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# Reliability estimation of maritime transportation: A study of two fuzzy reliability models



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# ABSTRACT

The progressive increase in marine vessel transportation, in recent years, is often a cause of congestion at sea and main cause of highly irregular vessel's travel time. This greatly affects scheduling of the facilities at the harbor and also related logistics. As a result, the reliability value of any marine vessel becomes a crucial factor in associated decision making. Modeling the uncertainty in estimation of marine vessel reliability has been a research interest for quite some time now. This paper investigates the problem in a different sense and tries to model the uncertainties using expert's opinions and their imprecise responses. Marine vessel transportation reliability is viewed in an entirely different perspective and framework. This paper initially proposed a transportation reliability estimation procedure considering 12 decision variables divided into three stages. Two realistic models based on fuzzy sets are subsequently developed; with two scenarios for the first model. The first model utilizes fuzzy arithmetic; whereas the second one is based on rule bases. The paper demonstrates how information based on experience of the experts on marine vessels could be used to obtain its reliability value. Both the models would be helpful where imprecision is an intrinsic attribute of the accessible data in case of sea going vessels.

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# 1. Introduction

Risk and reliability are significant operational issues of any transportation system. Sumalee and Kurauchi (2006), in a guest editorial of a special issue of Networks and Spatial Economics Journal focused on the reliability and emergency issues in transportation network analysis. In view of stringent international statutory requirements in the maritime transport segment, risk appraisal has become an important managerial tool for making critical decisions. Growing concern about the accidents, disasters and resulting environment impact has forced the attention on the hazards and associated risks involved in all the activities of every transportation system. Nowadays, a wide range of approaches and methods are available to estimate the risk and reliability. Both these are concerned with safety aspect of the system. Improved safety performances as regards to safety of the people, equipments and machineries, and protection of the environment are the prime aims. Risk and reliability assessment also helps in assessing the system's operational performance measures and life cycle costing. It as well aids in optimizing various processes in the system. Risk management approaches have been universally adopted in the maritime transportation sector with the key objective of providing improved safety and enhanced protection to the environment. Development of risk-based practices, over the years, addressed both quantitative and qualitative techniques, suitable for investigation in diverse applications and purposes.

Recently, Celik et al. (2010) developed a risk based modeling approach to enhance the execution process of shipping accident investigation. Specifically, the paper addressed a fuzzy extended fault tree analysis that combined the effects of organizational faults and shipboard technical system failures under a unique risk assessment scheme. Balmat et al. (2009) presented a fuzzy approach for the MAritime RISk Assessment (MARISA) applied to safety at sea. In this analysis, a fuzzy risk factor composed of a static risk factor and a dynamic risk factor has been considered. In a recent paper, (Balmat et al., 2011) developed a decision-making system to maritime risk assessment. Ren et al. (2009) proposed an offshore risk analysis method using fuzzy Bayesian network. Yang et al. (2009b) suggested a subjective security-based assessment and management framework for maritime security using a fuzzy evidential reasoning approach. Additional recent papers in risk assessment and safety in maritime transportation include (Eleve-Datubo et al., 2008; Hu et al., 2008; Koc, 2009; Liu et al., 2008a;

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Ting-rong et al., 2008; Yang et al., 2009a). Hadjimichael (2009) and Shyur (2008) studied risk assessment in aviation industry. Furthermore, other research investigations in marine transportation systems are (Grabowski et al., 2007; Kolowrocki and Soszynska, 2006; Kolowrocki et al., 2007; Liu et al., 2005; Sii et al., 2001; Wang, 2006; Wang et al., 2004).

The notion of the travel time reliability researched in transportation means, mostly rail and road transport, provided a diverse perspective in this area. Chang (2010), in his recent work, considered a way of assessing travel time reliability in transport appraisal and discussed two requirements for the evaluation and guidance of the appraisal. The requirements represented the measurement and valuation of travel time uncertainties. The gap between actual and planned journey times is used for the quantification and logic based choice model is developed to derive monetary values of travel time variation. Fosgerau and Karlstrom (2010) obtained the value of reliability in the scheduling of an activity of random duration i.e. travel under congested conditions. Hollander and Liu (2008) examined the methodological, statistical and computational aspects of estimation of travel time distribution by repeated simulation; while the paper (Margulici and Ban, 2008) recommended methodology for benchmarking the travel time estimates. Prabhu Gaonkar et al. (2011) presented a methodology of maritime transportation system reliability evaluation by the application of fuzzy sets and fuzzy logic techniques. Prabhu Gaonkar et al. (2013) considered the maritime travel time reliability in a possibilistic manner, and then the reliability optimization problem with budgetary constraints and stage-time limitations is formulated and solved. Contributions in similar direction in the literature are (Al-Deek and Emam, 2006; Batley, 2007; Ettema and Timmermans, 2006; Li et al., 2010; van Lint, 2008: van Lint et al., 2008: Wu et al., 2008) and Tzannatos (2005) looked into technical reliability of the Greek coastal passenger fleet. Fang and Das (2005) studied the survivability and reliability of damaged ships after collision and grounding.

