



A comparative thermoeconomic cost accounting analysis and evaluation of biogas engine-powered cogeneration

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ABSTRACT

This study presents an analysis of a biogas engine-powered cogeneration system using four different thermoeconomic methods. The most important parameter is the thermoeconomic cost of work produced by the gas engine for each method. The aim is to compare the results obtained from each of those methods. The first method is the exergetic cost theory, which introduced the exergetic cost concept to the thermoeconomic field for the first time. An incidence matrix is defined to show the interaction of flows and components within the system. Exergetic cost theory defines the main rules and delivers a result of 110.065\$/h for the work produced by the gas engine. A second method, modified productive structure analysis, is applied to the system and cost balance equations are formed for each component. Exergy destruction is clearly defined and tabulated. At the end of the analysis, the cost of gas engine work was found to be 85.536\$/h. A third new method described in published literature, Wonergy, is used to determine both the cost of work and the heat utilized in the cogeneration system. Wonergy gives the same thermoeconomic cost for the components which help to produce work. The smallest value obtained was 72.5\$/h. The fourth method, SPECO (specific exergy cost), was the final analytical method used on the system. It defines fuel and product rules to obtain auxiliary equations. The thermoeconomic cost of work produced from the gas engine was determined to be 141\$/h which was the highest value obtained in comparison to the others.

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1. Introduction

In the analysis of the production processes of complex energy systems, the economic profitability and the productivity displayed in resource consumption should both be considered. Performing this analysis, thermodynamics enables us to calculate the efficiencies of the subcomponents that make up the system and determine the locations and amounts of system irreversibilities that occur in the process. However, thermodynamic analysis cannot assess the overall production process in an economic context. Thermoeconomic analysis, by contrast, is a combined discipline that directly assesses the cost of consumed resources, i.e., money and system irreversibilities, within the total production process. While a thermoeconomic analysis shows a variety of ways to use resources more effectively, it also describes the concept of monetary irreversible cost as the economic impact of inefficiency, and it aims to increase the cost efficiency of production processes. Thus, in

a detailed thermoeconomic analysis, it becomes possible to understand the flows in the subcomponents and the entire production process, from the perspective of cost, from the raw material sources entering the system to the final products.

Thermoeconomic methods are generally divided into two groups: cost accounting [1–5], and optimization techniques [6–12]. Cost accounting is the process of determining the total cost of the production per unit of each output of a thermal system, such as electricity, steam, hot water, chilled water, etc., while optimization methods are applied to finding the optimum design or optimum set of operation conditions. All initial investment and operating costs for establishing and operating a thermal system should be allocated to the final product. Principally, there are two costs that must be defined for each product: (i) Direct costs, which include the cost of resources and materials that are clearly attributable to the product cost throughout the production process, and (ii) Indirect costs.

In a comprehensive thermoeconomic analysis, the aim of cost accounting is to establish a logical framework for evaluating the profitability, starting from the determination of the rational costs of the products, to organizing and evaluating the decisions made in

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accordance with this framework. Valero et al., in their initial work on exergetic cost accounting, developed the basic ideas of their thermoeconomic approach and presented a strong theoretical background. That study, which consisted of two parts, has been accepted as one of the pioneering studies in the thermoeconomic field. In the first part, they identified exergetic and thermoeconomic costs for a relatively simple thermal system and presented the basic conditions for conducting the thermoeconomic analysis of a more complex system [13]. In the second part, they developed the mathematical background for three different applications of the thermoeconomic analysis method described in the first part [14].

Exergetic Cost Theory (ECT) is one of the earliest cost accounting methodologies applied to energy conversion systems. The theory was first developed by Lozano and Valero [5], and the methodology presented in this theory is based on a set of analytical propositions. Previously, Valero et al. [13] had defined an incidence matrix that represented a system and interconnected the subcomponents with flows in the system. According to this very early study, the two main routes for calculating costs had been identified, and they had been evaluated in terms of the cost hexagon method. Vieira and Velásquez [15] conducted a thermoeconomic analysis of a thermal power plant using the exergetic cost theory in order to understand the cost history of internal flows in the system and to rationally evaluate the costs in question. Deng et al. [16] applied the exergetic cost theory, based on the structural theory of thermoeconomics, to a gas-fired micro-trigeneration system, which used a small-scale generator set driven by a gas engine and a new small-scale absorption chiller. They also presented a comparison between the methods of conventional energy-based economic analysis and exergetic cost analysis.

The Modified Productive Structure Analysis (MOPSA) is another well-known cost accounting method, and it was first developed theoretically by Kwak et al. [17]. This theory was presented by applying its synthetic propositions to the famous CGAM problem in order to investigate the cost structure of a predefined cogeneration system. The reason that we describe the proposals as “synthetic” [18] in the MOPSA method (this description is valid for all other cost accounting methodologies) is that they employ analytical judgments using both universal and mandatory principles (conservation of energy, generation of entropy or destruction of exergy) as well as extending our cost knowledge of the processes. The MOPSA method was also applied to a combined gas and steam cycle plant in order to estimate the unit exergetic cost of the electricity produced [19]. Bandyopadhyay et al. [20] performed a comprehensive exergetic and thermoeconomic analysis of an existing gas turbine plant and compared three cost-accounting methodologies, arriving at the conclusion that MOPSA is the best method for estimating the unit cost of the electricity produced.

As a relatively new thermoeconomic methodology developed by Kim [21], Wonerger is not as widely practiced as other cost accounting methods in published literature to the best of the authors' knowledge. This published work is one of the few studies in which the Wonerger method is applied in detail to a cogeneration system.

In energy conversion systems, defining the inputs and outputs of a subcomponent with the “fuel” and “product” approach and then recording all exergy flows through subcomponents using this method in a systematic way to establish exergy-based cost flows was first proposed by Lazzaretto et al., and this methodology has become known as the specific exergy costing method, or SPECO. This approach has been one of the most preferred thermoeconomic methods in available published literature due to its ease of application [22–29].

In this study, a comparative thermoeconomic cost-accounting analysis and assessment, including the four methodologies

mentioned above, is used for a biogas engine-powered cogeneration system in Gaziantep. The results from this study will be used in the thermoeconomic performance improvement and optimization of biogas-fueled cogeneration systems. This is the first study of its kind in Turkey since biogas engine-powered cogeneration became preferable in facilities where energy recovery from waste is possible as in the case of wastewater treatment systems. The results should provide a realistic and meaningful basis for thermoeconomic performance evaluation of these power systems, which may be useful in the analysis of similar systems.

2. System description

The biogas engine-powered cogeneration system presented in this work was established by Gaziantep Metropolitan Municipality Wastewater Works, and it started to produce electricity in 2006 using biogas produced from wastewater sludge. The total installed electricity generation and hot water capacity of the plant is 1.66 MWh and 135.11 tons/hr, respectively. Biogas produced through an anaerobic sludge digestion process is first transferred to a desulfurization unit for lowering its sulfur content to an acceptable legal value and then to a gas engine for electricity production. The total electricity produced by the biogas-powered gas engine is 1000 kWh, which is used within the wastewater treatment facility. A schematic diagram of the biogas engine-powered cogeneration unit in the wastewater treatment plant with all flow streams is shown in Fig. 1. The biogas engine in the cogeneration facility is a four stroke, spark ignition engine with 12 cylinders in a V configuration. It uses biogas that is produced by anaerobic digestion reactors. The annual electrical energy production is 8760 GWh, and the annual biogas consumption is nearly 3,400,000 m³ at its intended operating conditions, which means 61% of the biogas produced through anaerobic digesters is consumed by the on-site cogeneration system in the plant. In the cogeneration process, the biogas is first mixed with air before flowing through the intake valves of the gas engine. When the engine is started, an air-biogas mixture is injected into the compressor of the turbocharger unit. The compressor of the turbocharger is powered by a turbine mounted in the exhaust flow of the engine. The advantage of this is that none of the engine shaft output is used to drive the compressor, and only waste energy in the exhaust is used. The turbocharger is equipped with an intercooler to lower the compressed air-biogas mixture temperature. The exhaust gases leaving the turbine of the turbocharger enter the exhaust gas heat exchanger to transfer heat to the water, which circulates in a closed loop through the primary anaerobic digester unit to supply the necessary heat for the digestion process. The exhaust gas leaving the exhaust gas heat exchanger is sent to an exhaust filter which captures and reduces the CO₂ and CO emissions. The high temperature water flowing through the engine jacket of the gas engine is first used to heat the water from the primary digester units. It then enters the lubrication oil heat exchanger to cool the lubrication oil from the engine. Finally, it returns back to the gas engine after cooling the water by circulating it through an intercooler in a closed loop. Oil is used for lubrication and cooling purposes in the engine components. The temperature, pressure and mass flow rate data, and the energy and exergy rates in the biogas engine-powered cogeneration system are presented in Table 1 that is labeled using the nomenclature shown in Fig. 1.

3. Cost accounting methodologies

3.1. Exergetic Cost Theory (ECT)

This methodology requires the division of the system into units

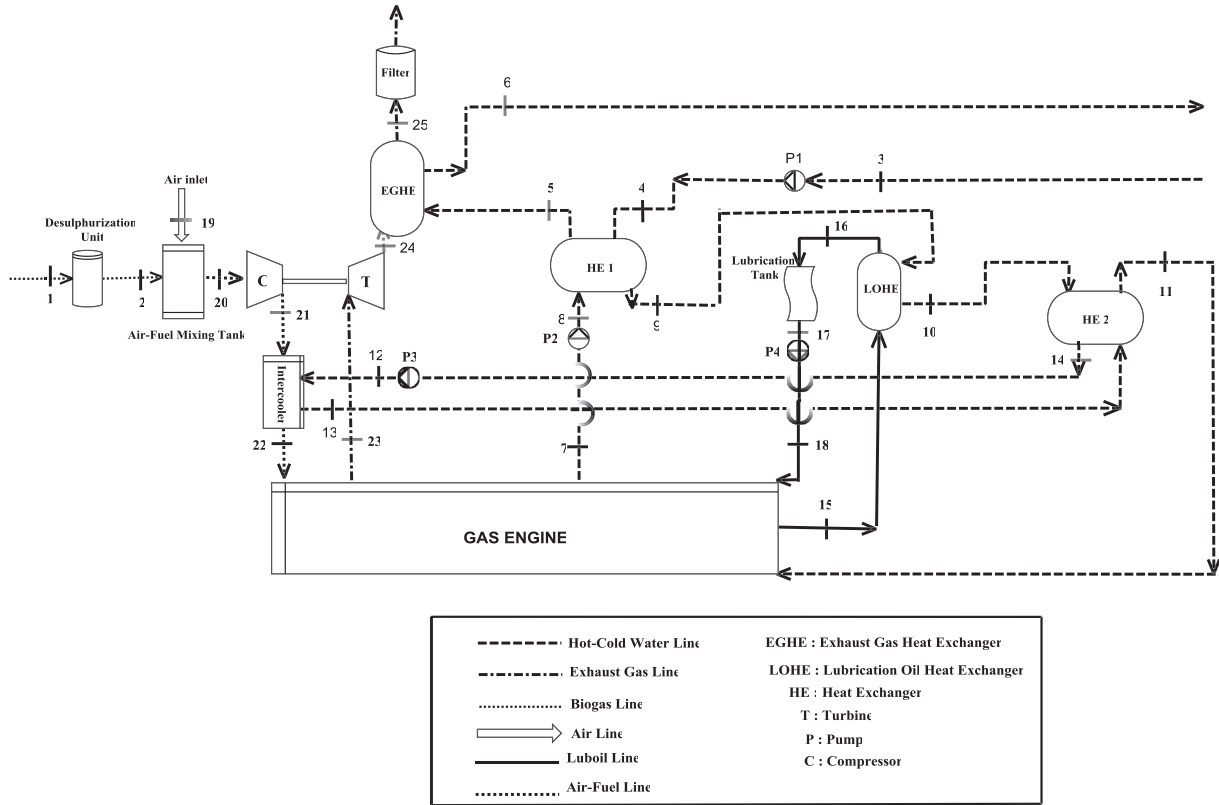


Fig. 1. Flow schematic of the biogas engine-powered cogeneration system.

