European Journal of Operational Research 267 (2018) 1051–1074

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Risk management for forestry planning under uncertainty in demand and prices[☆]

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ARTICLE INFO

Article history:
Received 26 October 2016
Accepted 14 December 2017
Available online 21 December 2017

Keywords:
OR in natural resources
Forestry planning
Optimization under uncertainty
Risk management
Consistency in risk-averse measures

ABSTRACT

The paper presents and compares approaches for controlling forest companies' risk associated with advance planning under variable future timber prices and demand. Decisions to be made in advance are which stands to cut and which new access roads to build in each period, while maximizing profit under manageable risk. We first developed a tighter, improved formulation of our earlier deterministic mixed 0–1 model (see Andalaft et al. (2003)), and its stochastic counterpart for a set of representative scenarios, an extension of our simplified risk-neutral version (see Alonso-Ayuso, Escudero, Guignard, Quinteros, and Weintraub (2011)). Using the expected value of the stochastic parameters might produce poor or even infeasible solutions if some extreme scenarios are realized. A stochastic model, however, enables the planner to make more robust decisions. In particular, being able to control risk in early periods is important, as firms tend to emphasize short term financial results. We tested two risk measures that extend the classical Conditional Value-at-Risk (CVaR) by controlling the risk at a subset of intermediate periods (time-inconsistent TCVaR) or at a subset of scenario groups (time-consistent ECVaR), with time consistency as given in Homem-de Mello and Pagnoncelli (2016) and others. We also combined TCVaR and ECVaR into what we call MCVaR. We analyzed the *planned* and *implementable* policies of all above risk measures in a broad computational experiment, on a large size realistic instance. The results show that ECVaR, TCVaR and MCVaR outperform the classical CVaR approach. MCVAR usually provides better solutions for the first periods with overall profit distribution similar to the other measures for the *planned* policy, TCVaR gives the highest profit results for the *implementable* policy, while ECVaR gives the highest profit at the end of the time horizon in both policies.

1. Introduction

Forest companies must plan the sustainable harvest of their resources over a given time horizon. Cut timber is then sold in specific local and international markets. They have to meet demand, primarily from pulp plants and sawmills. The main aim of the companies is to maximize profit while complying with environmental regulations. In previous studies we formulated and solved a specific problem addressing various issues that arise in forestry planning, namely, planning the harvest of forest land designated for timber production and the construction of access roads needed to transport the timber. Good surveys of forest-based supply chain planning cover such aspects as planting, cutting, construction of access roads for transportation. See Bredstrom, Lundgren, Rönnqvist, Carlsson, and Mason (2004), Marques, Borges, Sousa, and Pinho (2011), Pinho, Moreira, Veiga, and Boaventura-Cunha (2015), and Rönnqvist (2003), among others. Starting in the 70s, environmental and wildlife issues were increasingly considered in forest management models at different planning levels.

In the last 30 years the twin problems of planning harvesting and access road construction have been addressed jointly using mathematical optimization models and computational tools. The advantage of integrating the two processes in a single mixed 0–1 model was demonstrated in Jones, Hyde, and Meachan (1986), whose solutions are from 15 to 45% better than with models that optimized the processes separately.

☆ This research was partially funded by the Complex Engineering Systems Institute, ISCI (ICM-FIC: P05-004-F, CONICYT: FB0816), the projects MTM2012-36163-C06-06 (A. Alonso-Ayuso) and MTM2015-63710-P (A. Alonso-Ayuso and L.F. Escudero) from the Spanish Ministry of Economy and Competitiveness, and CONICYT grant FBO16 and Fondecyt project 1120318 (A. Weintraub).

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https://doi.org/10.1016/j.ejor.2017.12.022
0377-2217/

There exist relevant studies on the different phases of forestry planning, especially regarding access road construction and harvesting. The problem we deal with may be formulated in terms of a partition of the forest into harvesting units, called stands. For a chosen time horizon one must determine which stands will be cut in each period, which roads need to be constructed to access those stands and when, and what quantity of wood will be transported from one point to another. These decisions are made in Andalaft et al. (2003) and references therein, among others. Our approach did benefit from these earlier reports. Some ion to an optimization criterion, typically profit maximization. A model for solving the harvesting problem considering road building and adjacency is provided in Candia (2010), which constrains the possibility of harvesting adjacent stands for observing the maximum clearfell areas regulations.

Selling prices of forest products are a key element in forestry planning. Price fluctuations have a direct impact on profits from sales and figure prominently in the planners' decision-making. The role played by randomness in forestry planning is closely related to the length of the chosen time horizon. Planners who must make tactical decisions are therefore concerned about price variations during a time horizon of two to five years. Although the most relevant source of uncertainty is prices, uncertainty in tree growth, timber demand and losses due to fires is also significant. The approach developed in this paper analyzes decision-making under uncertainty in wood selling prices and demand. We assume that they can be modeled over time by means of a set of scenarios with different associated probabilities.

In mathematical terms, the deterministic version of the problem, which assumes that all parameters are known, may be formulated as a mixed 0–1 linear optimization model. Even this case is difficult to solve, due to its size and the presence of thousands of binary variables. Approaches for solving this problem have been described in Andalaft et al. (2003), Constantino and Martins (2017), Guignard, Ryu, and Spielberg (1998), Henningsson, Karlsson, and Rönnqvist (2007), and Weintraub and Navon (1976) of them use strengthening of the formulation and decomposition techniques such as Lagrangean relaxation to obtain very good solutions in reasonable computation times with low residual gaps. We should point out that the forest planning problems studied in Andalaft et al. (2003), Guignard et al. (1998), and Weintraub and Navon (1976) consider either the expected scenario or a single scenario.

A stochastic optimization model enables the planner to make more robust decisions by taking into account the stochastic behavior of the selling price and demand of timber. It considers a representative range of timber price scenarios over time, maximizing the expected value instead of merely analyzing a single (e.g., average) scenario as performed in the deterministic version of the problem. It is assumed that the realization of the scenarios at a given period is probabilistically conditioned by the realization of these scenarios in the earlier periods. So, the values of the decision variables at a given node in a multi-period scenario tree also depend on the realization of the uncertain parameters in the ancestors of the node. That is, the values of the variables depend on the values of the parameters and the value of the variables in the scenario groups with one-to-one correspondence with the nodes up to the period that the node belongs to, being a unique solution for those scenarios. So, the non-anticipativity principle introduced in Wets (1974) is satisfied. See e.g., Birge and Louveaux (2011), and Pflug and Pichler (2014) for the main concepts on stochastic optimization via scenario tree analysis.

There is a variety of papers incorporating risk and uncertainty into forest models, comparing it with a deterministic approach. A good survey and analysis is presented in Pasalodos-Tato et al. (2013), where different sources of risk and uncertainty are considered, namely, forest inventory, timber growth prediction, material hazards as fires, markets (timber prices), climate change, etc. Also different methodologies are shown for problems going from stand to regional levels. These include stochastic dynamic programs, fuzzy set theory, Monte Carlo techniques, scenario simulation, stochastic optimization with recourse, chance constrained optimization and Markov Chains, among others. Our work considers a multistage forest planning problem, when uncertainty is in timber prices and demand. Uncertainty is defined through a scenario representation, where each scenario is a realization of values of prices and demand for each period along the time horizon. We did present elsewhere (Alonso-Ayuso, Escudero, Guignard, Quinteros, & Weintraub, 2011) a risk neutral (RN) multi-period stochastic mixed 0–1 model based on a finite set of representative scenarios for the price uncertainty. The model is a stochastic counterpart of a tighter version of the deterministic model introduced in Andalaft et al. (2003). For problem solving we developed a matheuristic version of the decomposition algorithm called Branch-and-Fix Coordination (BFC) presented in Alonso-Ayuso, Escudero, and Ortuño (2003). The computational results outperformed those obtained by considering the expected value approach. The latter approach is a popular measure for solving stochastic optimization problems. It consists of replacing the scenario realizations of the uncertainty in the parameters with the average (i.e., expected) value. In problems with parameters with high variability, the results of the model using average values could be misleading; notice that a solution could be worse than that from the stochastic optimization, or even be infeasible if more-extreme scenarios are realized, for which the solutions based on expected values are not well covered. A progressive hedging decomposition approach is presented in Veliz, Watson, Weintraub, Wets, and Woodruff (2015) for solving the forest harvesting planning problem, where the model is based on the deterministic one given in Andalaft et al. (2003).

On the other hand, to our knowledge, Pagnoncelli and Piazza (2012) is the first study where forest harvesting planning with uncertain timber price is addressed by considering risk management, as opposed to a risk-neutral (RN) approach, in stochastic optimization. A stochastic dynamic programming approach as well as the popular (time-inconsistent[1]) classical Conditional Value-at-Risk (CVaR)[2] risk averse measure, see Pflug and Pichler (2016), and Rockafellar and Uryasev (2000), are used for determining the best harvesting policy; however, the logistic aspects of the problem (road construction, transportation, etc.) are not considered. (It is worth to point put that the work by the same authors in Piazza and Pagnoncelli (2014) is the risk neutral counterpart of the CVaR application presented in Pagnoncelli and Piazza (2012). The CVaR measure thus reduces the negative impact on the solution in low-probability high-loss scenarios in the forestry planning problem. Since the profit is for the whole time horizon, the risk reduction is performed up to the last period of the time horizon and, then, any scenario influences the VaR value and, as a consequence, it does the solution of any other scenario (so, CVaR does not have the time-consistency property as in defined in Homem-de Mello and Pagnoncelli (2016) and others). Recently, Eyvindson, Petty, and Kangas (2017) presented a two-stage stochastic program with recourse for the timing of the next forest inventory and the accompanying adjustments to the forest management plans including CVaR-based risk management.

A state-of-the-art review on robust optimization for forest harvesting planning is presented in Bajgiran, Zanjani, and Noureifath (2017), where it is assumed that there is not enough information about the probability distribution, nor a set of available scenar-

[1] For time-consistency and -inconsistency, see Section 2.3.

[2] Informally, CVaR at a given level $\beta \in [0, 1]$ is the expected profit of the β proportion of the worst scenarios, and then, risk-averse models propose a solution with the maximum CVaR.

Year	Exports	Prices
2000	100.0	100.0
2001	112.4	70.9
2002	122.6	67.3
2003	132.8	71.9
2004	155.4	82.2
2005	157.2	80.2
2006	161.3	89.5
2007	184.4	102.4
2008	188.0	108.9
2009	176.3	81.2
2010	163.9	118.1
2011	185.0	121.3
2012	181.7	102.9
2013	185.2	107.6
2014	194.6	109.4

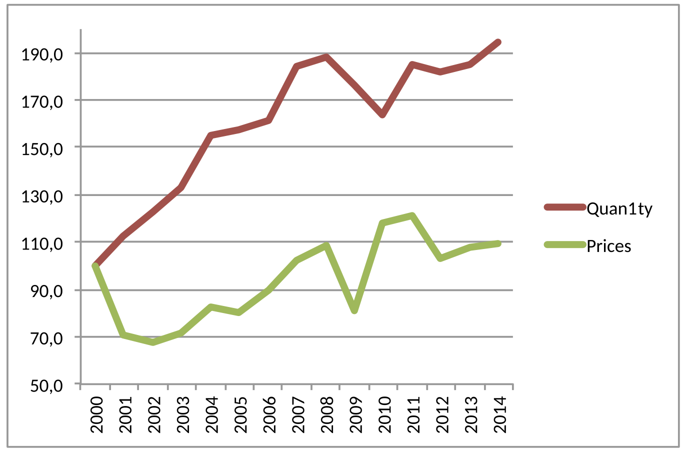


Fig. 1. Chilean forest exports index of wood quantity and prices. (base: Avr. year 2000=100).

ios to represent the stochasticity of the uncertain parameters. Specially, the uncertainty in distribution and inventory planning in the pulp production is handled in Carlsson, Flinsberg, and Rönnqvist (2014) by considering a rolling horizon in a robust optimization approach where the uncertainties are described as an arbitrary polytope and formulated as explicit constraints. However, a forest harvesting planning model is proposed in Bajgiran et al. (2017), where the robust-based cardinality constrained method Bertsimas and Sim (2004) is considered. Roughly, the known bounds of the intervals of the uncertain parameters in the functions (either the objective one or the constraints) are explicitly considered in the model, but only restricted to a modeler-defined fraction of the uncertain parameters, where iteratively only a fraction of a reference value inside the intervals is also considered in the model.

In this paper, we consider a real problem of harvesting and road building which was developed for Forestal Millalemu and reported in Andalaft et al. (2003). It handled 17 forest areas. In Alonso-Ayuso et al. (2011), uncertainty in prices and demands was incorporated in a scenario structure, considering risk neutral decision makers. It was solved for only one forest area, in a much simplified version of the original problem, considering 18 scenarios. In this work, the original problem is solved, not the simplified one, 2 and 3 forest areas are considered in the instances to experiment with and, in particular, it is assumed that a decision maker would want to minimize risk in earlier periods, which requires a significantly higher number of scenarios.

The case study under consideration is representative of the forest industry and it presents a realistic planning problem of timber harvesting and road building under uncertainty in Chile. Forestry is Chile's second largest source of exports, surpassed only by copper mining. According to data from INFOR (Instituto de Investigación Forestal de Chile – the Chilean Institute of Forest Research), the forest industry exports in 2014 exceeded for the first time the barrier of US$6 billions, registering a sum of US$ 6094.3 millions, which represents an increase of 6.7% over 2013. Such a figure confirms the magnitude of the industry and underlines the importance of providing its planners with efficient decision-making tools. See in Fig. 1 the evolution of the wood demand and price from 2000 to 2014.