All the above literature citations explored and examined the diverse facets of risk, safety and reliability quantitatively and qualitatively. This paper uses fuzzy sets approach in estimating maritime transportation reliability, which is quite different and not yet explored by the researchers working in this field. The paper looks into the problem in a different way. Firstly, it considers the uncertainties in variable data values demanded for estimating transportation reliability of marine vessel or ship in a sort of qualitative manner. Secondly, the variables are modeled using expert's opinions and their imprecise responses. Section 2 explains the perspective of the term reliability in context with maritime transportation, as presumed in this paper. Reliability estimation procedure is stated in the same section. Two models based on the fuzzy sets are developed in Section 3. Two different scenarios for

Table 1		
Various	Decision	Variables.

the first model have been formulated. Fuzzy set and fuzzy logic concepts are used as part of the model solution process. This novel work, in fact elucidates that the information based on experience of the experts on marine vessels could be used to find out its reliability value. The models would be helpful where imprecision is an intrinsic attribute of the accessible data in case of sea going vessels.

# 2. Marine vessel transportation reliability

Fuzzy reliability developed at the end of the last century has grown multifold and has numerous directional facets in its application. Various current advancements in the fuzzy reliability theory include (Gholizadeh et al., 2010; Huang et al., 2006; Liu et al., 2007; Liu et al., 2008b; Marano and Quaranta, 2010; Rotshtein, 2010; Viertl, 2009; Zhang and Huang, 2010). The marine vessel transportation reliability has been viewed in a different perspective and is modeled on the basis of qualitative variables in this paper. The subjective linguistic nature of the variables and acquisition of the same from the expert's responses necessitated the modeling and estimation in the form of fuzzy reliability.

Marine vessel transportation reliability is considered specifically with respect to three aspects: intended mission completion, timeliness and safety of the mission. These aspects depend on several decision variables and may vary as per the analysts. However, 12 most suitable decision variables have been considered in three stages, as given in Table 1. The categorization in these three stages scales down the problem to the smaller magnitude and also matches to the three aspects of the transportation reliability to a modest extent. These decision variables are finalized based on the experienced personnel working in marine transportation environment. Table 1 also shows the scale and the range of values that a variable can take. The linguistics terms defining the range of the variables have been stated later during second model development process.

The two models developed in this paper use similar reliability estimation procedure. It is depicted in Fig. 1. As seen, there are four levels of computation for *Stage I*, two levels for *Stage II* and three levels for *Stage III*. Final reliability of marine vessel transportation is obtained from the outputs of all three stages. Since modeling in this paper has been carried out in the fuzzy domain, the reliability output would be in the form of membership function. The output is then converted to estimate the crisp value of reliability by application of defuzzification method(s).

# 3. Model development and illustrations

This section explains the development process of two models along with a brief description on modeling the aggregation of

Stage	Decision variable	Scale	Variable notation
I	Experience of the navigation crew	0–30 years	X <sub>1</sub>
	Experience of the maintenance workforce	0–30 years	X <sub>2</sub>
	Effectiveness of the maintenance programs	0–10	X <sub>3</sub>
	Overall past operational history of the vessel	0–10	X4
	Unforeseen events	0–10	X <sub>5</sub>
II	Congestion at the source harbor	0–1 (congestion factor)	X <sub>6</sub>
	Congestion at the sea	0–1 (traffic intensity factor)	X <sub>7</sub>
	Congestion at the destination harbor	0–1 (congestion factor)	X <sub>8</sub>
III	Weather or environmental conditions	0–10	X <sub>9</sub>
	Delivery date of vessel	Year 1970-2009	X <sub>10</sub>
	Technological up-gradation of the vessel	0–10	X <sub>11</sub>
	Region, place or yard where the vessel was built	0–10	X <sub>12</sub>



Fig. 1. Marine vessel transportation reliability estimation procedure.

expert's responses in imprecision form by means of Gaussian membership function. The first model considers fuzzy arithmetic for vessel reliability estimation; whereas the second model utilizes fuzzy inference system from fuzzy logic. The Model–1 is illustrated with two scenarios. Symmetric and non-symmetric Gaussian membership functions are employed to represent the imprecision in the first and second scenarios of Model-1, respectively. The Model-2 uses Z-type, S-type and symmetric Gaussian membership functions.