Table 1

Biogas engine-powered cogeneration system data, thermodynamic properties, energy and exergy rates in the plant with respect to the state points in Fig. 1.

State No	Fluid	Pressure P (bar)	Temperature T (°C)	Mass flow rate \dot{m} (kg/sn)	Enthalpy h (kJ/kg)	Entropy s (kJ/kgK)	Specific energy e (kJ/kg)	Energy rate \dot{E} (kW)	Total exergy rate \dot{E}_x (kJ/kg)
0	Air	1.00	25.00	—	298.6	5.699	0.00	0.00	0.00
0'	Water	1.00	25.00	—	104.8	0.3648	0.00	0.00	0.00
0''	Biogas	1.00	25.00	—	-4650	11.62	—	—	—
0'''	Lub oil	1.00	25.00	—	46.66	0.1629	—	—	0.00
2	Biogas	1.02	30.10	0.129	-4638.0	11.64	12	1.548	4046.40
3	Water	5.25	75.80	20.88	317.7	1.025	212.9	4457.8	350.9
4	Water	6.20	75.80	20.88	317.8	1.025	213	4447.44	352.8
5	Water	6.10	82.80	20.88	347.1	1.108	242.3	5059.22	447.3
6	Water	3.40	88.00	20.88	368.7	1.169	263.9	5510.23	518.7
7	Water	2.80	88.40	15.61	370.4	1.174	265.6	4146.01	391.4
8	Water	7.60	88.50	15.61	371.2	1.1749	266.4	4158.5	400
9	Water	7.50	72.40	15.61	303.6	0.9837	198.8	3103.268	234.3
10	Water	7.30	77.90	15.61	326.7	1.05	221.9	3463.859	285.9
11	Water	7.20	78.50	15.61	329.2	1.057	224.4	3502.884	291.7
12	Water	4.55	50.00	11.28	209.7	0.7035	104.9	1183.27	49.83
13	Water	4.50	52.10	11.28	218.5	0.7306	113.7	1282.53	58.37
14	Water	1.10	50.00	11.28	209.4	0.7037	104.6	1179.8	47.53
15	Lub oil	4.69	100.6	20.0	202.9	0.627	156.24	3124.8	357.7
16	Lub oil	4.50	89.00	20.0	177.3	0.557	130.7	2612.8	259.4
17	Lub oil	1.00	85.00	20.0	168.3	0.533	121.6	2432.8	220.9
18	Lub oil	6.90	87.00	20.0	173.2	0.545	126.6	2530.8	248.7
19	Air	1.00	25.00	1.387	298.6	5.699	0.00	0.00	0.00
20	Air-fuel	1.00	25.00	1.50	298.6 ^a	5.699 ^a	0.00	0.00	4046.40
21	Air-fuel	1.90	116.9	1.50	391.2 ^a	5.786 ^a	92.6 ^a	126.95 ^a	4154.13
22	Air-fuel	1.90	51.00	1.50	324.7 ^a	5.599 ^a	26.14 ^a	35.83 ^a	4136.231
23	Exhaust gas	2.40	460.0	1.50	749.4	6.375	450.8	676.2	374.3
24	Exhaust gas	1.17	360.6	1.50	642.9	6.425	344.4	516.5	192.25
25	Exhaust gas	1.00	65	1.50	338.8	5.826	55.340	60.34	3.692

^a These values correspond to air not the air-fuel mixture; during analysis, air state properties and biogas state properties are calculated separately and their summations are used.

which may be adapted to a component or a set of components. This theory suggests the introduction of a new thermodynamic concept called exergy cost. For a given system whose limits, disaggregation level and production aim of the subsystems have been assigned, the exergy cost of a flow is defined as the amount of exergy needed to produce this flow. A single product and fuel for each component must be defined, and then the necessary calculations have to be performed. Firstly, exergetic costs are determined for each flow and then for each component. After that, cost balance equations for each component and cost allocation equations for external flows into the system are obtained. The solution can be reached when the two following considerations are taken into account: (1) If the fuel definition of a component includes a stream that goes through another component, and is used in it, the unit cost of the stream flowing into and out of the component is the same, and (2) if two or more streams are obtained from a component as the product, the unit cost of those streams are equal. Furthermore the fuel-product definitions are used to improve corresponding matrices. Starting from these matrices, and using the data from the design and operation of the power plant, it is possible to perform a thermoeconomic analysis of the plant.

3.1.1. Exergetic cost definitions

Some explanations are necessary that should be emphasized before implementing the analysis.

The incidence matrix, A ($n \times m$), is the representation of the connection between n subsystems with m flows through the system. It helps to find the destroyed exergy. For a given state, the exergy of a flow, $\dot{E}x$, and destroyed exergy, $\dot{E}x_{dest}$, corresponding to each subsystem can be shown as:

$$A \times \dot{E}x = \dot{E}x_{dest} \quad (1)$$

where $\dot{E}x$ is the exergy vector and $\dot{E}x_{dest}$ is the destroyed exergy vector. The incidence matrix of the system is shown in Table 2.

The aggregation level is the combination of subsystems which make up the whole system. It varies according to the limitation of the system and each aggregation level has its incidence matrix. Any system with a defined aggregation level is specified by an incidence matrix.

The fuel-product for any component or system -both fuel and product-is expressed in terms of exergy as $\dot{E}x_F$ or $\dot{E}x_P$. There can be a single exergy or the difference or sum of two or more exergetic flows. Table 3 shows the fuel-product definitions for our system. For example, consider the heat exchanger 1 (HE1) in Fig. 1 where the fuel and product is defined as the corresponding exergy differences ($\dot{E}x_8 - \dot{E}x_9$) and ($\dot{E}x_5 - \dot{E}x_4$).

Residue: Some flows are stated as completely or partially useless within the systems. For example, in coal-fired power plants ash from the boiler is the residue. In our system, stream 25 is considered to be a residue.

The efficiency and unit consumption of a system: Exergetic efficiency for any power plant is defined as $\eta \equiv \dot{E}x_F / \dot{E}x_P$, and unit exergetic consumption is $\kappa \equiv 1/\eta = \dot{E}x_P / \dot{E}x_F$. It is obvious that κ is greater than 1 given the implications of the second law of thermodynamics.

The exergetic cost is specified as, $\dot{E}x^*$, or the amount of exergy per unit time required to produce a flow. $\dot{E}x^*$, like $\dot{E}x$, is a thermodynamic function [14]. The costs of exergetic flows are determined according to the limits of the system; hence it is not possible to define an absolute exergetic cost for a flow. Where a flow exceeds the limits of a system, its exergetic cost is equal to its exergy, since

no exergy has so far been consumed to obtain it. For a defined subsystem, the exergetic cost of the fuel is equal to the exergetic cost of the product; $\dot{E}x_{F,i}^* = \dot{E}x_{P,i}^*$.

The exergetic cost balance equation for a system is formed with the help of its incidence matrix, A , as follows:

$$A \times \dot{E}x^* = 0 \quad (2)$$

where $\dot{E}x^*$ is the exergetic cost vector for dimension m , which is the number of flows through the system.

Unit exergetic cost is a flow's unit exergetic cost and it is defined by $\kappa^* = \dot{E}x^* / \dot{E}x$. A flow is characterized by its unit exergetic cost; hence, it is a property like exergy and temperature. Its value demonstrates the inefficiency of the production process, therefore greater unit exergetic cost emphasizes the more exergy destroyed to produce a flow. All the flows in the system have two unit exergetic costs, one is for fuel $\kappa_F^* = \dot{E}x_F^* / \dot{E}x_F$, the other is for product $\kappa_P^* = \dot{E}x_P^* / \dot{E}x_P$.

3.1.2. Exergetic cost determination for the biogas engine-powered cogeneration system

Basic information is given in the definition of exergetic cost. This information is put in order in the form of rules formulated by Valero [14]:

Rule 1: The fuel or product exergy of a component should be positive, $\dot{E}x_F, \dot{E}x_P > 0$. e.g. for component 7, LOHE:

$$\dot{E}x_{F,7} = \dot{E}x_{15} - \dot{E}x_{16} \quad (3a)$$

$$\dot{E}x_{15} > \dot{E}x_{16} \quad (3b)$$

$$\dot{E}x_{P,7} = \dot{E}x_{10} - \dot{E}x_9 \quad (3c)$$

$$\dot{E}x_{10} > \dot{E}x_9 \quad (3d)$$

Rule 2: In a component, the unit exergetic cost of each fuel is greater than or equal to one, $\kappa_F^* \geq \dot{E}x_F^* / \dot{E}x_F \geq 1$; also the streams that make up the fuel have the same unit exergetic cost.

e.g: for component 5, HE1:

$$\dot{E}x_{F,5} = \dot{E}x_8 - \dot{E}x_9 \quad (4a)$$

$$\kappa_8^* = \kappa_9^* \quad (4b)$$

$$\dot{E}x_8^* / \dot{E}x_8 = \dot{E}x_9^* / \dot{E}x_9 \quad (4c)$$

$$(1)\dot{E}x_8^* + (-x_5)\dot{E}x_9^* = 0 \quad (4d)$$

$$x_5 = \dot{E}x_8 / \dot{E}x_9 \quad (4e)$$

Rule 3: If a component has more than one product, all the products have the same unit exergetic cost in the absence of any external exergetic application. Furthermore, the unit exergetic cost of the product is always greater than the unit exergetic cost of the fuel for a generic component, $\kappa_P^* > \kappa_F^*$. In our system all components have a single product.

Rule 4: Residue or loss must appear explicitly as products in a subsystem; however, the exergetic cost of any loss or residue is equal to zero, $\dot{E}x_L^* = 0$; this situation results in exergetic cost equality for fuel and product, which is valid for any component,

Table 2

The incidence matrix, and fuel and product matrices showing the biogas engine-powered cogeneration system.

↓ Subsystem i	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	\dot{W}_{GE}	\dot{W}_T	\dot{W}_{P1}	\dot{W}_{P2}	\dot{W}_{P3}	\dot{W}_{P4}	←Flow j			
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	1	0	0	0	0	0	A	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	-1	0	0	0	0	0		
3	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0		
4	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0		
5	0	0	1	-1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
6	0	0	0	0	0	0	0	0	1	-1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		
9	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		
10	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0		
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	1	0	0	0	0	0	0	0		
The system	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	1	0	0	0	0	0	0		

↓ Subsystem i	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	\dot{W}_{GE}	\dot{W}_T	\dot{W}_{P1}	\dot{W}_{P2}	\dot{W}_{P3}	\dot{W}_{P4}	←Flow j			
1																										-1						A_F		
2																																		
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The system																			1								-1							

↓ Subsystem i	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	\dot{W}_{GE}	\dot{W}_T	\dot{W}_{P1}	\dot{W}_{P2}	\dot{W}_{P3}	\dot{W}_{P4}	←Flow j				
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The System																																			

Table 3
Fuel-product definition of the biogas engine-powered cogeneration system.

i	Component	Fuel	Product
1	Compressor	\dot{W}_T	21–20
2	Turbine	23–24	\dot{W}_T
3	Intercooler	13–12	21–22
4	EGHE	24–25	6–5
5	HE1	8–9	5–4
6	HE2	13–14	11–10
7	LOHE	15–16	10–9
8	P1	\dot{W}_{P1}	4–3
9	P2	\dot{W}_{P2}	8–7
10	P3	\dot{W}_{P3}	12–14
11	P4	\dot{W}_{P4}	18–17
12	Gas Engine	22	\dot{W}_{GE}
	The System	20–25	$\dot{W}_{GE} + (6-3)$

$$\dot{E}X_F^* = \dot{E}X_p^*$$

A few important points have to be discussed before the calculations. The line 23–24–25 in Fig. 1 is the exhaust gas line from the engine. Because no more exergy is spent to obtain these flows, their exergetic costs are equal to their exergies, as seen in Table 6. After the application of Rule 2 and Rule 3, a series of significant equations are obtained and are shown in Table 4 and Table 5. Eq. (2) is used to determine the exergetic costs and unit exergetic costs that are listed in Table 6. It is obvious that unit exergetic cost rises during the processes in the system, as more exergy is destroyed to produce the flows. It emphasizes the amount of fuel necessary for each exergetic component, and its value also demonstrates the production process inefficiency. It is understood from Table 6 that the unit exergetic costs show high increments up to 4.221 for the electrical work.