The main contributions of this work over what was previously published in the literature are as follows:

1. The real original model in Andalaft et al. (2003) is introduced instead of the simplified version in Alonso-Ayuso et al. (2011). The 18 scenario approach developed in Alonso-Ayuso et al. (2011) is extended to 144 scenarios in order to handle in a proper way considerations of risk at intermediate periods. This led to a much more difficult problem to solve, which forced us to strengthen the formulation.
2. We introduce methodology to handle risk at earlier periods. This is an important contribution, especially from a managerial point of view. Having negative results in earlier periods is more troublesome than just the consideration of a discount rate. In most firms short term financial results are vital when evaluating the performance of management, making longer term success secondary. Thus, reducing risk in early periods is important. Time-inconsistent and time-consistent versions of CVaR are dealt with in the forest harvesting planning problem, let us name them TCVaR (for Time-inconsistent CVaR) and ECVaR (for Expected CVaR). The former considers profit risk reduction at modeler-defined intermediate periods, instead of considering only profit up to the last period of the time horizon for the whole set of scenarios, and the latter considers the profit up to the last time period but individualized for given scenario groups. These measures provide better solutions for the first periods than the classical CVaR, while, as the results of the case study show, at least, the overall profit distribution remains similar to that obtained by the old measure.
3. An extensive computational experiment comparing the different approaches with different parameter settings has been performed on the stochastic version of the real problem presented in Andalaft et al. (2003). In particular, the measures EV (Expected Value), RN, ECVaR, TCVaR and MCVaR (a mixture of the last two) are tested. Both types of policies, namely the *planned* and *implementable* ones are considered for the risk averse measures.

TCVaR and MCVaR could be very interesting, particularly for long time horizons, since the risk reduction is forced at intermediate periods, although it may deteriorate the profit at the end of the time horizon and, then, they are suboptimal to ECVaR in that sense. The latter one (that is time-consistent) is very beneficial, see Homem-de Mello and Pagnoncelli (2016), Pflug and Pichler (2016), Rudloff, Street, and Valladao (2014), and Shapiro (2009), among others, since by construction it performs risk reduction in individual groups of scenarios (instead of considering its whole set). The rationale behind a time-consistent risk averse measure is that the solution value to be obtained in a given node, say n, of the multistage scenario tree, should not be influenced by the realizations of the uncertain parameters related to the nodes that are not in the ancestor path from the root to node n of the scenario tree. Notice that the group of scenarios in one-to-one correspondence with those other nodes cannot occur at node n and successors. See in Rudloff et al. (2014) an economic interpretation and practical consequences of a time-consistent risk averse measure, considering ECVaR as a pilot measure. In any case. it is up to the decision-maker to choose the measure to consider, so that if the aim is controlling the profit at intermediate periods at the price of a moderate reduction in profit up to the end of the time horizon, then MCVAR could be the measure of choice for the *planned* policy and TCVaR is the choice for the *implementable* policy. On the other hand, if the aim is to optimize the profit up to and including the last period then TCVaR and MCVaR could be suboptimal and ECVaR is the measure of choice.

The goal of the tactical forestry planning addressed in this work is to determine a policy for forest harvesting and access road construction that will maximize the expected profit in the scenarios. Given the uncertainty in wood price and demand along the time horizon, the constraints should be satisfied at each node of the scenario tree. Additionally, one will consider the time-consistent and -inconsistent versions of the risk-averse CVaR measure. The timber land under consideration is subdivided using geographic information systems (GIS) into units or stands for harvesting purposes.

The remainder of the paper is organized as follows. In Section 2, the multi-period mixed 0–1 stochastic problem is considered, with the risk neutral model presented in Section 2.1. Section 2.2 introduces our scheme for scenario tree generation to represent the uncertainty. Section 2.3 studies the ECVaR, TCVaR and MCVaR variants of the CVaR risk-averse measure. Section 3 describes the forest harvest planning problem as a case study to test the behaviour of the risk-averse measures examined in this work. Section 4 presents the main computational results comparing the risk averse approaches and their impact on the final solution. Finally, we discuss the main conclusions of the work and outline future research plans in Section 5. Appendix A presents the new mathematical formulation for the deterministic version of the problem to be dealt with in this work, and Appendix B describes some additional computational results.

2. Model building in stochastic optimization

2.1. Expected value and risk neutral models

For representing the uncertainty in wood demand and prices, a scenario analysis approach is used, where the scenario set can be visualized as a tree. Let $\mathcal{T}$ denote the set of periods in the time horizon, where $T = |\mathcal{T}|$ is the number of periods and let Ω be the finite set of representative scenarios. A *scenario* $\omega \in \Omega$ is a particular realization of the uncertain parameters along the time horizon, represented in the tree as a root-to-leaf path. A node of the tree represents the event where a realization of uncertain parameters and decision variables for a given period takes place. Notice that the group of scenarios that have the same realization of the uncertain parameters up to any given period have the same value for the decision variables up to the period and thus satisfy the well-known *nonanticipativity* principle. Notice that there is a one-to-one correspondence between nodes and scenario groups for the same period. Let n and $\mathcal{N}$ denote a node and the lexicographically numbered set of nodes $\{1, \dots, |\mathcal{N}|\}$ in the tree, respectively, and let $\mathcal{N}^t$ denote the subset of nodes that belong to period t, such that $\mathcal{N} = \cup_{t\in\mathcal{T}} \mathcal{N}^t$, $\mathcal{N}^t \cap \mathcal{N}^{t+1} = \emptyset$ for $t \in \mathcal{T} \setminus \{T\}$. To facilitate the presentation of the scheme for the scenario tree generation given in Section 2.2, let $\mathcal{N}^0 = \{0\}$, $\mathcal{N}^1 = \{1\}$ and $\mathcal{N}^{T+1} = \{|N^T| + 1\}$. Let also $\Omega^n \subseteq \Omega$ denote the subset of scenarios in one-to-one correspondence with node n in the tree. For scenario $\omega \in \Omega$ the weight w^ω represents the probability of its occurrence. Let $\tilde{\mathcal{A}}^n$ and $\tilde{\mathcal{S}}^n$ denote the sets of ancestor and successor nodes to node n (including itself in both of them), respectively, for $n \in \mathcal{N}$. Let $\mathcal{S}^n \subseteq \tilde{\mathcal{S}}^n$ denote the set of immediate successors of node $n \in \mathcal{N}$. Note: $\mathcal{N}^1$ is singleton, $\tilde{\mathcal{A}}^1 = \{1\}$ and $\tilde{\mathcal{S}}^n = \emptyset$ for $n \in \mathcal{N}^T$. Finally, let $\sigma(n)$ denote the immediate ancestor node of node n, for $n \in \mathcal{N} \setminus \{1\}$, where $n = 1$ is the root node of the scenario tree.

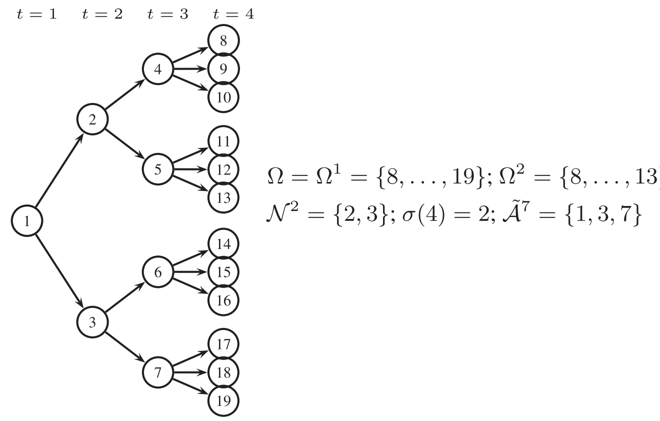


Fig. 2. A multi-period scenario tree.

As an example, let us consider the decision tree in Fig. 2. Each node, say n, represents a point in time where a decision can be made. Once a decision is made, some contingencies may occur (in this example the number of contingencies varies from two to three for periods 1 to 3), and information related to those contingencies is available at the beginning of the next period.

Without loss of generality, let us consider the following synthesized mixed 0–1 model that gives a compact view of the deterministic multi-period mixed 0–1 model (7)–(A.13) shown in Appendix A for this forestry planning problem,

$$
\begin{aligned}
z_{EV} = \max \; & \sum_{t\in\mathcal{T}} (a_1^t x^t + b_1^t y^t) \\
\text{subject to} \; & \sum_{t'\in\mathcal{T}:t'\le t} (A_t^{t'} x^{t'} + B_t^{t'} y^{t'}) = h^t, && \forall t \in \mathcal{T}, \\
& x^t \in \{0,1\}^{n_x(t)}, y^t \in \mathbb{R}^{n_y(t)}, && \forall t \in \mathcal{T},
\end{aligned} \tag{1}
$$

where EV stands for Expected Value, x^t and y^t are the $n_x(t)$ and $n_y(t)$ dimensional vectors of the 0–1 and continuous variables, respectively, a_1^t and b_1^t are the vectors of the coefficients of the objective function, $A_t^{t'}$ and $B_t^{t'}$ are the constraint matrices of period t for the 0–1 and continuous variables in its ancestor period t' (including itself), respectively, and h^t is the right-hand-side vector (rhs) for period t. Note: a_1^t and b_1^t can be considered as the unit profit related to the variables in vectors x^t and y^t, respectively.

Without loss of generality, let us consider the synthesized compact representation of the multi-period mixed 0–1 model for maximizing the expected value of the objective function in the set of scenarios Ω in the scenario tree, such that the so-called Risk Neutral (RN) model can be expressed as

$$
\begin{aligned}
z_{RN} = \max \; & \sum_{n\in\mathcal{N}} w^n (a_1^n x^n + b_1^n y^n) \\
\text{s.t.} \; & \sum_{q\in\mathcal{A}^n} (A_n^q x^q + B_n^q y^q) = h^n, && \forall n \in \mathcal{N}, \\
& x^n \in \{0,1\}^{n_x(n)}, y^g \in \mathbb{R}^{n_y(n)}, && \forall n \in \mathcal{N},
\end{aligned} \tag{2}
$$

where w^n is the weight or probability of node n in the scenario tree and is computed as $\sum_{\omega\in\Omega^n} w^\omega$; $\mathcal{A}^n \subseteq \tilde{\mathcal{A}}^n$ is the set of ancestor nodes to node n with nonzero elements in the constraints matrices of node n; x^n and y^n are the vectors of the 0–1 and continuous variables for node n, respectively; a_1^n and b_1^n are the vectors of the objective function coefficients for the 0–1 and continuous variables, respectively; A_n^q and B_n^q are the constraint matrices of node n for the variables in x^q and y^q in ancestor node q of node n, respectively; h^n is the rhs for node n; and $\mathrm{n}_x(n)$ and $\mathrm{n}_y(n)$ are the numbers of 0–1 and continuous variables, for $n\in\mathcal{N}$, respectively.

As an additional notation to be used throughout the rest of the work, let $t(n)$ denote the period from set $\mathcal{T}$ to which node n belongs. Notice that $\tilde{\mathcal{A}}^n$ for $n\in\mathcal{N}^T$ is the set of nodes for scenario $n\in\Omega$, so, for convenience, let $n=\omega$ for $n\in\mathcal{N}^T$.

2.2. Representing the uncertainty. Multi-period scenario tree generation scheme

It is beyond the scope of this work to present a methodology for multi-period scenario tree generation and reduction; see e.g., Heitsch and Römisch (2009), Leövey and Römisch (2015), and Pflug and Pichler (2015) for alternative ways of performing it. A rigorous development of scenario trees for future wood prices is extremely complex. We follow a proposal in Ríos, Weintraub, and Wets (2016) to generate the scenario tree for our case.

Let us consider that the tree structure represented in set $\mathcal{N}^t, \forall t\in\mathcal{T}$ and that the wood prices are mean-reverting as other commodities. Their stochastic behaviour can then be modeled through the stochastic differential equation,

$$dp(t)=\mu(\nu-p(t))d_t+\sigma p(t)dw(t),\quad p(0)=p_0,\quad t\geq 0,$$

where $p(t)$ is the price at period t, $p(0)$ is the present price (or an estimate) at time 0, μ is the speed of price convergence towards its long-term value ν, σ is the standard deviation and $w(t)$ is a Wiener process. The solution of the equation can be expressed as follows (see Ríos et al. (2016) for more details),

$$p(0)=\nu(1-e^{-\mu t})+p(0)\exp\left[-\left(\mu+\frac{\sigma^2}{2}t+\sigma w(t)\right)\right],$$

which corresponds to a displaced log-Gaussian process with mean and variance

$$E[p(t)]=\nu(1-e^{-\mu t})+p_0e^{-\mu t},$$
$$V[p(t)]=(p(0)e^{-\mu t})^2(e^{\sigma^2 t}-1).$$

Using data from 1988 to 2009, Ríos et al. (2016) proposes the following estimation for the coefficients of the process associated to sawtimber:

$$p_0=1.0749,\quad \nu=1.1998,\quad \mu=0.0462,\quad \sigma^2=0.0319.$$

For pulp-wood, again on the 1988–2009 database, the coefficients are as follows:

$$p_0=0.5501,\quad \nu=0.5623,\quad \mu=0.0979,\quad \sigma=0.0086.$$

Modeling export quality timber is not presented in Ríos et al. (2016); thus we considered, based on historical data, that export quality is 20% more expensive than sawtimber. Additionally, in order to consider some variability in the prices, a random perturbation of 10%, at most, has been introduced in the timber price for each market.

By following the ideas in Ríos et al. (2016) for building robust scenario trees, available information about price is used to conduct an analysis of the cumulative distribution functions (CDF) associated to the densities. The main step of the methodology applied to the case study from period $t=1$ to period $t=|\mathcal{T}|$ consists of obtaining a finite set of points that summarize the CDF, giving flexibility for considering specific segments of the domain, e.g., tail events. For that purpose, the CDF is split in a finite number of segments with a one-to-one correspondence with the nodes in set $\mathcal{N}^t, \forall t\in\mathcal{T}$; then, the information of each segment is assigned to a representative point. Formally, let ξ_t be a random variable with support Ξ_t, with probability density function $f_\xi(t)$ and cumulative distribution function $F_\xi(t)$. In addition, for the nodes $i\equiv|\mathcal{N}^{t-1}+1|$ and $j\equiv|\mathcal{N}^t|$ (i.e., the lexicographically ordered first and last nodes of period t, respectively), let $C^t=\{c_i,\dots,c_{j+1}\}$ be a partition of the interval [0, 1],

$$c_i=0,\ c_{j+1}=1,\ c_n<c_{n+1},\quad \forall n\in\{i,\dots,j\},\quad \text{and}$$
$$\bigcup_{n=i}^{j}[c_n,c_{n+1}]=[0,1].$$

Considering the value of F_ξ^{-1} evaluated in C^t, the c_n-quantiles of ξ_t, say Q_{c_n} are obtained, for $c_n\in\mathcal{C}^t, \forall n\in\mathcal{N}_t$, which define the partition of the support of ξ_t. Note: Depending on the set C^t, the approach allows the number of scenarios to be changed, as well as the segments of the support of ξ_t. Once the segments are defined, the expected value of the random variable for each one can be expressed as

$$\xi^n=E\left(\xi_t\,|\,Q_{c_n}\leq\xi_t\leq Q_{c_{n+1}}\right),\qquad \forall n\in\{i,\dots,j\}.$$

To generate the scenarios for the product demands, a similar scheme is considered by assuming that they are normally distributed. As in Andalaft et al. (2003), the model considers lower and upper bounds on the demand of each wood product to be offered in different markets. These bounds are different for each product q and market m at any node of the scenario tree. Therefore, for a given product q and market m, let us consider a node $n\in\mathcal{N}$ with its set $\mathcal{S}^n$ of the immediate successors in the tree, and o_s is the order in the set related to node $s\in\mathcal{S}^n$ from 1 to $k\equiv|\mathcal{S}^n|$, k being the last node in that set. The demand, say ζ^s in node $s\in\mathcal{S}^n$ can be approximated by a normally distributed random variable with mean μ^n and standard deviation σ^n, lower and upper bounds, say $Q_{\frac{o_s-1}{k}}$ and $Q_{\frac{o_s}{k}}$, $Q_{\frac{o_s}{k}}$ being the $\frac{o_s}{k}$-quantile of the normal distribution $N(\mu^n,\sigma^n)$. Note: Instead of the 0- and 1-quantiles of the normal distribution (which, in fact, are $-\infty$ and $+\infty$, respectively), we consider the 0.001- and 0.999-quantiles.

