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Research article

Disturbance-observer-based fuzzy model predictive control for nonlinear processes with disturbances and input constraints

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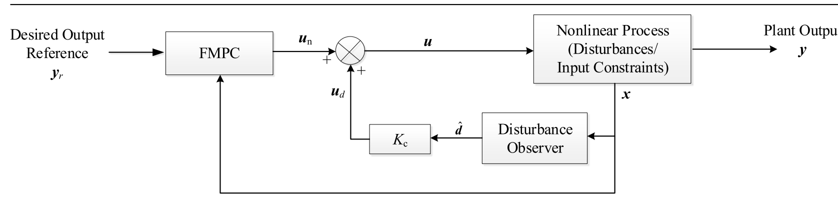
HIGHLIGHTS

- A systematic gap-based approach is proposed to develop an appropriate fuzzy model.
- Baseline fuzzy MPC ensures the asymptotic stability of nominal control system.
- Input constraints are satisfied by both the free and future control inputs in fuzzy MPC.
- Disturbance compensation gain is to remove the disturbance effect at steady state.
- Proposed control scheme suits both the matched and mismatched disturbance cases.

GRAPHICAL ABSTRACT



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ABSTRACT

This paper proposes a disturbance-observer-based fuzzy model predictive control (DOBFMPC) scheme for the nonlinear process subject to disturbances and input constraints. The proposed control scheme is composed of the baseline fuzzy model predictive control (FMPC) law designed on the Takagi–Sugeno fuzzy model and the disturbance compensation law. To build a fuzzy model of appropriate complexity and accuracy for the nonlinear process model, a systematic approach is developed via the gap metric to determine the linearization points. With FMPC, the asymptotic stability is theoretically proved, and the input constraints are satisfied by both the free control variables and the future control inputs in the form of the state feedback law. The disturbance compensation gain is designed such that the influence of the disturbance is removed from the output channels by the composite DOBFMPC law at the steady state. The application to a subcritical boiler–turbine system demonstrate the effectiveness of the proposed control scheme.

1. Introduction

Nowadays, industrial processes are faced with more and more stringent requirement on the safety and efficiency in operation. In order to meet the requirement, plants are often regulated by the conventional proportional–integral–derivative (PID) controller [1–3]. However, the PID controller becomes ineffective when the process behaviours, such as the tight input constraint, the severe nonlinearity over the wide operation range, and disturbances including external disturbances and model uncertainties, are considered [2–6]. Therefore, it is imperative to develop new control schemes to improve the control performance of the nonlinear process.

Model predictive control (MPC) can deal with the input constraint at the synthesis stage and is therefore becoming a prevailing process control method [7–9]. In [7], the dynamic matrix controller

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is developed based on the step response model of the nonlinear process. In [8,9], the generalized predictive controller is proposed for the nonlinear processes based on the controlled auto-regressive integrated moving average model. The simulation results demonstrate the linear MPCs can achieve better control performance compared with the PID controller. However, the linear MPC may no more meet the design specification in the wide operation range due to the nonlinearity of the process. In light of this, the nonlinear MPC controller with the nonlinear numerical optimization at each sampling instant is proposed [10,11]. In [12], the nonlinear MPC algorithm is developed based on the successive online linearization of the original nonlinear model. Although the control performance is improved, the nonlinear MPCs are quite time consuming due to the nonlinear optimization and the online linearization.

To reduce the computational effort, the Takagi–Sugeno (T–S) fuzzy model [13], which blends several local linear models with the membership function to approximate the nonlinear behaviour of the plant, has been widely used in the nonlinear controller design [14,15]. As an universal approximator, the T–S fuzzy model can approximate any smooth function to arbitrary accuracy with enough rules [16,17]. However, with the rule number increasing, the complexity of the fuzzy model and therefore the computational burden of the model-based controller will also increase. Hence, it should be careful to determine the linearization points. However, for the conventional fuzzy modelling, the linearization points are determined either by experience [18], or by investigating the nonlinearity of the plant at only a few common operation points [14,19]. Therefore, there is a lack of the systematic method to determine the linearization points for the T–S fuzzy modelling.

Based on the fuzzy model, the fuzzy MPC scheme is proposed for the nonlinear system [14,15,19]. In spite of the effectiveness of the fuzzy MPC, they alone cannot deal with the disturbance. To improve the robustness of the nonlinear control system to the disturbance, the disturbance-observer-based control (DOBC) is proposed [20–22]. In this scheme, a baseline controller is first independently designed on the nominal plant, and a disturbance compensation law is then added to counteract the adverse effect of the disturbance with the estimate by the disturbance observer. The compensation law is a 'patch' for the baseline controller to achieve the promising robustness, and disappears if there exists no disturbance, thus recovering the nominal control performance [23]. With the idea of DOBC method, the disturbance-observer-based MPC schemes are proposed to deal with the matched and the mismatched disturbance by combining MPC with the disturbance observer [24–27]. Although the schemes are effective, they are designed on the nonlinear model or the online linearization model, which increases the computational complexity of the control algorithms. This hence motives us to integrate the fuzzy MPC as the baseline controller of the DOBC. However, besides the lack of robustness as mentioned above, the existing fuzzy MPCs do not completely consider the input constraint, either. In [14,19], the control formulations only consider the input magnitude constraint, and in [15], the input constraint is imposed on the free control input only, ignoring the future control input in the form of state feedback law.

Inspired by all the analysis above, this paper proposes a disturbance-observer-based fuzzy model predictive control (DOBFMPC) scheme for the nonlinear process subject to the disturbance and input constraint. The T–S fuzzy model is first obtained with the nonlinear model by combining the local linear models at the linearization points. To represent the actual process, a disturbance model is then developed by integrating a disturbance term into the fuzzy model to accommodate the disturbance [28]. The DOBFMPC scheme is composed of the baseline fuzzy model predictive controller (FMPC) and the disturbance compensation law. The effectiveness of the proposed control scheme is demonstrated by the application to a 300 MW subcritical boiler–turbine system (BTS). The main contributions of this work are as follows:

(1) A systematic gap-based method is proposed to determine the linearization points by the nonlinearity analysis along a series of equilibrium points, such that a T–S fuzzy model of appropriate complexity and accuracy is built for the nonlinear process model. These will be presented in Section 2.

(2) The baseline FMPC law is designed such that the closed-loop system is asymptotically stable, while the input constraint is satisfied by both the free control variable and the future control input in the form of the state feedback law. These will be presented in Section 3.1 and Theorem 1.

(3) The other main theoretical contribution of this work is that the design method of the disturbance compensation gain is developed, so that the influence of the disturbance can be removed from the output channels by the composite DOBFMPC law at the steady state. The content will be shown in Section 3.2 and Theorem 2.

The rest of this paper is organized as follows. The Gap-based T–S fuzzy modelling technique for the nonlinear system is shown in Section 2. The DOBMPC scheme is developed in Section 3 and applied to the subcritical BTS in Section 4. Main conclusions are drawn in Section 5.

Notation: For any symmetric matrix A, $A > 0$ means A is positive definite. For any two symmetric matrices A and B, $A > B$ means that A–B is positive definite. I is the identity matrix of appropriate dimensions. A star $(*)$ in a matrix indicates the transposed elements in the symmetric position. The symbol $\otimes$ represents the Kronecker product. The mapping $\boldsymbol{\sigma}$ transforms a matrix into a column vector by transposing each row of the matrix from the first to the last, and $\boldsymbol{\sigma^{-1}}$ is the inverse mapping of $\boldsymbol{\sigma}.\boldsymbol{x}_{k+i|k}$ is the predicted states at time instant $k+i$ based on the current state $\boldsymbol{x}_k$. For normalized membership functions w_i, the antecedent variable z, and matrices X_i

$$w_i := w_i(z_k),\, X_z := \sum_i w_i X_i,\, X_z^{-1} := \left(\sum_i w_i X_i\right)^{-1};$$

$$w_{i+} := w_i(z_{k+1}),\, X_{z+} := \sum_i w_{i+} X_i,\, X_{z+}^{-1} := \left(\sum_i w_{i+} X_i\right)^{-1}.$$

2. Gap-based T–S fuzzy modelling

Consider the nonlinear plant described by

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} = C\boldsymbol{x} \end{cases} \tag{1}$$

where $\boldsymbol{x} \in \Re^n$, $\boldsymbol{u} \in \Re^m$ and $\boldsymbol{y} \in \Re^p$ denotes the state, control input and output vectors, $\boldsymbol{f}(\cdot)$ is a nonlinear vector function, and $C \in \Re^{p\times n}$ is a constant matrix.

The T–S fuzzy model for (1) is usually described as a collection of IF-Then rules of the following form

r_i: if z_k is Γ_i, then

$$\begin{cases} \boldsymbol{x}_{k+1} = A_i\boldsymbol{x}_k + B_i\boldsymbol{u}_k + \boldsymbol{R}_i \\ \boldsymbol{y}_k = C\boldsymbol{x}_k \end{cases},\, i = 1, 2, \ldots, N \tag{2}$$

where r_i denotes the ith rule, N the rule number, z the antecedent variable, Γ_i the ith fuzzy set, A_i and B_i the system matrices, $\boldsymbol{R}_i$ a constant vector and k the kth sampling instant.

In order to develop a satisfactory fuzzy model for the nonlinear system, a systematic gap-based approach is proposed in this paper to determine the linearization points in which the gap metric is applied to investigate the system nonlinearity [29]. Specifically, the proposed approach is as follows.

Step 1: Determine the equilibrium points of the nonlinear model (1) in an interval δ over the operation range $[z_{\min}, z_{\max}]$ and set the gap threshold ε [30].

Step 2: Select the first linearization point z_1 from the equilibrium points such that the maximum gap value in $[z_{\min}, z_1]$ equals to that in $[z_1, z_{\max}]$. If the maximum gap value is less than the gap threshold ε, the procedure ends. Otherwise, choose a point from $[z_{\min}, z_1]$ as $z_{1,LMP}$ between which and z_1 the gap value is ε, and meanwhile choose a point from $[z_1, z_{\max}]$ as $z_{1,RMP}$ between which and z_1 the gap value is also ε. Then, set $z_{1,LRP} = z_1$, $z_{1,LMP} = z_1$, $z_{1,RLP} = z_1$ and $z_{1,RMP} = z_1$ and go to *step* 3.

Step 3: Determine the ith left linearization point $z_{i,L}$ from $\left[z_{\min}, z_{i,\mathrm{LMP}}\right]$ $(i \geq 1)$ such that the gap value between it and $z_{i,LMP}$ is ε. If there exists no such point, go to Step 4; otherwise, choose a point from $\left[z_{\min}, z_{i,L}\right]$ as $z_{i+1,LMP}$ between which and $z_{i,L}$ the gap is ε, then set $i = i+1$, and repeat *Step* 3. If $z_{i+1,LMP}$ does not exist, go to *Step* 4.