Fortunately, hot water flows have small exergy consumptions, which benefits cogeneration. In fact Table 6 has various aspects of exergetic cost theory. One of them is the work given to the pumps that are assumed to be the fuels, so their exergetic costs are equal to their exergy. Flow 20 is the real fuel for the system, which is the air-fuel mixture that naturally has the same exergetic cost as its exergy because air - at dead state - does not have exergy or an exergetic cost.

After obtaining the unit exergetic costs of flows, the next step is to calculate the exergetic costs of components. The incidence matrix, *A*, which explicitly shows the inputs and outputs of the cogeneration system is not enough on its own. Therefore, the fuel and product matrices, *A_F*, and, *A_P* as seen in Table 2, are used to determine the exergetic cost for each component.

Table 4
Auxiliary equations resulting from the application of Rule 2 for Fig. 1 and Table 1.

$\dot{E}X_{13}^* / \dot{E}X_{13} = \dot{E}X_{12}^* / \dot{E}X_{12} \rightarrow \dot{E}X_{13}^* - x_3 \dot{E}X_{12}^* = 0$	$x_3 = \dot{E}X_{13} / \dot{E}X_{12}$
$\dot{E}X_8^* / \dot{E}X_8 = \dot{E}X_9^* / \dot{E}X_9 \rightarrow \dot{E}X_8^* - x_5 \dot{E}X_9^* = 0$	$x_5 = \dot{E}X_8 / \dot{E}X_9$
$\dot{E}X_{15}^* / \dot{E}X_{15} = \dot{E}X_{16}^* / \dot{E}X_{16} \rightarrow \dot{E}X_{15}^* - x_7 \dot{E}X_{16}^* = 0$	$x_7 = \dot{E}X_{15} / \dot{E}X_{16}$

Table 5
Auxiliary equations resulting from the application of Rule 3 for Fig. 1 and Table 1.

$\dot{E}X_{11}^* / \dot{E}X_{11} = \dot{E}X_{10}^* / \dot{E}X_{10} \rightarrow \dot{E}X_{11}^* - x_6 \dot{E}X_{10}^* = 0$	$x_6 = \dot{E}X_{11} / \dot{E}X_{10}$
$\dot{E}X_4^* / \dot{E}X_4 = \dot{E}X_3^* / \dot{E}X_3 \rightarrow \dot{E}X_4^* - x_8 \dot{E}X_3^* = 0$	$x_8 = \dot{E}X_4 / \dot{E}X_3$
$\dot{E}X_8^* / \dot{E}X_8 = \dot{E}X_7^* / \dot{E}X_7 \rightarrow \dot{E}X_8^* - x_9 \dot{E}X_7^* = 0$	$x_9 = \dot{E}X_8 / \dot{E}X_7$
$\dot{E}X_{12}^* / \dot{E}X_{12} = \dot{E}X_{14}^* / \dot{E}X_{14} \rightarrow \dot{E}X_{12}^* - x_{10} \dot{E}X_{14}^* = 0$	$x_{10} = \dot{E}X_{12} / \dot{E}X_{14}$
$\dot{E}X_{18}^* / \dot{E}X_{18} = \dot{E}X_{17}^* / \dot{E}X_{17} \rightarrow \dot{E}X_{18}^* - x_{11} \dot{E}X_{17}^* = 0$	$x_{11} = \dot{E}X_{18} / \dot{E}X_{17}$

Table 6
Exergetic costs obtained from the solution of equation $A \times \dot{E}X^* = 0$.

Flow No	Exergy $\dot{E}x$ (kW)	Exergetic cost $\dot{E}x^*$ (kW)	Unit exergetic cost κ^*
3	350.9	417.6	1.190
4	352.8	419.688	1.190
5	447.3	670.958	1.500
6	518.7	859.466	1.656
7	391.4	594.666	1.518
8	400	607.153	1.518
9	234.3	355.684	1.518
10	285.9	536.45	1.876
11	291.7	547.179	1.876
12	49.83	50.726	1.017
13	58.37	59.071	1.014
14	47.67	48.342	1.014
15	357.7	658.981	1.843
16	259.4	478.216	1.843
17	220.9	784	3.549
18	248.7	882	3.549
20	4046.4	4046.4	1.000
21	4154.13	4228.6	1.020
22	4136.231	4221.155	1.021
23	374.3	374.3	1.000
24	192.2	192.2	1.000
25	3.692	3.692	1.000
\dot{W}_{GE}	1000	4221.155	4.221
\dot{W}_T	159.75	182.2	1.140
\dot{W}_{P1}	2.088	2.088	1.000
\dot{W}_{P2}	12.488	12.488	1.000
\dot{W}_{P3}	3.384	3.384	1.000
\dot{W}_{P4}	98	98	1.000

$$A = A_F - A_P \tag{5}$$

$$\dot{E}X_F = A_F \times \dot{E}X \tag{6a}$$

$$\dot{E}X_F^* = A_F \times \dot{E}X^* \tag{6b}$$

$$\dot{E}X_p = A_p \times \dot{E}X \tag{6c}$$

$$\dot{E}X_p^* = A_p \times \dot{E}X^* \tag{6d}$$

The relationship between the thermodynamics and economics to reach the thermoeconomic costs is given in the next section.

3.1.3. Thermoeconomic cost definitions

To calculate the price of fuels crossing the system, and the amortization, maintenance and overhead costs of the subsystems contained in a bounded system with a prescribed aggregation level,

some necessary definitions [4] are given below:

The thermoeconomic cost, \dot{C} , of a flow is the amount of monetary units per hour required to obtain this flow.

The unit thermoeconomic cost, c^* , of a flow is the cost of each unit of exergy consumed in producing this flow:

$$\dot{C} = c^* \cdot \dot{E}x^* \tag{7}$$

The unit exergoeconomic cost, c , of a flow is the cost of each unit of exergy concerned with this flow:

$$\dot{C} = c \cdot \dot{E}x \tag{8}$$

The unit exergetic cost, κ^* , gives a relationship between the unit exergoeconomic cost and the unit thermoeconomic cost:

$$\dot{C} = c^* \cdot \kappa^* \tag{9}$$

Having defined these sets of costs, it is not difficult to form the thermoeconomic cost balance equations.

3.1.4. Thermoeconomic cost balance

The thermoeconomic cost balance is achieved by the addition of economics to exergy. The incidence matrix, and the cost of maintenance and operation of the production system, \dot{Z} , give indications about the thermoeconomic cost.

$$A \times \dot{C} + \dot{Z} = 0 \tag{10}$$

\dot{Z} , monetary units per hour, can be computed by applying the usual procedures for evaluating equipment costs [3,26]. The thermoeconomic costs of the flows through the system are determined by the application of Eq. (10). In the case of the components, if product and fuel are defined for the specified unit, it can be determined that the thermoeconomic cost of the product is equal to the thermoeconomic cost of the fuel used to generate it, plus the cost of maintenance and operation of the production system:

$$\dot{C}_p = \dot{C}_f + \dot{Z} \tag{11a}$$

The thermoeconomic costs for fuel and product are related by unit thermoeconomic and exergetic costs as given in Eqs. (11b) and (11c):

$$\dot{C}_f = c_f^* \cdot \dot{E}x_f^* = (c_f^* \cdot \kappa_f^*) \cdot \dot{E}x_f = c_f \cdot \dot{E}x_f \tag{11b}$$

$$\dot{C}_p = c_p^* \cdot \dot{E}x_p^* = (c_p^* \cdot \kappa_p^*) \cdot \dot{E}x_p = c_p \cdot \dot{E}x_p \tag{11c}$$

3.1.5. Exergetic cost theory analysis of the biogas engine-powered cogeneration

The relationship between exergy, exergetic cost, unit exergetic cost and thermoeconomic costs are shown in Fig. 2. It is obvious that exergetic analysis that must be linked to economics is enough to perform a thermoeconomic cost analysis.

In Fig. 2 the use of vector products is essential, otherwise the calculations will be wrong. The steps to calculate thermoeconomic costs from the beginning are as follows:

- i. Specify the incidence matrix, A , for the system and the exergetic values, $\dot{E}x$, of all the flows.
- ii. Define the fuel and product for all subsystems and form the corresponding matrices, A_f , and A_p .

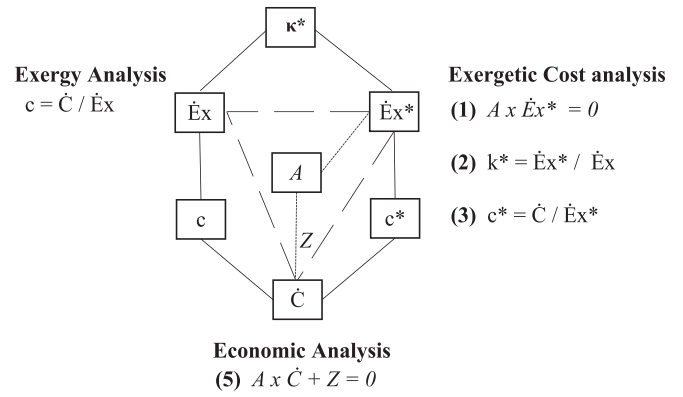


Fig. 2. Cost hexagon representing the relationships between costs.

- iii. Calculate the exergetic costs, $\dot{E}x^*$, of the flows by solving Eq. (2) then calculate the other exergetic costs, and $\dot{E}x_f^*$, $\dot{E}x_p^*$, for components and the unit exergetic costs, κ^* , κ_f^* , κ_p^* .
- iv. Find the amortization vector, \dot{Z} .
- v. Obtain the unit economic costs, c^* and c , and also the thermoeconomic cost \dot{C} .
- vi. Compute the thermoeconomic costs of fuel \dot{C}_f and product \dot{C}_p for each component.

To obtain more accurate results from the thermoeconomic analysis, the cost allocation of subsystems and other expenditures are obtained from the plant manager. The operating and maintenance costs are obtained by considering the entire cogeneration economic life, i.e. 25 years from 2006 to 2031. These costs are escalated by using the average nominal escalation rate, which is 5% in US dollars. The average capacity factor (τ) for the entire plant is 91.7%, which means that the system operates at full load for 8030 h out of the total available 8760 h per year. The total capital investment in the plant was 1.237 million US dollars. The purchased equipment costs (PEC), the hourly levelized costs of the capital investment (CI), the operating and maintaining (OM) costs, and the total costs of the components of the plant are given in Table 7.

The unit thermoeconomic cost of the air-fuel mixture is, $c_0 = c_{20} = c_{20}^* = 6.350\$/GJ$. Calculated thermoeconomic costs of flows are given in Table 8. It is interesting to see the highest thermoeconomic cost belongs to the flow 15, the lubrication oil flow, which is contrary to expectations because thoughts are naturally directed towards the work produced by the gas engine. LOHE is an extra component to transfer the heat from the lubrication oil to the water; therefore the cost of this process becomes expensive. In fact

Table 7

The cost rates associated with the initial capital investment, and the operating and maintenance costs for the components of the biogas engine-powered cogeneration.

Component	PEC (\$)	\dot{Z}^{CI} (\$/h)	\dot{Z}^{OM} (\$/h)	\dot{Z}^T (\$/h)
Compressor	38461.5	0.191	0.0095	0.2
Turbine	38461.5	0.191	0.0095	0.2
P1	2412.6	0.012	0.0006	0.0126
P2	1853.1	0.00923	0.0004615	0.00969
P3	1573.4	0.007837	0.000391	0.00822
P4	2237.81	0.0111	0.000557	0.01165
Intercooler	17482.5	0.087	0.0004354	0.09135
EGHE	1950	0.0975	0.000487	0.10237
HE1	8898.6	0.04432	0.002216	0.04653
HE2	8898.6	0.04432	0.002216	0.04653
LOHE	6993	0.0348	4.33×10^{-6}	0.0348
GE	419580.4	2.09	0.1045	2.194
System	–	–	–	2.95774

Table 8
Cost hexagon results for the system flows in Fig. 1.