# 3.1. Model-1

This model is developed on the basis of the arithmetic of fuzzy sets. The model considers a common scale '0 to 1' for all the decision variables ' $X_i$ '. Initially, estimates of each decision maker (expert) about every decision variable are converted

(or scaled) on to the 0–1 scale from respective customary scales (as shown in Table 1). This is to have a standard scale for every decision variable. The 0-1 scale may be viewed as: the values of ' $x_i$ ' range from 0 to 1 for each decision variable denoting 0 for the worst (pessimistic value) and 1 for the best (optimistic value) in context with the meaning of that variable. If not, the values need to be transformed appropriately using scaling or mirroring for that particular variable. Then, expert's scaled estimates are combined to obtain the parameters of Gaussian membership function. The Gaussian functions are most adequate choice of the membership functions for representing uncertainties in measurement (Kreinovich et al., 1992). The following theorem is proved by Kreinovich et al., (1992): If for a quantity 'X', several experts (say 'n') give their estimates as:  $a_1$ ,  $a_2$ ,  $a_3, \ldots, a_n$ , and they estimate the precision of their estimates correspondingly as:  $\delta_1, \delta_2, \delta_3, \dots, \delta_n$ , then the resulting membership

function is given by:

$$\mu(x) = e^{-\beta \left\{\frac{(x-\alpha)^2}{\beta^2}\right\}} \quad ; \text{ for } \beta > 0 \tag{1}$$

where

$$\delta = \frac{1}{\sqrt{\frac{1}{\delta_1^2} + \frac{1}{\delta_2^2} + \frac{1}{\delta_3^2} + \dots + \frac{1}{\delta_n^2}}} \text{ and } a = \frac{\frac{a_1}{\delta_1^2} + \frac{a_2}{\delta_2^2} + \frac{a_3}{\delta_3^2} + \dots + \frac{a_n}{\delta_n^2}}{\frac{1}{\delta_1^2} + \frac{1}{\delta_2^2} + \frac{1}{\delta_3^2} + \dots + \frac{1}{\delta_n^2}}$$

The membership function stated above is a symmetric Gaussian function that depends on two parameters namely; central value 'a' and deviation (i.e. delta) ' $\delta$ '. The non-symmetric Gaussian membership function depends on four parameters  $a_{lower}$ ,  $a_{upper}$  and  $\delta_{upper}$ ; where  $a_{lower} < a_{upper}$ . The first two parameters determine the shape of the left-most curve and subsequent two decide the shape of the right-most curve of non-symmetric Gaussian membership function. The parameter  $\beta$  of Eq. (1) decides the shape of the membership function, more specifically, fuzziness spread in the Gaussian membership function either becomes narrow or wide spread i.e. fuzziness is either less or more. This is decided by the analyst depending on his/her judgment about the variable.

Final fuzzy reliability estimation may change depending on the value of  $\beta$  and accordingly care must be taken while choosing the value of this parameter. It would be sensible to study the response behavior of the expert's with regards to accuracy of their judgment prior to deciding  $\beta$  value.

### Table 2

Expert input estimates for the Scenario 1.

Two scenarios have been described for illustration of Model–1. The first scenario considers two parameter Gaussian membership functions for each decision variable, while the second considers four parameters. Each expert opines his/her estimation about the decision variable in the respective scales as shown previously in Table 1. Seven experts have been considered giving their estimates about the decision variables in both the scenarios. Table 2 provides the input estimates for scenario 1 and Tables 3 and 4 for scenario 2. Table 3 gives lower variations i.e. estimates for leftmost curve whereas Table 4 gives upper variations i.e. estimates for right-most curve of non-symmetric Gaussian membership function. The aggregated parameters of Gaussian membership functions are acquired using theorem explained earlier. The resulting parameters thus obtained for both scenarios are tabulated in Table 5.

Considering value of  $\beta = 1/2$ , membership functions of each decision variable for three stages under consideration are shown in first three subplots of Fig. 2. Once membership functions resulting from expert's estimates are obtained for all the variables, the next step of Model–1 is to carry out fuzzy intersection (Klir and Yuan, 2001) of two decision variables (at Level – I) or output of a decision box from previous level and a variable (for other levels) of respective stages, as shown in Fig. 1. Fuzzy intersection of two fuzzy sets 'P' and 'Q', described by their membership functions  $\mu_P(x)$  and  $\mu_Q(x)$  leads to membership function:  $\mu_{Pn}Q(x) = min[\mu_P(x), \mu_Q(x)]$ . This is carried out at each level and output membership functions for each stage are thus acquired. It is rather possible that the output of a few or every decision box may be