Step 4: Determine the jth right linearization point $z_{j,\mathrm{R}}$ from $\left[z_{j,\mathrm{RMP}}, z_{\max}\right]$ $(j \geq 1)$ such that the gap value between it and $z_{j,\mathrm{RMP}}$ is ε. If there exists no such point, the procedure ends; otherwise, choose a point from $\left[z_{j,R}, z_{\max}\right]$ as $z_{j+1,RMP}$ between which and $z_{j,\mathrm{R}}$ the gap is ε, then set $j = j+1$, and repeat *Step* 4. If $z_{j+1,RMP}$ does not exist, the procedure ends.

Remark 1. Compared with [18,19], the gap-based approach is systematic, since it calculates all the gap values between any two linearized models at a series of equilibrium points other than only a few common operation points, and determines the linearization points with the gap threshold other than designates them by the experience. The basic idea behind this approach is to ensure that each local model (2) obtained by the linearization of the nonlinear model at the linearization points can well describe the dynamics of the nonlinear model over the corresponding local operation range.

With the Taylor series of the nonlinear model around the determined linearization points and the discretization technique [31], the local models (2) are obtained. Then the T–S fuzzy model is developed

$$\begin{cases}\boldsymbol{x}_{k+1} = A_z\boldsymbol{x}_k + B_z\boldsymbol{u}_k + \boldsymbol{R}_z \\ \boldsymbol{y}_k = C\boldsymbol{x}_k\end{cases} \tag{3}$$

where $A_z = \sum_{i=1}^{N} w_i A_i$ with the membership functions w_i, $w_i \geq 0$, $\sum_{i=1}^{N} w_i = 1$, and B_z and $\boldsymbol{R}_z$ are defined similarly.

Since the fuzzy model is impossible to be identical with the nonlinear model, and external disturbances, unmodelled dynamics, and parameter variations inevitably exist in practice, a disturbance model is developed as follows by integrating an extra disturbance term $\boldsymbol{d} \in \Re^s$ into the fuzzy model to lump the effect and thus represents the actual nonlinear process

$$\begin{cases}\boldsymbol{x}_{k+1} = A_z\boldsymbol{x}_k + B_z\boldsymbol{u}_k + \boldsymbol{R}_z + G_{dz}\boldsymbol{d}_k \\ \boldsymbol{y}_k = C\boldsymbol{x}_k\end{cases} \tag{4}$$

where $G_{dz} = \sum_{i=1}^{N} w_i * G_{d,i}$ with the disturbance gain matrix of the ith local model $G_{d,i}$.

The input constraints are denoted as

$$\begin{cases}\boldsymbol{u}_{\min} < \boldsymbol{u}_k < \boldsymbol{u}_{\max} \\ \Delta\boldsymbol{u}_{\min} < \Delta\boldsymbol{u}_k < \Delta\boldsymbol{u}_{\max}\end{cases} \tag{5}$$

Remark 2. In (4), if $G_{d,i} = \gamma B_i$, $\gamma \in \Re$, $i = 1, 2,\ldots, N$, i.e., the disturbance $\boldsymbol{d}$ enters the system through the same channels as the control input, $\boldsymbol{d}$ is the matched disturbance; otherwise, $\boldsymbol{d}$ is the mismatched disturbance [32].

To establish the proposed the control scheme, some assumptions are made as follows.

Assumption 1. The disturbance $\boldsymbol{d}$ is slowly time-varying and reaches an constant value at the steady state, i.e., $\lim_{k\to\infty} (\boldsymbol{d}_{k+1} - \boldsymbol{d}_k) = \lim_{k\to\infty} \Delta\boldsymbol{d}_k = 0$.

Assumption 2. The desired output trajectory reference is piecewise constant.

3. Disturbance-observer-based fuzzy model predictive control of nonlinear processes

The proposed DOBFMPC scheme for the nonlinear process subject to disturbances and input constraints is shown in Fig. 1. The baseline FMPC law $\boldsymbol{u}_n$ is designed based on the fuzzy model (3). The compensation law $\boldsymbol{u}_d$ is determined by combining the compensation gain K_c with the disturbance estimate $\hat{\boldsymbol{d}}$. By adding the compensation law to the baseline control law, the composite DOBFMPC law $\boldsymbol{u}$ is obtained, namely

$$\boldsymbol{u} = \boldsymbol{u}_\mathrm{n} + \boldsymbol{u}_\mathrm{d} = \boldsymbol{u}_\mathrm{n} + K_c\hat{\boldsymbol{d}}. \tag{6}$$

3.1. Baseline fuzzy model predictive control law

With the fuzzy model (3), we get

$$\begin{bmatrix} I - A_z & -B_z \\ C & 0 \end{bmatrix}\begin{bmatrix}\boldsymbol{x}_{\mathrm{e},k} \\ \boldsymbol{u}_{\mathrm{e},k}\end{bmatrix} = \begin{bmatrix}\boldsymbol{R}_z \\ \boldsymbol{y}_r\end{bmatrix} \tag{7}$$

where $\boldsymbol{x}_{e,k}$ and $\boldsymbol{u}_{\mathrm{e},k}$ are the equilibrium values of the state and input vectors to be determined; $\boldsymbol{y}_r$ is the desired output trajectory reference.

Then by subtracting (7) from (3), the following result is obtained

$$\begin{cases}\widehat{\boldsymbol{x}}_{k+1} = A_z\widehat{\boldsymbol{x}}_k + B_z\widehat{\boldsymbol{u}}_k \\ \widehat{\boldsymbol{y}}_k = C\widehat{\boldsymbol{x}}_k\end{cases} \tag{8}$$

where $\widehat{\boldsymbol{x}}_k = \boldsymbol{x}_k - \boldsymbol{x}_{\mathrm{e},k}$, $\widehat{\boldsymbol{u}}_k = \boldsymbol{u}_k - \boldsymbol{u}_{\mathrm{e},k}$ and $\widehat{\boldsymbol{y}}_k = \boldsymbol{y}_k - \boldsymbol{y}_r$ are the shifted state, input and output vectors, respectively.

As in [33], the following infinite-horizon objective function is applied

$$J_{0,k}^{\infty} = \sum_{i=0}^{\infty}\left[\widehat{\boldsymbol{x}}_{k+i|k}^T Q\widehat{\boldsymbol{x}}_{k+i|k} + \widehat{\boldsymbol{u}}_{k+i|k}^T R\widehat{\boldsymbol{u}}_{k+i|k}\right] \tag{9}$$

where Q and R are positive define symmetric weighting matrices for the shifted states and inputs, respectively. Then the baseline FMPC law is determined by the following proposed theorem and the detailed proof is given in Appendix A.

Theorem 1. *Consider the fuzzy system (3) under the input constraints (5) in which $\Delta\boldsymbol{u}_{\min} < 0$ and $\Delta\boldsymbol{u}_{\max} > 0$. If there exist a control move $\widehat{\boldsymbol{u}}_{k|k}$, matrices Y_i, G_i, slack matrices $Q_{ij}^l = \left(Q_{ji}^l\right)^T$, symmetric matrices $\tilde{S}_i > 0$, Q_{ii}^l and an upper bound of the infinite-horizon object function γ, $i, j, l = 1, 2, \ldots, N$, such that the following semidefinite programming problem is feasible:*

$$\min_{\widehat{\boldsymbol{u}}_{k|k}, Y_i, G_i, Q_{ij}^l, \tilde{S}_i, \gamma} \gamma \tag{10}$$

s.t (11)–(18)

then a free control input $\boldsymbol{u}_{\mathrm{n},k} = \widehat{\boldsymbol{u}}_{k|k} + \boldsymbol{u}_{\mathrm{e},k}$ and a nonparallel distributed compensation (non-PDC) law $\boldsymbol{u}_{\mathrm{n},k+i|k} = -Y_z G_z^{-1}\widehat{\boldsymbol{x}}_{k+i|k} + \boldsymbol{u}_{\mathrm{e},k}$, $i = 1$ can be determined such that both of them satisfy the input

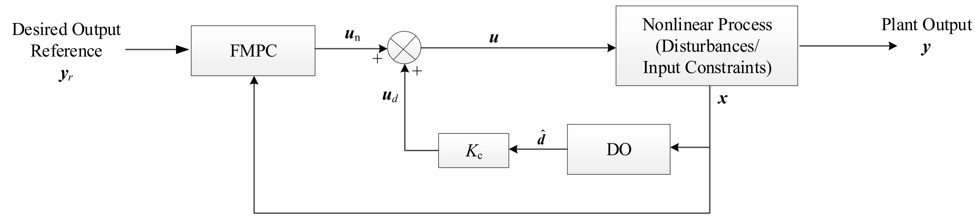

Fig. 1. The DOBFMPC scheme for the nonlinear process.

constraints and the resulting closed-loop system is asymptotically stable.

$$n_l = \begin{bmatrix} 1 & (*) & (*) & (*) \\ A_{l,k}\hat{\boldsymbol{x}}_{k|k} + B_{l,k}\hat{\boldsymbol{u}}_{k|k} & \tilde{S}_l/2 & 0 & 0 \\ Q^{1/2}\hat{\boldsymbol{x}}_{k|k} & 0 & \gamma I & 0 \\ R^{1/2}\hat{\boldsymbol{u}}_{k|k} & 0 & 0 & \gamma I \end{bmatrix} > 0, l = 1, 2, \ldots, N \tag{11}$$

$$r^l_{ii} \geq Q^l_{ii}, i, l = 1, 2, \ldots, N \tag{12}$$

$$r^l_{ij} + r^l_{ji} \geq Q^l_{ij} + Q^l_{ji}, j > i, i, j, l = 1, 2, \ldots, N \tag{13}$$

$$\Psi_l = \begin{bmatrix} Q^l_{11} & Q^l_{12} & \cdots & Q^l_{1N} \\ Q^l_{21} & Q^l_{22} & \cdots & Q^l_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Q^l_{N1} & \cdots & Q^l_{N(N-1)} & Q^l_{NN} \end{bmatrix} > 0, l = 1, 2, \ldots, N \tag{14}$$

$$\boldsymbol{u}_{\min} - \boldsymbol{u}_{e,k} \leq \hat{\boldsymbol{u}}_{k|k} \leq \boldsymbol{u}_{\max} - \boldsymbol{u}_{e,k} \tag{15}$$

$$\Delta\boldsymbol{u}_{\min} + \boldsymbol{u}_{n,k-1} - \boldsymbol{u}_{e,k} \leq \hat{\boldsymbol{u}}_{k|k} \leq \Delta\boldsymbol{u}_{\max} + \boldsymbol{u}_{n,k-1} - \boldsymbol{u}_{e,k} \tag{16}$$

$$q_{1,i} > 0, i = 1, 2, \ldots, N \tag{17}$$

$$q_{2,i} > 0, i = 1, 2, \ldots, N \tag{18}$$

where

$$r^l_{ij} = \begin{bmatrix} G_i + G_i^T - \tilde{S}_i & (*) & (*) & (*) \\ A_iG_j - B_iY_j & \tilde{S}_l & 0 & 0 \\ Q^{1/2}G_i & 0 & \gamma I & 0 \\ R^{1/2}Y_i & 0 & 0 & \gamma I \end{bmatrix}, i, j, l = 1, 2, \ldots, N \tag{19}$$

$$q_{1,i} = \begin{bmatrix} G_i + G_i^T - \tilde{S}_i & (*) \\ Y_i & W_1 \end{bmatrix}, i = 1, 2, \ldots, N$$

$$q_{2,i} = \begin{bmatrix} G_i + G_i^T - \tilde{S}_i & (*) \\ Y_i & W_2 \end{bmatrix}, i = 1, 2, \ldots, N$$

and $\tilde{S}_i = S_i/\gamma$, W_1, *and* W_2 *are diagonal matrices with the components* $W_{1,jj} = \bar{\boldsymbol{u}}_j^2$, *and* $W_{2,jj} = \left(\bar{\boldsymbol{u}}_{d,j}/2\right)^2$, *and* $\bar{\boldsymbol{u}} = \min\left\{\left|\boldsymbol{u}_{\max} - \boldsymbol{u}_{e,k}\right|, \left|\boldsymbol{u}_{\min} - \boldsymbol{u}_{e,k}\right|\right\}$, $\bar{\boldsymbol{u}}_d = \min\left\{\left|\Delta\boldsymbol{u}_{\max}\right|, \left|\Delta\boldsymbol{u}_{\min}\right|\right\}$, $i = 1, 2, \ldots, p, j = 1, 2, \ldots, m$.