Flow No	Exergy $\dot{E}x$ (kW)	Exergetic cost $\dot{E}x^*$ (kW)	Unit exergetic cost, κ^*	Unit thermo-economic cost, c^* (\$/GJ)	Unit exergoeconomic cost, c (\$/GJ)	Thermo-economic cost \dot{C} (\$/h)
3	350.9	417.6	1.190	26.553	31.570	39.880
4	352.8	419.688	1.190	26.553	31.570	40.080
5	447.3	670.958	1.500	26.044	39.067	62.910
6	518.7	859.466	1.656	26.110	43.300	80.855
7	391.4	594.666	1.518	25.193	38.235	53.876
8	400	607.153	1.518	25.193	38.200	55.008
9	234.3	355.684	1.518	25.193	38.203	32.224
10	285.9	536.45	1.876	35.135	65.671	67.592
11	291.7	547.179	1.876	35.135	65.653	68.944
12	49.83	50.726	1.017	34.553	35.175	6.310
13	58.37	60.0	1.014	32.520	34.292	7.206
14	47.67	48.342	1.014	33.884	34.362	5.897
15	357.7	658.981	1.843	50.792	93.136	119.934
16	259.4	478.216	1.843	50.792	93.201	87.035
17	220.9	784	3.549	25.031	88.727	70.56
18	248.7	882	3.549	25.031	88.661	79.38
20	4046.4	4046.4	1.000	6.350	6.350	92.5
21	4154.13	4228.6	1.020	7.030	7.156	107.017
22	4136.231	4221.155	1.021	6.940	7.082	105.461
23	374.3	374.3	1.000	23.517	23.517	31.690
24	192.2	192.2	1.000	26.531	26.531	18.357
25	3.692	3.692	1.000	34.985	34.985	0.465
\dot{W}_{GE}	1000	4221.155	4.221	7.243	30.573	110.065
\dot{W}_T	159.75	182.1	1.140	23.641	23.529	13.532
\dot{W}_{P1}	2.088	2.088	1	25	25	0.188
\dot{W}_{P2}	12.488	12.488	1	25	25	1.124
\dot{W}_{P3}	3.384	3.384	1	25	25	0.304
\dot{W}_{P4}	98	98	1	25	25	8.820

Table 9
Fuel cost results for the components in Fig. 1.

Component No	Exergy $\dot{E}x$ (kW)	Exergetic cost $\dot{E}x^*$ (kW)	Unit exergetic cost, κ^*	Unit thermo-economic cost, c^* (\$/GJ)	Unit exergoeconomic cost, c (\$/GJ)	Thermo-economic cost \dot{C} (\$/h)
1	159.75	182.1	1.139	20.641	23.529	13.532
2	182.1	182.1	1.000	20.336	20.336	13.332
3	8.54	9.274	1.121	26.807	29.111	0.895
4	188.508	188.508	1.000	26.364	26.364	17.892
5	165.7	251.47	1.517	25.167	38.194	22.784
6	10.70	10.729	1.002	33.890	33.982	1.309
7	98.3	180.766	1.838	50.554	92.966	32.899
8	2.088	2.088	1	25	25	0.188
9	12.488	12.488	1	25	25	1.124
10	3.384	3.384	1	25	25	0.304
11	98	98	1	25	25	8.820
12	4136.23	4221.155	1.020	6.939	7.082	105.461

this situation emphasizes that not just exergetic cost, but also the unit thermo-economic cost, plays a role in determining the total cost for each flow and component. Furthermore, it is seen from Table 8 that work given to the pumps has low cost because it is produced in the system. Although significant differences are observed in the thermo-economic costs of the flows, the unit thermo-economic costs are more stable in the cogeneration system.

Fuel costs for the components of the system are shown in Table 9. Work produced from the turbine results from the utilisation of the exhaust gas line; hence, the exergetic cost of fuel is accepted to be equal to its exergy for this device. The same situation is valid for the EGHE. It also uses exhaust gas as fuel. The compressor uses the turbine work; therefore the turbine product is the compressor's fuel. However, only their exergetic costs are the same, because of exergy destruction, not all of the produced work can be applied to the air-gas mixture. This result is concluded from the exergetic calculations during the analysis. The unit thermo-economic costs of the pumps are known beforehand, and their

exergetic costs are equal to their exergy, which was previously remarked upon.

Table 10 illustrates the product cost results of system components. As in the fuel case, the highest thermo-economic cost belongs to the gas engine. By observing Table 10 it is obvious that for component 3, the intercooler, there is a failure because of the job this component is supposed to do. The exergy of the product is greater than the exergy of the fuel, which is impossible according to the exergetic cost theory, but its main purpose is to cool the air-fuel mixture not to heat the water circulating in the closed loop through the HE1. During the application of the theory rules this point must be addressed. The end result of the exergetic cost theory, cost of the gas engine work flow is specified in Table 11.

3.2. Modified Productive structure analysis (MOPSA)

In this method, an exergy costing system without flow cost calculations was proposed by Kwak et al. [19]. Cost-balance

Table 10
Product cost results for the components in Fig. 1.

Component No	Exergy $\dot{E}x$ (kW)	Exergetic cost $\dot{E}x^*$ (kW)	Unit exergetic cost, κ^*	Unit thermo-economic cost, c^* (\$/GJ)	Unit exergoeconomic cost, c (\$/GJ)	Thermo-economic cost \dot{C} (\$/h)
1	107.73	182.1	1.690	22.144	37.431	14.517
2	159.75	182.1	1.139	20.641	23.529	13.532
3	17.9	9.274	2.437	46.605	24.146	1.556
4	71.4	188.508	2.640	39.755	104.960	26.979
5	94.5	251.47	2.661	25.218	67.107	22.830
6	5.8	10.729	1.849	35.003	64.750	1.352
7	51.6	180.766	3.503	54.348	190.396	35.368
8	1.9	2.088	1.098	26.607	29.239	0.2
9	8.6	12.488	1.452	25.179	36.563	1.132
10	3.300	3.384	1.025	36.117	37.037	0.44
11	27.8	98	3.525	25	88.129	8.82
12	1000	4221.155	4.221	7.055	29.783	107.221

equations, which are valid for any component of the cogeneration system, are obtained by assigning a unit exergy cost to each disaggregated exergy in the stream at any state. Subsequently, a set of equations for the unit exergy costs is formed for the system. The interactions between components in the costing process are naturally enclosed in the set of equations, i.e., a particular exergy cost in one component contributes to the evaluation of the exergy cost in another component. The monetary evaluations of various exergy costs, as well as the production cost of the electricity, are acquired by solving the set of equations for the unit exergoeconomic costs. In addition, the lost costs for each component of the system, which are connected to entropy generation processes, can be obtained by solving the cost-balance equations.

3.2.1. Thermo-economic cost balance for MOPSA

A general exergy-balance equation, applicable to any unit of a cogeneration system can be formulated by utilizing the first and second law of thermodynamics. To emphasize the exergy losses due to heat transfer, the exergy-balance equation for the non-adiabatic components is modified. The general exergy balance equation for this theory can be written as follows:

$$\begin{aligned} \dot{E}x^{CHE} + \left(\sum_{inlet} \dot{E}x_j^T - \sum_{outlet} \dot{E}x_j^T \right) + \left(\sum_{inlet} \dot{E}x_j^M - \sum_{outlet} \dot{E}x_j^M \right) \\ + T_0 \left(\sum_{inlet} \dot{S}_j - \sum_{outlet} \dot{S}_j + \dot{Q}_{CV} / T_0 \right) \\ = \dot{W} \end{aligned} \tag{12}$$

The term $\dot{E}x^{CHE}$ in Eq. (12) denotes the rate of chemical exergy flow of fuel in the plant, and the term \dot{Q}_{CV} shows the heat transfer interaction between a component and the environment. Specifying a unit exergy cost for every decomposed exergy stream, the cost-balance equation corresponding to the exergy-balance equation given in Eq. (12) is shown below:

$$\begin{aligned} \dot{E}x^{CHE} c_0 + \left(\sum_{inlet} \dot{E}x_j^T - \sum_{outlet} \dot{E}x_j^T \right) c_T + \left(\sum_{inlet} \dot{E}x_j^M - \sum_{outlet} \dot{E}x_j^M \right) c_M + \left(\sum_{inlet} \dot{E}x_j^M - \sum_{outlet} \dot{E}x_j^M \right) d_M + \\ \left(\sum_{inlet} \dot{E}x_j^T - \sum_{outlet} \dot{E}x_j^T \right) d_T + T_0 \left(\sum_{inlet} \dot{S}_j - \sum_{outlet} \dot{S}_j + \dot{Q}_{CV} / T_0 \right) c_S + \dot{Z} = \dot{W} c_W \end{aligned} \tag{13}$$

Table 11
Costs of work produced by the biogas engine according to the “Exergetic Cost Theory”.

Cost of Work Produced by Biogas Engine		
Component	Unit exergoeconomic cost c_W (\$/GJ)	Thermo-economic cost \dot{C}_W (\$/h)
Gas Engine	30.573	110.065

The unit exergoeconomic costs according to thermal and mechanical exergies, c_T, c_M are for gas streams and d_T, d_M are for the water and lubrication oil streams. The term \dot{Z} comprises all the financial charges associated with owning and operating the plant components as mentioned in the exergetic cost theory. In this method the total exergy is divided into thermal and mechanical parts. Eqs. (12) and (13) are the two main equations used in MOPSA analysis and Eq. (12) yields the productive structure of the system.

3.2.2. Cost balance equations based on MOPSA for the biogas engine-powered cogeneration

The MOPSA cost balance equations for each component in our cogeneration system, shown in Fig. 1, can be derived from the general cost balance equation given in Eq. (13). When the cost balance equation is used for a component, a new unit cost must be attributed to the component's principal product [20], whose unit cost is expressed in bold. For example, the compressor in the system is used to increase the mechanical exergy of air; the method assigns a new unit cost of c_{1M} to the mechanical exergy of air, the component's principal product. After the unit cost is assigned to the main product of the components, the cost balance equations for the biogas engine-powered cogeneration are formulated as follows:

- (1) Compressor

$$\begin{aligned} & (\dot{E}x_{20}^T - \dot{E}x_{21}^T)c_T + (\dot{E}x_{20}^M - \dot{E}x_{21}^M)c_{1M} + T_0(\dot{S}_{20} - \dot{S}_{21})c_S + \dot{Z}_1 \\ & = \dot{W}_C c_{W_T} \end{aligned} \quad (14)$$

(2) Turbine

$$\begin{aligned} & (\dot{E}x_{23}^T - \dot{E}x_{24}^T)c_T + (\dot{E}x_{23}^M - \dot{E}x_{24}^M)c_M + T_0(\dot{S}_{20} - \dot{S}_{21})c_S + \dot{Z}_2 \\ & = \dot{W}_T c_{W_T} \end{aligned} \quad (15)$$

(3) Intercooler

$$\begin{aligned} & (\dot{E}x_{21}^T - \dot{E}x_{22}^T)c_{3T} + (\dot{E}x_{21}^M - \dot{E}x_{22}^M)c_M + (\dot{E}x_{12}^T - \dot{E}x_{13}^T)d_T \\ & + (\dot{E}x_{12}^M - \dot{E}x_{13}^M)d_{M+} \\ & T_0(\dot{S}_{21} - \dot{S}_{22} + \dot{S}_{12} - \dot{S}_{13} + \dot{Q}_3 / T_0)c_S + \dot{Z}_3 = 0 \end{aligned} \quad (16)$$