For computing the demand bounds of any node, say $s\in\mathcal{S}^n$, the parameters of the normal distribution to be used $N(\mu^s,\sigma^s)$ are computed as follows: The mean μ^s is the conditional expectation of the random variable ζ^n in the interval defined by the bounds $\left(Q_{\frac{o_s-1}{k}},Q_{\frac{o_s}{k}}\right)$, i.e.,

$$\mu^s=E\left(\zeta^n\,|\,Q_{\frac{o_s-1}{k}}\leq\zeta^n\leq Q_{\frac{o_s}{k}}\right),$$

and, for the computational experience reported in Section 4, the standard deviation σ^s is set to $0.3\mu^s$. Note: The parameters for the distribution function of the demand at the root node are a modeler-defined data.

As an example, let us consider the three-period scenario tree depicted in Fig. 3 where $\mathcal{T}=\{1,2,3\}$, the root node is $n=1$ and its immediate successor set $\mathcal{S}^1$ is $\{s=2,\dots,5\}$. For a decision maker-driven data with $\mu=2000$ and $\sigma=600$, the lower and upper bounds for the demand at the nodes in period $t=2$ are as follows:

Node 2: $Q_{0.001}=146$ and $Q_{\frac{1}{4}}=1505$.	Node 3: $Q_{\frac{1}{4}}=1505$ and $Q_{\frac{2}{4}}=2000$.
Node 4: $Q_{\frac{2}{4}}=2000$ and $Q_{\frac{3}{4}}=2495$.	Node 5: $Q_{\frac{2}{4}}=2495$ and $Q_{0.999}=3854$.

The normal distribution associated with the random variable ξ^3 used for computing the bounds on the demand for the immediate successor node set $\mathcal{S}^3$ has a mean equal to: $\mu^3=E\left(\zeta^1\,|\,1505\leq\zeta^1\leq 2000\right)=1811$.

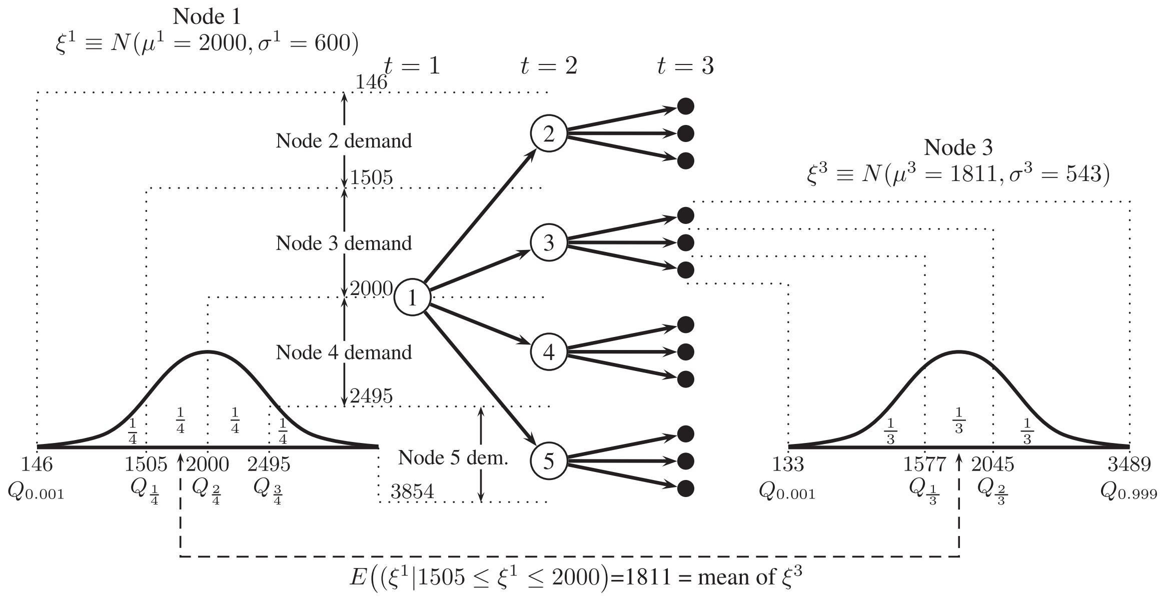


Fig. 3. Demand scenario tree generation.

2.3. CVaR-based risk averse management

The aim of the RN model (2) is simply to maximize the expected value of the objective function without any hedging against the uncertainty in the parameters. For that reason, the main criticism that can be made to this very popular mean measure is that it ignores the variability of the objective function value in the set of scenarios and, in particular, the *left* tail of the undesirable scenarios. There are, however, other approaches that, additionally, deal with risk management; see in Alonso-Ayuso et al. (2014) a comprehensive computational comparison of the most popular time-inconsistent risk-averse measures. Well-known theoretical research suggests that the measures based on quantiles are good functions for risk management. Among them, the Value-at-Risk (VaR) and Conditional VaR (CVaR), see Guigues and Sagastizbal (2013), Homem-de Mello and Pagnoncelli (2016), Pflug (2000), Pflug and Pichler (2016), Rockafellar and Uryasev (2000), and Shapiro, Dencheva, and Ruszczynski (2009). have become a benchmark for many applications in the financial, supply chain, energy, transportation and productions planning sectors, among others.

Definition 1. $\text{VaR}_\beta(X, \Omega, \mathcal{P})$ of a solution X for a given set of scenarios Ω, each one with a probability of occurrence in $\mathcal{P}$, is the highest value, say α, such that the probability of occurrence of any of those scenarios with a profit smaller than α is lower than β, where $\beta \in (0, 1)$ is a modeler-defined parameter.

Notice that the advantage of the VaR measure over the traditional maxmin measure is obvious, since it specifies the bound β on the probability of the occurrence of a scenario whose profit is below α. It does not however consider how bad the scenarios with a profit below α can be. As an alternative, the classical $\text{CVaR}_\beta(X, \Omega, \mathcal{P})$ measure for linear models was introduced in Rockafellar and Uryasev (2000), being expressed as

$$z = \max_{\alpha,x,y} \alpha - \frac{1}{\beta}\sum_{w\in\Omega} w^\omega \left\{\alpha - \sum_{q\in\tilde{\mathcal{A}}^\omega}(a_1^q x^q + b_1^q y^q)\right\}_+ . \tag{3}$$

In this section, we present two modifications of model RN (2) that allow risk management by taking into account the profit for those non-desirable scenarios, i.e., those with profit below α, by considering extensions of the classical CVaR. As many other risk averse approaches in the literature, CVaR reduces the probability of a negative impact of the model's solution in the unwanted scenarios. However, it does not consider scenarios with higher profit than α. On the contrary, decision makers usually look for a trade-off between risk minimization and profit maximization. For this reason, the risk measures are usually combined with the optimization of the expected value of the objective function, leading to the following mean-risk model, see Schultz and Tiedemann (2006),

$$z_{CVaR} = \max\left[\gamma\sum_{n\in\mathcal{N}} w^n(a_1^n x^n + b_1^n y^n) + \rho\left(\alpha - \frac{1}{\beta}\sum_{w\in\Omega} w^\omega \upsilon^\omega\right)\right] \tag{4a}$$

$$\text{s.t.} \quad \sum_{q\in\mathcal{A}^n}(A_n^q x^q + B_n^q y^q) = h^n \quad \forall n\in\mathcal{N} \tag{4b}$$

$$\alpha - \sum_{q\in\tilde{\mathcal{A}}^\omega}(a_1^q x^q + b_1^q y^q) \le \upsilon^\omega \quad \forall\omega\in\Omega \tag{4c}$$

$$x^n \in \{0,1\}^{\mathrm{n}_x(n)}, y^n \in \mathbb{R}^{\mathrm{n}_y(n)} \quad \forall n\in\mathcal{N} \tag{4d}$$

$$\upsilon^\omega \in \mathbb{R}_+ \quad \forall\omega\in\Omega \tag{4e}$$

$$\alpha \in \mathbb{R}, \tag{4f}$$

where α is a continuous variable that computes the $VaR_\beta\big((x,y),\Omega,\mathcal{P}\big)$ associated with each solution, υ^ω is a non-negative variable that collects the difference (if it is positive) between α and the profit for scenario ω. Note that υ^ω is strictly positive only for scenario ω with profit below α, γ is a non-negative parameter for the expect function value and $\rho > 0$ is also a weight factor.

Function (4a) is the composite function of the RN expected function (i.e., profit), the weighted VaR and the weighted negative expected shortfall on reaching the VaR profit. The constraint system (4b) is the RN scenario node-based system. Constraints (4c) define VaR and the shortfall of each scenario profit on reaching it. The bounds (4d)–(4f) give the variables' domain.

Observe that the CVaR model (4) maximizes the expected profit and the highest profit (i.e., VaR) at the end of the time horizon for which the weighted expected scenario shortfall on reaching that VaR is minimized. That very popular risk reduction measure may have two big drawbacks depending on the modeler-defined aim to consider. First, observe that the VaR profit and the related scenario expected shortfall are evaluated for the whole set of periods (i.e., at the end) of the time horizon. And, second, all scenarios in the tree are considered in the same set for obtaining the VaR profit without differentiating between categories (i.e., groups).

It could therefore be interesting to consider a modeler-defined intermediate subset of periods for avoiding low-probability high-loss scenarios up to those periods in the time horizon. In that case, the classical CVaR is not the most appropriate risk averse measure. As an alternative, we propose the TCVaR measure, see Section 2.3.1, in which the objective function considers the profit VaR and related expected shortfall up to each of the chosen periods in the scenarios.

Another alternative is the ECVaR measure, see Section 2.3.2, where a set of scenario groups from the whole set is considered in one-to-one correspondence with the nodes that belong to a modeler-defined subset of periods. The profit risk reduction for each of those groups is considered for obtaining the profit VaR and the related expected shortfall up to the last period in the time horizon. The measure complies with the time-consistency definition given in Homem-de Mello and Pagnoncelli (2016), Kozmik and Morton (2015), Rudloff et al. (2014), and Shapiro (2009), among others, whose rough statement in Kozmik and Morton (2015) says: 'At each state of the system, optimality of a decision policy should not involve states which cannot happen in the future'. It is the definition to be used throughout this work.

A computational comparison of the measures TCVaR, ECVaR and MCVaR is reported in Section 4. Among other results (such as the profit distribution) it shows the following results for each measure (say, m) and up to the chosen periods in the time horizon for both policies, *planed* and *implementable*: Number of scenarios out of set Ω where the measure has the highest profit among the three measures (let us name such set Ω_m), and the expected difference between the highest profit among those measures and the profit given by the measure in the scenario subset $\Omega\backslash\Omega_m$. That information may support the decision making when focusing on the profit performance for the periods of interest.

2.3.1. Time-inconsistent multi-period TCVaR measure

The CVaR model (4) only performs risk management for the profit but, in many contexts, the decision maker wants to manage the risk for other functions such as the one that measures the environmental impact. Let $\mathcal{F}$ denote the set of functions where risk management could be performed in the value $\sum_{n\in\tilde{\mathcal{A}}^\omega}(a_f^n x^n + b_f^n y^n)$, $\forall f\in\mathcal{F}$ for the set of scenarios $\omega\in\Omega$. The model also measures and controls the risk at the end of the time horizon. However, since the value of the function f under risk-control for each scenario is calculated as the sum of the values of the function at the nodes $\tilde{\mathcal{A}}^\omega$, this approach does not prevent very bad results at any of the stages. Notwithstanding, it could eventually be compensated by good results in the others stages.

