variety of well-established MADM models is implemented enabling the performance of a variety of decision attitudes.

## 2. Resource Leveling Problem

Achieving smooth resource profiles is a major goal during project implementation. RLP is a technique that adjust activities' start and finish dates according to resource constraints aiming to balance their demand with the available supply (PMBOK 2013). Fluctuations in resource usage histograms represent additional costs in a resource usage profile. An ideal resource usage profile has a continuous and plateau-shaped resource histograms where instances of both over and under-allocation are minimized. Resource leveling is required to meet limitations of resource availability by reducing the peak of daily resource requirements and determine the minimum amount of resources necessary to accomplish a project by a specified finish date. To model the RLP, an acyclic activity-on-node (AoN) representation of a project network is used. The network consists of  $n$  activities (nodes) and precedence relations (arcs) between activities. We denote  $d_i$  and  $f_i$  the duration and the finish time of activity  $i$ , respectively, and assume that node 1 ( $n$ ) is the dummy activity of the project beginning (completion) with no ingoing (outgoing) arcs. The RLP can then be defined as an optimization mathematical programming problem according to Eqs. (1)–(5).

$$\min M_x = \sum_{t=1}^{f_n} u_t^2 \quad (1)$$

subject to

$$f_i \leq f_j - d_j \quad \text{for all precedence relations } (i, j) \quad (2)$$

$$f_1 = 0, \quad d_1 = 0, \quad d_n = 0 \quad (3)$$

$$f_n \leq f_n^p \quad (4)$$

$$\sum_{i \in P_t} u_{it} \leq U \text{ for } t = 1, \dots, f_n \quad (5)$$

The solution procedure aims to find a feasible schedule so that the moment  $M_x$  around the horizontal axis of the resource usage histogram is minimized (Eq. (1)). Eq. (2) assures that precedence relations are not violated and Eq. (3) establishes that the project starts at time moment zero (0). A single resource  $R$  is assigned to each activity, and the usage of that resource is assumed to remain constant throughout the progress of the activity. Resource availability  $U$  is also constant during execution of the project. The sum of resource usage  $u_{it}$  of all ongoing activities  $P_t$  in any time period  $t$  cannot exceed  $U$  (Eq. (5)).

## 3. Methodological framework

### 3.1. Decision flowchart

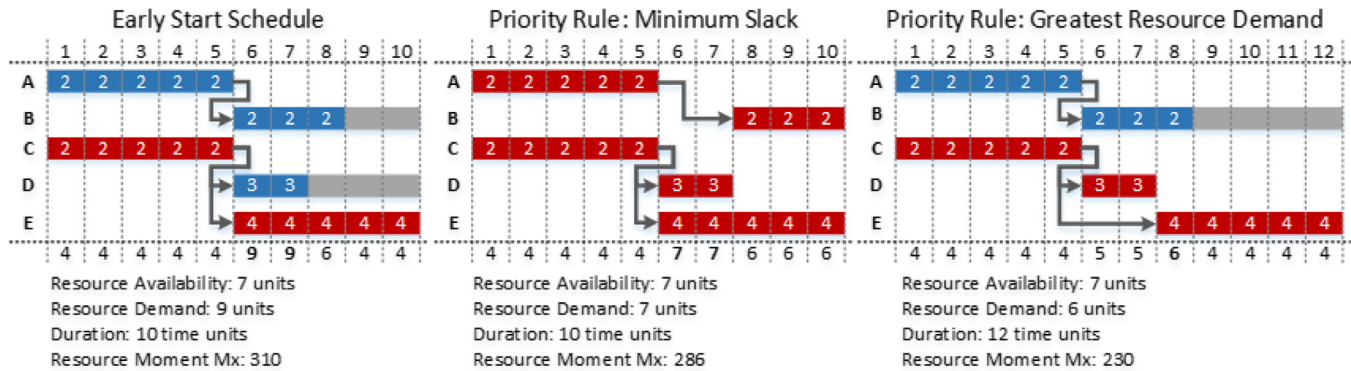
The proposed framework aims to elicit priorities, when priority-rules heuristic procedures are implemented, in order to improve the shape of the resource usage histogram (Fig. 1). For example, MS-Project allows users to set priorities for specific tasks to control how they are leveled in relation to one another. Priorities are specified either as numbers (0–1000) or as linguistic values (lowest to highest) with the “highest” (or 1000) priority corresponding to “Do not level.” Because task priorities have an impact on the schedule, it is possible to affect leveling by altering assigned priorities. The final schedule of activities and CPM calculation is accomplished by MS-Project. However, the lack of coherent procedures for establishing priorities and the intuitional consideration of a variety of activities delay selection rules, can result in the appearance of black boxes during the prioritization phase. Given that solutions to the RLP aim to rank a discrete set of alternatives (activities) under the consideration of a range of decision criteria (priority rules), this paper examines how a variety of MADM models perform in eliciting project activity priorities and providing flexibility during the resource allocation resolution procedure. On the contrary standard heuristic approaches have no prior knowledge about the search space and the specific characteristics of each problem, and so they need to be run several times in order to achieve a near optimal solution.

### 3.2. Activities prioritization criteria-rules

To support PMs during the prioritization process, a variety of priority rules exist that assist activity selection during the heuristic procedure (Table 1). Priority rules determine the order in which activity will be selected during the heuristic procedure, ranking them in a priority list with respect to their dependencies. As such, an activity cannot be selected before its predecessor activity. Selecting priority rules during the project scheduling phase is

**Table 1**  
Major priority rules.

Activity based rules	Network based rules	Critical path based rules	Resource based rules
Shortest Processing Time (SPT)	Most Immediate Successors (MIS)	Earliest Start Time (EST)	Greatest Resource Demand (GRD)
Longest Processing Time (LPT)	Most Total Successors (MTS)	Earliest Finish Time (EFT)	Greatest Cumulative Resource Demand (GCRD)
Random (RND)	Least Non-Related Jobs (LNRJ)	Latest Start Time (LST)	Resource Equivalent Duration (RED).
	Greatest Rank Positional Weight (GRPW)	Latest Finish Time (LFT)	Weighted Resource Utilization and Precedence (WRUP).
	Greatest Rank Positional Weight* (GRPW*)	Minimum Slack (MSLK)	



**Fig. 2.** Educational example illustration.

based on the project’s characteristics; rules are chosen to facilitate the scheduling process depending on the assumptions and the experience of the project scheduling team. According to the literature, priority rules are classified into four general categories based on the type of information needed to construct those priority lists (Demeulemeester & Herroelen, 2002). Namely, these are (a) Activity-based rules, (b) Network-based rules, (c) Critical Path-based rules and (d) Resource-based rules.