Remark 3. In Theorem 1, the infinite-horizon control inputs are divided into the free control variables and the feedback control law to improve the system performance [14,33–35]. Since the computational effort increases with the number of the free control inputs, only the first control input is set as the free control variable [15]. In addition, to relax the conservatism of the stability condition, the feedback law adopts the non-PDC law and the nonquadratic Lyapunov function is applied [36].

Remark 4. The roles of the linear matrix inequalities (LMIs) (11)–(18) are as follows: (11) ensures γ is the upper bound of the infinite-horizon objective function (9); (12)–(13) together with (14) guarantee the asymptotic stability of the closed-loop system; (15) and (17) ensure the satisfaction of the magnitude constraint for the free control variables and the future control inputs in non-PDC law, respectively; (16) and (18) deal with the incremental constraint of the free control variables and the future control inputs, respectively.

3.2. Disturbance-observer-based fuzzy model predictive control law

Since the disturbance $\boldsymbol{d}_k$ is unknown, a disturbance observer is designed [37]

$$\begin{cases} \hat{\boldsymbol{d}}_k = \boldsymbol{\tau}_k + L\boldsymbol{x}_k \\ \boldsymbol{\tau}_{k+1} = \boldsymbol{\tau}_k - L\left[\left(A_z - I\right)\boldsymbol{x}_k + B_z\boldsymbol{u}_k + \boldsymbol{R}_z + G_{dz}\hat{\boldsymbol{d}}_k\right] \end{cases} \tag{20}$$

where the symbol "$\wedge$" represents the estimation, $\boldsymbol{\tau} \in \Re^s$ is the internal state vector and L is the observer gain matrix to be determined. Then the dynamics of the disturbance estimation error is

$$\tilde{\boldsymbol{d}}_{k+1} = \left(I - LG_{dz}\right)\tilde{\boldsymbol{d}}_k + \Delta\boldsymbol{d}_k \tag{21}$$

where the symbol "$\sim$" denotes the estimation error.

Suppose Assumption 1 is satisfied, and L is designed to be

$$L = \left(I - \Lambda\right)G^+_{dz}$$

where $G^+_{dz} = \left(G^T_{dz}G_{dz}\right)^{-1}G^T_{dz}$ is the left pseudo-inverse of G_dz and $\Lambda = \text{diag}\{\lambda_1,\ldots,\lambda_s\}$ with $|\Lambda_j| < 1, j = 1,\ldots,s$, the error system (21) is then asymptotically stable, which indicates the disturbance estimate $\hat{\boldsymbol{d}}$ tracks the disturbance $\boldsymbol{d}$ asymptotically [32].

Next, we develop the design method for the disturbance compensation gain in the following Theorem 2, and thus the DOBFMPC law (6) can be determined. The detailed proof of Theorem 2 is given in Appendix B.

Theorem 2. *Suppose the asymptotic tracking of the disturbance estimate is guaranteed, and Assumption 2 is satisfied. If there exists a vector* $\boldsymbol{\alpha} \in \Re^{m*s}$ *satisfying the following linear equations*

$$\left(B_i \otimes I\right)\boldsymbol{\alpha} = \boldsymbol{\sigma}\left(-G_{d,i}\right), i = 1, 2, \ldots, N. \tag{22}$$

Then considering the fuzzy system (4) with the composite DOBFMPC law (6), if the disturbance compensation gain

$$K_c = \boldsymbol{\sigma}^{-1}\left(\boldsymbol{\alpha}\right), \tag{23}$$

the effect of the disturbance can be removed from the output channels at the steady state.

Remark 5. The design method of the disturbance observer gain in Theorem 2 is applicable to both the matched and mismatched disturbance cases [23]. In particular, in the matched disturbance case, it can be directly obtained that $K_c = -\gamma \mathrm{I}$.

4. Simulation example

In this section, the proposed DOBFMPC scheme is applied to a 300 MW subcritical BTS. The dynamics of the BTS is described as a fourth-order nonlinear model [38]:

$$\begin{cases} \dot{x}_1 = -0.01x_1 + 2.623x_2u_1 \\ \dot{x}_2 = 0.0428(0.0138{x_3}^2 + 0.15x_3 + 4.17)\sqrt{x_3 - x_2} \\ \quad -0.0451x_2u_1 \\ \dot{x}_3 = \dfrac{20237.25x_4 - 1669.8x_1 - 318.28}{175.39(1.06x_3 - 5.16)(-1.434{x_3}^2 + 13.578x_3 + 1547)} \\ \quad - \dfrac{73.8(0.0138{x_3}^2 + 0.15x_3 + 4.17)\sqrt{x_3 - x_2}}{175.39(1.06x_3 - 5.16)} \\ \dot{x}_4 = -0.005x_4 + 0.005u_2 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \tag{24}$$

where, the state variables x_1, x_2, x_3 and x_4 denote the electric power P (MW), throttle steam pressure P_{T} (MPa), drum steam pressure P_{D} (MPa) and the flow rate of pulverized coal entering the furnace D_{cf} (kg/s), respectively; the control inputs u_1 and u_2 are the turbine throttle valve opening u_{T} and the flow rate of the feed coal u_B (kg/s), respectively; the outputs y_1 and y_2 are the electric power P (MW) and throttle pressure P_{T} (MPa), respectively.

Because of the physical limits of the actuator, there exist constraints on both the magnitude and change rate of the control inputs. According to the actuator specification and the operation demand, the input constraints are as follows:

$$\begin{cases} 0 \le u_1 \le 1 \\ 0 \le u_2 \le 45 \\ -0.015 \le \dot{u}_1 \le 0.015 \\ -2 \le \dot{u}_2 \le 2 \end{cases} \tag{25}$$

4.1. T–S fuzzy model of the subcritical BTS

In order to develop the T–S fuzzy model of the BTS, the proposed gap-based approach is applied to determine the linearization point, in which P is chosen to be the premise variable and meanwhile represent the equilibrium point, since all the other state variables depend on it. The operation range under consideration is confined to [150 MW, 300 MW], and the interval δ is set as 5 MW to fully investigate the nonlinearity of the BTS dynamics. The gap threshold ε is chosen to be 0.06, and three linearization points are determined, namely [170 MW, 225 MW, 280 MW], as shown in Fig. 2. The distance between adjacent linearization points are equal, since the gap values increase nearly linearly with the distance between equilibrium points. With the sampling time 1 s, the local discrete models (2) at the linearization points are obtained, and then the T–S fuzzy model (3) is built with the membership functions shown in Fig. 3.

Fig. 4 shows the simulation results of the T–S fuzzy model and the nonlinear model, the plant data and the plant set points for two consecutive days. It is noted that the simulation result of the nonlinear model agrees well with the plant data, and the behaviours of the fuzzy model and the nonlinear model are nearly identical over the wide operation range. Therefore, the nonlinear model well describes the dynamics of the BTS, and the fuzzy model has high approximation accuracy for the nonlinear model. It is then concluded that the fuzzy model can capture the dynamics of the BTS in a wide operation range, which is verified by subgraphs (a.3) and (b.3). Besides, it is obvious that the PID controller adopted in the subcritical power plant has a very poor performance, since the overshoot for the throttle pressure reaches up to 1.8 MPa as shown in the subgraph (b.4). Therefore, it is necessary to propose a new control scheme to improve the control performance of the BTS.

4.2. Dobfmpc of the subcritical bts

In this subsection, the DOBFMPC scheme is applied to the subcritical BTS. The sampling time is 1 s, and in the input constraints (5), the bounds $\boldsymbol{u}_{\min} = [0;0]$, $\boldsymbol{u}_{\max} = [1;45]$, $\boldsymbol{u}_{\min} = [-0.015;\ -2]$, and $\boldsymbol{u}_{\max} = [0.015;2]$ according to (25). The weighting matrices $Q = \mathrm{diag}\,(1, 50, 50, 1)$ and $R = \mathrm{diag}\,(1, 1)$. Besides, the disturbance gain matrices $G_{d,i} = B_i$, $i = 1, \ldots, N$, i.e., the matched disturbance case is considered, and $\Lambda = \mathrm{diag}\{0.5, 0.5\}$, $\boldsymbol{z}_0 = [0;0]$.

In order to test the control performance of the DOBFMPC over a wide operation range, the first case is designed in which the BTS is under a ramp-type load change. The case assumes from $t = 200$ s to $t = 750$ s, the set points of the electric power and the throttle pressure change linearly from (225 MW, 14.95 MPa) to (280 MW, 17.47 MPa) and then from $t = 1500$ s to $t = 2600$ s, change linearly to (170 MW, 11.99 MPa). The load change rate is set to 0.1 MW/s, namely 2% of the nominal load per minute, according to [39] and the practice. The proposed control strategy is compared with another two controllers:

(1) the regular PID controller adopted in the power plant. The controller parameters are tuned at the (225 MW, 14.95 MPa) point, i.e., the middle point of the variation range, to expect a good tracking performance.

(2) the nonlinear MPC (NMPC) based on the same fuzzy model as the DOBFMPC scheme [40]. The prediction and control horizon lengths are set as 14 and 3 respectively by trial and error.

The simulation results are depicted in Fig. 5. The figure shows that the DOBFMPC scheme tracks the set points of the power and the throttle pressure near perfectly over the wide operation range. Based on the same fuzzy model, the NMPC scheme also has a good performance, which is just a little worse than that of DOBFMPC scheme due to the small overshoot of the throttle pressure. However, the stability of the closed-loop system with NMPC is easily affected by the lengths of the prediction and control horizons; if they were not well tuned, the response would oscillate or even diverge. On the other hand, the performance of the PID controller over the wide operation range is the worst among the three schemes, because the PID controller tuned at a fixed operation point does not work effectively any more when the load changes, and unlike the MPC algorithm, the input constraints cannot be handled at the control calculation stage, either. Figs. 6 and 7 display the control inputs and their increments, respectively. It is evident that the input constraints are respected for all controllers. This holds for all the following cases in which the incremental inputs are no more displayed for simplicity.