It may thus happen that bad results at intermediate nodes in the solution of model (4) drive the decision maker to a situation of no-return. To avoid such situations, risk management can be performed, as stated above, at some intermediate time periods; let us denote $\tilde{\mathcal{T}}_f\subseteq\mathcal{T}$ the subset of periods where function f is under risk control (note that for singleton set $\tilde{\mathcal{T}}_f=\{T\}$, the classical CVaR is obtained). So, model (4) can be extended to consider multi-period, multi-function risk management, resulting in the following model

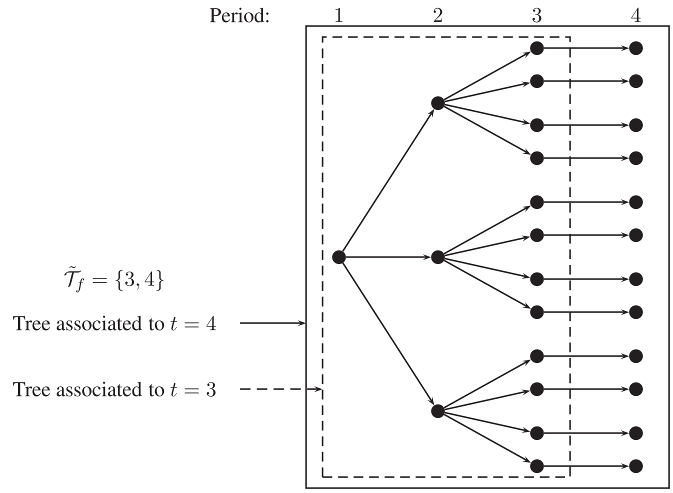


Fig. 4. The multi-period scenario tree and the related subtree for $t=3$.

related to the TCVaR measure,

$$z_{TCVaR} = \max\Big[\gamma\sum_{n\in\mathcal{N}} w^n(a_1^n x^n + b_1^n y^n) + \sum_{f\in\mathcal{F}}\sum_{t\in\tilde{\mathcal{T}}_f}\rho_f^t\Big(\alpha_f^t - \frac{1}{\beta_f^t}\sum_{n\in\mathcal{N}^t} w^n v_f^n\Big)\Big] \tag{5a}$$

$$\text{s.t.} \sum_{q\in\mathcal{A}^n}(A_n^q x^q + B_n^q y^q) = h^n \qquad \forall n\in\mathcal{N} \tag{5b}$$

$$\alpha_f^t - \sum_{q\in\tilde{\mathcal{A}}^n}(a_f^q x^q + b_f^q y^q) \le v_f^n \qquad \forall n\in\mathcal{N}^t,\ t\in\tilde{\mathcal{T}}_f,\ f\in\mathcal{F} \tag{5c}$$

$$x^n\in\{0,1\}^{n_x(n)}, y^n\in\mathbb{R}^{n_y(n)} \qquad \forall n\in\mathcal{N} \tag{5d}$$

$$v_f^n\in\mathbb{R}_+ \qquad \forall n\in\mathcal{N}^t,\ t\in\tilde{\mathcal{T}}_f,\ f\in\mathcal{F} \tag{5e}$$

$$\alpha_f^t\in\mathbb{R} \qquad \forall t\in\tilde{\mathcal{T}}_f,\ f\in\mathcal{F}, \tag{5f}$$

where α_f^t is $VaR_{\beta_f^t}$ for function $f\in\mathcal{F}$ up to period $t\in\tilde{\mathcal{T}}_f$ in the time horizon for the whole set of scenarios, i.e., set $\mathcal{N}^t$, v_f^n is a non-negative variable that gives the shortfall of the scenario group in one-to-one correspondence with node n for reaching VaR $\alpha_f^{t(n)}$ up to period $t(n)$, and ρ_f^t and β_f^t for $t\in\tilde{\mathcal{T}}_f$ are modeler-defined parameters, such that ρ_f^t weighs the risk reduction importance for the pair (t, f) and β_f^t is related to the importance given to the expected shortfall on reaching α_f^t in the scenarios.

Function (5a) is like (4a) but now the risk reduction-based part takes the weighted VaR and the related expected shortfall of the value of each function up to the chosen periods where the risk reduction is to be performed. Constraints (5c) define the function-based VaR in those periods. They also define the related shortfall of the function-value of the scenario group in one-to-one correspondence with a node that belongs to the chosen periods.

As an illustration, consider the four-period scenario tree in Fig. 4 and consider that $\tilde{\mathcal{T}}_f=\{3,4\}$, then TCVaR model (5) performs risk management by minimizing the CVaR associated to the two types of depicted scenario subtrees.

Among other remarks on the VaR and CVaR measures, Pflug (2000) is the first work, as far as we know, that observes that CVaR is a coherent measure according to the standards setup in Artzner, Delbaen, Eber, and Health (1999), see also Artzner, Delbaen, Eber, Health, and Ku (2007), since it satisfies the properties of translation invariance, positive homogeneity, monotonicity and convexity.

2.3.2. Time-consistent multi-period ECVaR measure

One desirable constrained qualified property for a solution of a multi-period model is time-consistency in the sense that the solution to be obtained for any node, say n, of the scenario tree and its successor node set $\tilde{\mathcal{S}}^n$ in the related submodel 'solved' at period $t(n)$ should have the same value as the solution obtained for that node and its successors in the original model 'solved' at period t=1. It is worth pointing out that the above mentioned constrained qualification consists of requiring that the value of the variables in the ancestor nodes to be considered in the model 'to solve' at period $t(n)$ for the subtree rooted with node n is precisely the one obtained in the original model 'solved' at period $t = 1$. In the words of Rudloff et al. (2014), *an optimal policy is time consistent if and only if the future planned decisions are actually going to be implemented*.

It is well known that the RN model (2) is time-consistent, while the risk measures CVaR and TCVaR are not. It was shown in Homem-de Mello and Pagnoncelli (2016) that the time-consistency property of CVaR depends on parameter β_f^t: the smaller it is, the higher the probability of consistency of CVaR, in the sense that the difference between the solution obtained for the original problem and the solution for the problem 'solved' at any other period decreases.

As a time-consistent alternative to the TCVaR model (5), the risk management can be performed at any node $n \in \mathcal{N} : t(n) < T$, considering the scenarios in the associated subtree with root in node n of the original scenario tree. Let $\tilde{\mathcal{T}}_f \subset \mathcal{T}$ denote the subset of periods where the risk reduction in the value of the function indexed with f is to be performed, for $f \in \mathcal{F}$. Observe that any group of scenarios, say Ω^n, is in one-to-one correspondence with the related node n in the tree. The ECVaR model for performing the required risk reduction in the forest harvest planing problem is a straightforward extension of the synthesized model considered in Homem-de Mello and Pagnoncelli (2016) for a general case. It can be expressed as

$$z_{\text{ECVaR}} = \max\Big[\gamma \sum_{n\in\mathcal{N}} w^n (a_1^n x^n + b_1^n y^n) + \sum_{f\in\mathcal{F}}\sum_{t\in\tilde{\mathcal{T}}_f} \rho_f^t \sum_{n\in\mathcal{N}^t}\Big(w^n\alpha_f^n - \frac{1}{\beta_f^t}\sum_{\omega\in\Omega^n} w^\omega v_f^\omega\Big)\Big] \tag{6a}$$

$$\text{s.t.} \sum_{q\in\mathcal{A}^n} (A_n^q x^q + B_n^q y^q) = h^n \qquad \forall n \in \mathcal{N} \tag{6b}$$

$$\alpha_f^n - \sum_{q\in\tilde{\mathcal{A}}^\omega} (a_f^q x^q + b_f^q y^q) \le v_f^\omega \qquad \forall \omega \in \Omega^n,\ n \in \mathcal{N}^t,\ t \in \tilde{\mathcal{T}}_f,\ f \in \mathcal{F} \tag{6c}$$

$$x^n \in \{0,1\}^{\mathrm{n}_x(n)}, y^n \in \mathbb{R}^{\mathrm{n}_y(n)} \qquad \forall n \in \mathcal{N} \tag{6d}$$

$$v_f^\omega \in \mathbb{R}_+ \qquad \forall \omega \in \Omega^n,\ n \in \mathcal{N}^t,\ t \in \tilde{\mathcal{T}}_f,\ f \in \mathcal{F}$$

$$\alpha_f^n \in \mathbb{R} \qquad \forall n \in \mathcal{N}^t,\ t \in \tilde{\mathcal{T}}_f,\ f \in \mathcal{F}, \tag{6f}$$

where α_f^n is $VaR_{\beta_f^t}$ for function $f \in \mathcal{F}$ at the end of the time horizon for the set of scenarios in Ω^n, and v_f^ω is a non-negative variable that gives the shortfall of scenario ω for reaching α_f^n, for $\omega \in \Omega^n$.

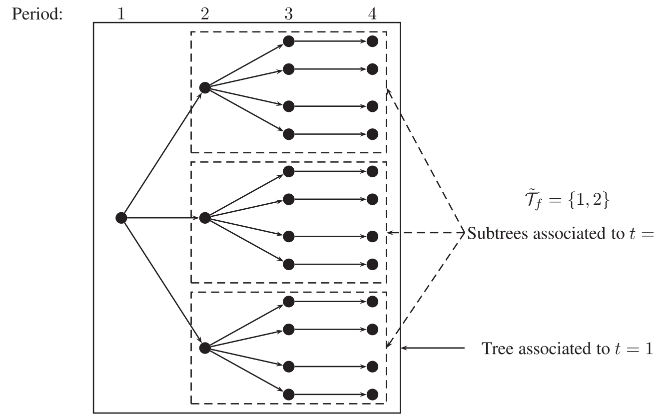


Fig. 5. Multi-period scenario tree and the related subtree for $t = 2$.

Function (6a) is like (5a) but now the risk reduction-based part takes the weighted VaR and the related expected shortfall of the value of each function up to the last period in the time horizon in the chosen scenario groups. Constraints (6c) define the function-based VaR in those scenario groups in one-to-one correspondence with a node that belongs to any of the chosen periods. They also define the related shortfall of the function-value of each scenario that belongs to the VaR-related group. It is worth pointing out that model (6) is close to the average VaR proposals introduced in Gaivoronski and Plug (2005), Pflug and Pichler (2016), and Pflug and Ruszczynski (2005).

As an illustration, for the scenario tree in Fig. 5, consider that $\tilde{\mathcal{T}}_f = \{1, 2\}$, then the ECVaR model (6) performs the risk management by minimizing the CVaR associated to the whole set of scenarios for $t = 1$, and the set of scenarios Ω^n in the $|\mathcal{N}^t| = 3$ depicted subtrees, each rooted with node n, for $n \in \mathcal{N}^t$, for $t = 2$.

The ECVaR measure as presented in model (6) belongs to the family of *Expected Conditional Risk averse Measures* (ECRMs) considered in Homem-de Mello and Pagnoncelli (2016), where the time-consistency property of those measures is proved, according to the definition introduced there. Notice that the proof only requires the measure to have the properties of translation-invariance and monotonicity. See some variants in Bion-Nadal (2008), Carpentier, Chancelier, Cohen, De Lara, and Girardeau (2012), Collado, Papp, and Ruszczynski (2012), De Lara and Leclere (2016), Pflug and Pichler (2016), Pflug and Pichler (2015), Rudloff et al. (2014), Ruszczyski (2010), and Shapiro (2009). The particularization of the definition in our context is as follows:

Let $\big(\hat{x}^q, \hat{y}^q\ \forall q \in \mathcal{N},\ \hat{v}_f^\omega\ \forall \omega \in \Omega \text{ and } \hat{\alpha}_f^n\ \forall n \in \mathcal{N}^t,\ t \in \tilde{\mathcal{T}}_f,\ f \in \mathcal{F}\big)$ denote any of the optimal solutions of the original ECVaR model (6), and z_{ECVaR}^n is the solution value in that model where only the terms related to the subtree $\tilde{\mathcal{A}}^n \cup \tilde{\mathcal{S}}^n$ are considered. It can be expressed as

$$z_{\text{ECVaR}}^n = \gamma \sum_{q\in\subset\tilde{\mathcal{A}}^n\cup\tilde{\mathcal{S}}^n} w^q (a_1^q \hat{x}^q + b_1^q \hat{y}^q) + \sum_{f\in\mathcal{F}} \sum_{q\in\tilde{\mathcal{S}}^n : t(q)\in\tilde{\mathcal{T}}_f} \rho_f^{t(q)}\Big(w^q\hat{\alpha}_f^q - \frac{1}{\beta_f^{t(q)}}\sum_{\omega\in\Omega^q} w^\omega \hat{v}_f^\omega\Big)$$

For any node $n \in \mathcal{N}$, let us define the ECVaR^n submodel from (6) as follows:

- The scenario subtree that supports the ECVaR^n submodel includes the nodes in set $\tilde{\mathcal{A}}^n$ (from the original scenario tree) plus the subtree rooted with node n whose nodes are in set $\tilde{\mathcal{S}}^n$.
- The input data of the submodel are the same as in model (6) for any scenario tree of any scenario set Ω.

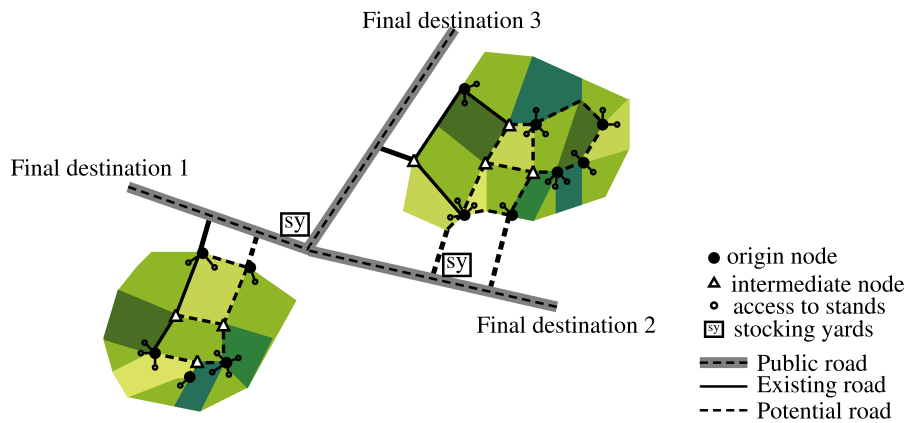


Fig. 6. Areas and (potential and existing) logistic structure.

- Finally, in the submodel the variables in vectors x^q and y^q $\forall q \in \tilde{\mathcal{A}}^n \setminus \{n\}$ are fixed to the values in $\hat{x}^q$ and $\hat{y}^q$, respectively

Let z_{ECVaR^n} denote the value of an optimal solution of the ECVaR^n submodel. Therefore, the *ECVaR measure is a time-consistent one,* since the following assertion is true:

$$z^n_{\mathrm{ECVaR}} = z_{\mathrm{ECVaR}^n}.$$

Another consistent CVaR-based measure that may ben considered as an alterative to model ECVaR (6) is the nested risk measure introduced in Kozmik and Morton (2015), Philpott, de Matos, and Finardi (2013), and Shapiro (2009). The nested mechanism of the CVaR submodels for each period in set $\tilde{\mathcal{T}}_f$, $f \in \mathcal{F}$, however, makes its decomposition more difficult.

Note: ECVaR, as well as any other ECRM, is usually very suitable for using decomposition algorithms (such as Escudero, Monge, and Romero-Morales (2015), and Zou, Ahmed, and Sun (2016), among others) for solving large-scale instances.

An interesting question is: What version, either the time-consistent one or the time-inconsistent version of a risk averse measure, performs better for risk management? The computational experiments whose main results are reported in the next section may help to answer that question.