Activity-based rules introduce three different priority rules: SPT, or Shortest Processing Time; LPT, or Longest Processing Time; and RND, or Random. The first two are based on activity durations while the last priority rule is used when there is limited knowledge and information about the project. In these cases, RND is used as a benchmark against which the other priority rules may be compared (Hartmann, 1999).

Network-based rules include priority rules based on a project network diagram. There are five different rules based on the precedence relationships of project activities: MIS, or Most Immediate Successors; MTS, or Most Total Successors, LNRJ, or Least Non-Related Jobs; GRPW, or Greatest Rank Positional Weight; and GRPW\*, or Greatest Rank Positional Weight\*. The first two network-based priority rules use direct precedence relations of the activities to create a priority list. The third rule determines the activities that can be scheduled in parallel with the considered activity. The last two priority rules weight the precedence relations with the duration of the corresponding activities. The difference between them is that GRPW considers the durations sum of the activities immediate successors while GRPW\* examines all its successors durations sum (Demeulemeester & Herroelen, 2002)

Critical path-based rules consists of priority rules based on a project’s critical path. These include EST, or Earliest Start Time; EFT, or Earliest Finish Time; LST, or Latest Start Time; LFT, or Latest Finish Time; and MSLK, or Minimum Slack. These rules can be calculated at the start of the scheduling process.

The fourth category includes priority rules based on the resource utilization of project activities. These are GRD, or Greatest Resource Demand; GCRD, or Greatest Cumulative Resource Demand; RED, or Resource Equivalent Duration; and WRUP, or Weighted Resource Utilization and Precedence. WRUP, proposed by

Ulusoy and Özdamar (1995) and Özdamar (1999), is the weighted sum of the number of successors (weight = 0.7) and the average resource utilization for all resource types (weight = 0.3).

The selected heuristic rules are considered to be the most important and most frequently used among seventy three priority rules evaluated in Klein (2000). Any priority rule application leads to a schedule construction by selecting and programming activities prioritization according to the criterion defined by each rule. For example, applying the MSLK rule means that with respect to the precedence relations, the first activity selected to be delayed is the one having the smallest slack. In that manner a list of prioritized activities is the output. Similarly, applying the rule GRD results to a different list, as it selects activity in each loop with respect to the precedence relations and the largest demand for resources. Fig. 2 illustrates an educational example of the above rules implementation to a project with a resource constraint at seven units. Each priority rule implementation provides resolution to the RLP, however the derived schedules are different. The first outcome resolves RLP without provoking overrun to the initial time window, while the second expands the projects’ duration by two time units. Thus the indirect-costs will remain constant while possible penalty clausng risk is avoided with the derived MSLK-based schedule. Nevertheless, with respect to resources allocation GRD-based schedule presents smoother resource allocation profile and it demands one unit less than the resource availability. As a result the project minimizes under-allocation resource usage issues and moreover it can adapt easier to address unpredictable resources shortage during project execution. Moreover the MSLK-based schedule can considered risk-averse since technically none of the activities can be delayed without producing over-allocation issues.

### 3.3. Multi-Attribute Decision Models

These models provide sophisticated tools for handling semi-constructed decision problems where a variety of alternatives are to be evaluated under the consideration of a variety of criteria and they are suitable for selecting and ranking candidate solutions. The performed framework aims to establish activities prior-

ities using MADM models. During the years a variety of MADM approaches have been developed (e.g. Figuera, Greco, & Ehrgott, 2005; Hwang & Yoon 1981). In general, these models can be categorized as compensatory or non-compensatory decision models (Yoon & Hwang 1995). The above distinction is made on the basis of decision models' allowance trade-offs among attributes. In a compensatory decision model inferior performances of an alternative to cost criteria (descend criterion type) can be counterbalanced by additional superiority to benefit criteria (ascend criterion type). On the contrary non-compensatory decision models provide procedures that allows Decision Makers' (DMs) to handle the level of the permitted trade-offs. With respect to the current framework a variety of approaches should be provided to PMs to enable them with the ability to define their decision attitude with respect to the analysis environment. In particular, this paper examines a variety of both compensatory and non-compensatory decision models implementation to resolve RLP through a real case example. Weighted Sum Method (WSM), Analytic Hierarchy Process (AHP) (Saaty, 1980), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) (Brans, Vincke, & Mareschal, 1986) are performed as compensatory models representatives. On the other hand Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS) (Huang & Yoon, 1981), Ordered Weighted Average (OWA) with linguistic quantifiers (Malczewski, 2006) and Hybrid Weighted Average (HWA) (Xu & Da, 2003) stands as non-compensatory approaches.

### 3.3.1. Weighted Sum Method

The WSM is the most commonly used MADM model and may be expressed as follows: given a decision problem with  $m$  alternatives (activities) and  $n$  criteria (priority rules), the score of each alternative is derived from the weighted sum of its standardized performance to the analysis criteria (Eq. (6)). Given that the performances of alternatives to the analysis criteria are estimated on different scales of measurement, standardization – also known as utilization or normalization – allows the user to obtain comparable and consistent scales of preference intensity for comparison. This is achieved through maximum score, vector, or score-range normalization approaches (Hwang, Lai, & Liu, 1993; Yoon & Hwang, 1995) presented in Eqs. (7)–(9), where  $x_{ij}$  is the performance of  $i$ th alternative to the  $j$ th criterion and  $\max/\min_j$  the maximum/minimum observed performance the same criterion ( $i = 1, \dots, m$ ;  $j = 1, \dots, n$ ). Then WSM score is estimated according to Eq. (6).

$$U_i^{wsm} = \sum_{j=1}^n u_{ij} = \sum_{j=1}^n w_j \times r_{ij} \quad (6)$$

$$r_{ij} = \frac{x_{ij}}{x_j^{max}} \quad (7)$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (8)$$

$$r_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}} \quad (9)$$

Where  $u_{ij}$  is the weighted standardized performance of the  $i$ th alternative to the  $j$ th criterion for each  $i = 1, 2, \dots, m$ ,  $r_{ij}$  the standardized performance and  $w_j$  is the criterion weight. The best alternative has the highest overall value (Chang & Yeh, 2001).