Decision variable	Expert n	umber												
	1		2		3		4		5		6		7	
	<i>a</i> <sub>1</sub>	$\delta_1$	<i>a</i> <sub>2</sub>	$\delta_2$	<i>a</i> <sub>3</sub>	$\delta_3$	<i>a</i> <sub>4</sub>	$\delta_4$	a <sub>5</sub>	$\delta_5$	<i>a</i> <sub>6</sub>	$\delta_6$	a <sub>7</sub>	$\delta_7$
$X_1$	19	4	23	2	23	1	20	3	21	3	24	2	22	4
X <sub>2</sub>	23	2	26	1	25	2	22	3	24	1	20	3	21	3
X <sub>3</sub>	6	2	5	3	6	2	7	1	7	2	6	3	8	1
X4	7	2	7	1	6	3	5	3	6	1	5	2	8	1
X <sub>5</sub>	8	3	6	4	7	2	5	1	6	1	7	1	6	2
X <sub>6</sub>	0.5	0.2	0.5	0.1	0.6	0.2	0.7	0.2	0.6	0.3	0.8	0.1	0.8	0.1
X <sub>7</sub>	0.6	0.2	0.7	0.2	0.5	0.3	0.7	0.1	0.8	0.1	0.7	0.2	0.6	0.3
X <sub>8</sub>	0.7	0.1	0.7	0.2	0.6	0.3	0.5	0.2	0.6	0.1	0.8	0.1	0.5	0.3
X <sub>9</sub>	5	2	6	3	7	1	5	3	4	4	4	4	8	1
X <sub>10</sub>	1996	4	1995	6	1998	4	1999	3	1999	2	1998	4	1997	5
X <sub>11</sub>	6	2	7	2	5	3	5	3	8	1	5	2	6	1
X <sub>12</sub>	7	3	5	3	4	4	5	3	6	2	7	2	8	2

#### Table 3

Expert input lower variation estimates for the Scenario 2.

Decision variable	Expert n	umber												
	1		2		3		4		5		6		7	
	<i>a</i> <sub>1</sub>	$\delta_1$	<i>a</i> <sub>2</sub>	$\delta_2$	<i>a</i> <sub>3</sub>	$\delta_3$	<i>a</i> <sub>4</sub>	$\delta_4$	<i>a</i> <sub>5</sub>	$\delta_5$	<i>a</i> <sub>6</sub>	$\delta_6$	a <sub>7</sub>	$\delta_7$
$X_1$	22	5	20	2	17	3	19	4	16	1	18	3	18	2
X <sub>2</sub>	18	2	23	5	21	4	24	3	22	2	20	1	19	3
X3	5	2	6	3	4	1	4	2	5	3	4	2	6	1
X <sub>4</sub>	6	3	8	1	7	1	7	2	6	1	8	1	6	2
X <sub>5</sub>	5	3	8	1	5	2	7	2	6	2	6	1	6	3
X <sub>6</sub>	0.4	0.3	0.5	0.3	0.5	0.2	0.4	0.2	0.4	0.1	0.4	0.3	0.5	0.3
X <sub>7</sub>	0.3	0.3	0.4	0.3	0.6	0.2	0.5	0.1	0.6	0.3	0.3	0.1	0.4	0.2
X <sub>8</sub>	0.5	0.1	0.6	0.2	0.6	0.1	0.5	0.3	0.7	0.2	0.7	0.1	0.4	0.3
$X_9$	4	2	6	1	5	3	6	3	4	2	7	1	6	2
$X_{10}$	1992	3	1995	2	1994	4	1993	2	1992	3	1994	1	1991	1
X <sub>11</sub>	5	1	5	3	4	3	7	1	6	1	4	2	6	3
X <sub>12</sub>	7	2	6	2	4	2	5	3	7	1	5	3	6	2

Table 4	
Expert input upper variation estimates for the Scenario 2.	