(4) EGHE

$$\begin{aligned} & (\dot{E}x_5^T - \dot{E}x_6^T)d_{4T} + (\dot{E}x_5^T - \dot{E}x_6^T)d_M + (\dot{E}x_{24}^T - \dot{E}x_{25}^T)c_T \\ & + (\dot{E}x_{24}^M - \dot{E}x_{25}^M)c_{M+} \\ & T_0(\dot{S}_{24} - \dot{S}_{25} + \dot{S}_5 - \dot{S}_6 + \dot{Q}_4 / T_0)c_S + \dot{Z}_4 = 0 \end{aligned} \quad (17)$$

5) HE-1

$$\begin{aligned} & (\dot{E}x_4^T - \dot{E}x_5^T)d_{5T} + (\dot{E}x_8^T - \dot{E}x_9^T)d_T \\ & + (\dot{E}x_4^M - \dot{E}x_5^M + \dot{E}x_8^M - \dot{E}x_9^M)d_{M+} \\ & T_0(\dot{S}_4 - \dot{S}_5 + \dot{S}_8 - \dot{S}_9 + \dot{Q}_5 / T_0)c_S + \dot{Z}_5 = 0 \end{aligned} \quad (18)$$

6) HE-2

$$\begin{aligned} & (\dot{E}x_{10}^T - \dot{E}x_{11}^T)d_{6T} + (\dot{E}x_{13}^T - \dot{E}x_{14}^T)d_T + (\dot{E}x_{10}^M - \dot{E}x_{11}^M \\ & + \dot{E}x_{13}^M - \dot{E}x_{14}^M)d_{M+} \\ & T_0(\dot{S}_{10} - \dot{S}_{11} + \dot{S}_{13} - \dot{S}_{14} + \dot{Q}_6 / T_0)c_S + \dot{Z}_6 = 0 \end{aligned} \quad (19)$$

7) LOHE

$$\begin{aligned} & (\dot{E}x_9^T - \dot{E}x_{10}^T)d_{7T} + (\dot{E}x_{15}^T - \dot{E}x_{16}^T)d_T + (\dot{E}x_9^M - \dot{E}x_{10}^M + \dot{E}x_{15}^M \\ & - \dot{E}x_{16}^M)d_{M+} \\ & T_0(\dot{S}_9 - \dot{S}_{10} + \dot{S}_{15} - \dot{S}_{16} + \dot{Q}_7 / T_0)c_S + \dot{Z}_7 = 0 \end{aligned} \quad (20)$$

8) P-1

$$\begin{aligned} & (\dot{E}x_3^T - \dot{E}x_4^T)d_T + (\dot{E}x_3^M - \dot{E}x_4^M)d_{8M} + T_0(\dot{S}_3 - \dot{S}_4)c_S + \dot{Z}_8 \\ & = \dot{W}_{P1}d_w \end{aligned} \quad (21)$$

9) P-2

$$\begin{aligned} & (\dot{E}x_7^T - \dot{E}x_8^T)d_T + (\dot{E}x_7^M - \dot{E}x_8^M)d_{9M} + T_0(\dot{S}_8 - \dot{S}_9)c_S + \dot{Z}_9 \\ & = \dot{W}_{P2}d_w \end{aligned} \quad (22)$$

10) P-3

$$\begin{aligned} & (\dot{E}x_{14}^T - \dot{E}x_{12}^T)d_T + (\dot{E}x_{14}^M - \dot{E}x_{12}^M)d_{10M} + T_0(\dot{S}_{14} - \dot{S}_{12})c_S + \dot{Z}_{10} \\ & = \dot{W}_{P3}d_w \end{aligned} \quad (23)$$

11) P-4

$$\begin{aligned} & (\dot{E}x_{17}^T - \dot{E}x_{18}^T)d_T + (\dot{E}x_{17}^M - \dot{E}x_{18}^M)d_{11M} + T_0(\dot{S}_{17} - \dot{S}_{18})c_S + \dot{Z}_{11} \\ & = \dot{W}_{P4}d_w \end{aligned} \quad (24)$$

12) GE

$$\begin{aligned} & \dot{E}x_{C0}^{CHE} + (\dot{E}x_{22}^T - \dot{E}x_{23}^T)c_T + (\dot{E}x_{22}^M - \dot{E}x_{23}^M)c_M \\ & + (\dot{E}x_{11}^T - \dot{E}x_7^T + \dot{E}x_{18}^T - \dot{E}x_{15}^T)d_T + \\ & (\dot{E}x_{11}^M - \dot{E}x_7^M + \dot{E}x_{18}^M - \dot{E}x_{15}^M)d_M + T_0(\dot{S}_{22} - \dot{S}_{23} + \dot{S}_{11} - \dot{S}_7 \\ & + \dot{S}_{18} - \dot{S}_{15} + \dot{Q}_{12} / T_0)c_S + \dot{Z}_{12} = \dot{W}_{GE}c_w \end{aligned} \quad (25)$$

Both gas and water exergy streams pass through three components in the cogeneration system. Inlet and outlet flows are highlighted in Eqs. (16) and (17). Significant pressure changes don't occur in the heat exchangers but considerable heat losses must be taken into account, as the series of equations from (16) to (20) emphasize. These instructions are necessary to formulate the complete exergy and cost balances for the system. Twelve cost balance equations from twelve units of the plant are reproduced with 17 unknown unit exergoeconomic costs c_{1M} , c_{W_T} , c_{3T} , d_{4T} , d_{5T} , d_{6T} , d_{7T} , d_{8M} , d_{9M} , d_{10M} , d_{11M} , c_w , c_T , c_M , c_S , d_T and d_M . Five more cost balance equations can be obtained for the junctions of thermal

and mechanical exergies of the gas and water streams.

Gas streams

$$(\dot{E}x_{21}^T - \dot{E}x_{22}^T)c_{3T} = (\dot{E}x_{21}^T - \dot{E}x_{22}^T)c_T \quad c_{3T} = c_T \quad (26)$$

$$(\dot{E}x_{20}^M - \dot{E}x_{21}^M)c_{1M} = (\dot{E}x_{20}^M - \dot{E}x_{21}^M)c_M \quad c_{1M} = c_M \quad (27)$$

Water streams

$$(\dot{E}x_5^T - \dot{E}x_6^T)d_{4T} + (\dot{E}x_4^T - \dot{E}x_5^T)d_{5T} + (\dot{E}x_{10}^T - \dot{E}x_{11}^T)d_{6T} + (\dot{E}x_9^T - \dot{E}x_{10}^T)d_{7T} = \quad (28)$$

$$\begin{aligned} &(-\dot{E}x_6^T + \dot{E}x_4^T - \dot{E}x_{11}^T + \dot{E}x_9^T)d_T \\ &(\dot{E}x_3^M - \dot{E}x_4^M)d_{8M} + (\dot{E}x_7^M - \dot{E}x_8^M)d_{9M} + (\dot{E}x_{14}^M - \dot{E}x_{12}^M)d_{10M} \\ &+ (\dot{E}x_{17}^M - \dot{E}x_{18}^M)d_{11M} = \\ &(\dot{E}x_3^M - \dot{E}x_4^M + \dot{E}x_7^M - \dot{E}x_8^M + \dot{E}x_{14}^M - \dot{E}x_{12}^M + \dot{E}x_{17}^M - \dot{E}x_{18}^M)d_M \end{aligned} \quad (29)$$

Another cost balance equation suitable for the exergy-balance for the boundary of the system can be expressed as follows:

$$\begin{aligned} &(\dot{E}x_{20}^T - \dot{E}x_{25}^T)c_T + (\dot{E}x_{20}^M - \dot{E}x_{25}^M)c_M + (\dot{E}x_3^T - \dot{E}x_6^T)d_T \\ &+ (\dot{E}x_3^M - \dot{E}x_6^M)d_M + \\ &T_0(\dot{S}_{20} - \dot{S}_{25} + \dot{S}_3 - \dot{S}_6)C_S + \dot{Z}_{bound} = 0 \end{aligned} \quad (30)$$

\dot{Z}_{bound} is the plant construction cost, which is one-third or two-thirds of the equipment costs. In these calculations it is taken to be one-third of the total cogeneration. A unit cost that defines the aim of the component is used to form the cost balance equations. Different unit costs are assigned depending on the type of fluids. Entropy generation acts in the same way everywhere, so its unit cost is unique. Table 12 shows thermal and mechanical exergy flow rates and entropy flow rates at various state points shown in Fig. 1. These flow rates are calculated based on the values of measured properties such as pressure, temperature and the mass flow rate at various points (see Table 1).

Various exergy flow rates passing the boundary of each unit in the cogeneration system are presented in Table 13. Positive values of exergies show the product exergy flow rate, on the other hand negative values indicate the resource flow rate or fuel. In this situation, the product of the unit is the added exergy while the source is consumed. Entropy produced in each component is assumed to be the product of the exergy-balance equation. Summation of the exergy flow rates of resources and products is equal to zero for each unit and for the total system. This zero sum shows that exergy

Table 13
Exergy balances for each component in the system of Fig. 1 according to the MOPSA.

Component	$\dot{E}x^W$ (kW)	$\dot{E}x^T$ (kW)	$\dot{E}x^M$ (kW)	\dot{S} (kW)	$\dot{E}x^{CHE}$ (kW)
Compressor	-155.21	20.059	88.06	47.091	0.000
Turbine	159.750	-90.900	-92.150	23.300	0.000
Intercooler	0.000	-10.278	-1.01	11.288	0.000
EGHE	0.000	-91.408	-25.81	117.218	0.000
HE1	0.000	-70.90	-0.34	71.24	0.000
HE2	0.000	-1.88	-2.98	4.86	0.000
LOHE	0.000	-46	-0.739	46.739	0.000
P1	-2.088	0.088	2.0	0.000	0.000
P2	-12.488	1.100	7.49	3.898	0.000
P3	-3.384	0.010	3.383	0.000	0.000
P4	-98	14.500	13.330	70.17	0.000
GE	1000	-480.829	-12.357	3539.586	-4046.4
System	888.58	-756.438	-21.123	3935.38	-4046.4

Table 12
Property values, thermal and mechanical exergy flows and entropy production rates at various state points in the biogas engine-powered cogeneration system.

State No	Fluid	Pressure P (bar)	Temperature T (°C)	Mass flow rate \dot{m} (kg/sn)	Entropy generation rate \dot{s} (kJ/kgK)	Thermal exergy \dot{E}^T (kW)	Mechanical exergy $\dot{E}x^M$ (kJ/kg)
0	Air	1.00	25.00	—	—	0.00	0.00
0'	Water	1.00	25.00	—	—	0.00	0.00
0''	Biogas	1.00	25.00	—	—	—	—
0'''	Lub oil	1.00	25.00	—	—	0.00	0.00
2	Biogas	1.02	30.10	0.129	1.501	0.013	0.39
3	Water	5.25	75.80	20.88	21.402	342.00	8.89
4	Water	6.20	75.80	20.88	21.402	341.90	10.89
5	Water	6.10	82.80	20.88	23.135	436.60	10.68
6	Water	3.40	88.00	20.88	24.408	513.60	5.020
7	Water	2.80	88.40	15.61	18.326	388.60	2.810
8	Water	7.60	88.50	15.61	18.341	389.70	10.30
9	Water	7.50	72.40	15.61	15.355	224.10	10.17
10	Water	7.30	77.90	15.61	16.39	276.00	9.86
11	Water	7.20	78.50	15.61	16.49	282.00	9.70
12	Water	4.55	50.00	11.28	7.935	47.00	2.83
13	Water	4.50	52.10	11.28	8.240	55.37	3.00
14	Water	1.10	50.00	11.28	7.937	47.49	0.18
15	Lub oil	4.69	100.6	20.00	12.542	349.4	8.337
16	Lub oil	4.50	89.00	20.00	11.158	251.5	7.908
17	Lub oil	1.00	85.00	20.00	10.676	220.9	0.00
18	Lub oil	6.90	87.00	20.00	10.918	235.4	13.33
19	Air	1.00	25.00	1.387	7.904	0.00	0.00
20	Air-fuel	1.00	25.00	1.50	9.311	0.00	0.00
21	Air-fuel	1.90	116.9	1.50	9.469	20.059	88.06
22	Air-fuel	1.90	51.00	1.50	9.155	1.771	88.06
23	Exhaust gas	2.40	460.0	1.50	9.562	262.00	112.30
24	Exhaust gas	1.17	360.60	1.50	9.637	172.1	20.15
25	Exhaust gas	1.00	65	1.50	8.739	3.692	0

Table 14
Cost flow rates for each component in the system of Fig. 1 according to the MOPSA.