### 3.3.2. Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a decision making model developed by Saaty (1980). It is used in decision problems where there are multiple conflicting attributes. Its objective is to

identify the best alternative by ranking all of the proposed solutions when all of the problem criteria are considered simultaneously. The AHP involves the implementation of four steps. The first step is to form hierarchical structures (hierarchies) in order to analyze the decision making problem by its constituent parts. The problem is defined in a hierarchical breakdown structure where the overall analysis objective creates the top level of the hierarchy, followed by levels of sub-objectives and decision criteria. Alternative scenarios are found at the bottom level. Problem criteria are identified by expert judgments based on the characteristics of the decision at hand. The second step is to establish the framework for criterion weight elicitation. This estimation is achieved by forming pairwise comparison matrices (denoted as  $A$ ) that allow decision analysis criteria evaluation by comparing the importance of child nodes with respect to their parent nodes. Pairwise comparisons are made using the linguistic terms of a DMs judgments, which are quantified using a numerical scale of 1 to 9. The value of  $a_{ij}$  of  $A$  represents the importance of criterion  $i$  over criterion  $j$ , with their respective importance elicited from the principal eigenvector of the pairwise comparisons matrix. The third step is to assess the standardized performances of alternatives. This is achieved by forming an alternatives pairwise comparison matrix for every decision criterion. Finally, the alternatives are ranked according to their normalized additive value (Eq. (6)), where  $w_j$  is the overall criterion weight with respect to the analysis's primary objective. Each pairwise comparison matrix in the AHP method undergoes consistency testing, leading to stable results. The AHP method, therefore, models the decision problem as a hierarchical structure that illustrates the problem in a simple way.

### 3.3.3. PROMETHEE

The PROMETHEE is a method used to construct outranking relations between a set of alternatives to find the optimum solution (Brans et al., 1986). This framework is able to combine both information between-criteria, using criterion weights, and information within-criterion, using the amplitude of the deviations between each pair ( $i, k$ ) of alternatives over  $N$  analysis criteria (Eq. (10)) (Brans & Mareschal, 2005). To address the scaling effect, PROMETHEE provides six different types (Usual, U-Shape, V-Shape, Level, V-Shape with indifference and Gaussian) of generalized criteria (preference functions)  $F_j$  to estimate the indifference area of one alternative over the other under the consideration of criterion  $j$  (Eq. (11)).

$$d_j(i, k) = x_{ij} - x_{kj} \quad (10)$$

$$P_j = F_j(d_j(i, k)) \quad (11)$$

The degree of preference of alternative  $i$  over alternative  $k$  for all the analysis criteria is then derived according to Eq. (12). When  $\pi(i, k)$  tends to zero,  $\pi$  indicates a weak preference of  $i$  over  $k$  while values closer to 1 indicate a strong preference (Brans & Mareschal, 2005). To construct the outranking graph, PROMETHEE calculates the outgoing and incoming flow for each node according to Eqs. 13 and 14:

$$\pi(i, k) = \sum_{j=1}^n w_j P_j(i, k) \quad (12)$$

$$\Phi^+(i) = \frac{1}{n-1} \sum_{k=1}^m \pi(i, k) \quad (13)$$

$$\Phi^-(i) = \frac{1}{n-1} \sum_{k=1}^m \pi(k, i) \quad (14)$$

The PROMETHEE method is capable of providing both partial (PROMETHEE I) and full (PROMETHEE II) rankings of alternatives.



Partial ranking is obtained using pairwise comparisons of the outgoing and incoming alternative flows. Through partial ranking, the method can conclude that one alternative is preferred (P) over the other (Eq. (15)), that the two alternatives are indifferent (I) (Eq. (16)) or that the two alternatives are incomparable (R) (Eq. (17)).

$$iPk = \begin{cases} \Phi^+(i) > \Phi^+(k) & \text{and} & \Phi^-(i) < \Phi^-(k) \\ \Phi^+(i) = \Phi^+(k) & \text{and} & \Phi^-(i) < \Phi^-(k) \\ \Phi^+(i) > \Phi^+(k) & \text{and} & \Phi^-(i) = \Phi^-(k) \end{cases} \quad (15)$$

$$ilk = \{ \Phi^+(i) = \Phi^+(k) \text{ and } \Phi^-(i) = \Phi^-(k) \} \quad (16)$$

$$iRk = \begin{cases} \Phi^+(i) > \Phi^+(k) & \text{and} & \Phi^-(i) > \Phi^-(k) \\ \Phi^+(i) < \Phi^+(k) & \text{and} & \Phi^-(i) < \Phi^-(k) \end{cases} \quad (17)$$

PROMETHEE II is able to rank alternatives by estimating each alternative's net flow as the difference between positive and negative flows (Eq. (18)) and then ordering those net flows. In contrast to partial ranking, PROMETHEE II does not provide information regarding possible incomparabilities. The alternative with the highest net flow is considered to be the most preferred alternative (Eq. (19)), while alternatives with equal net flows are considered indifferent (Eq. (20)).

$$\Phi^{net} = \Phi^+(i) - \Phi^-(k) \quad (18)$$

$$iPk = \Phi(i) > \Phi(k) \quad (19)$$

$$ilk = \Phi(i) = \Phi(k) \quad (20)$$

### 3.3.4. TOPSIS

TOPSIS is a method for ranking alternatives according to their distance from the positive and negative ideal solutions. TOPSIS is based on the concept that the most preferable alternative should be one that is as close as possible to the ideal solution and as far as possible from the anti-ideal solution (Yoon & Hwang, 1995). The separation measure is obtained using the Euclidean distance metric. To implement TOPSIS, six steps are required. The performance of alternatives should first be standardized to obtain a comparable scale for measurement. Then, the weighted normalized matrix  $u_{ij}$  can be obtained according to Eq. (21), where  $w_j$  is the weight of criterion  $j$ .

$$u_{ij} = w_j * r_{ij} \quad (21)$$

As long as the weighted normalized matrix is formed, both ideal and anti-ideal solutions can be reached. In particular, the ideal solution is obtained by the alternatives' highest scores in benefit attributes set  $I'$  and the lowest scores in the cost attributes set  $I''$  (Eq. (22)). Correspondingly, the anti-ideal solution is obtained in an analogous way according to Eq. (23) (Yoon & Hwang, 1995).

$$A^+ = \{u_1^+, \dots, u_m^+\} = \{(\max_i u_{ij} | j \in I'), (\min_i u_{ij} | j \in I'')\} \quad (22)$$

$$A^- = \{u_1^-, \dots, u_m^-\} = \{(\min_i u_{ij} | j \in I'), (\max_i u_{ij} | i \in I'')\} \quad (23)$$

Once both ideal and anti-ideal solutions are estimated, the distances of alternative solutions from them are calculated using the Euclidean distance metric (Eqs. 24 and 25). Finally, a ranking of alternatives is obtained through the estimation of relative closeness  $c_i$  using Eq. (26).