Decision variable	Expert n	umber												
	1		2		3		4		5		6		7	
	$a_1$	$\delta_1$	<i>a</i> <sub>2</sub>	$\delta_2$	<i>a</i> <sub>3</sub>	$\delta_3$	<i>a</i> <sub>4</sub>	$\delta_4$	<i>a</i> <sub>5</sub>	$\delta_5$	<i>a</i> <sub>6</sub>	$\delta_6$	a <sub>7</sub>	$\delta_7$
X <sub>1</sub>	24	3	25	3	24	2	21	4	20	5	23	3	22	5
X <sub>2</sub>	22	5	26	2	26	1	25	2	25	1	21	3	23	3
X3	8	2	7	2	6	3	6	2	8	1	7	3	7	1
X <sub>4</sub>	8	1	9	1	9	1	8	2	7	1	9	1	9	1
X5	8	2	9	1	6	2	9	1	7	1	9	1	8	2
X <sub>6</sub>	0.7	0.1	0.6	0.1	0.7	0.3	0.5	0.2	0.8	0.1	0.5	0.3	0.6	0.3
X <sub>7</sub>	0.6	0.2	0.7	0.1	0.7	0.2	0.8	0.1	0.8	0.2	0.7	0.3	0.6	0.3
X <sub>8</sub>	0.8	0.1	0.9	0.1	0.9	0.1	0.8	0.2	0.8	0.1	0.9	0.1	0.7	0.2
X <sub>9</sub>	5	4	8	2	8	1	7	3	6	3	9	1	9	1
X <sub>10</sub>	1998	3	1997	3	1999	1	1998	3	1999	2	1997	1	1997	2
X <sub>11</sub>	6	3	7	2	6	2	9	1	7	3	7	1	8	1
X <sub>12</sub>	8	1	7	3	6	1	6	2	9	1	8	1	9	1

 Table 5

 Resulting parameters of the Gaussian membership functions.

Decision variable	Scenario	o 1	Scenario 2							
	а	δ	a <sub>lower</sub>	$\delta_{lower}$	<i>a<sub>upper</sub></i>	$\delta_{upper}$				
<i>X</i> <sub>1</sub>	0.7555	0.0245	0.5746	0.0247	0.7804	0.0391				
$X_2$	0.8118	0.0198	0.6781	0.0247	0.8359	0.0201				
$X_3$	0.7056	0.0580	0.4869	0.0580	0.7299	0.0580				
$X_4$	0.6776	0.0518	0.7139	0.0466	0.8480	0.0400				
$X_5$	0.6129	0.0522	0.6636	0.0580	0.8316	0.0459				
X <sub>6</sub>	0.6777	0.0509	0.4243	0.0717	0.6767	0.0528				
X <sub>7</sub>	0.7140	0.0580	0.4216	0.0594	0.7299	0.0580				
X <sub>8</sub>	0.6776	0.0518	0.5978	0.0518	0.8500	0.0426				
X <sub>9</sub>	0.6920	0.0621	0.5963	0.0580	0.8418	0.0532				
X <sub>10</sub>	0.7296	0.0318	0.5941	0.0150	0.7240	0.0149				
X <sub>11</sub>	0.6598	0.0580	0.5767	0.0528	0.7709	0.0518				
X <sub>12</sub>	0.6448	0.0934	0.6238	0.0671	0.7886	0.0432				

subnormal membership functions. The subplots 4, 5 and 6 of Fig. 2 depict output of fuzzy intersection of the respective decision variables at *Stage I*, *II* and *III* respectively. Likewise, further, modeling outputs of the three stages as fuzzy intersection, membership function for marine vessel transportation reliability is obtained, which is as shown in the last subplot of Fig. 2.

Similar reliability estimation procedure is accomplished for the second scenario. Readers may observe the nature of non-symmetric Gaussian membership functions resulting from the individual expert's estimates for decision variables, from first three subplots of Fig. 3. The lower side and the upper side variations for each decision variable are different, moreover, there is a range of ' $x_i$ ' values with a certainty zone i.e. ' $x_i$ ' values taking membership function value as one. Fig. 3 also illustrates the output membership functions of *Stage 1, II* and *III* and the concluding output i.e. membership function of marine vessel transportation reliability.

As crisp reliability value is more significant to the practitioners, defuzzifiers (Klir and Yuan, 2001) are used to obtain the same from resulting membership functions. Many defuzzifiers have been proposed in the literature (Klir and Yuan, 2001). Defuzzifiers used in this paper are the Centroid and the Mean of Maximum (MOM). Centroid method calculates the weighted average of the output fuzzy set, whereas MOM defuzzification calculates the average of all variable values with maximum membership degrees. Centroid is calculated using Eq. (2).

$$x^* = \frac{\int \mu_i(x) \bullet x \, dx}{\int} \mu_i(x) dx \tag{2}$$

where  $x^*$  is the defuzzified output,  $\mu_i(x)$  is the aggregated membership function and x is the output variable. The MOM takes the mean of those points where the membership function is at a maximum. Estimated reliability values obtained for both the scenarios are given in Table 6.