Fig. 8 demonstrates the tracking performance of the DOBFMPC for the set points in Fig. 4. The output trajectories are almost identical with the set values, indicating that the DOBFMPC much outperforms the PID controller.

On the other hand, in order to further illustrate the effect of the proposed systematic fuzzy modelling method, two other disturbance-observer-based MPC schemes are proposed to compare with the DOBFMPC. One (DOBLMPC) is built on the second local linear model only instead of the fuzzy model, and the other (DOBCFMPC) applies the conventional fuzzy modelling technique by designating the equilibrium points [160 MW 210 MW 260 MW] as the linearization points. It is supposed that at $t = 50$ s the set points change instantly from (240 MW, 15.68 MPa) to (200

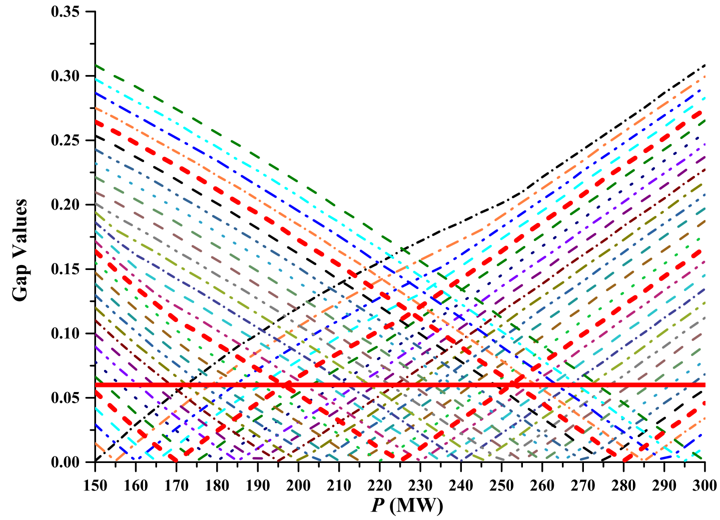

Fig. 2. The gap values between linearized models at equilibrium points.

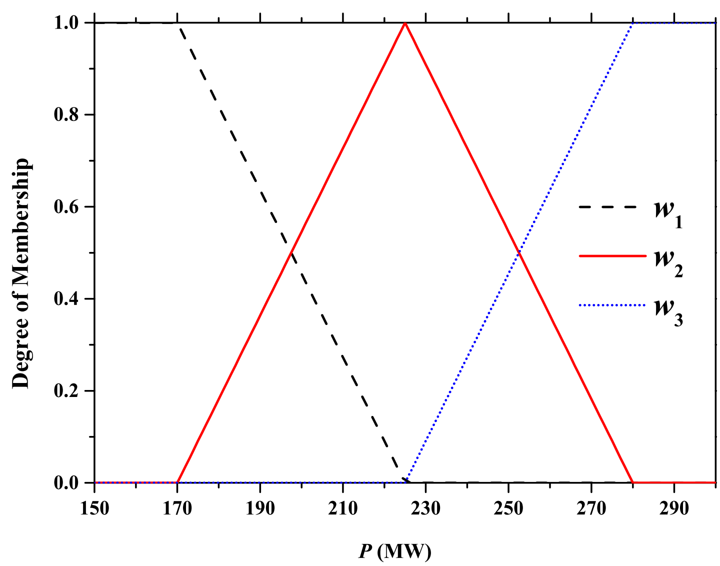

Fig. 3. Membership functions of the fuzzy sets.

MW, 13.67 MPa) intimating the load rejection of the BTS. The simulation results of the three schemes are represented in Fig. 9. It can be seen that the DOBFMPC scheme has the best performance, which can quickly track the set points of the power output and the throttle pressure. The DOBCFMPC has a better performance than the DOBLMPC, since both the overshoot and the settling time of the power output of the DOBCFMPC are smaller than those of the DOBLMPC, and the performances of the throttle pressure are almost identical. Therefore, it is concluded that the proposed control scheme can achieve an improved performance with the systematic gap-based fuzzy modelling method.

Next, the third case is constructed to demonstrate the attenuation ability of the DOBFMPC scheme for external disturbances by comparing with the FMPC. In this case, it is supposed that the plant is working at (200 MW, 13.67 MPa) point, and at $t = 50$ s, two unknown constant disturbances, 0.1 and −5 kg/s, are imposed on u_T and u_B, respectively. The performances of the two controllers are shown in Fig. 10. It is evident that the DOBFMPC can effectively deal with the disturbance and recover the prescribed operation point without offset, while with the FMPC, the system settles at a new operation point. The difference of the control performances between the DOBFMPC and the FMPC is in that the DOBFMPC counteracts the effect of the disturbance on the outputs with the disturbance compensation law. Fig. 11 displays the estimates of the disturbance term on the left. The estimates are not exactly equal to the disturbance at the steady state, since there exists the approximation error between the fuzzy model and the nonlinear model, which is also included into the disturbance term. The estimates of the error are shown on the right of Fig. 11. It can be seen the errors are nearly zero, which again demonstrates the high approximation accuracy of the fuzzy model.

The fourth case is to investigate the robustness of the DOBFMPC against the model mismatch caused by the system parameter perturbation. This case assumes the plant is working at (200 MW, 13.67 MPa) point, and a −5% variation of the coal calorific value (about a decrease of 1 MJ/kg), which leads to the change of the dynamics of the nonlinear model since some of its parameters depend on the coal calorific value [38]. Like the third case, the DOBFMPC is compared with the FMPC. The result shown in Fig. 12 demonstrates that the DOBFMPC is robust to the variation of the coal calorific

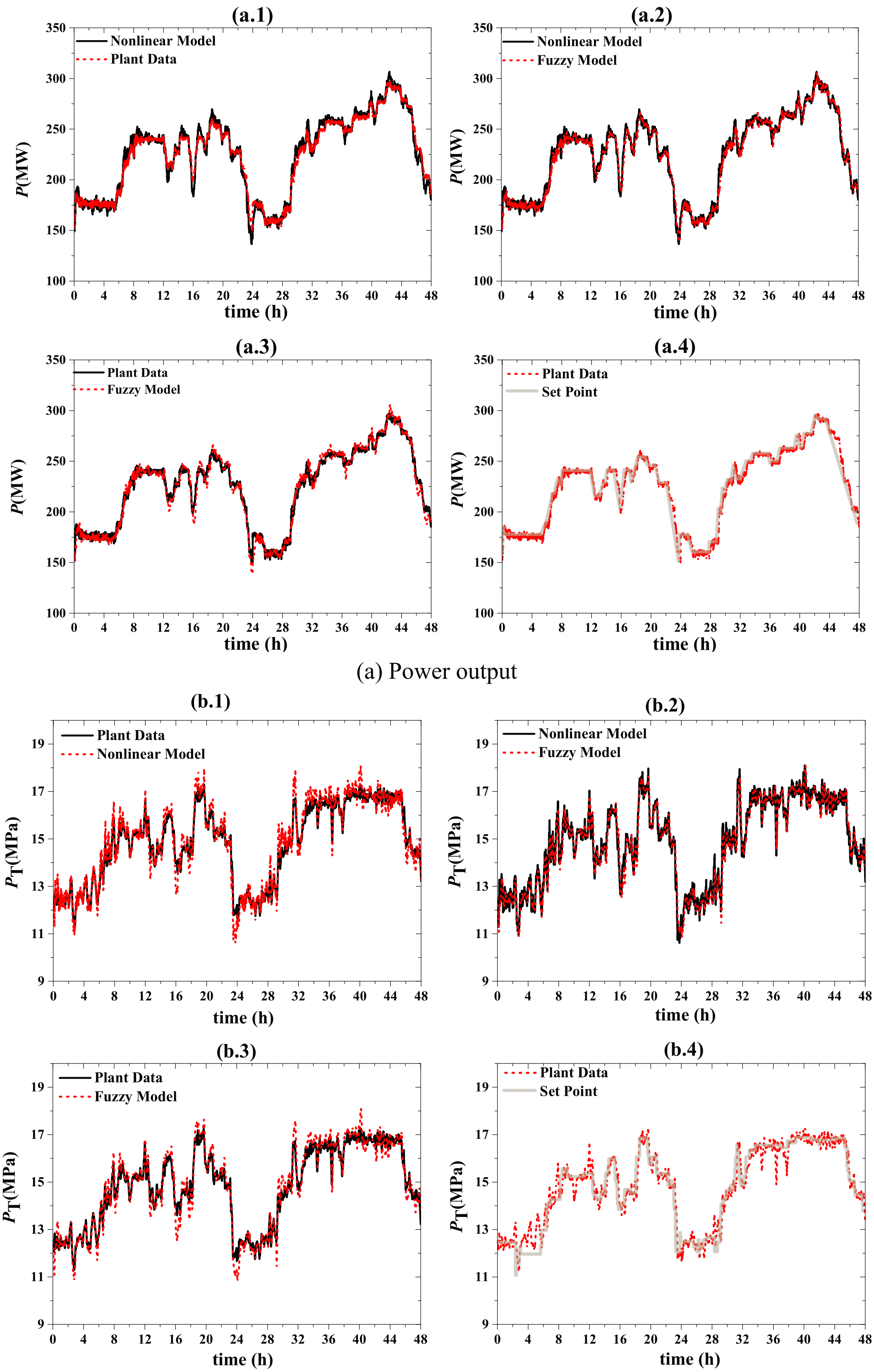

Fig. 4. The simulation results of the fuzzy model, the nonlinear model, plant data and the set points.

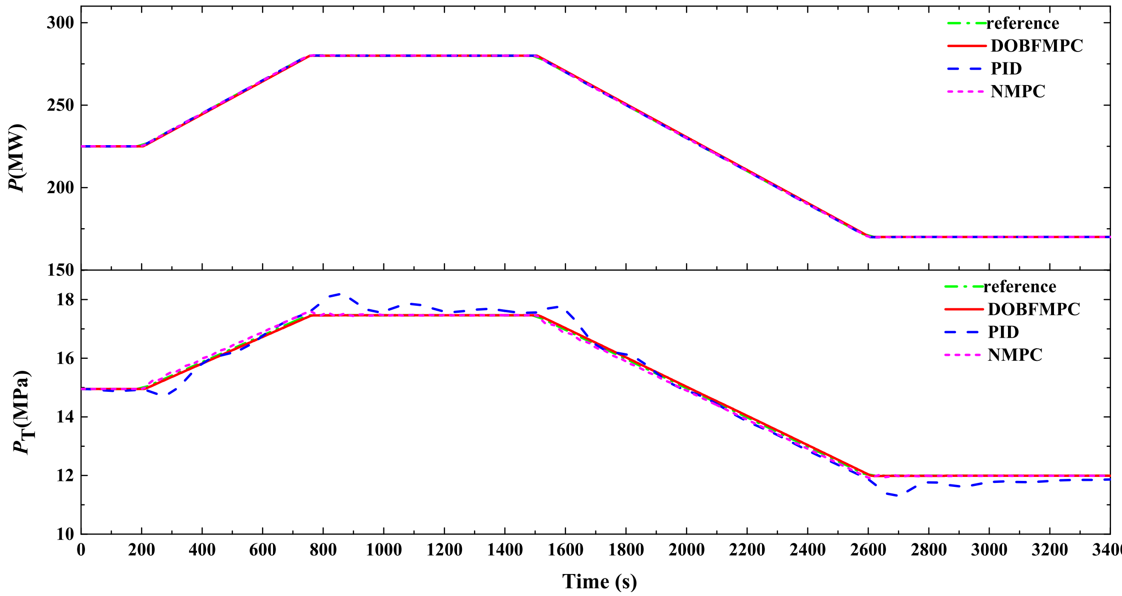

Fig. 5. Outputs of the BTS for a ramp-type load change: 225–280–170 MW.