3. The case study

The above risk averse measures have been tested in the real forestry problem presented in Andalaft et al. (2003). For that purpose, its deterministic mixed 0–1 model has been tightened and extended to the stochastic version in order to be able to include the uncertainty in timber prices and demand (see the deterministic version in Appendix A). The known data used in the experiment are based on Andalaft et al. (2003) while the multi-period scenario tree for representing the uncertainty due to the variability of the timber price and demand along the time horizon is generated according to the scheme presented in Section 2.2. In Section 3.1, a detailed description of the forestry planning problem is shown, and then, in Section 3.2 the main characteristics of the three instances used for testing are presented.

3.1. The forestry problem

Consider the following management planning problem in the timber industry. The firm under consideration owns plantation lands that are divided into areas. Within each area there are different stands, considered homogeneous as defined by age of trees, soil quality (site index), and volume available per hectare (see Fig. 6). All areas are planted with pine trees, which mature at age 22–28. The stands that can be harvested during the time horizon are therefore known. Growth-simulator models developed by the forest firms are used to estimate timber yields in future periods. In this kind of problems, the time horizon considered is usually two to five years (in the computational experience three years are considered).

On the demand side, timber production goes to export, to sawmills, and to pulp plants, as logs. While in reality there are many different products, defined mainly by log length and diameter, at this level of planning we define only a few basic aggregate products, referred to as export, sawmill, and pulp. Usually a higher-level quality can be used for lower-level purposes, at a loss in sale revenue. For example, the pulp mill takes any type of timber, while only export quality can be exported. The main goal of the planning process is to match the supply of standing timber with demand for timber product of specific grades, lengths, and diameters, and, thus, reducing losses in revenues due to down-grading and non-profitable additional cutting.

The problem also considers the logistics of producing and delivering those timber products. Most timber areas are near paved public roads, but in order to get access to the different stands in each area, inside the areas private roads are needed. At the beginning of the time horizon there are potential roads, i.e., roads that can be built, as well as existing roads. In any later period there are roads already built and projected ones. In addition to taking into account the existence or nonexistence of roads, one also has to consider their surface quality. First, private roads can be built of either dirt or gravel, and this has an impact on operations. Gravel roads are more expensive to build, but lead to lower transportation costs and can be used year-round, while dirt roads are only useful in the dry summer. Next, road building and upgrading should be carried out in proper sequence so as to be consistent, timed with stand harvesting, as well as to avoid excessive road building. In addition, road building can only be carried out in summer.

Harvested timber can be stocked from summer to winter in stocking yards; it makes sense to keep in the stocking yards from summer to winter some of the timber harvested in stands accessed via dirt roads, which can only be harvested in summer. The stocking yards are located where there exist gravel road connections to the area exit, so that timber harvested in summer can also be sent to destinations in winter.

Finally, consider the production and delivery of timber demand. Aggregate demands are projected to future periods, often as lower and upper bounds and so are the expected prices. Cable logging (or towers) carry out harvesting for steep areas, while skidders harvest flat terrain. Timber hauling is carried out by truck to such destinations as ports, pulp plants, sawmills, or stocking yards. Harvesting

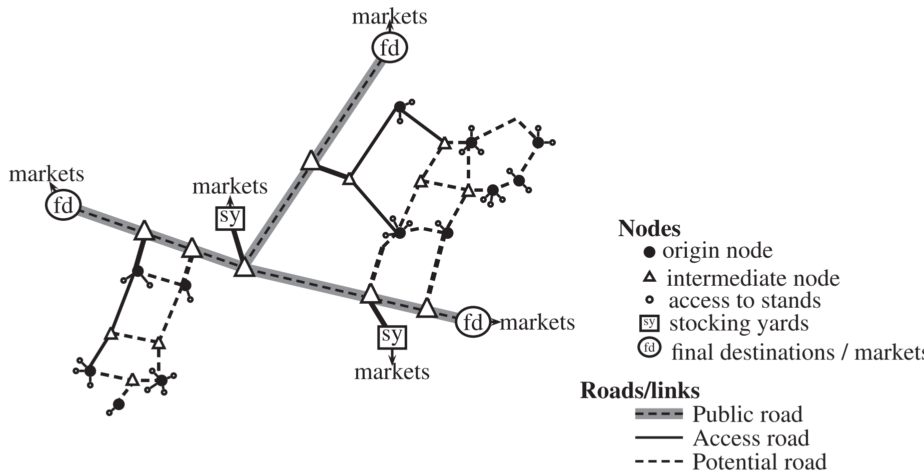


Fig. 7. Logistic network.

machinery and crews are usually subcontracted with yearly contracts. There is no clear way to evaluate the fixed costs needed to install the harvesting machinery process, so firms replace this cost by a policy of harvesting at least 10 or 15 hectares for larger stands, and harvest the whole stand for smaller areas.

To summarize, the basic decisions to be considered in each period are as follows:

- stands to be harvested;
- roads to be built (in gravel or dirt) and roads to be upgraded from dirt to gravel;
- amount of timber production, by aggregate product for harvesting to satisfy demand;
- amount of timber transported to destinations or stocked from summer to winter, if applicable.

As an example, a logistic infrastructure for the problem is depicted in Fig. 6. Observe that there are public roads for transporting the material from the two harvesting areas and two stocking yards. Each harvesting area is accessible through an existing gravel road, but there are other possible access roads. Not all stands are accessible through the existing roads and additional roads are needed to be able to access them.

The logistic structure can be modeled as a network where the nodes can be defined as follows:

- Stands: stands can be represented by nodes in the network associated to an access point.
- Origins: Each access point to a stand is linked to an origin node, such that one or more stands are accessible from each origin point, but the stands are only accessible from one origin.
- Stocking yards.
- Final destinations.
- Intermediate points: road junctions (linking different pieces of roads, public or private).

Notice that products are sent to the markets from the final destination nodes or directly from the stocking yards.

The set of links in the network includes all roads in the model (public and private, existing or potential for the latter) and the connections between origins and stands. Fig. 7 shows the network associated with the logistic structure depicted in Fig. 6.

See Appendix A for the detailed mathematical formulation of the deterministic version of the problem. Note that this model incorporates all the features of the real problem presented in Andalaft et al. (2003) that were deleted in the simplified version of Alonso-Ayuso et al. (2011). These include : (a) The three basic timber products, rather than one; (b) The two types of road categories, dirt and gravel, with the possibility of upgrading from dirt to gravel; and (c) The possibility of transferring Summer production to Winter deliveries via use of stocking yards. In order to solve this more complex model, given the high number of scenarios considered, as shown next, its need to be tightened.

In this deterministic model, all parameters are assumed to be known at the beginning of the time horizon, including timber prices and demand along the time horizon. Wood demand and prices can, however, vary along the time horizon, see Fig. 1. Notice the volatility of the uncertain parameters which are, therefore, very difficult to predict. In order to be able to solve the far more complex resulting stochastic model, the deterministic model presented in the Appendix A is a tightened version of that in Andalaft et al. (2003) (see constraints (7g) and (7h) and (12b)–(12i)). This allows carrying out the broad experiments reported in Section 4.

The instances in Section 4 used for testing the risk averse reduction measures are based on Andalaft et al. (2003), in which a deterministic version of the problem was solved using real data from the forest company *Forestal Millalemu*. It consisted of 17 forests, geographically separated, each connected through public roads to demand nodes. It produced three wood qualities (for export, sawmills and pulp plants) that were sent to different destinations, either final markets or processing plants. In this work, different instances have been created by selecting subsets of the areas in order to obtain small but yet realistic examples where the stochastic version could be solved in a reasonable computing time. The time horizon considered is three years and each year is divided into two seasons (summer and winter). $\mathcal{T}$, therefore, includes six time periods (also called stages). The first time period is considered to be summer.

Table 1
Instance description.

Ins.	na	$\mathcal{S}$	$\mathcal{C}$	$\mathcal{I}$	$\mathcal{L}^P$	$\mathcal{L}^{Ed}$	$\mathcal{L}^{Eg}$	Ha	$\mathcal{Q}$	$\mathcal{M}$
i1	2	11	0	15	7	7	0	627.4	3	7
i2	3	14	0	20	7	10	0	694.0	3	7
i3	2	21	0	33	8	16	9	216.1	3	7

3.2. Description of the instances

The main characteristics of the instances considered in this work are shown in Table 1. The headings are as follows: *na*, number of areas; $\mathcal{S}$, number of stands; $\mathcal{C}$, number of stocking yards; $\mathcal{I}$, number of nodes; $\mathcal{L}^P$, number of potential roads; $\mathcal{L}^{Ed}$ and $\mathcal{L}^{Eg}$,

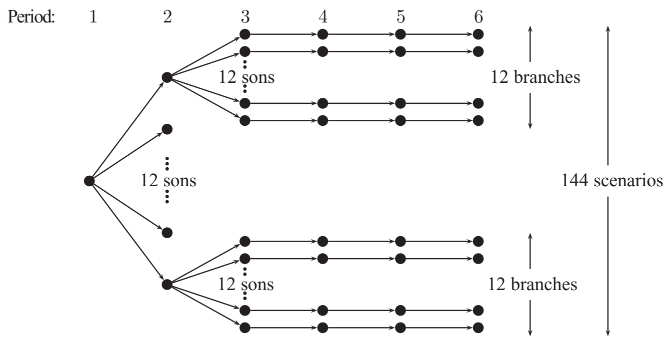


Fig. 8. Forestry scenario tree.

Table 2
Model dimensions.

	Deterministic model (1)			RN model (2)				
Ins.	*m*	*nc*	*n*01	*m*	*nc*	*n*01	$\lvert\Omega\rvert$	$\lvert\mathcal{N}\rvert$
i1	1531	2399	220	158,919	234,070	22,880	144	589
i2	1916	3311	268	198,288	323,598	27,592	144	589
i3	2365	4063	350	245,806	397,136	35,825	144	589

number of existing roads in dirt and in gravel, respectively; *Ha*, total forest surface; $\mathcal{Q}$, number of harvest products; and $\mathcal{M}$, number of markets.

The scenario tree structure corresponds to a $1 \times 12 \times 12 \times 1 \times 1 \times 1$ model, where the second stage has 12 nodes, each of the second-stage nodes has 12 sons, and the nodes in the rest of the stages have only one son, resulting in $12 \times 12 = 144$ scenarios, see Fig. 8. To build the data for the sons of a node, 4 price scenarios and 3 demand scenarios have been combined in order to obtain 12 combinations.

The size of the mathematical formulation of the deterministic model (only one scenario) and the compact formulation of the risk neutral (RN) approach of the stochastic model are shown in Table 2. The headings are as follows: *m*, number of constraints, *nc*, number of continuous variables, *n*01, number of binary variables, $|\Omega|$, number of scenarios in the scenario tree, and $|\mathcal{N}|$, number of nodes in the scenario tree. Note: Risk management is considered for only the objective function (the net present value of the profits), i.e., set $\mathcal{F}$ is a singleton.

The dimensions of the mathematical formulations for the risk management models TCVaR (5) and ECVaR (6) are very similar to the dimensions of the RN model (2). Model (5) adds one α-variable for each stage for which the risk management is performed (i.e., $|\tilde{\mathcal{T}}|$ variables), one ν-variable for each node in the stages in $\tilde{\mathcal{T}}$ (i.e., $k = \sum_{i\in\tilde{\mathcal{T}}} |\mathcal{N}^t|$), and the related k constraints. Model (6) adds k α-variables, $|\Omega|$ ν-variables, and the related $|\Omega|$ constraints. In the instances of the experiment, there are fewer than 300 new constraints and continuous variables. Taking into account the dimensions of model (2) (see Table 2), there are less than 0.2% and 0.1% increments in the number of constraints and variables, respectively.

4. Results

The analysis of the computational performance of the CVaR versions and its comparison with RN is organized as follows: Section 4.1 presents the comparison of the results for the deterministic (i.e., Expected value, EV) model (1) of the forest harvesting planning and the related RN model (2). Section 4.2 performs a comparison between the *planned* policy of TCVaR and ECVaR models (5) and (6), and MCVaR, a mixture of both, versus the RN model. The objective is to analyze the impact of those measures on the solution. The comparison is performed on the expected profit, VaR and CVaR (for specific values of the parameters in the last two) as well as on the profit distribution along the scenarios at periods $t = 1, 2, 6$ related to the measures RN, CVaR, ECVaR, TCVaR and MCVaR. The comparison between these results takes as a reference (with value 100) the expected profit of the WS (Wait-and-See) model, since it is an upper bound on the expected profit in any of the measures under consideration. It is obtained from the optimal profit of the scenarios considered individually (i.e., it is the RN profit where the non-anticipativity constraints are relaxed). Section 4.3 performs a similar analysis of the risk measures dealt with in this work, but the *implementable* policy is considered (explaining the rolling horizon scheme that is used), instead of the *planned* one. The differences between both types of policies are emphasized at the end of the section and also in Section 5. Additionally, Appendix A discusses the impact of the changes for different parameters in the risk averse measures.

The computational experiments were conducted in the HW/SW platform given by a workstation under the Linux operating system (version Ubuntu GNU/linus 14.04.1) with 64 bits, 2 processors Intel(R) Xeon(R) CPU E5-2630 @ 2.3 gigahertz, 64 gigabyte of RAM DDR3 1600 megahertz ECC and 24 virtual cores. The model has been implemented with GAMS 24.3.2. The optimization uses one of the state-of-the-art commercial optimization engines, CPLEX 12.6.1; the optimality gap has been set to 2%.

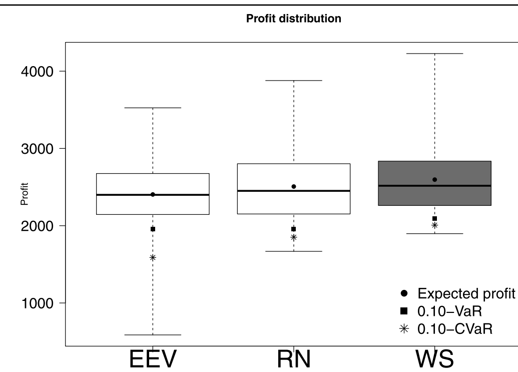
Note: The reporting of the results of the experiment is made by taken benefit of the use of boxplots. It is a standardized way that, in our case, allows to display the summary of the distribution of the scenario profit quantified in the following five statistical measures: minimum, first quartile, median, third quartile, and maximum. The central box spans from the first quartile to the third one. The segment inside the rectangle shows the median. The *whiskers* above and below the box show the locations of the minimum and maximum. Additionally, the mean (expected profit), and the computed VaR (c-VaR) and CVaR (c-CVaR) are shown in each boxplot. Note: In all experiments, it has been set up $\gamma = 1$.