$$D_i^+ = \sqrt{\sum_{j=1}^n (u_{ij} - u_i^+)^2} \quad (24)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (u_{ij} - u_i^-)^2} \quad (25)$$

$$c_i = \frac{D_i^-}{(D_i^+ + D_i^-)} \quad (26)$$

### 3.3.5. Ordered Weighted Average (OWA)

The OWA can illustrate the DM's risk attitude by introducing an aggregation operator based on linguistic quantifiers. The method uses two different types of weights: criterion weights are defined by the DM during problem quantification and order weights are obtained by an aggregation operator when considering the DM's degree of optimism. This operator can be a Regular Increasing Monotone (RIM) quantifier, as shown in Eq. (27) (Yager, 1996). Here,  $Q(r)$  is a fuzzy set in interval  $[0,1]$ . By adjusting parameter  $\alpha$ , we can generate different sets of weights regarding a DM's risk attitude. For  $\alpha = 1$ , the risk attitude of the DM is considered indifferent. As  $\alpha$  tends to 0, risk acceptance increases, indicating a very optimistic DM. As  $\alpha$  tends to infinity, the DM's risk acceptance decreases and the DM has a pessimistic attitude (risk aversion) (Malczewski, 2006). Given those considerations, order weights are obtained according to Eq. (28).

$$Q(r) = r^a, a > 0 \quad (27)$$

$$u_j = \left( \sum_{k=1}^j u_k \right)^a - \left( \sum_{k=1}^{j-1} u_k \right)^a \quad (28)$$

As soon as both order  $u_j$  and criterion weights  $w_j$  are calculated, the OWA index of each alternative is calculated by ordering the criteria values in a decreasing order. Criterion weights are then analogously reordered and, finally, the alternative rankings are obtained according to Eq. (29) (Yager, 1988). The OWA can be implemented when considering different risk scenarios by applying different parameters of  $\alpha$ .

$$OWA_i = \sum_{j=1}^n \left( \frac{u_j u_j}{\sum_{j=1}^n u_j u_j} \right) z_{ij} \quad (29)$$

### 3.3.6. Hybrid Weighted Average (HWA)

The HWA is a method similar to OWA, considering both criterion values and criterion importance (Xu & Da, 2003). HWA is an aggregation operator and can be computed using Eq. (30). Here,  $v_j$  is the order weights calculated in the same way as the OWA,  $b_j$  is the biggest value of set  $nw_i a_i$  in ascending order,  $W = (w_1, w_2, \dots, w_n)^T$  denote the criterion weights,  $a_i (i = 1, 2, \dots, n)$ , and  $n$  is a balance parameter. If  $W = (w_1, w_2, \dots, w_n)^T$  tends to  $(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$ , then  $nw_i a_i = a_i$  and the HWA is equal to the OWA (Mirabi, Mianabadi, Zarghami, Sharifi, & Mostert, 2014; Xu & Da, 2003).

$$HWA_{v,w}(a_1, a_2, \dots, a_n) = \sum_{j=1}^n v_j b_j \quad (30)$$

## 4. Framework implementation

To illustrate how MADM methods address RLP, we apply these approaches to a case study regarding the construction of a solar park (100 kW capacity) in the region of Thessaloniki in Northern Greece. The area of the proposed construction location 19,856 m<sup>2</sup> and the actual construction size of the installation is 1600 m<sup>2</sup>.

**Table 2**  
Solar park project WBS and scheduling information.

Level 1 project	Level 2 work packages	Level 3 activities	Relationship with predecessor activity	Relationship with successor activity	Duration
Solar park project	1. Landscaping	1.1 Site preparation and leveling	–	1.2SS	2
		1.2 Fence installation	1.1SS	2.1SS	1
	2. Earthworks	2.1 Base holes excavation	1.2SS	2.2SS + 1d;3.1FS	1
		2.2 Ditches excavation	2.1SS + 1d	2.3SS + 1d;2.4SS+1d;5.1SS	5
		2.3 Concrete implementation	2.2SS + 1d	5.8FS	2
		2.4 Pillar excavations	2.2SS + 1d	5.8FS	2
	3. Bases construction	3.1 Bases construction	2.1FS	3.2FS	1
		3.2 Bases audit	3.1FS	5.6FS	1
	4. Panel delivery	4.1 Panel delivery		5FS	0
		5. Electrical infrastructure	5.1 Grounding	2.2SS	5.2SS + 1d;5.3FF
	5.2 DC cables installation		5.1SS + 1d	5.3SS + 1d;5.4SS;5.9FS	7
	5.3 AC cables installation		5.2SS + 1d;5.1FF	5.7FS	6
	5.4 Security System cables		5.2SS	6.1SF	1
	5.5 Panel & Inverter delivery			5.6FS;5.7FS	1
	5.6 Panel installation		3.2FS;5.5FS	5.9FS	2
	5.7 Inverter installation		5.3FS;5.5FS	5.9SS	1
	5.8 Pillars installation		2.3FS;2.4FS	5.9FS	1
	5.9 Connection of modules		5.2FS;5.7SS;5.6FS;5.8FS	6.1FS;7.1FS	2
	6. Security system		6.1 Surveillance system installation	5.4SF;5.9FS	7.1SS
	7. Testing	7.1 Test runs	5.9FS;6.1SS	8.1FS + 1d	1
	8. Project completion	8.1 Connection with national grid	7.1FS + 1d	8.2FS	1
		8.2 Closing proceedings	8.1FS	9.1FS	2
	9. Project delivery	9.1 Project takeover instruction	8.2FS	–	0

While a feasibility study should be conducted as well – covering the project's technical characteristics and implementation methodology – this paper focuses on the project management framework so those elements are out of scope and consequently not included. In the earliest stages of the project life cycle, the project scope and success criteria have to be established and agreed upon by all stakeholders. During the planning phase of the project, all necessary work required to complete the project, according to its scope, should be identified and documented in the Work Breakdown Structure (WBS) and WBS Dictionary. WBS is a hierarchical decomposition of a project's scope, including all the work that needs to be executed to complete the project, achieve the project's objectives and meet its success criteria. Table 2 presents the WBS, activities, scheduling limitations and duration.

The Early Start Schedule (ESS) constructed by CPM has a total duration of 16 time units, but this is not considered to be a feasible schedule because resource availability was not considered. The ESS is shown in Fig. 3. To solve this issue and construct a feasible project schedule with regards to resource availability ( $R=5$ ), activities must be shifted (delayed) in time. To solve these overallocations, MS-Project offers resource leveling options to its users. Selecting the “Standard” option through the leveling window, MS-Project provides a project schedule with respect to resource constraints. This feasible project schedule, after the leveling procedure, has a minimum duration of 22 time units. During this process, the priority rules that select the order of activities to be scheduled are unknown. Fig. 4 shows the project schedule after MS-Project's standard leveling.