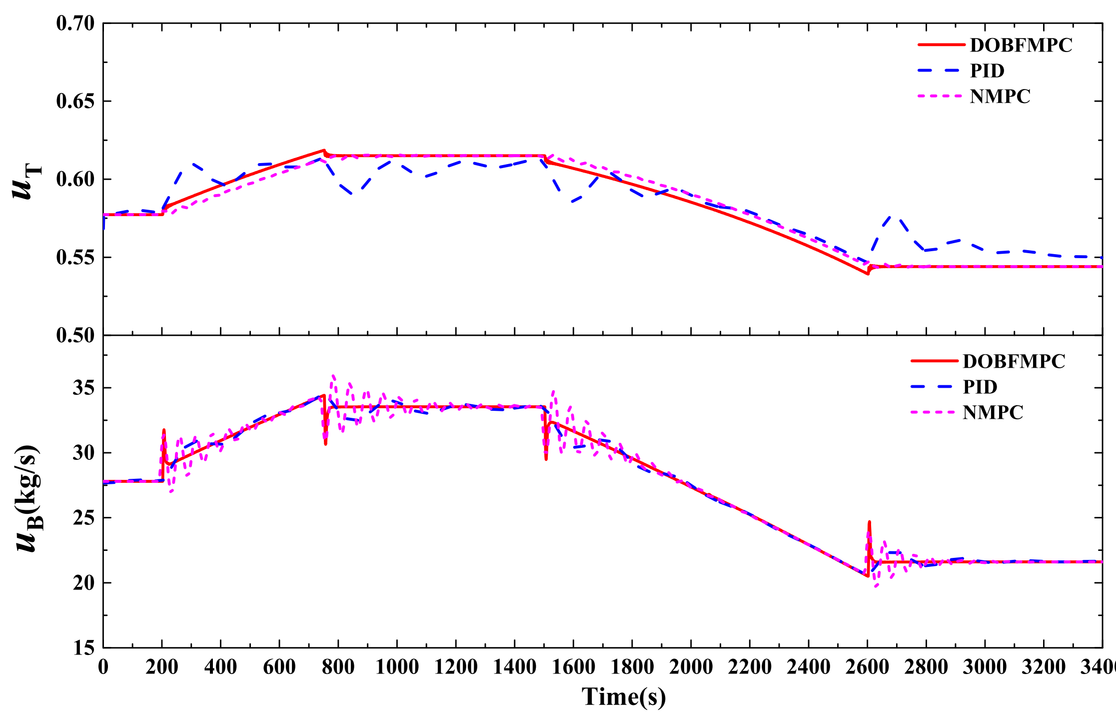

Fig. 6. Inputs of the BTS for a ramp-type load change: 225–280–170 MW.

value, while with the FMPC, both the output power and throttle pressure deviate from the previous operation point. Hence, the critical role of the compensation law on the system robustness is well demonstrated by the comparison of the simulation results of the DOBFMPC and the FMPC in this case.

It should be noted that all the cases above are carried out with the matched disturbance. In order to validate that the proposed control scheme is also applicable to the case of the mismatched disturbance, this final case is designed in which the disturbance gain matrices displayed in Appendix C are adopted. In this case $s = 4$ and $m = 2$, hence $\boldsymbol{d}$ is the mismatched disturbance. Besides, with the system matrices of the local models in Appendix C, it is verified that $G_{d,i}$ satisfies the condition (22), which ensures the existence of the disturbance compensation gain matrix. In this case, it is also supposed the plant is working at (200 MW, 13.67 MPa) point, and at $t = 50$ s, two unknown constant disturbances, 0.1 and −5 kg/s, are imposed on u_T and u_B, respectively. The performances of the BTS with the DOBFMPC and the FMPC are shown in Fig. 13. It is evident the plant is still well controlled by the DOBFMPC scheme under the mismatched disturbance, and the importance of the compensation law for the disturbance rejection is demonstrated as well.

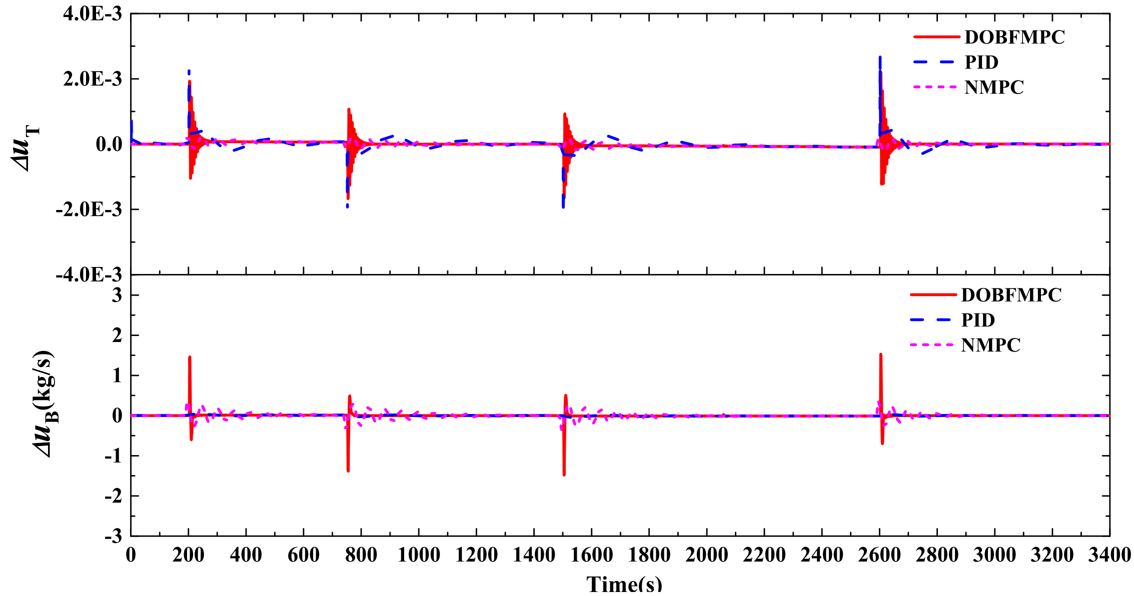

Fig. 7. Incremental inputs of the BTS for a ramp-type load change: 225–280–170 MW.

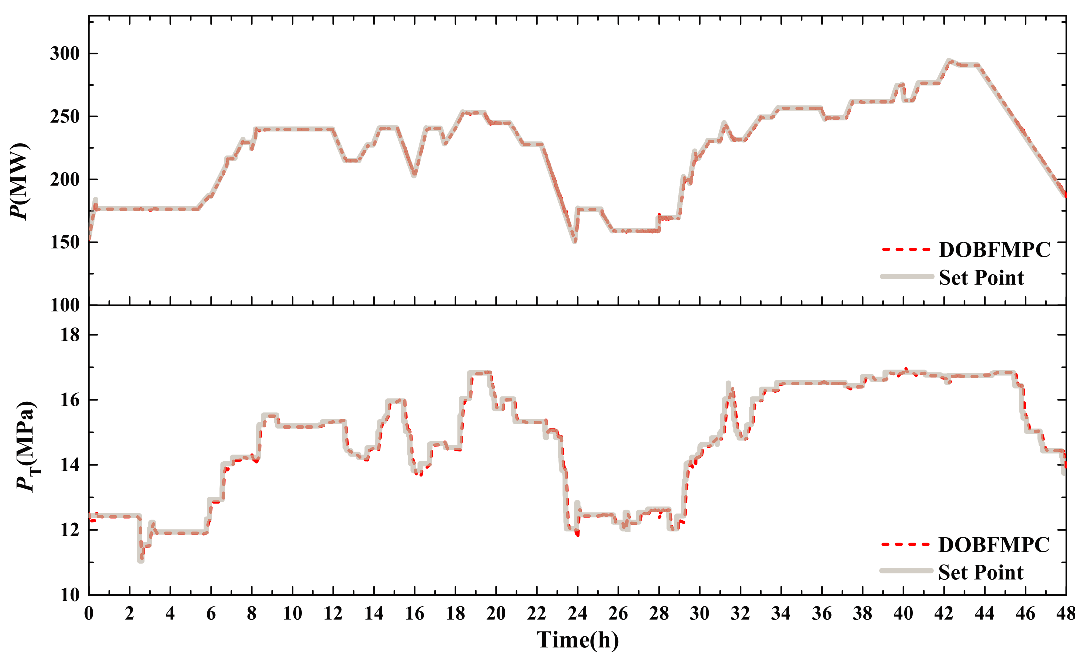

Fig. 8. Tracking performance of DOBFMPC for the plant set points of the BTS.

5. Conclusions

This paper presents a disturbance-observer-based fuzzy model predictive control (DOBFMPC) scheme for nonlinear processes with disturbances and input constraints. The T–S fuzzy model of appropriate approximation accuracy and complexity is developed with the proposed systematic gap-based method. The disturbance model then represents the nonlinear process by integrating a disturbance term into the fuzzy model to lump the disturbance effect. The baseline FMPC is synthesized on the fuzzy model such that the asymptotic stability is guaranteed, and the input constraints are satisfied by both the free control variables and the future control inputs in the form of non-PDC law. With the disturbance estimate by the disturbance observer, the disturbance compensation law ensures that the effect of the disturbance on the outputs of the closed-loop system is removed by the composite DOBFMPC law at the steady state. Besides, the proposed control scheme is a general case and suitable to deal with both the matched and mismatched disturbance. Case studies carried out on a 300 MW subcritical BTS fully evaluate the proposed control scheme.