Component	\dot{C}_W or \dot{D}_W (\$/h)	\dot{C}_T (\$/h)	\dot{C}_M (\$/h)	\dot{D}_T (\$/h)	\dot{D}_M (\$/h)	\dot{C}_0 (\$/h)	\dot{C}_S (\$/h)	\dot{Z} (\$/h)
Compressor	-17.107	0.034	17.340	0.000	0.000	0.000	-0.067	-0.200
Turbine	17.390	-0.154	-17.004	0.000	0.000	0.000	-0.032	-0.200
Intercooler	0.000	0.0383	0.000	0.113	-0.0227	0.000	-0.0386	-0.090
EGHE	0.000	0.430	-3.042	5.238	-2.258	0.000	-0.266	-0.102
HE1	0.000	0.000	0.000	0.364	-0.148	0.000	-0.170	-0.046
HE2	0.000	0.000	0.000	1.274	-1.215	0.000	-0.013	-0.046
LOHE	0.000	0.000	0.000	0.445	-0.294	0.000	-0.117	-0.034
P1	-0.187	0.000	0.000	0.00185	0.1977	0.000	0.000	-0.0126
P2	-1.123	0.000	0.000	0.0203	1.1223	0.000	-0.010	-0.0096
P3	-0.304	0.000	0.000	0.000185	0.312	0.000	0.000	-0.0082
P4	-8.820	0.000	0.000	0.2682	8.7312	0.000	-0.1684	-0.011
GE	85.536	0.442	4.710	0.957	4.705	-92.5	-1.656	-2.194
Boundary	0.000	0.010	0.000	3.182	1.544	0.000	-2.812	-1.914
System	75.385	0.8003	2.004	12.491	12.674	-92.500	-5.35	-4.877

balances have been perfectly executed.

Table 13 presents the productive structure of the system. The most significant entropy production (exergy destruction) takes place in the gas engine. Nearly 25% of the fuel chemical exergy is converted to work and the other part is destroyed. Also, heat exchangers are remarkable entropy generators because of lost heat. As stated in the exergetic cost theory, not all of the work produced in the turbine is used in the compressor, there is exergy destruction. In the system, the lost exergy corresponds to 81.579% of the total exergy input, and is calculated to be 3935.38 kW.

Table 14 denotes the cost flow rates corresponding to the subsystems' exergy flow rates and the construction costs. The unit exergoeconomic cost of biogas is 6.350\$/GJ which was demonstrated before. The same sign order for the products and resources' cost flow rates was found, which was used to define the exergy balances in Table 13. Entropy generation cost in a component shows the consumed cost. The summation of the cost flow rates for each unit in the system is equal to zero satisfying true cost balances.

At the end of the MOPSA calculations, pump 4 indicates a high mechanical thermoeconomic cost in Table 14, which means the inverse of the exergetic cost theory is not profitable. It is obvious that using exhaust gas to heat the water through the EGHE does not seem advantageous. Finally, the cost of the gas engine work flow, according to the modified productive structure analysis, is given in Table 15.

Table 15 indicates that the MOPSA cost results are smaller than the results obtained from the ECT analysis. The reasons for this will be explained in detail in the results and discussion section.

3.3. The Wonerger method

This methodology was introduced by Kim [21] for the fields of cost allocation, cost optimization and cost analysis. In this method various energies combining enthalpy and exergy are integrated in "Wonerger", a fusion of worth and energy. Wonerger is defined as an energy that can equally evaluate the worth of each product. The first law of thermodynamics emphasizes the energy balance

Table 15
Costs of work produced by the biogas engine-powered cogeneration according to the MOPSA.

Cost of Work Produced by Biogas Engine-Powered Cogeneration		
Component	Unit Exergoeconomic cost c_W (\$/GJ)	Thermoeconomic cost \dot{C}_W (\$/h)
Gas Engine	23.760	85.536

equation for the *i*-th component and overall system as follows:

$$\dot{W}_i + \dot{E}_{Q,i} = \dot{E}_{F,i} - \dot{E}_{P,i} - \dot{E}_{L,i} \tag{31}$$

$$\dot{W} + \dot{E}_Q = \dot{E}_F^{CHE} - \dot{E}_L \tag{32}$$

where \dot{W} and \dot{E}_Q are the quantity of total work and heat, which are the desirable final products of a cogeneration system; $\dot{E}_{F,i}$ is the fuel inlet, $\dot{E}_{P,i}$ is the product output and $\dot{E}_{L,i}$ is the heat lost into the environment. The tabulated values of these energy equations are listed in Table 16. The exergy balance equation for the *i*-th component and overall system is written as follows:

$$\dot{W}_i + \dot{E}x_{Q,i} = \dot{E}x_{F,i} - \dot{E}x_{P,i} - \dot{E}x_{L,i} \tag{33}$$

$$\dot{W} + \dot{E}x_Q = \dot{E}x_F^{CHE} - \dot{E}x_L \tag{34}$$

where $\dot{E}x_Q$ is the quantity of hot water exergy. Table 17 shows the values of each the terms in Eqs. (33) and (34).

The energy balance equations agree with the exergy balance equations. Therefore, combining energy and exergy in Wonerger, and then modifying the symbols of \dot{E} and $\dot{E}x$ with \dot{K} , the Wonerger balance equation can be written as follows:

$$\dot{W}_i + \dot{K}_{Q,i} = \dot{K}_{F,i} + \dot{K}_{P,i} - \dot{K}_{L,i} \tag{35}$$

$$\dot{W} + \dot{K}_Q = \dot{K}_F - \dot{K}_L \tag{36}$$

Table 16
The energy balance of the biogas engine-powered cogeneration according to the Wonerger method.

Component	\dot{W} (kW)	\dot{E}_Q (kW)	E_F^{CHE} (kW)	\dot{E}_F (kW)	\dot{E}_P (kW)	\dot{E}_L (kW)
Compressor	-155.21	-	-	-	-155.21	-
Turbine	159.75	-	-	159.75	-	-
Intercooler	-	-	-	99.26	-99	0.26
EGHE	-	-	-	456.16	451	5.16
HE1	-	-	-	1055.27	-611.85	443.42
HE2	-	39	-	103	-	64
LOHE	-	-	-	512	-360	152
P1	-2.088	-	-	-	-2.088	-
P2	-12.488	-	-	-	-12.488	-
P3	-3.384	-	-	-	-3.384	-
P4	-98	-	-	-	-98	-
GE	1000	-	2308.06	-	-592.42	715.64
System	888.58	39	2308.06	2385.44	-2385.44	1380.48

Table 17

The exergy balance of the biogas engine-powered cogeneration system according to the Wonerger method.

Component	\dot{W} (kW)	\dot{E}_{XQ} (kW)	Ex_F^{CHE} (kW)	\dot{E}_{XF} (kW)	\dot{E}_{XP} (kW)	\dot{E}_{XL} (kW)
Compressor	-155.21	-	-	-	-107.73	47.48
Turbine	159.75	-	-	182.05	-	22.3
Intercooler	-	-	-	17.9	-8.241	9.65
EGHE	-	-	-	188.505	-71.4	117.105
HE1	-	-	-	165.7	-94.5	71.2
HE2	-	5.8	-	10.84	-	5.04
LOHE	-	-	-	98.3	-51.6	46.7
P1	-2.088	-	-	-	-2.088	-
P2	-12.488	-	-	-	-12.488	-
P3	-3.384	-	-	-	-3.384	-
P4	-98	-	-	-	-98	-
GE	1000	-	4046.4	-	-213.7	2832.655
System	888.58	5.8	4046.4	663.345	-663.345	-3152.02

The cost-balance equation requires that the sum of the output costs must be equal to the sum of the input costs for the whole system.

$$c_w \dot{W} + c_Q \dot{E}_Q = \dot{C}_0 + \dot{C}_{CO_2} + \dot{Z}_{ID} + \sum \dot{Z} \quad (37)$$

where c_w and c_Q are the unit exergoeconomic cost of work and heat produced by the system, \dot{C}_{CO_2} is the environmental pollution cost flow - such as carbon emission, \dot{Z}_{ID} is the sum of the indirect cost flow passing from the outside to the system, and \dot{Z} is the capital investment. The small environmental pollution cost flow for our system is neglected, and there is no indirect cost flow passing from the outside to the system so, \dot{C}_{CO_2} and values \dot{Z}_{ID} are equal to zero.

The Wonerger method divides the system into three components: the common components [c] associated with work and heat production, the work-only components [W] associated with work production, and the heat-only components [Q] associated with heat production. According to these classifications, the common components are HE1, P1, P2, P3 and P4; the work-only components are the compressor, the turbine, the gas engine, the EGHE, the LOHE, and the heat-only component is the HE2 in the biogas engine-powered cogeneration system. Here, the EGHE can be a common component or a work component; however, our system is part of a wastewater treatment plant where it is more reasonable that the EGHE contributes to work. It is obvious from the Wonerger equations, Tables 16 and 17, that the summation of $\dot{K}_{F,i}$ and $\dot{K}_{P,i}$ for all components is exactly zero, as expressed in Eq. (39). Thus, multiplying $(\dot{K} + \dot{K}_W + \dot{K}_Q = \sum K_{F,i} + K_{P,i})$ by the wonergetic unit cost c_K , and adding the term to Eq. (37), that the wonergetic cost balance equation can be formed:

$$c_w \dot{W} + c_Q \dot{E}_Q = \dot{C}_0 + \dot{Z}_W + \dot{Z}_Q + c_K (\dot{K} + \dot{K}_W + \dot{K}_Q) \quad (38)$$

This method asserts that the input cost flow in the common components is fairly distributed to work and heat by Wonerger, the input cost flow in the work-only components is entirely distributed for producing work, and the input cost flow in the heat-only components is completely distributed for producing heat. According to this idea, Eq. (38) for all components is split up into Eq. (40) for [c] components, Eq. (41) for [W] components and Eq. (42) for [Q] components.

$$\dot{K} + \dot{K}_W + \dot{K}_Q = 0 \quad (39)$$

$$0 = \dot{C}_0 + c_K \dot{K} \quad (40)$$

$$c_w \dot{W} = \dot{Z}_W + c_K \dot{K}_W \quad (41)$$

$$c_Q \dot{E}_Q = \dot{Z}_Q + c_K \dot{K}_Q \quad (42)$$

where each value of \dot{K}_W and \dot{K}_Q is positive and \dot{K} is negative. The signs for these values can be checked from the values of \dot{E}_F , \dot{E}_P , \dot{E}_{XF} , \dot{E}_{XP} and from Tables (16) and (17). If these equations are rearranged, the exergoeconomic cost of work, c_w , the exergoeconomic cost of heat, c_Q , the work cost flow rate, \dot{C}_W , and the heat cost flow rate, \dot{C}_Q , can be determined as follows:

$$c_w = k_W \cdot \frac{\dot{C}_0}{k_W \cdot \dot{W} + k_Q \cdot \dot{E}_Q} + \frac{\dot{Z}_W}{\dot{W}} \quad (43)$$

$$\dot{C}_W = c_w \dot{W} = \dot{K}_W \cdot \frac{\dot{C}_0}{\dot{K}_W + \dot{K}_Q} + \dot{Z}_W \quad (44)$$

$$c_Q = k_Q \cdot \frac{\dot{C}_0}{k_W \cdot \dot{W} + k_Q \cdot \dot{E}_Q} + \frac{\dot{Z}_Q}{\dot{E}_Q} \quad (45)$$

$$\dot{C}_Q = c_Q \dot{E}_Q = \dot{K}_Q \cdot \frac{\dot{C}_0}{\dot{K}_W + \dot{K}_Q} + \dot{Z}_Q \quad (46)$$

where $k_W = \dot{K}_W / \dot{W}$ and $k_Q = \dot{K}_Q / \dot{E}_Q$. \dot{K} is the Wonerger input and k is the Wonerger input ratio. Equations (43)–(46) help solve the cost problem for the system. Numerical values used in wonergetic calculations include: the heat input of the biogas, \dot{E}_F^{CHE} , of 2308.06 kW, the total electricity product, \dot{W} , of 888.58 kW, and the heat product, \dot{E}_Q , of 39 kW. The cost of components was calculated and the results are shown in Table 18. As understood from Table 18, producing work is much more expensive than producing heat because the amount of Wonerger input for work is greater than heat. Of course this is the normal situation for cogeneration systems since heat is a by-product.