4.1. Computational comparison between the deterministic EV and RN models

Let us start with a comparison between the traditional deterministic EV model (1), where the uncertain parameters have been replaced with their expected values, and the RN model (2). Let the well-known Expected profit of the Expected Value (EEV) be obtained by applying the EV solution to the scenarios. Let also WS (Wait-and-See) denote the expected profit obtained by solving the independent scenario models, which is an upper bound on the expected profit of the original RN model. Notice that the WS solution does not usually satisfy the non-anticipativity constraints (NAC). The methodology for obtaining the EEV is very well established for the two-stage setting, see Birge and Louveaux (2011), but it is not for the multistage one, see Escudero, Garín, and Pérez (2007). Alternatively, we propose the following methodology for obtaining EEV in a rolling horizon type of calculation (see Agustín, Alonso-Ayuso, Escudero, and Pizarro (2012) for more details): (1) The solution for the first stage is taken from the EV solution, (2) Once the solution up to stage $t-1$ is fixed, $|\mathcal{N}^t|$ independent scenario subtrees remain, (3) The EV solution is independently obtained for the scenario subtrees, whose root nodes are the nodes in set $\mathcal{N}^t$, so that the solution for each root node is fixed to its EV solution, (4) The models where only the stages $T-1$ and T are involved are mixed 0–1 two-stage problems, where the first stage nodes belongs to set $\mathcal{N}^{T-1}$, and finally they are solved. At the end of the process there is a solution for each scenario, such that EEV is the weighting of the solution values of the scenarios as calculated by the procedure.

Table 3
EV model (1) and RN model (2): Results and profit distribution for instance i2.

	EEV	RN	WS
Solution value	2415	2507	2597
Greatest scen. solution value	3525	3879	4228
Median	2399	2451	2517
c-VaR	1959	1959	2091
c-CVaR	1589	1851	2008
Smallest scen. solution value	587	1668	1897
CPU time t_0 (secs)	11	33810	125



The main results related to the RN, WS and EEV solutions for instance i2 are shown in Table 3. No results are reported for instances i1 and i3, since the EV solution becomes infeasible at some stage of the procedure. For the EEV, RN and WS solutions, c-VaR is the 0.10-quantile of the profit vector for the set of scenarios (that is, 10% of the scenarios have a profit lower than c-VaR, and c-CVaR gives the expected profit for those scenarios whose profit is lower than c-VaR). We can observe that the EEV solution value (in this case, the expected profit) is 5.5% smaller than the RN profit but only 2.2% smaller than the computed VaR profit. However, the computed CVaR and the smallest scenario-related profit obtained in the EEV approach are very poor, comparing with the ones obtained by the RN model (2). Therefore, the RN maximization of the expected profit in the scenarios along the time horizon also provides better results in terms of risk than the EEV results for the traditional EV approach.

The boxplots associated with the distribution of profits along the different scenarios for the EEV, RN and WS solutions are also shown in Table 3. (Notice that the latter is taken as the reference). From those boxplots, it can be concluded that the solution provided by the EEV approach is worse than the one provided by the RN model (2), given the worse values of the statistical measures. It highlights the CVaR and the lower tail of the profit distribution, with very poor values for the worst-case scenarios.

4.2. Computational comparison between the RN, TCVaR, ECVaR and MCVaR models. A planned policy

The main input for the comparison to be performed between the models RN (2), TCVaR (5), ECVaR (6) and MCVaR is as follows:

- TCVaR performs the risk management in two points, namely, an intermediate period and the end of the time horizon, so, say, $\tilde{\mathcal{T}}_1 = \{3, 6\}$. The parameters β_1^t for those periods have been set up to 0.10.
- ECVaR performs the risk management in each of the given subtrees in the scenarios tree. Considering the tree structure depicted in Fig. 8, period $t = 2$ is the only intermediate one where the risk can be controlled by ECVaR, besides of course $t = 1$, then, $\tilde{\mathcal{T}}_1 = \{1, 2\}$. The parameters β_1^1 and β_1^2 have been set up to 0.10 and 0.18, respectively. Observe that the $|N^t| = 12$ nodes in $t = 2$ have 12 sons each, thus each weight is 0.0833. On the other hand, it has been decided to perform the experiment with a greater value for the β-parameters in order to penalize not just one scenario per subtree, but two at least.
- MCVaR combines the TCVaR and ECVaR models such that the risk management is carried out by using ECVaR for the scenario groups in one-to-one correspondence with the nodes in period 2 and TCVaR for the nodes in periods 3 and 6. That is, MCVaR performs the risk management at period 6 but individualized for each group of scenarios in period 2 and, simultaneously, it performs a risk management for the whole set of scenarios up to periods 3 and 6.

Notice that the traditional CVaR (4) is a particular case of TCVaR for $\tilde{\mathcal{T}}_1 = \{6\}$ and ECVaR for $\tilde{\mathcal{T}}_1 = \{1\}$. These risk management approaches are biobjective mean-risk models. All of them include in the objective function different weights for each time period in $\tilde{\mathcal{T}}_1$, where the risk is controlled. It is well known in multiobjective optimization that each combination of these weights gives a efficient solution; see in Appendix B the results of our testing with different combinations.

The results of the best tested combination in our experiment are shown in Table 4, where the expected profit, computed VaR and computed CVaR up to the end of periods $t = 1, 2, 6,$ are presented. Notice that most of the studies in the literature focus the analysis in the comparison of the profit distribution at the end of the time horizon ($t = 6$ in the forest harvesting problem). However, taking into account that, usually, (1) the available information changes along the time and, therefore, the model will be re-optimized once new information is available, and (2) decision-makers may want to obtain good decisions at the beginning of the time horizon (and, in this case, periods $t = 1, 2$ correspond to the first year), it seems convenient to improve profits at these first periods as well as the profit at the end of the time horizon.

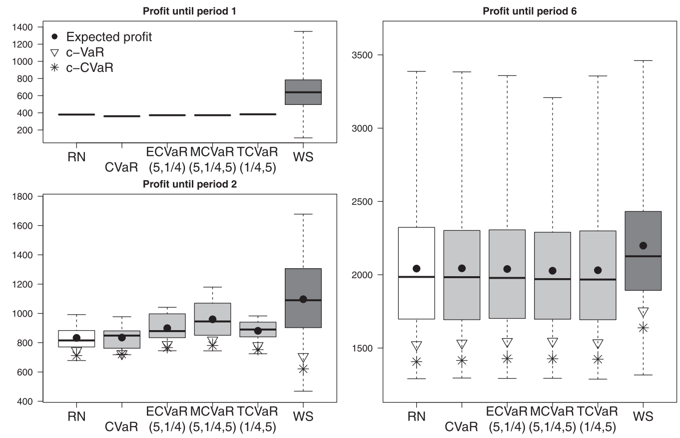
As can be observed in Table 4, all stochastic models practically provide the same expected profit at the end of the time horizon ($t = 6$), while c-CVaR and c-VaR are slightly higher for the risk averse approaches. However, the risk averse models provide an expected profit for $t = 1, 2$ (especially, for the former) much higher than RN as well as a significant improvement in c-VaR and c-CVaR. In short, it can be observed that the risk averse measures move the profits to the early periods. On the other hand, the expected profit is only slightly deteriorated at the end of the time horizon ($t = 6$) compared with the RN profit. Finally, it is not a surprise that the classical CVaR measure provides higher expected profit (at the end of the time horizon) than the other risk averse measures, due to the fact of the weaker risk reduction that is considered.

Besides these statistics, it is also important to consider the profit distribution in the set of scenarios, instead of just the expected one. For that purpose, next to each table, the boxplots are shown for the profit distribution at the end of the three periods that are considered for risk reduction in the instances. The set of boxplots on the right figure of each table shows the profit distribution at the end of the time horizon ($t = 6$), the lower figure in the left part shows the profit at the end of $t = 2$ (corresponding to the end of the first year of harvesting) and the upper left figure corresponds to the profit at the end of $t = 1$ (corresponding to

Table 4
Results and profit distribution for the different approaches. *Planned* policy.

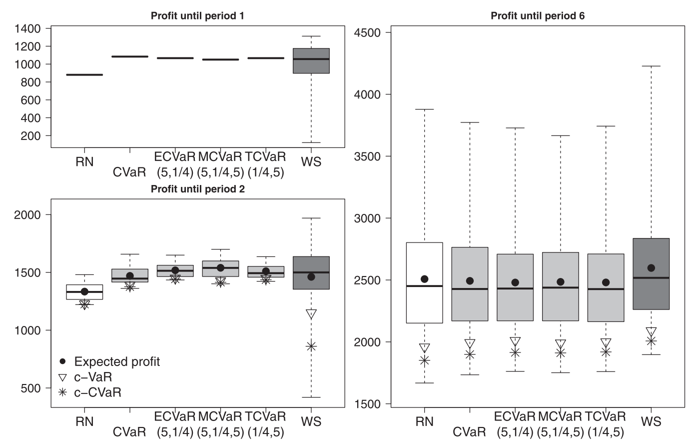
Instance i1

t	Stat.	RN	CVaR	ECVaR	MCVaR	TCVaR	WS
				$\rho_1^1\ \rho_1^2$	$\rho_1^2\ \rho_1^3\ \rho_1^6$	$\rho_1^3\ \rho_1^6$	
				$5\ \frac{1}{4}$	$5\ \frac{1}{4}\ 5$	$\frac{1}{4}\ 5$	
1	Expected	17.3	16.4	16.9	16.9	17.4	28.7
2	Expected	37.9	38.0	40.9	43.6	40.1	49.9
	c-VaR	33.9	33.0	35.8	37.2	35.5	32.1
	c-CVaR	32.4	32.8	34.8	35.5	34.2	28.3
6	Expected	92.9	93.0	92.8	92.2	92.4	100.0
	c-VaR	69.3	69.8	70.3	70.3	69.9	79.8
	c-CVaR	64.0	64.4	64.9	64.9	64.8	74.5



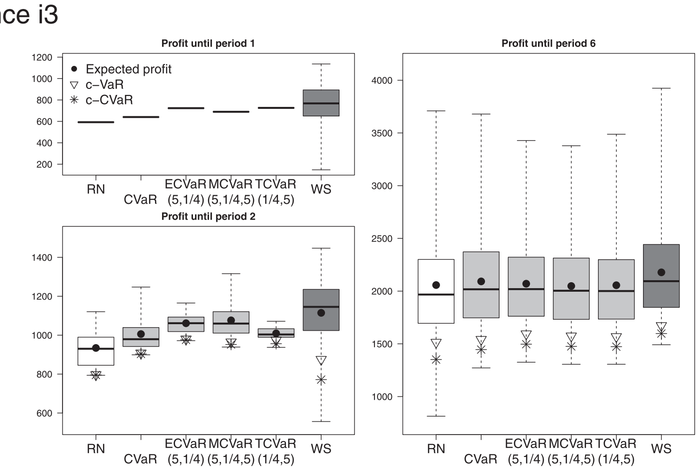
Instance i2

t	Stat.	RN	CVaR	ECVaR	MCVaR	TCVaR	WS
				$\rho_1^1\ \rho_1^2$	$\rho_1^2\ \rho_1^3\ \rho_1^6$	$\rho_1^3\ \rho_1^6$	
				$5\ \frac{1}{4}$	$5\ \frac{1}{4}\ 5$	$\frac{1}{4}\ 5$	
1	Expected	33.9	41.7	41.1	40.4	41.1	38.5
2	Expected	51.4	56.6	58.5	59.3	58.2	56.3
	c-VaR	47.3	53.5	55.9	55.0	55.8	44.3
	c-CVaR	47.2	53.0	55.5	54.5	55.3	33.1
6	Expected	96.6	96.0	95.5	95.7	95.5	100.0
	c-VaR	75.5	76.8	77.5	76.7	77.1	80.5
	c-CVaR	71.3	73.1	73.7	73.6	73.9	77.3



Instance i3

t	Stat.	RN	CVaR	ECVaR	MCVaR	TCVaR	WS
				$\rho_1^1\ \rho_1^2$	$\rho_1^2\ \rho_1^3\ \rho_1^6$	$\rho_1^3\ \rho_1^6$	
				$5\ \frac{1}{4}$	$5\ \frac{1}{4}\ 5$	$\frac{1}{4}\ 5$	
1	Expected	27.2	29.4	33.2	31.7	33.3	35.2
2	Expected	42.9	46.2	48.7	49.4	46.3	51.2
	c-VaR	36.6	41.7	45.0	44.3	44.8	40.2
	c-CVaR	36.5	41.5	44.8	43.7	43.9	35.4
6	Expected	94.5	96.1	95.0	94.0	94.4	100.0
	c-VaR	69.5	70.8	73.2	72.3	72.0	76.8
	c-CVaR	62.1	66.4	68.7	67.8	67.6	73.4



the first semester in the time horizon). Note that in the latter case, the solution proposed by all the stochastic optimization models is the same for all scenarios (the boxplot is just a dot), since the non-anticipativity principle is satisfied, while the WS approach provides (by construction) a different profit for each scenario. It can also be observed that at the end of $t = 2$, ECVaR, TCVaR and MCVaR lead to a better solution than RN, since not only the expected profits are higher (as shown in the tables), but also the improvement is due to the profit distribution in the set of scenarios.

Let us consider the results for instance i2. It can be observed that all the stochastic measures practically provide the same expected profit and profit distribution at the end of the time horizon ($t = 6$). However, ECVaR, MCVaR and TCVaR are advancing high profit to period $t = 1$, where the ECVaR and TCVaR profits are higher than in MCVaR. Additionally, the expected profit as well as the other statistics (median, quartiles, minimum and maximum) for $t = 2$ in MCVaR are higher than in the other three measures. It should be noted that the boxplot in TCVaR shows that this

Table 5
Comparison of the three measures: ECVaR, MCVaR and TCVaR. *Planned* policy.