# 3.2. Model-2

This model is based on the fuzzy inference system (Bojadziev and Bojadziev, 2007; Kahraman, 2006; Ross, 2004) that uses metaphor of drawing a conclusion via opinions of the panel of experts. This method is often called as Fuzzy Expert System in the literature. The system initially calculates the degree to which input data match the antecedents and then evaluates consequent based on the matching degree. It then combines the conclusion inferred by all antecedent-consequent pairs to obtain a final conclusion. Lastly, a fuzzy conclusion is converted to crisp output using defuzzifiers. Illustration of the Model-2 utilizes the following: (i) development of the membership functions for decision variables and decision box outputs (ii) deciding linguistics for the decision variables (iii) framing fuzzy antecedents-consequents (iv) obtaining and combining the expert's estimates for decision variables (v) application to the marine vessel transportation reliability estimation procedure depicted in Fig. 1.

Three different types of membership functions based on the suitability of their shapes in the problem context have been utilized for modeling the linguistic scale. Z-type membership function is a spline curve based function of 'X' with parameters ' $c_1$ ' and ' $c_2$ ' that locate the extremes of the sloped portion of the curve. S-type membership function is as well spline based curve that provides mapping on to 'X' and have parameters ' $d_1$ ' and ' $d_2$ ' which locate the extremes of the sloped portion of the curve. The third type of the membership function which is used for representing three linguistic variables is symmetric Gaussian membership function. The parameters of this function are central value 'a' and deviation (i.e. delta) ' $\delta$ '. The respective membership function expressions are as below:

$$\mu_{Z-type}(x) = \begin{cases} 1 & ; \quad x \le c_1 \\ 1 - 2\left(\frac{x - c_1}{c_2 - c_1}\right)^2 & ; \quad c_1 \le x \le \frac{c_1 + c_2}{2} \\ 2\left(\frac{x - c_2}{c_2 - c_1}\right)^2 & ; \quad \frac{c_1 + c_2}{2} \le x \le c_2 \\ 0 & ; \quad x \ge c_2 \end{cases}$$
(3)



Fig. 2. Membership functions of decision variables, outputs of three stages and reliability for the Scenario 1.

$$\mu_{S-type}(x) = \begin{cases} 0 & ; \quad x \le d_1 \\ 2\left(\frac{x-d_1}{d_2-d_1}\right)^2 & ; \quad d_1 \le x \le \frac{d_1+d_2}{2} \\ 1-2\left(\frac{x-d_2}{d_2-d_1}\right)^2 & ; \quad \frac{d_1+d_2}{2} \le x \le d_2 \\ 1 & ; \quad x \ge d_2 \end{cases}$$
(4)

$$\mu_{Gaussian}(X) = e^{-\frac{(X-\alpha)^2}{2\beta^2}}$$
(5)

Table 7 gives the membership function parameters. It may be noted that for all three Gaussian membership functions, left most and right most parts are modeled using  $a_{lower}$ ,  $\delta_{lower}$ ,  $a_{upper}$  and  $\delta_{upper}$ ; where  $a_{lower} < a_{upper}$ . The parameters of all the membership functions are chosen so as to satisfy the design guidelines or conditions of the membership functions. Various methods are in use for parameters elicitation by the analyst with the interaction with the experts (Klir and Yuan, 2001). Like Model-1, the values of ' $x_i$ ' range from 0 to 1 for each decision variable, denoting 0 for the worst (pessimistic value) and 1 for the best (optimistic value) in context with the meaning of that variable. Variable  $X_i = X_2$ ,  $X_7$ ,  $X_{10}$ and all other variables are considered as  $X_i$ . As a decision pair of two variables are considered at a time; notation  $X_i$  have been used here for variables  $X_2$ ,  $X_7$  and  $X_{10}$ . The variable  $O_k$  represents output of each decision box, with 'k' taking a value of each decision box. Five linguistic terms are considered for each decision variable. Details of the same are presented in Table 8. As seen, all the

variables have been considered as qualitative type and the fuzzy membership function scale varies from 'not desirable' to 'most desirable' for each of the variable. The quantitative variables namely  $X_1$ ,  $X_2$  and  $X_{10}$  are also modeled using subjective linguistic terms so as to maintain the uniformity in designing the model. Table 9 shows the fuzzy antecedents and consequents, that evaluates the output for the decision pairs. The variety of decision pairs based on Fig. 1 are: (i) ' $X_i - X_j$ ' for Level – I (ii) ' $X_i - O_k$ ' for Levels – II, III and IV (iii) ' $O_{1.4} - O_{2.2}$ ' for output of *Stage I* and *II* (iv) ' $O_{3.3} - O_I$  and *II* (iv) ' $O_{3.3} - O_I$  and *II* value of reliability.