To remedy this issue, MS-Project provides the “Priority Standard” leveling option, which allows users to insert their own weights and thus determine the priority list of project activities for scheduling. The proposed methodology considers priority rules from the literature as problem criteria and project activities as the alternative solutions. The derived results represent the priority of each activity to be used by MS-Project to run the Priority Standard Leveling process that generates the project schedule. The priority rules for solving RLP with resource constraints select the order of activities to be scheduled; this is done during the heuristic procedure from a priority list consisting of the project activities. We

consider nine priority rules as problem sub-criteria within the four general criteria (Problem Criteria).

These priority rules are treated as decision criteria and their weights are defined according to the DM's experience and judgment. Criterion weights are obtained according to AHP by comparing decision criteria in pairs to estimate their relative importance. Table 3 presents the analysis criteria and their weights. After defining all criteria, sub-criteria and their weights as well as alternative solutions, the decision matrix is obtained (Table 4) and the problem becomes quantified. Activities performance to the priority rules reveals that there is no superiority of an activity to all the decision rules thus neither a dominant solution nor a dominant activities ranking exists. If it had an obvious solution would have arisen. Additionally, there is no unique activities performance to any of the examined priority rules which could result to its exclusion from the analysis. To solve the problem, each MADM method is implemented to generate a priority number for each activity. Thus, six different priority lists will be estimated to use them in the priority standard leveling operation in MS-Project. The results of each priority list will be compared and discussed to derive conclusions.

## 5. Results and discussion

To differentiate the results obtained by WSM with respect to AHP, criterion weights have a more generalized form. All the additive utility approaches are implemented by estimation of the weighted normalized sum of the attributes row data. The TOPSIS is performed as the Euclidean distance separations from both the ideal and the anti-ideal solutions summation. For the PROMETHEE II, a preference function is defined for each criterion, which represents the DM's preference of an activity when compared to all other activities. DMs use their judgment and prior experience to define the type of each problem criterion and quantify each criterion's function thresholds in accordance with project data. Table 5 provides information about the type of each criterion and the thresholds used for the solar park project.

The preference index for each alternative can be calculated from the pairwise comparison of all alternatives over all problem criteria. To derive the priority of each activity, the PROMETHEE II

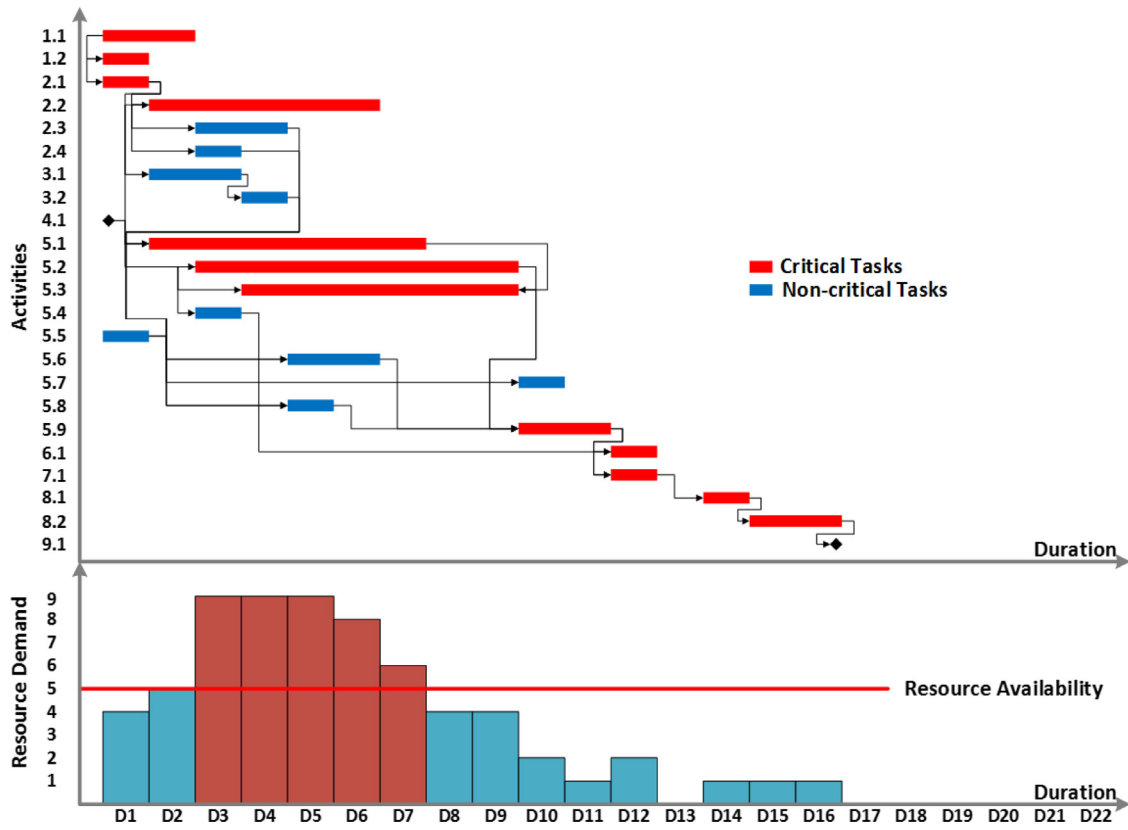


Fig. 3. Early start schedule before leveling.

complete ranking method is used by calculating the net flow for each criterion. The OWA addresses the risk and uncertainty attitudes of DMs in the decision making process. A linguistic quantifier ( $Q(p) = p^a, a > 0$ ) is used to illustrate three different scenarios based on different DM risk attitudes: Extremely Optimistic, Neutral and Extremely Pessimistic. To compute the OWA score for each

scenario, a descending order of activity scores is used to reorder the criterion weights. Similar to the OWA, the HWA considers both the positions of alternatives in the order preference and criterion weights. Order weights are calculated in the same way as the OWA. The balance coefficient  $n$  equals 10. Table 6 provides the results derived from the examined methodological frameworks and, with