	ECVaR		MCVaR		TCVaR	
t	*#best*	*dev.best*	*#best*	*dev.best*	*#best*	*dev.best*
Instance i1						
1	0	1.88	**144**	**0.00**	**144**	**0.00**
2	24	10.12	**108**	**1.35**	24	13.09
3	11	4.29	**98**	**1.80**	42	5.86
6	22	0.64	**71**	**1.71**	54	0.75
Instance i2						
1	**144**	**0.00**	0	1.52	**144**	**0.00**
2	60	3.74	**72**	**1.70**	36	3.54
3	47	1.84	**54**	**1.18**	51	1.90
6	**64**	1.98	34	**1.18**	50	1.67
Instance i3						
1	0	0.34	0	5.01	**144**	**0.00**
2	**84**	8.87	60	**4.46**	0	8.57
3	**70**	4.70	57	**2.28**	17	4.62
6	**79**	2.64	40	3.28	25	**2.48**

approach provides a more concentrated profit distribution than the other two risk averse measures, and notice that less variability in future profits is also a desirable property.

In summary, the main conclusions that can be drawn from the experiment reported in Table 4 are as follows:

- All the stochastic models provide similar expected profit at the end of the time horizon ($t = 6$). The profit distribution is also very similar for the four models, although the risk averse measures have better (i.e., smaller) lower profit tails.
- The risk averse measures usually provide a better profit than the risk neutral measure for periods $t = 1, 2$.

Table 5 shows some other results for periods $t = 1, 2, 3, 6$ on the same experiment shown in Fig. 4. They are closely related to the comparison of the three risk averse measures, namely m =ECVaR, MCVaR, TCVaR. Remember that $\tilde{\mathcal{T}} = \{1, 2\}$ for ECVaR, $\tilde{\mathcal{T}} = \{1, 2, 3\}$ for MCVaR and $\tilde{\mathcal{T}} = \{3, 6\}$ for TCVaR. We use the following notation: $profit^{\omega}_{m,t}$, profit obtained in scenario ω by measure m up to period t, for $\omega \in \Omega$; $best^{\omega}_t = max_{\{m\}}\{profit^{\omega}_{m,t}\}$, highest profit among the three measures in scenario ω up to period t, for $\omega \in \Omega$; and $\Omega_m \subseteq \Omega$, subset of scenarios where the profit $profit^{\omega}_{m,t}$ obtained by measure m was not the highest $best^{\omega}_t$ up to period t. The headings are as follows for each measure m and up to each period t: $\#best$, number of scenarios where measure m has the highest profit up to period t out of the $|\Omega| = 144$ scenarios in the experiment; $dev.best$, expected difference (in percentage) between the highest profit $best^{\omega}_t$ and the profit $profit^{\omega}_{m,t}$ obtained by measure m up to period t, among the subset of scenarios Ω_m (i.e., the set of scenarios where measure m does not provide the best profit). It can be expressed as $dev.best = 100\frac{\sum_{\omega \in \Omega_m} w^{\omega}(best^{\omega}_t - profit^{\omega}_{m,t})}{\sum_{\omega \in \Omega_m} w^{\omega} best^{\omega}_t}$. Notice that the highest the number $\#best$ and the smaller the difference $dev.best$, the higher the quality of measure m up to period t.

The conclusion that can be drawn from the experiment is that MCVaR advances the profit to the first periods and ECVaR is usually the most profitable (at the end of the time horizon).

4.3. Computational comparison between the RN, TCVaR, ECVaR and MCVaR models. An implementable policy

In this section the comparison that is performed in Section 4.2, in particular, for the risk averse models (2), (5) and (6) is expanded to consider the suggestion made in Rudloff et al. (2014). It consists of presenting a rolling horizon scheme for an *implementable* policy in the RN, TCVaR, ECVaR and MCVaR risk measures.

Implementable RN solution

The RN solution for period $t = 1$ is the one obtained from the original RN model (2). By fixing that value, the model is decomposed into $|\mathcal{N}^2| = 12$ independent submodels with 12 scenarios each. After that, solving each submodel results in the rest of the RN solution.

Implementable TCVaR solution

The first step considers the original model (5) supported by the full scenario tree. The subset of risk averse periods is $\tilde{\mathcal{T}} = \{3, 6\}$ as for the *planned* policy, and $\rho^6_1 = 0.25$ and $\rho^3_1 = 5$. The risk reduction on the profit is performed in the whole scenario set Ω. The profit is taken into account up to the end of the time horizon for $t = 6$ and only up to the nodes in subset $\mathcal{N}^t$ for $t = 3$. So, there are two places for risk reduction.

After fixing the solution for period $t = 1$ to the value obtained from solving the original model (5) in the first step, the second one considers $|\mathcal{N}^2| = 12$ independent submodels (5). Each submodel is supported by a scenario subtree rooted with node n, for $n \in \mathcal{N}^2$ as for RN, but now $\tilde{\mathcal{T}} = \{4, 6\}$ (note that the scheme has advanced one period in the implementation), $\rho^6 = 5$ and $\rho^4 = 0.25$. The risk reduction is performed in each scenario subset Ω^n, for $n \in \mathcal{N}^6$, where the profit is taken into account up to the end of the time horizon, and only up to the nodes in subset $\mathcal{N}^4$. Thus, by construction, there are 24 places for risk reduction.

Implementable ECVaR solution

The first step considers the original model (6) also supported by the whole scenario tree. The subset of risk averse periods is $\tilde{\mathcal{T}} = \{1, 2\}$ as for the *planned* policy, and $\rho^1_1 = 5$ and $\rho^2_1 = 0.25$. The risk reduction on the profit is performed in the whole scenario set Ω for $t = 1$ as well for each scenario subset Ω^n for $n \in \mathcal{N}^t$ for $t = 2$. The profit in both risk reduction steps is taken into account up to the end of the time horizon. So, there are 13 places for risk reduction.

After fixing the solution for period $t = 1$ to the value obtained from solving the original model (6) in the first step, the second one considers $|\mathcal{N}^2| = 12$ independent submodels (6). Each submodel is supported by a scenario subtree rooted with node n, for $n \in \mathcal{N}^2$ as for RN and TCVAR, but now $\tilde{\mathcal{T}} = \{2\}$. (Note that the pilot case only allows singleton scenario groups for $t > 2$). The risk reduction is performed in each scenario subset Ω^n, for $n \in \mathcal{N}^2$ as in the first step, where the profit is taken into account up to the end of the time horizon, but, for being coherent, $\rho^2_1 = 5$ instead of 0.25. Thus, there are 12 places for risk reduction.

Implementable MCVaR solution

The first step considers the mixture of the original models (6) and (5), so-called MCVaR, supported by the whole scenario tree. The subset of risk averse periods is $\mathcal{T} = \{1, 2, 3\}$ as for the *planned* policy, and $\rho^1_1 = 5$ and $\rho^2_1 = \rho^3 = 0.25$. The risk reduction on the profit is performed in the whole scenario set Ω for $t = 1$ in the ECVaR-part of the joint MCVaR model, for each scenario subset Ω^n for $n \in \mathcal{N}^t$ in period $t = 2$ in the ECVaR-part of the joint model, and for the whole set Ω for $t = 3$ in the TCVaR-part of the joint model. The profit is taken into account up to the end of the time horizon for periods $t = 1, 2$ and only up to the nodes in subset $\mathcal{N}^t$ for $t = 3$. So, by construction, there are also 14 places for risk reduction.

After fixing the solution for period $t = 1$ to the value obtained from solving the joint original model solved in the first step, the second one considers $|\mathcal{N}^2| = 12$ independent submodels as a mixture of the related models (6) and (5). Each submodel is supported by a scenario subtree rooted with node n, for $n \in \mathcal{N}^2$ as for RN, TCVaR and EVCaR, but now $\mathcal{T} = \{2, 4\}$ (note that the scheme has advanced one period in the implementation), and $\rho_1^2 = 5$ and $\rho_1^4 = 0.25$. The risk reduction is performed in each scenario subset Ω^n for $n \in \mathcal{N}^t$ in period $t = 2$, where the profit is taken into account up to the end of the time horizon, and only up to the nodes in subset $\mathcal{N}^t$ for $t = 4$. Thus, there are also 24 places for risk reduction.

Remark. The second steps of the time horizon-based schemes for obtaining the *implementable* TCVaR and MCVaR solutions in the pilot case are identical. However, the solutions could be different, since both schemes start with different solutions for period $t = 1$, they have been obtained in their first steps.

Observe from the results shown in Table 6 that MCVaR is usually the best measure for all periods, but the last one ($t = 6$) where ECVaR is generally the most profitable one.

From the analysis of the results shown in Table 7 for the *implementable* policy, we can observe that ECVaR is clearly the champion in the periods $t = 2, 3, 6$ for instance i3, TCVaR is clearly the champion in period $t = 1$ for the three instances, and MCVaR is the champion in periods $t = 2, 3$ for instances i1 and i2 and close to the other measures in the rest of the experiments.

Comparing the expected profit results and profit distribution shown in Table 4 (*planned* policy) and Table 6 (*implementable* policy), for the three risk averse CVaR measures, it can be observed that MCVaR is not the champion measure in the latter as it was in the former. See in the next section a discussion about the opportunity of considering the *implementable* policy versus the *planned* one and the meaning of the latter.

5. Discussion and outline of future research plans

In this work, we have formulated and solved a multi-period stochastic mixed 0–1 model for planning forest harvesting and road building. We consider uncertainties in timber prices and demand along the time horizon, by analyzing a finite set of discrete scenarios, as already shown, contrary to the traditional approach that uses average (i.e., expected) values for the uncertain parameters. Contrary also to the most frequent approaches in the stochastic optimization literature, the model proposed in this work considers that the parameters' uncertainty is not independent periodwise, but it is based on the probability distribution of the parameters' realizations, which depend on the realization of the same or other uncertain parameters in the previous periods.

The main contribution of the work has been to extend the risk management on the solution of the risk neutral (RN) model for the forest harvesting problem, by considering two versions of the very popular CVaR risk averse measure in a stochastic model. It can be observed that the main advantage of the risk averse measures is that they advance the profit to early periods at the price of a very small deterioration (in the experiment, at least) in the expected profit at the end of the time horizon. That is, risk averse measures provide higher profits at early periods than EV and RN, in addition to reducing the variability of the profits for unwanted scenarios (i.e., low-probability scenarios with a profit in unwanted quantiles).

One version of the risk averse measure is the time-consistent ECVAR. It allows risk reduction on the values of the given function (in this case, the harvesting profit) for the whole time horizon in the scenario groups in one-to-one correspondence with the nodes in the scenario tree that belong to a modeler-defined subset of periods. The other version is the time-inconsistent TCVaR. It allows risk reduction to be performed on the values of the objective function in the whole set of scenarios up to a modeler-defined subset of periods of the time horizon. Risk management can thus be performed at intermediate periods, a very useful additional tool for a decision-maker who needs to plan for a long time horizon. The performance of both risk averse measures has been analyzed and compared with the traditional EV and RN measures, by considering the solutions obtained for a set of large instances of the forestry planning problem. We have also experimented with the so-called MCVAR measure, a mixture of the other two CVaR versions dealt with in the work. An interesting advantage of TCVaR and MCVaR over ECVAR is that they can be used in all periods. On the contrary, by construction, the periods of the time horizon whose nodes do not have successor subtrees cannot not be used for building the groups of scenarios in ECVaR. Observe that in the instances we have experimented with, risk reduction can be performed for the periods t=2,3,4,5,6 by using TCVaR, while it can only be performed for periods 1 and 2 when using ECVaR. Notice that, by construction, TCVaR for $t = 6$ is the same as ECVaR for $t = 1$. At any rate, our provisional conclusion is that the mixture MCVaR takes advantage of both risk averse measures in the *planned* policy. Its obvious drawback is the computing time. See below our future plan to address this issue.

On the other hand, as expected given the tightness of the RN model (2), the computing time that is required by plain use of the state-of-the-art MIP solver CPLEX is very reasonable (up to 4 hours, approx) for the HW/SW platform that we have used for such large-sized instances (up to 250,000 constraints, 400,000 continuous variables and 36,000 0–1 variables). Notice that the WS and EV measures required up to 123 and 11 seconds, respectively. Models TCVaR (5), ECVaR (6) and the mixture MCVaR, however, require a much higher computing effort (very frequently reaching the time limit of 24 hours, even though the optimality gap allowed for the MIP solver was 2%), see Appendix B.

Another important issue that was dealt with in this work is the computational analysis of the *implementable* policy versus the *planned* one when deciding about the accuracy of the assessment on the quality of the risk measures under consideration. The *planned* policy is a more frequent approach in the literature than the *implementable* one for assessing the goodness of models, risk measures and even decomposition algorithm for problem solving. Probably, it is due to the difficulty to handle risk averse measures for multistage problems in the latter policy. Notice that the analysis based on the *planned* policy could only be correct if it is assumed that the *planned* decisions (solution of future nodes along the time horizon) are to be implemented. Since, by construction, a time inconsistent model does not guarantee that, then, performing post-optimization analysis on the *planned* policy is misleading. We have reported the main results from the solution of the risk measures RN, TCVaR, ECVaR and MCVaR by considering the *implementable* policy versus the *planned* one. In the former policy we have used a rolling horizon scheme that is very similar to the one presented in Agustín et al. (2012), albeit only for the RN measure in that work.

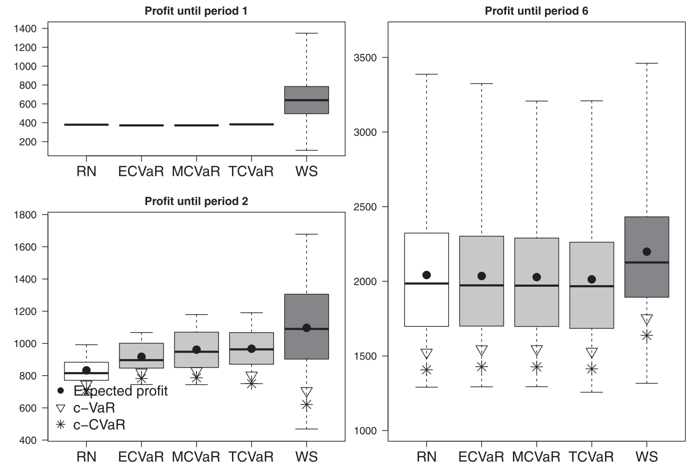
The overall conclusion from the results of our experiment is: In the *planned* policy, MCVaR could be the measure of choice if the main goal of the decision maker is the profit advancement to early periods, and ECVaR is usually the most profitable one (at the end of the time horizon) and, then, it is the measure of choice if the main goal of the decision maker is to obtain the highest profit. And, in the *implementable* policy: TCVaR is the measure that most advances the profit to the first period, and ECVaR continuous being the most profitable measure (at the end of the time horizon).