For illustration, estimates of seven expert's are combined; the experts input responses and resulting estimate of each parameter is presented in Table 10. The technique, same as the one utilized in the earlier model is used here as well for combining individual expert's estimates into distinct group estimate. Each decision pair from Fig. 1 is evaluated to further obtain the output of each decision box. At the start, fuzzy matching is carried out using decision variable value for the antecedent part. Then, fuzzy inference step is invoked for each antecedent-consequent pairs to produce a conclusion on their matching degree. This is done using two inference methods (Ross, 2004) namely 'clipping method' and 'scaling method'. Both these methods generate an inferred conclusion by suppressing the membership function of the consequent. The extent to which membership function is suppressed by these methods depends on the matching degree and these methods do the suppressing work differently. Clipping method cuts off the top part of the membership function whose



Fig. 3. Membership functions of decision variables, outputs of three stages and reliability for the Scenario 2.

Table 6Reliability estimates for the Scenarios 1 and 2.

Scenario	Defuzzifiers						
	Centroid	MOM					
1 2	0.7671 0.6941	0.7600 0.7000					

value is higher than the matching degree; where, scaling method scales down the membership functions proportional to the matching degree. After obtaining the inferred conclusion, final crisp reliability value is estimated using defuzzifiers. Centroid and MOM defuzzifiers are used in the illustration here. Fig. 4 depicts four different sub-plots of the surface views, depending on inference method and defuzzifier combinations. Surface view illustrates a three dimensional curve representing the mapping from decision pairs to output. For the reason that in Model-2, the curve represents two inputs and one output case, the entire mapping results can be displayed in one sub-plot. Table 11 shows the outputs of each decision pair (i.e. decision boxes as shown in Fig. 1), and the final reliability estimates. These estimates are obtained using the two defuzzifiers with both inference methods as stated above. The solution of this model is obtained using the Fuzzy Toolbox of MATLAB software.

Table 7Membership function parameters.

Xi	Xj	O <sub>k</sub>	Membership function	Parameters
$S_1$	$T_1$	А	Z-type	$c_1 = 0.05$
$S_2$	T <sub>2</sub>	В	Gaussian	$c_2=0.2$ $a_{lower}=0.2$ $\delta_{lower}=0.06$ $a_{upper}=0.3$ $s_{aupor}=0.06$
S <sub>3</sub>	T <sub>3</sub>	С	Gaussian	$\delta_{upper} = 0.06$ $a_{lower} = 0.45$ $\delta_{lower} = 0.075$ $a_{upper} = 0.55$
$S_4$	T <sub>4</sub>	D	Gaussian	$\delta_{ m upper} = 0.075$ $a_{ m lower} = 0.7$ $\delta_{ m lower} = 0.06$ $a_{ m upper} = 0.8$
$S_5$	T <sub>5</sub>	Е	S-type	$\delta_{upper} = 0.06$ $d_1 = 0.8$ $d_2 = 0.95$

# 4. Conclusion

This paper developed two models for reliability estimation of the marine vessel transportation operation based on the information obtained from the experts in the field. Concepts from the fuzzy set theory and fuzzy logic are used to carry out the estimation procedure and obtain the reliability value. Both the

# Table 8 Linguistic variables notations.

Decision variable	For variables $X_2$ , $X_7$ , $X_{10}$		Use T <sub>1</sub> , T <sub>2</sub> , T <sub>3</sub> , T <sub>4</sub> , T <sub>5</sub>								
	For all other variables		Use <i>S</i> <sub>1</sub> , <i>S</i> <sub>2</sub> , <i>S</i> <sub>3</sub> , <i>S</i> <sub>4</sub> , <i>S</i> <sub>5</sub>								
	<i>S</i> <sub>1</sub>	<i>S</i> <sub>2</sub>	S <sub>3</sub>	<i>S</i> <sub>4</sub>	S <sub>5</sub>						
	$T_1$	<i>T</i> <sub>2</sub>	<i>T</i> <sub>3</sub>	$T_4$	<i>T</i> <sub>5</sub>						
X <sub>1</sub> X <sub>2</sub> X <sub>3</sub> X <sub>4</sub> X <sub>5</sub> X <sub>6</sub> X <sub>7</sub> X <sub>8</sub> X <sub>9</sub> X <sub>10</sub> X <sub>11</sub>	Very low Very low Very low effectiveness Worst Very high possibility Densely congested Very high traffic density Densely congested Very rough Too old Insignificant	Low Low effectiveness Bad High possibility High crowd High traffic High crowd Bad Old Minor	Average Average Average effectiveness Average Average possibility Average crowd Average traffic Average crowd Middle or okay Middle age Medium level	High Highy effective Good Low possibility Low crowd Low traffic Low crowd Good Recent Major	Very high Very high Very much effective Excellent Very low possibility No crowd at all Very low traffic No crowd at all Excellent Brand new Very high level						

# Table 9Fuzzy antecedents and consequents.

X <sub>i</sub>	X <sub>j</sub> or O <sub>k</sub>											
	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>							
S <sub>1</sub>	А	А	В	С	С							
S <sub>2</sub>	А	В	В	С	D							
S <sub>3</sub>	В	В	С	С	D							
S <sub>4</sub>	С	С	С	D	E							
S <sub>5</sub>	С	D	D	E	E							

# Table 10

Expert input estimates and resulting combined estimates.