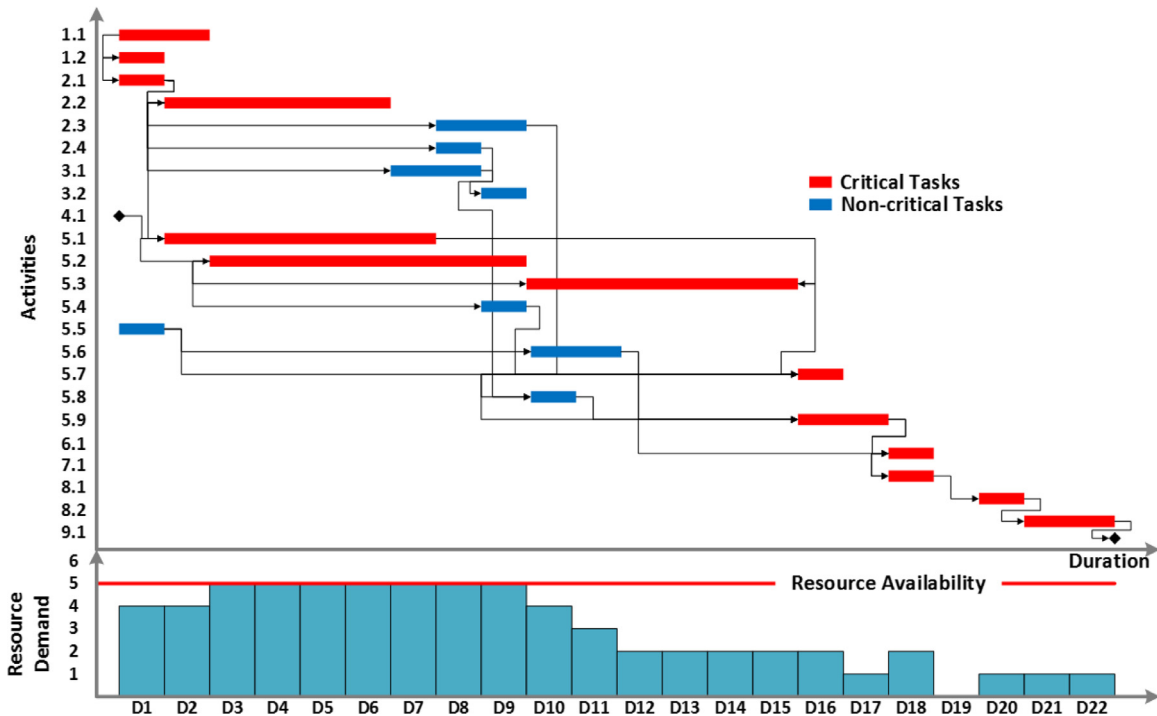


Fig. 4. Early start schedule after leveling using MS-project.

**Table 3**  
Decision criteria weights.

Criterion	Criterion weights	Sub-criterion	Sub-criterion weights	Global weights
Activity based criteria	0,10	LPT	1	0,10
Network based criteria	0,36	MIS	0,16	0,06
		MTS	0,30	0,11
		GRPW	0,54	0,20
		EST	0,25	0,04
Critical path based criteria	0,15	MLSK	0,75	0,11
		GRD	0,17	0,06
		GCUMRD	0,39	0,15
		WRUP	0,44	0,17

respect to the OWA and HWA, three risk scenarios are also examined.

The derived results provide different lists of activities scores. The latter are adjusted accordingly (within the range 0–1000) using the maximum score normalization option (Eq. (7)) to be consistent with the norm of inserting priorities in MS-Project. Maximum score procedure has been chosen given its ability to retain constant the relative order of magnitude between natural and standardized performances. These priorities are inserted into MS-Project and the resource leveling procedure generates one project schedule for every considered approach. To measure resource efficiency, four different resource moments have been calculated (Jaejun, Kyughwan, Namyong, & Yungasang, 2005). The level of resource fluctuations throughout the project life span is calculated using the moment of variance ( $M_x$ ), which is the squared sum of resource usage from a time period (Eq. (31)), and moment of time ( $M_v$ ), which is the squared sum of the difference of resource usages of two sequential time periods (Eq. (32)). To measure the uncertainty of demand and supply for future resources, the  $M_y$  index is used, which is calculated by the sum of the product of the utilized resource times their level of availability (Eq. (33)). The RR index evaluates the rate of resource usage in comparison with the maximum amount of resources utilized (Eq. (34)). The results of these indexes, along with the total project duration for each method were compared with the MS-Project standard leveling results to derive conclusions.

$$M_x = \sum_{i=1}^n (D_i)^2 \tag{31}$$

$$M_v = \sum_{i=1}^n (D_{i-1} - D_i)^2 \tag{32}$$

$$M_y = \sum_{i=1}^n (D_i \times i) \tag{33}$$

$$RR = \sum_{i=1}^n \left( \frac{D_i}{n \times \max D_i} \right) \tag{34}$$

According to Table 7, the standard leveling method in MS-Project performs the best according to the  $M_x$  and  $M_v$  moments. The latter metric indicates that software's default option provides better results in reducing resource usage fluctuations during a project's duration, which is reasonable because these results are achieved in a schedule with a longer duration by 2 time units. However, if the decision is to be made within an uncertain environment for future resources, the pessimistic scenarios of the OWA and HWA provide a higher score because the DM does not accept any risks in these scenarios, instead seeking the minimum uncertainty for his project. Given the short project duration; the resource constraint, which has been set to five resource units; and the dependence of relationships between activities, the RR index is almost identical for all of the above methods. Considering the total project duration, the MS-Project standard leveling method yields a

**Table 4**  
Activities performance to the priority rules under consideration (Decision matrix).

alt	LPT	MIS	MTS	GRPW	EST	MLSK	GRD	GCUMRD	WRUP
	max	max	max	max	min	min	max	max	max
1.1	2	1	19	3	0	0	2	3	0,76
1.2	1	1	18	2	0	0	1	2	0,76
2.1	1	2	17	8	0	0	1	8	1,46
2.2	5	3	16	14	1	0	5	20	2,16
2.3	2	1	7	3	2	4	2	3	0,76
2.4	1	1	7	2	2	5	1	2	0,76
3.1	2	1	8	3	1	3	2	3	0,76
3.2	1	1	7	3	3	3	1	3	0,76
5.1	6	2	7	19	1	0	12	38	1,52
5.2	7	3	6	16	2	0	14	29	2,22
5.3	6	1	6	7	3	0	12	13	0,82
5.4	1	1	4	2	2	10	1	2	0,76
5.5	1	2	6	4	0	6	1	4	1,46
5.6	2	1	7	4	4	3	2	4	0,76
5.7	1	1	5	3	5	0	1	3	0,76
5.8	1	1	6	3	4	4	1	3	0,76
5.9	2	2	4	4	5	0	2	4	1,46
6.1	1	1	3	2	6	0	1	2	0,76
7	1	1	3	2	6	0	1	2	0,76
8.1	1	1	2	3	7	0	1	3	0,76
8.2	2	1	1	2	8	0	2	2	0,76