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Conflict of interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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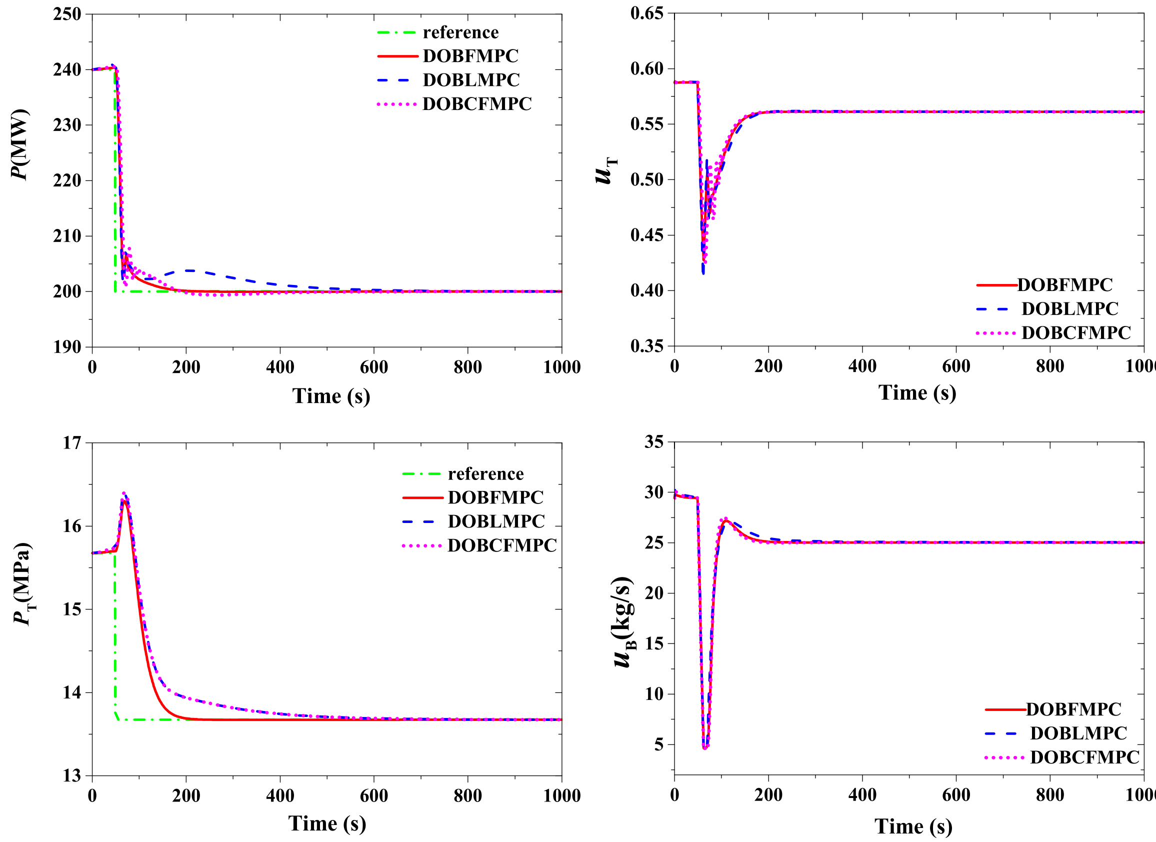

Fig. 9. Performances of the BTS for a step load change: 240–200 MW: (left) outputs; (right) inputs.

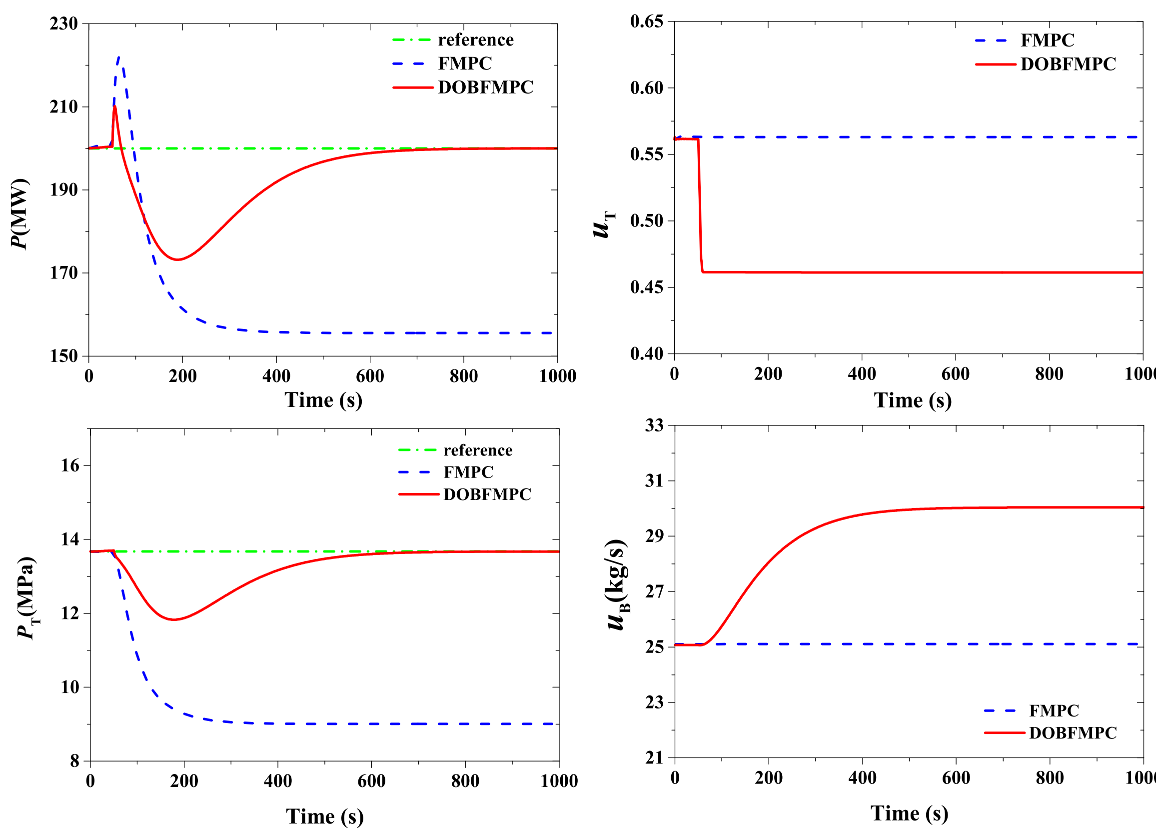

Fig. 10. Performance of the BTS with external disturbances: (left) outputs; (right) inputs.

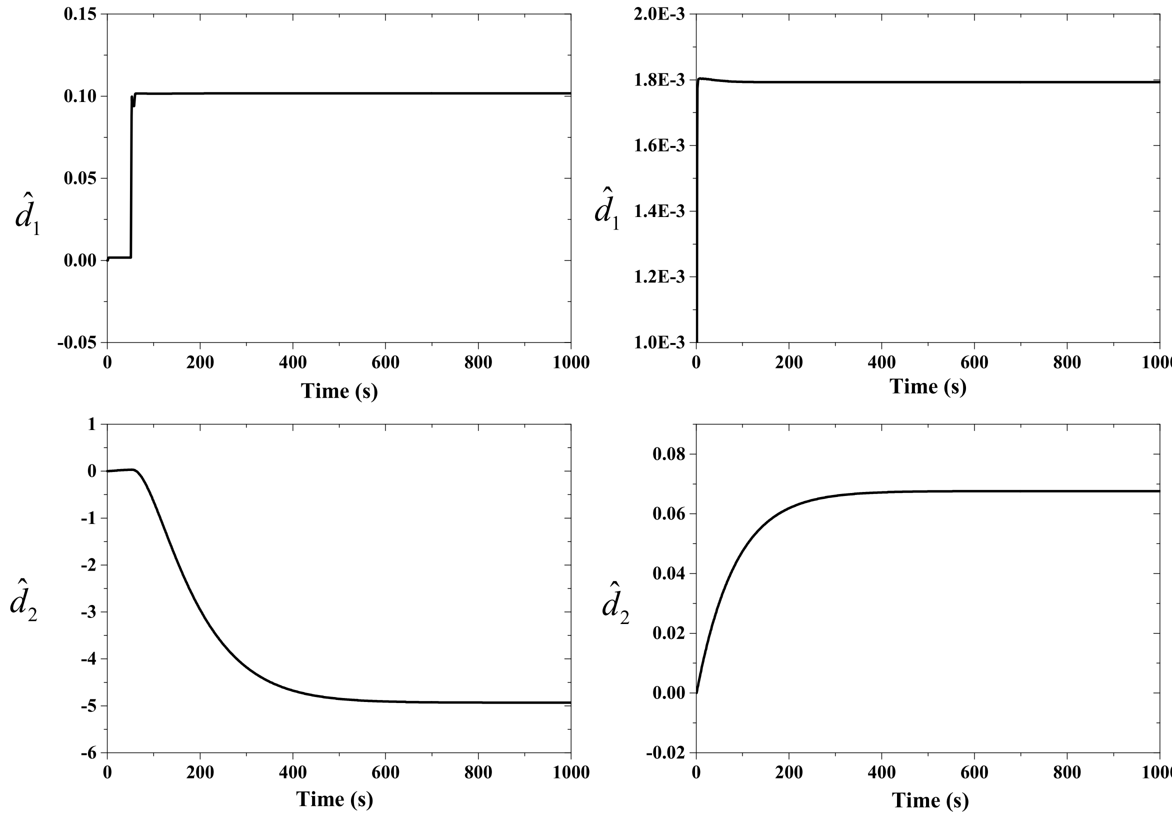

Fig. 11. Estimates of the disturbance term: (left) with external disturbances; (right) without external disturbances.