At the end of the Wonerger analysis, the cost of the work flow produced by the biogas engine-powered cogeneration is shown in Table 19.

3.4. Specific exergy Costing method (SPECOC)

Specific exergy costing is a systematic and general methodology

Table 18
Cost allocations for biogas engine-powered cogeneration using the Wonerger method.

Component	\dot{K} (kW)	\dot{W}, \dot{Q} (kW)	k %	\dot{C}_0 (\$/h)	\dot{Z}_{ID} (\$/h)	\dot{Z} (\$/h)	c (\$/GJ)	\dot{C} (\$/h)
Common []	-35.11	-	-	-	0	0.133	94.222	-
Work only [W]	24.27	888.58	2.731	92.451	0	0.491	20.138	64.419
Heat only [Q]	10.84	39	27.794	-	0	0.046	202.542	28.436

Table 19
Costs of work produced by the biogas engine-powered according to the Wonerger method.

Cost of Work Produced by Biogas Engine		
Component	Unit Exergoeconomic cost c_w (\$/GJ)	Thermoeconomic cost \dot{C}_W (\$/h)
Gas Engine	20.138	72.5

to calculate efficiencies and costs in thermal systems [22] such as cogeneration. This method models the fuel and product definitions as the exergy additions to, and removal from, a component [16]. The cost rates are obtained by using basic principles of business administration. The specific exergy costing method includes three steps. The first is the identification of energy and exergy flows through the system components. The calculated energy and exergy for each state of the flows are given in Table 1. The second is an explanation of the fuel and product rule, the main principle of SPECO, to determine the exergoeconomic cost of the flows. Finally, the exergetic cost balance and auxiliary equations for the subsystems are derived.

3.4.1. Fuel and product rule

The fuel rule is related to the removal of exergy from a flow through a component. The definition of fuel should include the difference between the inlet and outlet flow. It indicates that the total cost regarding the removal of exergy is the same as the cost at which the extracted exergy is provided to the same flow in the next component. The product rule is concerned with the supply of exergy flow through the component. It states that each exergy unit is provided to any flow corresponding to products with the same exergoeconomic cost. This cost can be obtained using the cost balance or other equations formed by applying the fuel rule. Fig. 3 is a schematic of a component in any system to explain fuel, product and auxiliary costing equations.

All flows represented in Fig. 3 are exergy streams. The exit stream of the second flow is smaller than its inlet stream, so the fuel (F) rule introduces Eqs. (49) and (50).

$$c_{2in} = c_{2out} \tag{49}$$

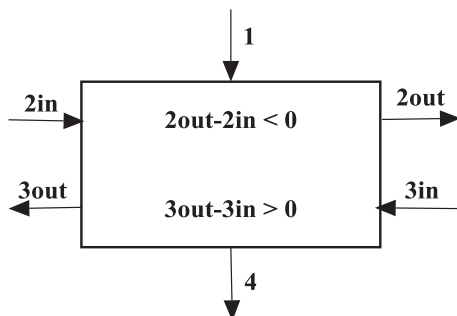


Fig. 3. Inlet and outlet exergetic streams for a component in any system.

$$c_4 = \frac{\dot{C}_{3out} - \dot{C}_{3in}}{\dot{E}x_{3out} - \dot{E}x_{3in}} \tag{50}$$

The requirements of the fuel (F) and product (P) rule are defined, then the cost balance equations are formed in the next section.

3.4.2. Cost balance equations for SPECO

According to the inlet and outlet flows, which carry fuel exergy into a component or product exergy from it, the cost balance equations are written as follows:

$$\dot{C}_F = c_F \dot{E}x_F \tag{51}$$

$$\dot{C}_P = c_P \dot{E}x_P \tag{52}$$

$$\dot{C}_W = c_W \dot{W} \tag{53}$$

$$\dot{C}_Q = c_Q \dot{Q} \tag{54}$$

Here c_{in} , c_e , c_w , c_Q define exergoeconomic costs per unit of exergy for inlet, outlet, work and heat flows, and \dot{C}_{in} , \dot{C}_e , \dot{C}_W , \dot{C}_Q are thermoeconomic costs related to the corresponding exergy streams. Generally, SPECO defines the cost balance equation for the i -th subsystem [22]

$$c_{F,i} \dot{E}x_{F,i} - c_{P,i} \dot{E}x_{P,i} \pm c_{W,i} \dot{E}x_{W,i} \pm c_{Q,i} \dot{E}x_{Q,i} + \dot{Z}_i = 0 \tag{55}$$

SPECO cost balance equations for each component in the cogeneration system, shown in Fig. 1, can be derived from the cost balance equation given in Eq. (55). Table 20 shows these equations.

The cost balance equations shown in Table 20 do not include heat losses in the components because SPECO injects these costs into products. Auxiliary equations helped to solve the cost balance equations. There are some assumptions for the system; one of them is giving the same unit exergoeconomic cost to all work produced or applied in the systems except the gas engine, since it is the main producer of the work that is converted into electricity. The cost of the water flow from the anaerobic digester part of the wastewater treatment plant, c_3 , is already known. Equipment costs, \dot{Z} , were calculated beforehand.

Table 20 leads to the formulation of Table 21. The costs of flows through the system are calculated and the results are given in Table 21. Unit exergoeconomic costs of work flows belonging to pumps are assumed to be known, thus it is easy to find their thermoeconomic costs. Table 22 and Table 23 illustrate values of fuel and product thermoeconomic costs for each component. It is apparent from the tables that the most expensive unit in the system, according to the SPECO analysis is the gas engine due to the fuel and product prices. The cost of the work flow of the gas engine in SPECO perspective is given in Table 24.

4. Results and discussion

Four methodologies, based on exergy and economy, are examined in this study to analyse an existing cogeneration system which

Table 20
Exergetic cost balances and corresponding auxiliary equations for the components in Fig. 1.

Component	Cost balance equation	Auxiliary equation
Compressor	$c_{20}\dot{E}x_{20} - c_{21}\dot{E}x_{21} + c_e\dot{W}_C + \dot{Z}_1 = 0$	$c_{20} = 6.35 \frac{\$}{GJ}$
Turbine	$c_{23}\dot{E}x_{23} - c_{24}\dot{E}x_{24} - c_e\dot{W}_T + \dot{Z}_2 = 0$	$c_{23} = c_{24}$
Intercooler	$c_{21}\dot{E}x_{21} - c_{22}\dot{E}x_{22} + c_{12}\dot{E}x_{12} - c_{13}\dot{E}x_{13} + \dot{Z}_3 = 0$	$c_{21} = c_{22}$
EGHE	$c_{24}\dot{E}x_{24} - c_{25}\dot{E}x_{25} + c_5\dot{E}x_5 - c_6\dot{E}x_6 + \dot{Z}_4 = 0$	$c_{24} = c_{25}$
HE1	$c_8\dot{E}x_8 - c_9\dot{E}x_9 + c_4\dot{E}x_4 - c_5\dot{E}x_5 + \dot{Z}_5 = 0$	$c_8 = c_9$
HE2	$c_{10}\dot{E}x_{10} - c_{11}\dot{E}x_{11} + c_{13}\dot{E}x_{13} - c_{14}\dot{E}x_{14} + \dot{Z}_6 = 0$	$c_{13} = c_{14}$
LOHE	$c_{15}\dot{E}x_{15} - c_{16}\dot{E}x_{16} + c_9\dot{E}x_9 - c_{10}\dot{E}x_{10} + \dot{Z}_7 = 0$	$c_{15} = c_{16} = c_{17}$
P1	$c_3\dot{E}x_3 - c_4\dot{E}x_4 + c_e\dot{W}_{P1} + \dot{Z}_8 = 0$	$c_3 = 31.570 \frac{\$}{GJ}$
P2	$c_7\dot{E}x_7 - c_8\dot{E}x_8 + c_e\dot{W}_{P2} + \dot{Z}_9 = 0$	–
P3	$c_{14}\dot{E}x_{14} - c_{12}\dot{E}x_{12} + c_e\dot{W}_{P3} + \dot{Z}_{10} = 0$	–
P4	$c_{17}\dot{E}x_{17} - c_{18}\dot{E}x_{18} + c_e\dot{W}_{P4} + \dot{Z}_{11} = 0$	–
GE	$c_{22}\dot{E}x_{22} - c_{23}\dot{E}x_{23} + c_{11}\dot{E}x_{11} - c_7\dot{E}x_7 + c_{18}\dot{E}x_{18} - c_{15}\dot{E}x_{15} - c_W\dot{W} + \dot{Z}_{12} = 0$	$\dot{W}_{GE} = 1000 \text{ kW}$

Table 21
Cost results of the biogas engine-powered cogeneration flows according to the SPECO.

Flow No	Exergy $\dot{E}x$ (kW)	Unit exergetoeconomic cost, c (\$/GJ)	Thermoeconomic cost \dot{C} (\$/h)
3	350.9	31.570	39.880
4	352.8	31.550	40.086
5	447.3	40.242	64.894
6	518.7	43.300	80.855
7	391.4	41.660	58.876
8	400	41.721	60.078
9	234.3	41.721	35.190
10	285.9	66.111	68.044
11	291.7	65.833	69.314
12	49.83	34.612	6.209
13	58.37	31.661	6.653
14	47.53	31.661	5.417
15	357.7	93.136	119.934
16	259.4	93.136	86.974
17	220.9	93.136	74.065
18	248.7	92.5	173.592
20	4046.4	6.35	92.5
21	4154.13	7.211	107.591
22	4136.231	7.211	107.128
23	374.3	23.517	31.690
24	192.2	23.517	18.357
25	3.692	23.517	0.465
\dot{W}_C	155	27.152	15.15
\dot{W}_T	159.75	27.152	15.615
\dot{W}_{GE}	1000	39.166	141.0
\dot{W}_{P1}	2.088	25	0.188
\dot{W}_{P2}	12.488	25	1.124
\dot{W}_{P3}	3.384	25	0.304
\dot{W}_{P4}	98	25	8.820

Table 22
Fuel cost results for the components of the biogas engine-powered cogeneration system according to the SPECO analysis.

Component	$\dot{E}x_F$ (kW)	Unit Exergetoeconomic Cost c (\$/GJ)	Thermoeconomic Cost \dot{C} (\$/h)
Compressor	155.0	27.152	15.15
Turbine	182.1	23.517	15.416
Intercooler	8.54	14.40	0.444
EGHE	188.508	26.364	17.892
HE1	165.7	41.721	24.888
HE2	10.84	31.67	1.236
LOHE	98.3	26.364	32.96
P1	2.088	25	0.188
P2	12.488	25	1.124
P3	3.384	25	0.304
P4	98	25	8.820
GE	4136.23	7.211	107.128

Table 23
Product cost results for the components of the biogas engine-powered cogeneration system according to the SPECO analysis.