Decision variable	Expert number													Resulting combined	
	1		2		3 4		5		6		7		estimate		
	<i>a</i> <sub>1</sub>	$\delta_1$	<i>a</i> <sub>2</sub>	$\delta_2$	a <sub>3</sub>	$\delta_3$	<i>a</i> <sub>4</sub>	$\delta_4$	a <sub>5</sub>	$\delta_5$	a <sub>6</sub>	$\delta_6$	a <sub>7</sub>	$\delta_7$	а
$X_1$	25	1	24	1	22	3	26	2	26	1	24	2	25	1	0.8309
X <sub>2</sub>	24	3	23	2	26	1	27	1	25	3	23	4	24	2	0.8574
X3	8	2	8	1	7	3	6	3	8	1	9	1	8	1	0.8149
X <sub>4</sub>	9	1	7	2	8	1	9	1	8	2	7	2	8	1	0.8316
X <sub>5</sub>	8	1	8	2	7	3	9	1	8	1	7	2	7	2	0.8101
X <sub>6</sub>	0.7	0.2	0.8	0.1	0.9	0.1	0.7	0.2	0.7	0.3	0.9	0.1	0.8	0.2	0.8360
X <sub>7</sub>	0.9	0.1	0.7	0.2	0.9	0.1	0.7	0.1	0.8	0.1	0.7	0.2	0.8	0.1	0.8091
X <sub>8</sub>	0.7	0.3	0.7	0.2	0.8	0.2	0.8	0.2	0.7	0.1	0.8	0.1	0.9	0.1	0.7906
X <sub>9</sub>	9	1	9	1	8	2	7	2	8	1	8	1	7	3	0.8355
X <sub>10</sub>	2003	2	2002	3	2003	3	2004	2	2002	2	2003	2	2002	3	0.8458
X <sub>11</sub>	7	2	7	2	8	2	7	3	8	1	9	1	7	3	0.8093
X <sub>12</sub>	8	1	7	3	7	2	9	1	8	1	7	2	8	1	0.8084

models were illustrated with suitable examples. The process shown in this paper is indeed an unconventional approach as far as reliability assessment is concerned. Besides this, viewing transportation reliability in the framework of three aspects and in turn, twelve diverse variables is entirely novel thought. The problems such as insufficiency or unavailability of data in the marine vessel transportation reliability evaluation could be well tackled using the methodology demonstrated in this paper. The first model considered fuzzy arithmetic for vessel reliability estimation; whereas the second model incorporated fuzzy inference system with rule bases. Both notable models would be useful wherever imprecision is an inherent characteristic of the marine vessel transportation reliability attributes. Furthermore, in the case where expert's responses are gathered in a subjective or linguistic form, the transportation reliability estimation could also be accomplished using fuzzy set-based techniques.



Fig. 4. Surface views: (a) clipping and centroid, (b) clipping and MOM, (c) scaling and centroid, (d) scaling and MOM.

Table 11 Reliability estimates of the model-2.

Stage	Decision pair	Pair output	Defuzzified output			
			Clipping and centroid	Clipping and MOM	Scaling and centroid	Scaling and MOM
I	$X_1 - X_2$	011	0.773	0.750	0.794	0.700
	X <sub>3</sub> -0 <sub>1.1</sub>	012	0.745	0.750	0.750	0.700
	$X_{4}-0_{1,2}$	0 <sub>1.3</sub>	0.739	0.750	0.751	0.700
	X5-01.3	01.4	0.733	0.750	0.742	0.700
II	X <sub>6</sub> -X <sub>7</sub>	O <sub>2.1</sub>	0.755	0.750	0.758	0.700
	X8-02.1	O <sub>2.2</sub>	0.739	0.750	0.742	0.700
III	X9-X10	O <sub>3.1</sub>	0.762	0.750	0.783	0.700
	X <sub>11</sub> -O <sub>3.1</sub>	O <sub>3.2</sub>	0.742	0.750	0.748	0.700
	X <sub>12</sub> -O <sub>3.2</sub>	O <sub>3.3</sub>	0.734	0.750	0.741	0.700
Intermediate	01.4-02.2	O <sub>I and II</sub>	0.73	0.750	0.730	0.700
Final	0 <sub>3.3</sub> -0 <sub>I</sub>	Reliability	0.728	0.750	0.725	0.700
	and II	value				

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