**Table 5**  
PROMETHEE preference functions to the analysis criteria.

Criterion	Criterion type	Preference function
LPT	U-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } d \leq 2 \\ 1, & \text{if } d > 2 \end{cases}$
MIS	V-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } d \leq 0 \\ d, & \text{if } 0 < d \leq 1 \\ 1, & \text{if } d > 1 \end{cases}$
MTS	V-shape with indifference criterion	$F_j(d) = \begin{cases} 0 & \text{if } d \leq 2 \\ (d-2)/4 & \text{if } 2 < d \leq 6 \\ 1 & \text{if } 6 < d \end{cases}$
GRPW	V-shape with indifference criterion	$F_j(d) = \begin{cases} 0 & \text{if } d \leq 3 \\ (d-3)/4 & \text{if } 3 < d \leq 7 \\ 1 & \text{if } 7 < d \end{cases}$
EST	U-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } -d \leq 2 \\ 1, & \text{if } -d > 2 \end{cases}$
MLSK	U-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } -d \leq 3 \\ 1, & \text{if } -d > 3 \end{cases}$
GRD	U-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } d \leq 3 \\ 1, & \text{if } d > 3 \end{cases}$
GCUMRD	V-shape criterion	$F_j(d) = \begin{cases} 0, & \text{if } d \leq 0 \\ d/7, & \text{if } 0 < d \leq 7 \\ 1, & \text{if } d > 7 \end{cases}$
WRUP	Level criterion	$F_j(d) = \begin{cases} 0 & \text{if } d \leq 0.69 \\ 1/2 & \text{if } 0.69 < d \leq 1.51 \\ 1 & \text{if } 1.51 < d \end{cases}$

project schedule of 22 time units. On the other hand, implementation of MADM methods reduces the total project duration by 2 time units, which amounts to a 10% shorter project duration. Moreover it is noted that of the examined models present the same performance to the analysis metrics (duration and moments). The ex-



**Table 6**  
Estimated activities priorities.

ID	WSM	AHP	TOPSIS	PROMETHEE	OWA			HWA		
					Optimistic	Neutral	Pessimistic	Optimistic	Neutral	Pessimistic
1.1	486	494	325	320	999	494	215	550	388	155
1.2	442	448	304	290	999	448	144	550	350	69
2.1	631	631	404	672	999	631	193	560	524	69
2.2	925	922	704	914	783	922	970	825	816	362
2.3	351	348	164	89	750	348	215	330	291	155
2.4	300	296	143	82	750	296	144	290	243	69
3.1	376	374	185	203	875	374	215	385	301	155
3.2	335	334	172	173	700	334	193	385	291	69
4.1	–	–	–	–	–	–	–	–	–	–
5.1	973	978	1000	998	1000	978	1000	1000	1000	603
5.2	1000	1000	862	1000	1000	1000	859	850	864	517
5.3	595	607	412	588	999	607	859	550	544	345
5.4	223	211	69	0	611	211	0	290	165	0
5.5	425	409	156	280	999	409	193	560	330	69
5.6	365	367	189	189	700	367	285	145	350	1000
5.7	342	347	191	161	999	347	193	550	291	69
5.8	309	308	149	75	600	308	193	330	262	69
5.9	467	466	205	324	999	466	285	560	369	155
6.1	305	310	173	100	998	310	144	550	301	69
7.1	305	310	173	100	998	310	144	550	301	69
8.1	309	315	178	106	998	315	193	550	311	69
8.2	299	307	170	59	998	307	0	550	291	0
9.1	–	–	–	–	–	–	–	–	–	–

**Table 7**  
Comparative analysis results.

Method	Mx	Mv	My	RR	Project duration
MS-Standard	260	28	551	0.63	22
WSM	282	30	521	0.66	20
AHP	282	30	521	0.66	20
PROMETHEE	282	30	521	0.66	20
TOPSIS	282	30	521	0.66	20
OWA-Extremely optimistic	282	30	521	0.66	20
OWA-Neutral	282	30	521	0.66	20
OWA-Extremely pessimistic	280	32	486	0.66	20
HWA-Extremely optimistic	282	30	521	0.66	20
HWA-Neutral	282	30	521	0.66	20
HWA-Extremely pessimistic	280	32	486	0.66	20

tent of the examined project and the limited options for activities scheduling restricts the area of the possible solutions. However, this should not lead to the conclusion that the derived schedules are same. In particular, with the obvious exception of MS-standard result, four different schedules occur since, as it is shown in Table 6, different activities are selected for delay by each decision model. In particular, WSM, AHP, OWA-optimistic and OWA-neutral result to the same outcome, PROMETHEE, TOPSIS, HWA-neutral produce a common schedule, a third solution is obtained by the pessimistic versions of OWA and HWA while HWA-optimistic implementation results to its own Gantt chart. Finally, the results of most MADM methods are noteworthy because in some cases they yield better scores in one or more criteria.

## 6. Conclusions

The framework presented discuss a basic dilemma when examining RLP solutions. In summary, DMs have to decide either on a more efficient resource utilization or on a reduction of project duration. To properly decide, they should also consider the project management environment to address the impacts of these two options. Therefore, DMs must consider the best option to level their projects based on their own thoughts and priorities. For organizations, these impacts are felt in financial terms such as cost overruns due to poor resource utilization or a longer project du-

ration. To address these considerations correctly, more information should also be considered. For example, constant resource utilization provides an opportunity for an organization to have a smoother flow of procurement materials, thereby achieving low purchase prices from its suppliers. However, a reduction in project duration provides an opportunity for organizations to avoid indirect costs and costs due to delays, as well as the ability to approve more projects for execution sooner than before. In conclusion, the choice dilemma between resource utilization and project duration for organizations needs to be resolved through the establishment of the decision attitude that will be followed under resources availability, time and budget constraints and even more a risk-prone attitude.

With respect to criteria and sub-criteria implemented the integration of MADM models to the RLP problem can be considered sufficiently generic since it considers the most common priority rules for shifting activities. However, it can be easily adapted to meet DMs preferences with respect to other priority rules implementation. For example in cases where fewer rules are to be implemented the analysis can be utilized by assigning zero weights to those in excess. Additionally the consideration of different priority rules can be assisted since MS-Project tasks form is capable of supporting a variety of indexes estimations. The in parallel performance of MADM synergy and standard procedures facilitating by MS-Project revealed that the first can be effective. Thus, the solution process gain benefit from the low computational effort demanded by MADM models.