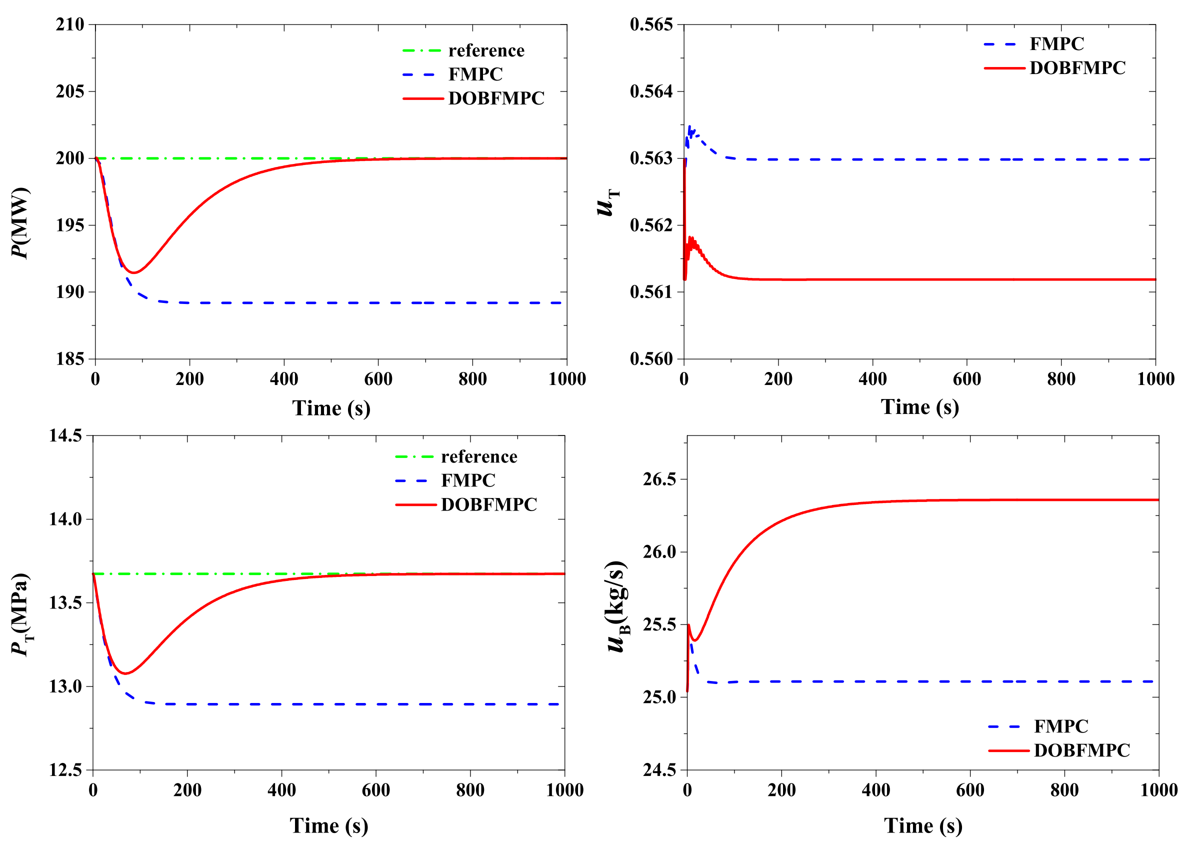

Fig. 12. Performance of the BTS with the model mismatch: (left) outputs; (right) inputs.

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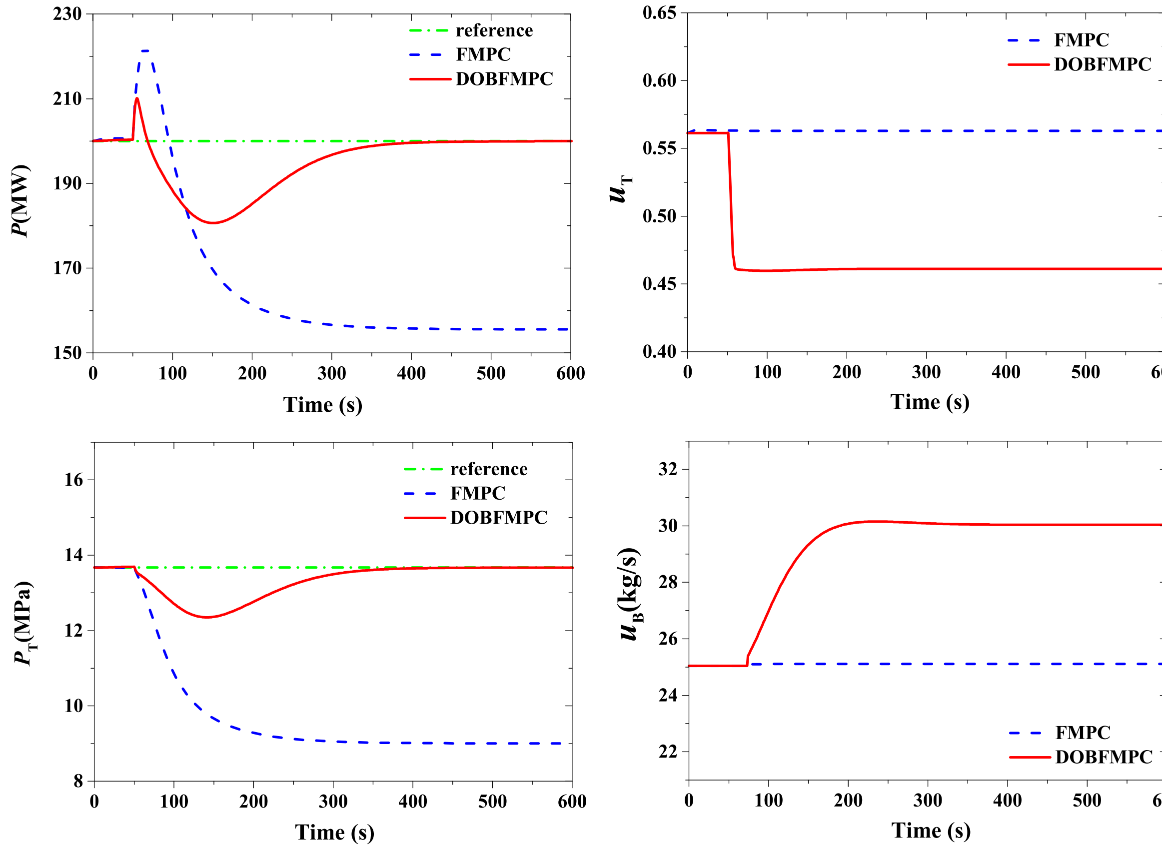

Fig. 13. Performance of the BTS with external disturbances in the mismatched case: (left) outputs; (right) inputs.

Appendix A

The proof of Theorem 1 consists of three parts. The first part proves the condition (11) ensures γ is an upper bound of the infinite-horizon objective function, thus minimizing $J^{\infty}_{0,k}$ is equivalent to the minimization of γ subject to (11). The second part proves conditions (12)–(14) guarantee the asymptotic stability of the closed-loop system. The third part proves both the free control variables and the future control inputs in non-PDC law satisfy the input constraints with conditions (15)–(18).

Part I: The upper bound of the infinite-horizon objective function

With the division of infinite-horizon control inputs, the objective function (9)

$$J^{\infty}_{0,k} = J^{1}_{0,k} + J^{\infty}_{1,k} = \left(\hat{\boldsymbol{x}}^T_{k|k} Q \hat{\boldsymbol{x}}_{k|k} + \hat{\boldsymbol{u}}^T_{k|k} R \hat{\boldsymbol{u}}_{k|k}\right) + \sum_{i=1}^{\infty} \left(\hat{\boldsymbol{x}}^T_{k+i|k} Q \hat{\boldsymbol{x}}_{k+i|k} + \hat{\boldsymbol{u}}^T_{k+i|k} R \hat{\boldsymbol{u}}_{k+i|k}\right) \tag{A.1}$$

Suppose a nonquadratic Lyapunov function [41]

$$V_k = \hat{\boldsymbol{x}}^T_k S_z^{-1} \hat{\boldsymbol{x}}_k \tag{A.2}$$

satisfies

$$V_{k+i+1|k} - V_{k+i|k} < -\left[\hat{\boldsymbol{x}}^T_{k+i|k} Q \hat{\boldsymbol{x}}_{k+i|k} + \hat{\boldsymbol{u}}^T_{k+i|k} R \hat{\boldsymbol{u}}_{k+i|k}\right], \tag{A.3}$$

then the closed-loop system is guaranteed to be asymptotically stable [33,35].

Summing (A.3) from $i = 1$ to $i = 8$, and with $V_{\infty|k} = 0$, we have

$$J^{\infty}_{0,k} < \hat{\boldsymbol{x}}^T_{k|k} Q \hat{\boldsymbol{x}}_{k|k} + \hat{\boldsymbol{u}}^T_{k|k} R \hat{\boldsymbol{u}}_{k|k} + \left(A_z \hat{\boldsymbol{x}}_{k|k} + B_z \hat{\boldsymbol{u}}_{k|k}\right)^T \times S_z^{-1} \left(A_z \hat{\boldsymbol{x}}_{k|k} + B_z \hat{\boldsymbol{u}}_{k|k}\right) \tag{A.4}$$

Next we define a scalar variable γ and suppose that

$$\hat{\boldsymbol{x}}^T_{k|k} Q \hat{\boldsymbol{x}}_{k|k} + \hat{\boldsymbol{u}}^T_{k|k} R \hat{\boldsymbol{u}}_{k|k} + \left(A_z \hat{\boldsymbol{x}}_{k|k} + B_z \hat{\boldsymbol{u}}_{k|k}\right)^T S_z^{-1} \left(A_z \hat{\boldsymbol{x}}_{k|k} + B_z \hat{\boldsymbol{u}}_{k|k}\right) < \gamma, \tag{A.5}$$

such that γ is an upper bound of the infinite-horizon objective function $J^{\infty}_{0,k}$. Then minimizing $J^{\infty}_{0,k}$ is turned into the minimization of γ subject to (A.5).

With the definition $\tilde{S}_z^{-1} = \gamma^{-1} S_z^{-1}$ and Schur complement [40], (A.5) can be further expressed as follows

$$\begin{bmatrix} 1 & (*) & (*) & (*) \\ A_z \hat{\boldsymbol{x}}_{k|k} + B_z \hat{\boldsymbol{u}}_{k|k} & \tilde{S}_z & 0 & 0 \\ Q^{1/2} \hat{\boldsymbol{x}}_{k|k} & 0 & \gamma I & 0 \\ R^{1/} \hat{\boldsymbol{u}}_{k|k} & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{A.6}$$

which is equivalent to $\sum_{l=1}^{N} w_l n_l > 0$.

Therefore, the condition (11) guarantees γ is the upper bound of the infinite-horizon objective function

Part II: Stability condition

With the non-PDC feedback control law

$$\boldsymbol{u}_{n,k+i|k} = -Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k} + \boldsymbol{u}_{e,k},\ i \geq 1, \tag{A.7}$$

the closed-loop system is

$$\hat{\boldsymbol{x}}_{k+i+1|k} = A_z \hat{\boldsymbol{x}}_{k+i|k} - B_z Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k} = \left(A_z - B_z Y_z G_z^{-1}\right) \hat{\boldsymbol{x}}_{k+i|k}. \tag{A.8}$$

Substituting (A.7) and (A.8) into the stability condition (A.3)

$$(A_z G_z - B_z Y_z)^T \tilde{S}_{z+}^{-1} (A_z G_z - B_z Y_z) - G_z^T \tilde{S}_z^{-1} G_z + G_z^T Q G_z / \gamma + Y_z^T R Y_z / \gamma < 0. \tag{A.9}$$

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With the fact that

$$G_z^T \tilde{S}_z^{-1} G_z \geq G_z^T + G_z - \tilde{S}_z, \tag{A.10}$$

thus (A.9) is satisfied if

$$(A_zG_z - B_zY_z)^T \tilde{S}_{z+}^{-1} (A_zG_z - B_zY_z) + G_z^T Q G_z/\gamma + Y_z^T R Y_z/\gamma < G_z^T + G_z - \tilde{S}_z \tag{A.11}$$

which can be expressed as follows

$$\begin{bmatrix} G_z^T + G_z - \tilde{S}_z & (*) & (*) & (*) \\ A_zG_z - B_zY_z & \tilde{S}_{z+} & 0 & 0 \\ Q^{1/2}G_z & 0 & \gamma I & 0 \\ R^{1/2}Y_z & 0 & 0 & \gamma I \end{bmatrix} > 0. \tag{A.12}$$

According to the definition of r_{ij}^l in (19), (A.12) is equivalent to

$$\sum_{l=1}^{N} w_{l+} \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j r_{ij}^l > 0. \tag{A.13}$$

With conditions (12)–(14), the left side of (A.13)

$$\geq \sum_{l=1}^{N} w_{l+} \left(\sum_{i=1}^{N} w_i^2 Q_{ii}^l + \sum_{i=1}^{N} \sum_{j>i}^{N} w_i w_j \left(Q_{ij}^l + Q_{ji}^l\right) \right)$$

$$\geq \sum_{l=1}^{N} w_{l+} \left[w_1 I w_2 I \cdots w_N I\right] \Psi_l \left[w_1 I w_2 I \cdots w_N I\right]^T > 0.$$

Therefore, the closed-loop fuzzy control system is asymptotically stable.