Component	\dot{E}_{Xr} (kW)	Unit Exergoeconomic Cost c (\$/GJ)	Thermoeconomic Cost \dot{C} (\$/h)
Compressor	107.73	38.91	15.091
Turbine	159.75	27.152	15.615
Intercooler	17.9	7.184	0.463
EGHE	71.4	62.095	15.961
HE1	94.5	72.921	24.808
HE2	5.8	60.823	1.27
LOHE	51.6	176.862	32.854
P1	1.9	30.116	0.206
P2	8.6	38.824	1.202
P3	3.300	66.666	0.792
P4	27.8	87.280	8.735
GE	1000	39.166	141

Table 24
Costs of work produced by the biogas engine-powered cogeneration according to the SPECO.

Cost of Work Produced by Biogas Engine		
Component	Unit Exergoeconomic cost c_w (\$/GJ)	Thermoeconomic cost \dot{C}_w (\$/h)
Gas Engine	39.166	141

uses a biogas engine. All methods initially carried out a thermodynamic analysis. Energetic and exergetic values of streams flowing through the system in a given state were obtained using a computer program. System data, thermodynamic properties, energy and exergy rates in the plant with respect to state points in Fig. 1 are given in Table 1. Thermoeconomic analyses start with the exergetic analysis, therefore necessary exergetic definitions are provided. The exergy balance for each component of the cogeneration system was formed. It was pointed out that product exergy should be smaller than the fuel exergy for every component. Thermoeconomic instructions follow these definitions. Theorems propose cost balance equations using the combination of exergy balance with the operating and maintenance costs for each component in the system. Firstly, the equipment costs of the units, \dot{Z} , are calculated before, by a suitable cost approach. Attention is directed towards the cost of the work produced by the gas engine since it is the most important parameter obtained by the thermoeconomic analyzing methods to achieve the maximum profit. A comparison of the values obtained from each method is given in Table 25.

Table 25 seems to show generally closer values. On the other hand, the comparison of some of the values is difficult to understand. For example, the unit exergoeconomic cost evaluated using MOPSA is significantly smaller than the estimated value for SPECO, even though the analysis produced the same work, 1000 kW, for both of them. Such differences occur due to the application principles of the theorems. ECT brought a very useful concept to thermoeconomics: exergetic cost. An incidence matrix is formed that describes the inlet and outlet flows of the overall system components. The exergetic costs of the flows are obtained with the help of the incidence matrix. Unit exergetic cost, κ^* , gives clues about the

amount of exergetic consumption. This concept requires making a detailed analysis. Unit exergoeconomic costs are determined from the thermodynamics and economic relationship. Equations in the cost hexagon in Fig. 2 clearly reveal this relationship. Finally, the exergetic cost theory results in a thermoeconomic cost for the gas engine work of 110.065\$/h.

MOPSA theory examines the exergy in two parts: mechanical and thermal. Mechanical exergy deals with pressure while thermal exergy deals with temperature. This procedure needs comprehensive calculations. Table 12 shows thermal and mechanical exergy flows, and entropy production rates, at various state points in the system. The general cost balance equation shown in Eq. (13) is applied to each subsystem to appoint a new unit exergoeconomic cost according to the subsystem's principal product. To achieve this result, detailed equations are performed for gas and water streams. The calculations emphasize that the cost structure of the system is mostly influenced by the entropy production of each component. MOPSA may be the method that shows this fact most obviously, due to the table calculations performed. By the end of the process, the measured exergoeconomic cost production rate of the work from the biogas engine was 85.536 \$/h, which is smaller than the result found from ECT. The turbine work is the fuel for the compressor in the ECT principles, which ignore the exergy destruction from turbine to compressor. ECT determines unit exergoeconomic costs for all flows; however, MOPSA defines only two unit exergoeconomic costs, one for gas streams and the other for water streams. Also, the unit exergetic cost for MOPSA is eclipsed by other methods. Therefore the calculated results show differences.

Wonerger is the combination of the energy and exergy flows in a power plant. There are different Wonerger assumptions depending on differences which are accepted [8]. The methodology considered in this study, accepts the exergy as Wonerger, furthermore, components making up the system are classified by their tasks. The general cost balance equation for the Wonerger theory is written to include a wonergetic unit cost, c_k . This wonergetic cost balance equation is separated into three parts according to the unit classifications. A careful analysis makes clear that, the level of Wonerger input, k , plays the most important role in determining the costs. The calculated thermoeconomic cost of work produced from the gas engine is 72.5 \$/h. It is the smallest value of the comparison

Table 25
Comparison of the cost flows obtained for the work produced by the gas engine.

Cost accounting method	Unit Exergoeconomic cost c_w (\$/GJ)	Thermoeconomic cost \dot{C}_w (\$/h)
ECT	30.573	110.065
MOPSA	23.760	85.536
WONERGER	20.138	72.5
SPECO	39.166	141

values in Table 25. This result points focuses attention on work production because the Wonergy method attributes responsibility not just to the gas engine, but also to it being shared among other components.

SPECO defines the fuel and product application rules, which are the basis for calculating the costs. These rules suggest auxiliary equations to make calculations easier. Table 20 represents exergetic cost balances and the corresponding auxiliary equations for each component in the system. The thermoeconomic cost accompanies of the work flow of the gas engine is 141 \$/h. It is the highest value in Table 24 because SPECO don't perform detailed calculations and assumptions are permitted.

The main measuring devices, calibrated range, accuracy or relative error of various instruments involved in the study for various parameters are listed in Table 26. An error analysis based on the accuracies of the direct measurements is conducted to determine the maximum possible errors of these deduced parameters, such as the fuel exergy, product exergy, unit exergy consumption and unit marginal exergy cost of product of every component. The adopted analysis method is the differential method of propagating errors based on Taylor's theorem. It gives the maximum error Δy of a function $y = f(x_1, x_2, x_3, \dots, x_n)$ as follows:

$$\Delta y = \sqrt{\sum \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2}$$

As a result, the maximum relative root mean square errors of every component are tabulated in Table 27. It can be seen that the greatest possible error is $\pm 1.854\%$ due to the flow through the intercooler. It should be clearly noted that the estimated errors in the measurements of the derived quantities do not significantly influence the final results.

The exergetic cost theory is more detailed than the other theories because it calculates the cost of each flow in subsystems. This detailed analysis does not ignore any cost incurred in subsystems, so it can be said that there is a superior viewpoint in ECT over other methods. As can be seen from the open literature, ECT has been applied to cogeneration systems in which different fuels are used [13,14,21] and it has been observed that the thermoeconomic cost of the produced work varies markedly according to the complexity of the systems and the type of fuel used. On the other hand, although ECT evaluates the cost of the flows in the subsystems in a very detailed way it assumes the exergy destruction as another "product" in the system. Therefore, this method does not separate the actual production cost of a subsystem from the cost of its exergy destruction.

Considering the other cost allocation methods, it should be noted that the MOPSA is a method that defines the cost of exergy destruction in a subsystem in the most understandable way. In the analysis of the CGAM system, it can be seen that MOPSA yields the highest work cost compared to other cost allocation methods [17,21]. However, it is not the case in this study. This is partly due to the fact that the biogas engine powered cogeneration system in this study uses renewable fuel (biogas), as well as the system is more complex than the CGAM system for example, depending on the number of subsystems and the state of the flows.

Table 26
Specification of the different measuring devices.

Instrument	Parameter	Calibrated range	Accuracy
Platinum resistance temperature sensor	Water inlet and outlet temperature of a unit	0–100	± 0.01 °C
Platinum resistance temperature sensor	Fuel gas temperature	0–100	± 0.10 °C
k-type thermocouple	Exhaust gas temperature	0–600	± 1.00 °C

Table 27
Summary of the estimated relative errors.

Components	Maximum relative errors	
	Fuel $\dot{E}x_f$	Product $\dot{E}x_p$
Compressor	$\pm 1.437\%$	$\pm 0.042\%$
Turbine	$\pm 1.261\%$	$\pm 1.437\%$
Intercooler	$\pm 1.854\%$	$\pm 1.427\%$
EGHE	$\pm 1.218\%$	$\pm 0.122\%$
HE1	$\pm 0.133\%$	$\pm 0.312\%$
HE2	$\pm 0.934\%$	$\pm 1.724\%$
LOHE	$\pm 0.0\%$	$\pm 0.426\%$
P1	$\pm 0.0\%$	$\pm 0.0\%$
P2	$\pm 0.0\%$	$\pm 0.0\%$
P3	$\pm 0.0\%$	$\pm 0.0\%$
P4	$\pm 0.0\%$	$\pm 0.0\%$
GE	$\pm 0.248\%$	$\pm 0.0\%$

Although it is a relatively new method, it can be said that the Wonergy is one of the most useful methods that can be easily applied to a cogeneration system. This method does not calculate the costs separately for each flow in the subsystems as the ECT method does, or it does not explicitly calculate the exergy destruction costs of the subsystems as in the MOPSA method. However, it separately calculates the costs of work and heat generated in a cogeneration system, and in a sense, Wonergy method reveals the purpose of the cogeneration system in a holistic manner.

The SPECO method can make very flexible assumptions among the other methods we have evaluated above, thus making it easier to apply to a cogeneration system. This method does not perform a detailed analysis such as ECT, nor does it calculate the costs of the exergy destructions of the subsystems like MOPSA nor does the direct cost analyses of work and heat which are the main outputs of the cogeneration system like Wonergy. Due to this structure of the method, SPECO is the method that calculated the highest costs in this study.

5. Conclusions

Cost accounting is a procedure to calculate both fuel and product costs for components in power-producing systems. Thermoeconomics gives hints to calculate these costs from an exergy and economics perspective. A cogeneration system produces work and heat. Since work has more importance, determining the thermoeconomic cost of it in detail becomes obligatory. In this study, different methodologies were applied to an existing cogeneration system having 1000 kW of power production capacity. The first method is ECT, which researches the system in a very detailed way. This method views destructions as products and performs its calculations from this perspective. The second method, MOPSA, investigates destructions more clearly as entropy generation units and it determines the cost of the main product, work. On the other hand it gives distinct costs to destroyed exergies. Hence the cost of the work flow from the engine obtained by MOPSA is smaller than that obtained by ECT. The third method, Wonergy, deals with the cost of both work and heat produced by the system. In this manner,

this method may be more reasonable for cogeneration systems and it produces the smallest thermoeconomic cost for work generated by the engine. The fourth and last method, SPECO, makes some assumptions from the fuel product rule perspective. It also considers destructions as products, like ECT; however, ECT gives both unit exergoeconomic cost, dealing with exergy, and unit thermoeconomic cost, dealing with exergetic cost, to each flow, and it analyzes the system more rigorously. Thus, SPECO calculations ended with the highest thermoeconomic cost for work flow from the biogas engine.

Nomenclature

Uppercase letters

C	Compressor
T	Turbine
EGHE	Exhaust Gas Heat Exchanger
HE	Heat Exchanger
LOHE	Lubrication Oil Heat Exchanger
P	Pump
GE	Gas Engine
A	Incidence matrix of a system
\dot{C}	Thermoeconomic cost of a flow, \$/h
\dot{C}	Thermoeconomic cost vector of a system, \$/h
\dot{E}_x	Flow exergy of, kW
\dot{E}_x	Flow exergy vector, kW
\dot{E}_x^*	Exergetic cost, kW
\dot{E}_x^*	Exergetic cost vector, kW
\dot{E}_{x_F}	Fuel exergy, kW
\dot{E}_{x_F}	Fuel exergy vector, kW
$\dot{E}_{x_F}^*$	Fuel exergetic cost, kW
\dot{E}_{x_P}	Product exergy, kW
\dot{E}_{x_P}	Product exergy vector, kW
$\dot{E}_{x_P}^*$	Product exergetic cost, kW
K	Wonerger, kW
K	Wonergetic input ratio
R	Residue or lost, kW

Lowercase letters

C	Unit exergoeconomic cost, \$/GJ
c*	Unit thermoeconomic cost, \$/GJ
i	Generic index associated to components
j	Generic index associated to flows
m	Number of flows of a system
n	Number of components of an installation

Subscripts

dest	Destruction
F	Fuel
I	Components
P	Product

Greek letters

κ	Exergetic unit consumption
κ^*	Exergetic unit cost

η Exergetic efficiency

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