RLP solution approaches are dominated by the performance of heuristic algorithms which have no prior knowledge about the search space and the specific characteristics of each problem, and they need to run several times in order to achieve a near optimal solution. The proposed approach enrich the solution phase with specific knowledge about the problem by incorporating DMs preferential system. The framework is illustrated through a real case study which has been chosen with respect to its extent and thus it can also serve as an educational example easy to follow. Finally, the proposed framework would gain added value by the development of an MS-Project add-in for supporting MADM models performance. Moreover the implementation of stochastic approaches will enforce the frameworks' ability to assist sensitivity analysis.

## References

- Anagnostopoulos, K. P., & Koulinas, G. K. (2010). A simulated annealing hyperheuristic for construction resource levelling. *Construction Management and Economics*, 28(2), 163–175.
- Bandelloni, M., Tucci, M., & Rinaldi, R. (1994). Optimal resource leveling using non-serial dynamic programming. *European Journal of Operational Research*, 78(2), 162–177.
- Brans, J., Vincke, P., & Mareschal, B. (1986). How to select and how to rank projects: The PROMETHEE method. *European Journal of Operational Research*, 24(2), 228–238.
- Brans, J. P., & Mareschal, B. (2005). PROMETHEE methods. In multiple criteria decision analysis: State of the art surveys. In J. Figuera, S. Greco, & M. Ehrgott (Eds.), *International series in operations research management science* (pp. 133–162). Springer.
- Burgess, A. R., & Killebrew, J. B. (1962). Variation in activity level on a cyclic arrow diagram. *Industrial Engineering*, 13(2), 76–83 March–April.
- Chang, Y., & Yeh, C. (2001). Evaluating airline competitiveness using multi-attribute decision making. *Omega*, 29, 405–415.
- Demeulemeester, E., & Herroelen, W. (2002). *Project scheduling: A research handbook*. Leuven, Belgium: Kluwer academic Publishers.
- Figuera, J., Greco, S., & Ehrgott, M. (2005). *Multiple criteria decision analysis: State of the art surveys*. Springer.
- Harris, R. B. (1990). Packing method for resource leveling (PACK). *ASCE Journal of Construction Engineering and Management*, 116(2), 331–350.
- Hartmann, S. (1999). *Project scheduling under limited resources - models, methods and applications*. USA: Springer.
- Hiyassat, M. A. S. (2001). Applying modified minimum moment method to multiple resource leveling. *ASCE Journal of Construction Engineering and Management*, 127(3), 192–198.
- Hwang, C.-L., Lai, Y.-J., & Liu, T.-Y. (1993). A new approach for multiple objective decision making. *Computers & Operations Research*, 20(8), 889–899.
- Huwang, C., & Yoon, K. (1981). In *multi attribute decision making: Methods and applications*. New York, USA: Springer.
- Jaeeun, K., Kyughwan, K., Namyong, J., & Yungasang, Y. (2005). Enhanced resource leveling technique for project scheduling. *Journal of Asian Architecture and Building Engineering*, 4(2), 461–466.
- Klein, R. (2000). *Scheduling of resource constrained projects*. Boston: Kluwer Academic Publishers.
- Koulinas, G. K., & Anagnostopoulos, K. P. (2011). Construction resource allocation and leveling using a threshold accepting-based hyperheuristic algorithm. *Journal of Construction Engineering and Management*, 138(7), 854–863.
- Koulinas, G. K., Kotsikas, L., & Anagnostopoulos, K. P. (2014). A particle swarm optimization based hyper-heuristic algorithm for the classic resource constrained project scheduling problem. *Information Sciences*, 277, 680–693.
- Kyriklidis, C., & Dounias, G. (2016). Evolutionary computation for resource leveling optimization in project management. *Integrated Computer-Aided Engineering*, 23(2), 173–184.
- Kyriklidis, C., Vassiliadis, V., Kirytopoulos, K., & Dounias, G. (2014). Hybrid nature-inspired intelligence for the resource leveling problem. *Operational Research*, 14(3), 387–407.
- Leu, S. S., Yang, C. H., & Huang, J. C. (2000). Resource leveling in construction by genetic algorithm-based optimization and its decision support system application. *Automation in Construction*, 10(1), 27–41.
- Li, H., & Demeulemeester, E. (2016). A genetic algorithm for the robust resource leveling problem. *Journal of Scheduling*, 19(1), 43–60.
- Malczewski, J. (2006). Ordered weighted averaging with fuzzy quantifiers: GIS-based multicriteria evaluation for land-use suitability analysis. *International Journal of Applied Earth Observation and Geoinformation*, 8(4), 270–277.
- Mirabi, M., Mianabadi, H., Zarghami, M., Sharifi, M.-B., & Mostert, E. (2014). Risk-based evaluation of wastewater treatment projects: A case study in Niasar city, Iran. *Resources, Conservation and Recycling*, 93, 168–177.
- Neumann, K., & Zimmermann, J. (2000). Procedures for resource leveling and net present value problems in project scheduling with general temporal and resource constraints. *European Journal of Operational Research*, 127(2), 425–443.
- Özdamar, L. (1999). A genetic algorithm approach to a general category project scheduling problem. *IEEE Transactions on Systems, Man and Cybernetics*, 28(1), 44–59.
- PMBOK Guide, P. M. (2013). *A guide to the project management body of knowledge* (5th ed.). Newtown Square, USA: Project Management Institute, Inc..
- Ponz-Tienda, J., Yepes, V., Pellicer, E., & Moreno-Flores, J. (2013). The resource leveling problem with multiple resources using an adaptive genetic algorithm. *Automation in Construction*, 29, 161–172.
- Saaty, T. (1980). *The analytical hierarchy process*. New York, USA: McGraw-Hill.
- Shtub, A., Bard, J., & Globerson, S. (2005). *Project management: Processes, methodologies and economics engineering, technology and implementation*.
- Son, J., & Skibniewski, M. J. (1999). Multiheuristic approach for resource leveling problem in construction engineering: Hybrid approach. *ASCE Journal of Construction Engineering and Management*, 125(1), 23–31.
- Ulusoy, G., & Özdamar, L. (1995). Survey on the resource-constrained project scheduling problem. *IIE Transactions*, 27(5), 574–586.
- Xu, Z., & Da, Q. (2003). An overview of operators for aggregating information. *International Journal of Intelligent Systems*, 18(9), 953–969.
- Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man and Cybernetics*, 18, 183–190.
- Yager, R. R. (1996). Quantifier guided aggregation using OWA operators. *International Journal of Intelligent Systems*, 11, 49–73.
- Yoon, K. P., & Hwang, C.-L. (1995). *Multiple attribute decision making: An introduction* (pp. 07–104). Sage University Paper Series on Quantitative Applications in the Social Sciences. Sage.