Part III: Input constraints

With (A.3) and (A.5), we get

$$\hat{\boldsymbol{x}}_{k+i|k}^T \tilde{S}_z^{-1} \hat{\boldsymbol{x}}_{k+i|k} \leq 1, i \geq 1. \tag{A.14}$$

On the other hand, with (17)

$$\begin{bmatrix} G_z + G_z^T - \tilde{S}_z & (*) \\ Y_z & W_1 \end{bmatrix} > 0,$$

from which, the following result is obtained with (A.10) and (A.14)

$$\left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right)^T (W_1)^{-1} \left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right) \leq 1, i \geq 1.$$

Since W_1 is diagonal and the component $W_{1,jj} = \bar{\boldsymbol{u}}_j^2$,

$$\left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right)_j^2 < \bar{\boldsymbol{u}}_j^2.$$

With the non-PDC law

$$\boldsymbol{u}_{n,k+i|k} = -Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k} + \boldsymbol{u}_{e,k}, i \geq 1,$$

we have

$$\left| \left(\boldsymbol{u}_{n,k+i|k} - \boldsymbol{u}_{e,k}\right)_j \right| < \left|\bar{\boldsymbol{u}}_j\right|, i \geq 1.$$

With the definition

$$\bar{\boldsymbol{u}} = \min \left\{ \left|\boldsymbol{u}_{\max} - \boldsymbol{u}_{e,k}\right|, \left|\boldsymbol{u}_{\min} - \boldsymbol{u}_{e,k}\right| \right\},$$

the following inequality holds

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u}_{n,k+i|k} \leq \boldsymbol{u}_{\max}, i \geq 1. \tag{A.15}$$

In addition, from (15)

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u}_{n,k|k} \leq \boldsymbol{u}_{\max}. \tag{A.16}$$

Thus, the input magnitude constraints are satisfied by both the free control input and the future control inputs in the non-PDC law.

Next, we deal with the incremental input constraints.

With (18),

$$\left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right)^T (W_2)^{-1} \left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right) \leq 1, i \geq 1.$$

Since W_2 is diagonal and the component $W_{2,jj} = \left(\bar{\boldsymbol{u}}_{d,j}/2\right)^2$,

$$\left| \left(Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i+1|k} - Y_z G_z^{-1} \hat{\boldsymbol{x}}_{k+i|k}\right)_j \right| < \bar{\boldsymbol{u}}_{d,j}, i \geq 1.$$

With the non-PDC law, we get

$$\left| \left(\Delta \boldsymbol{u}_{n,k+i|k}\right)_j \right| < \bar{\boldsymbol{u}}_{d,j}, i \geq 1.$$

On the other hand, with the definition

$$\bar{\boldsymbol{u}}_d = \min \left\{ \left|\Delta \boldsymbol{u}_{\max}\right|, \left|\Delta \boldsymbol{u}_{\min}\right| \right\},$$

the following holds

$$\Delta \boldsymbol{u}_{\min} < \Delta \boldsymbol{u}_{n,k+i|k} < \Delta \boldsymbol{u}_{\max}, i \geq 1. \tag{A.17}$$

In addition, from (16)

$$\Delta \boldsymbol{u}_{\min} < \Delta \boldsymbol{u}_{n,k|k} < \Delta \boldsymbol{u}_{\max}. \tag{A.18}$$

Thus, the incremental input constraints are satisfied by both the free control input and the future control inputs in the non-PDC law.

Appendix B

Proof. By substituting (6) into (4), one yields the closed-loop system

$$\begin{cases} \boldsymbol{x}_{k+1} = A_z \boldsymbol{x}_k + B_z \left(\boldsymbol{u}_{n,k} + K_c \hat{\boldsymbol{d}}_k\right) + \boldsymbol{R}_z + G_{dz} \boldsymbol{d}_k \\ \boldsymbol{y}_k = C \boldsymbol{x}_k \end{cases} \tag{B.1}$$

and hence

$$\boldsymbol{x}_{k+1} = A_z \boldsymbol{x}_k + B_z \boldsymbol{u}_{n,k} + \boldsymbol{R}_z + \left(B_z K_c + G_{dz}\right) \boldsymbol{d}_k - B_z K_c \tilde{\boldsymbol{d}}_k \tag{B.2}$$

On the other hand, with (22)

$$\left(B_z \otimes I_n\right) \boldsymbol{\alpha} = \boldsymbol{\sigma} \left(-G_{dz}\right),$$

and then

$$B_z \boldsymbol{\sigma}^{-1} (\boldsymbol{\alpha}) + G_{dz} = 0.$$

Therefore, (B.2) becomes with (23)

$$\boldsymbol{x}_{k+1} = A_z \boldsymbol{x}_k + B_z \boldsymbol{u}_{n,k} + \boldsymbol{R}_z - B_z K_c \tilde{\boldsymbol{d}}_k. \tag{B.3}$$

Denote $\tilde{\boldsymbol{d}}_\infty$ as the disturbance estimation error at the steady state. Since the disturbance estimate $\hat{\boldsymbol{d}}$ tracks the disturbance $\boldsymbol{d}$ asymptotically, i.e., $\tilde{\boldsymbol{d}}_\infty = 0$, the steady-state closed-loop dynamics is

$$\begin{cases} \boldsymbol{x}_\infty = A_z \boldsymbol{x}_\infty + B_z \boldsymbol{u}_{n,\infty} + \boldsymbol{R}_z \\ \boldsymbol{y}_\infty = C \boldsymbol{x}_\infty \end{cases}. \tag{B.4}$$

Subtracting the steady-state (7) from (B.4) results in

$$\begin{cases} \boldsymbol{x}_\infty - \boldsymbol{x}_{e,\infty} = A_z \left(\boldsymbol{x}_\infty - \boldsymbol{x}_{e,\infty}\right) + B_z \left(\boldsymbol{u}_{n,\infty} - \boldsymbol{u}_{e,\infty}\right) \\ \boldsymbol{y}_\infty - \boldsymbol{y}_r = C \left(\boldsymbol{x}_\infty - \boldsymbol{x}_{e,\infty}\right) \end{cases}. \tag{B.5}$$

in which the existence of $\left(\boldsymbol{x}_{e,\infty}, \boldsymbol{u}_{e,\infty}\right)$ is guaranteed with Assumption 2. On the other hand, since the nominal system is proved to be asymptotically stable in Appendix A, there exists a stabilizing linear feedback control law at steady state $\hat{\boldsymbol{u}}_\infty = -K\hat{\boldsymbol{x}}_\infty$, i.e., $\boldsymbol{u}_{n,\infty} - \boldsymbol{u}_{e,\infty} = -K\left(\boldsymbol{x}_\infty - \boldsymbol{x}_{e,\infty}\right)$ [42–44]. Substituting the feedback law into (B.5) yields

$$\left(A_{z,\infty} - B_{z,\infty}K - I\right)\left(\boldsymbol{x}_\infty - \boldsymbol{x}_{e,\infty}\right) = 0 \tag{B.6}$$

For (B.6), the eigenvalues of $\left(A_{z,\infty} - B_{z,\infty}K\right)$ are within the unit circle, thus the only solution is $\boldsymbol{x}_\infty = \boldsymbol{x}_{e,\infty}$ [42–44]. Therefore, $\boldsymbol{y}_\infty = \boldsymbol{y}_r$ with (B.5), that is, the influence of the disturbance can be removed from the output channels at steady state.

Appendix C

The system matrices of the local linear models of T–S fuzzy model are:

$$A_1 = \begin{bmatrix} 0.9047 & 1.2073 & 0.1361 & 0.0021 \\ -0.0003 & 0.8108 & 0.1802 & 0.0043 \\ -0.0024 & 0.2579 & 0.7247 & 0.0366 \\ 0 & 0 & 0 & 0.9835 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 29.4094 & 8E-06 \\ -0.4645 & 3E-05 \\ -0.1133 & 0.0003 \\ 0 & 0.0165 \end{bmatrix}, \quad R_1 = \begin{bmatrix} -16.033 \\ 0.2041 \\ 0.0477 \\ 0 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} 0.9047 & 1.2689 & 0.1590 & 0.0018 \\ -0.0002 & 0.79360 & 0.2007 & 0.0036 \\ -0.0019 & 0.2017 & 0.7849 & 0.0288 \\ 0 & 0 & 0 & 0.9835 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 36.6575 & 8E-06 \\ -0.5732 & 2E-05 \\ -0.1095 & 0.0003 \\ 0 & 0.0165 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -21.2495 \\ 0.2152 \\ 0.04289 \\ 0 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} 0.9047 & 1.3396 & 0.1838 & 0.0018 \\ -0.0002 & 0.7784 & 0.2185 & 0.0033 \\ -0.0016 & 0.1747 & 0.8153 & 0.0253 \\ 0 & 0 & 0 & 0.9835 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 42.7875 & 8E-06 \\ -0.6634 & 2E-05 \\ -0.1108 & 0.0002 \\ 0 & 0.0165 \end{bmatrix}, \quad R_3 = \begin{bmatrix} -26.4724 \\ 0.21404 \\ 0.01403 \\ 0 \end{bmatrix}.$$

The disturbance gain matrices for the mismatched disturbance simulation case are as follows:

$$G_{d,1} = \begin{bmatrix} -30.3081 & -58.8188 & -88.2282 & -117.6376 \\ 2.0468 & 0.9288 & 1.3933 & 1.8578 \\ -32.1687 & 0.2247 & 0.3377 & 0.4507 \\ -1652.9 & -0.0991 & -0.1157 & -0.1322 \end{bmatrix},$$

$$G_{d,2} = \begin{bmatrix} -37.4442 & -73.3149 & -109.9724 & -146.6299 \\ 1.5051 & 1.1463 & 1.71959 & 2.29282 \\ -24.9509 & 0.21744 & 0.3266 & 0.4358 \\ -1652.9 & -0.0991 & -0.1156 & -0.1322 \end{bmatrix},$$

$$G_{d,3} = \begin{bmatrix} -43.5685 & -85.5750 & -128.3625 & -171.1500 \\ -1.2753 & 1.3266 & 1.9899 & 2.6532 \\ -21.7545 & 0.2202 & 0.3308 & 0.4413 \\ -1625.9 & -0.0991 & -0.1156 & -0.1322 \end{bmatrix}.$$

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