In our future research plan, we will develop a decomposition methodology for reducing the computational effort needed for solving the instances while dealing with the risk averse measures. Notice that the model requires groups of cross scenario constraints (as many groups as the number of nodes in the subset

Table 6
Results and profit distribution for the different approaches. *Implementable* policy.

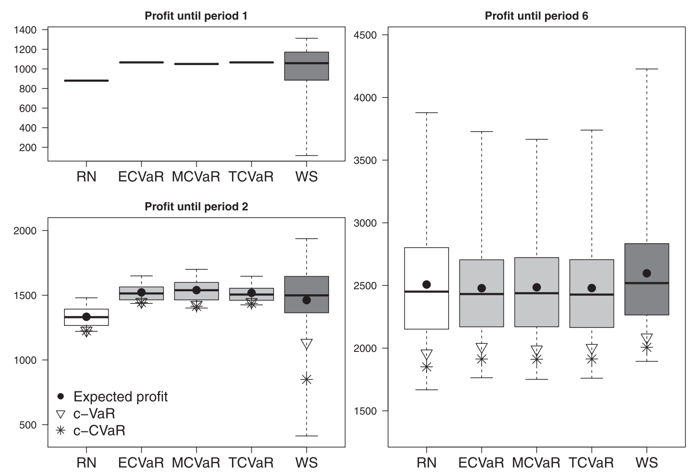
Instance i1

t	Stat.	RN	ECVaR	MCVaR	TCVaR	WS
1	Expected	17.3	16.9	16.9	17.4	28.7
2	Expected	37.9	41.7	43.7	44.0	49.9
	c-VaR	33.9	37.3	37.7	36.5	32.1
	c-CVaR	32.4	35.6	35.8	34.1	28.3
6	Expected	92.9	92.6	92.2	91.6	100.0
	c-VaR	69.3	70.3	70.4	69.6	79.8
	c-CVaR	64.0	64.9	64.9	64.3	74.5



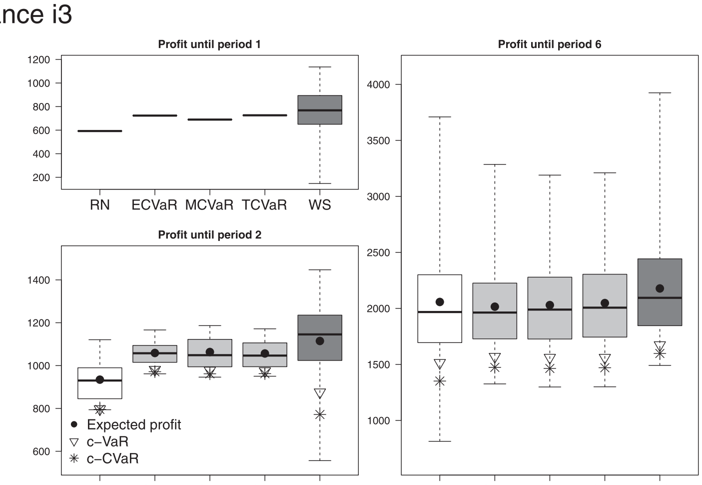
Instance i2

t	Stat.	RN	ECVaR	MCVaR	TCVaR	WS
1	Expected	33.9	41.1	40.5	41.1	38.5
2	Expected	51.4	58.7	58.6	58.7	56.3
	c-VaR	47.3	56.0	55.3	55.9	44.3
	c-CVaR	47.2	55.9	54.6	55.9	33.1
6	Expected	96.6	95.6	95.9	94.4	100.0
	c-VaR	75.5	77.2	76.6	75.0	80.5
	c-CVaR	71.3	73.8	73.6	70.7	77.3



Instance i3

t	Stat.	RN	ECVaR	MCVaR	TCVaR	WS
1	Expected	27.2	33.2	31.7	33.3	35.2
2	Expected	42.9	48.6	48.8	48.5	51.2
	c-VaR	36.6	45.0	44.9	44.7	40.2
	c-CVaR	36.5	44.6	44.2	44.2	35.4
6	Expected	94.5	92.5	93.2	94.0	100.0
	c-VaR	69.5	72.1	71.7	71.7	76.8
	c-CVaR	62.1	67.7	67.2	67.5	73.4



of periods considered), such that the nice structure of the scenario tree based constraints is destroyed. And, so, typical decomposition algorithms cannot be used with an affordable computational effort for solving large-sized problems. Hence, given the large dimensions of the instances and the problems' complexity, it is unrealistic to seek for an optimal solution. Our research effort on developing suitable decomposition algorithms will be twofold: On the one hand, our effort will be concentrated on specific versions of Lagrangean Decomposition (Guignard, 2003; Guignard & Kim, 1987), the so-called scenario Cluster Dualization and

Table 7
Comparison of the three measures: ECVaR, MCVaR and TCVaR. *Implementable* policy.

Instance i1						
	ECVaR		MCVaR		TCVaR	
t	#best	dev.best	#best	dev.best	#best	dev.best
1	0	2.77	0	2.77	**144**	**0.00**
2	12	7.07	48	**3.14**	**84**	3.62
3	40	**2.55**	**69**	2.83	35	3.55
6	**72**	**0.50**	43	0.86	29	1.67
Instance i2						
	ECVaR		MCVaR		TCVaR	
t	#best	dev.best	#best	dev.best	#best	dev.best
1	**144**	**0.00**	0	1.52	**144**	**0.00**
2	48	3.03	**72**	**1.87**	24	2.71
3	44	1.66	**49**	**1.18**	51	1.66
6	**56**	1.85	34	**1.12**	54	1.73
Instance i3						
	ECVaR		MCVaR		TCVaR	
t	#best	dev.best	#best	dev.best	#best	dev.best
1	0	0.34	0	5.01	**144**	**0.00**
2	**72**	3.95	60	2.79	12	**2.40**
3	**54**	1.65	39	1.76	51	**1.54**
6	**58**	3.37	33	1.85	53	**0.85**

Lagrangean Relaxation (CDLR) algorithms (Escudero, Garín, & Unzueta, 2017b) for providing strong upper bounds on the solution value of the problem, and on a type of Lagrangean heuristic for obtaining (hopefully, good) feasible solutions with guaranteed goodness gap for the TCVaR measure. On the other hand, given the structure of the scenario tree, and considering that the scenario groups belong to a modeler-defined subset of periods, the ECVaR measure has a suitable structure to be exploited. Then, the algorithm could be chosen from a computational comparison between different decomposition methodologies, such as an scenario CDLR, a version of the Progressive Hedging methodology (Rockafellar & Wets, 1991; Veliz et al., 2015) called Regularized Cluster Progressive (Escudero, Garín, Monge, & Unzueta, 2017) , and an stepwise dependent non-Markovian-based Stochastic Nested Decomposition (Aldasoro, Escudero, Merino, Monge, & Pérez, 2015; Escudero, Monge, & Romero-Morales, 2017c; 2015; Zou et al., 2016). The benchmark testbed will include problems in forestry planning from this work, supply chain management from Aldasoro et al. (2015), and Escudero et al. (2017c), preparedness resource allocation planning in humanitarian logistics from Escudero et al. (2017a), rapid transit network design planning from Cadarso, Escudero, and Marín (2016), and electricity generation and transmission capacity expansion planning from Alonso-Ayuso, Escudero, and Martín-Campo (2016).

Acknowledgement

The authors would like to thank the three anonymous reviewers for their help on clarifying some of the concepts presented in the manuscript and strongly improving its presentation, mainly suggesting the analysis of the *implementable* policy for deciding about the accuracy of the assessment on the quality of the risk measures under consideration.

Appendix A. Deterministic forestry model

In this appendix, a tighter formulation of the deterministic mixed 0–1 model in Andalaft et al. (2003) is presented. It allows to solve the related stochastic version for the large-sized instances in the experiment whose computational experience is presented in Section 4. This new version of the model includes a redefinition of the variables associated to road building and upgrading and, on the other hand, the step variables-based model is used instead of the impulse variables-based one). Additionally, some constraints are reformulated in order to tighten the model, specifically the constraint systems (6g) and (6h) and (11b)–(11i).

Notation

The parameters and variables are denoted with capital and small letters, respectively.

Sets

- $\mathcal{T}=\{1,\dots,T\}$, set of time periods (summer and winter seasons).
- $\mathcal{T}^S,\mathcal{T}^W$, set of summer and winter time periods, respectively, such that $\mathcal{T}=\mathcal{T}^S\cup\mathcal{T}^W$, $\mathcal{T}^S\cap\mathcal{T}^W=\emptyset$.
- $\mathcal{Q}=\{q_1,q_2,\dots,q_K\}$, harvest product. It can be considered an ordered set, such that product q has higher quality than product q' provided that $q<q'$. A higher-level quality can be used for lower level purposes, at a loss in sale price.
- $\mathcal{S}$, set of stands.
- $\mathcal{G}=(\mathcal{I},\mathcal{L})$, where $\mathcal{I}$ is the set of nodes in the network and $\mathcal{L}$ is the set of links.
- $\mathcal{I}$, set of nodes in the road network, $\mathcal{I}=\mathcal{I}^O\cup\mathcal{I}^I\cup\mathcal{I}^F\cup\mathcal{C}$, where $\mathcal{C}$ is the set of stocking yards, $\mathcal{I}^O$ is the set of origin nodes (it has some stands associated), $\mathcal{I}^I$ is the set of intermediate nodes (a junction in the roads network), and $\mathcal{I}^F$ is the set of final destination nodes (it is directly connected to the markets).
- $\mathcal{S}_i$, set of stands associated with origin node i, $\mathcal{S}_i\subset\mathcal{S}$, $\forall i\in\mathcal{I}^O$.
- $\mathcal{L}$, set of links (potential or existing) in the road network. A link is an edge linking two consecutive nodes in the road network.
- $\mathcal{R}=\{d,g\}$, road standards, d being for dirt and g for gravel.
- $\mathcal{L}^E,\mathcal{L}^P$, set of existing and potential links, respectively.
- $\mathcal{L}^{Ed},\mathcal{L}^{Eg}$, set of existing links in dirt and gravel, respectively. Note 1: All public roads exist from the beginning of the time horizon and they are in gravel. Note 2: Existing roads in dirt (i.e., private roads) can be upgraded to gravel. Note 3: Existing or potential dirt links cannot be used in winter, so, they should be upgraded to gravel in case they are to be used.
- $\mathcal{M}$, set of markets.
- $\mathcal{M}_i$, set of markets served from node i (where i is a final destination or stocking yard), $\forall i\in\mathcal{I}^F\cup\mathcal{C}$. Note: $\mathcal{M}_i\subseteq\mathcal{M}$.
- $\Gamma(i)$, adjacency set of node i, for $i\in\mathcal{I}$. Note: $j\in\Gamma(i)\iff\{i,j\}\in\mathcal{I}\iff i\in\Gamma(j)$.

Constraint-related parameters

- B_s^{qt}, amount of timber (m^3) of quality q produced per ha in stand s if harvested in period t, (this parameter is determined through a growth simulator), $\forall s\in\mathcal{S},q\in\mathcal{Q},t\in\mathcal{T}$.
- $\overline{A}_s$, upper bound in the area (ha) of stand s that can be harvested, $\forall s\in\mathcal{S}$.
- $\underline{A}_s$, lower bound in the area (ha) of stand s to be harvested in any time period, if any, $\forall s\in\mathcal{S}$.
- N_s, maximum number of periods that stand s can be harvested. Note 1: It depends on A_s, such that combined with $\underline{A}_s$ it tries to concentrate the harvesting of a stand in a reasonable number of time periods with a minimum area to be harvested, at least. Note 2: A possible value for N_s could be $\left\lceil\frac{\overline{A}_s}{\underline{A}_s}\right\rceil$, $\forall s\in\mathcal{S}$.
- U_{ijr}^t, flow capacity (cubic meters) on link $\{i,j\}$ built in standard r available in period t, $\forall\{i,j\}\in\mathcal{L},r\in\mathcal{R},t\in\mathcal{T}$. Note: The flow in a link can be in both directions.
- $\underline{Z}_m^{qt},\overline{Z}_m^{qt}$, lower and upper bound on demand (cubic meters) of product k at destination m in period t, $\forall q\in\mathcal{Q},m\in\mathcal{M},t\in\mathcal{T}$.
- C_c, capacity (cubic meters) of stocking yard c, $\forall c\in\mathcal{C}$.
- τ_d and τ_g, latency (i.e., number of periods) required for making available a potential link in dirt and in gravel, respectively, since the time period that is decided to build it. Note: If a link

is available in any of the first $\tau_d - 1$ ($\tau_g - 1$) periods, then it is assumed that the decision to build it is made before the beginning of the time horizon.

Objective function parameters at their et present value (NPV)

- R_m^{qt} and S_m^{qt}, unit selling price and unit penalization cost for unmet demand of timber of quality q, respectively, in market m in period t, $\forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T}$. Note: $S_m^{qt} >> R_m^{qt}$.
- P_s^t, unit harvesting cost per ha in stand s in period t, $\forall s \in \mathcal{S}, m \in \mathcal{M}, t \in \mathcal{T}$.
- $\overline{P}_i^{qt}$, unit production cost per cubic meters of timber of quality q in node i in period t, $\forall i \in \mathcal{I}^O, q \in \mathcal{Q}, t \in \mathcal{T}$.
- D_{ijr}^{qt}, unit transportation cost of timber of quality q through link $\{i, j\}$ in standard r in period t, $\forall \{i, j\} \in \mathcal{L}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T}$.
- $\overline{D}_{im}^{qt}$, unit transportation cost of timber of quality q from node i to market m in period t, $\forall i \in \mathcal{I}^F \cup \mathcal{C}, m \in \mathcal{M}_i, q \in \mathcal{Q}, t \in \mathcal{T}$.
- H_{ijr}^t, cost of building link $\{i, j\}$ in standard r, $\forall \{i, j\} \in \mathcal{L}^P, r \in \mathcal{R}$.
- $\overline{H}_{ij}^t$, cost of upgrading link $\{i, j\}$ from standard dirt to gravel in period t, $\forall \{i, j\} \in \mathcal{L}^P \cup \mathcal{L}^{Ed}, t \in \mathcal{T}$.
- $\hat{H}_c^t$, unit stocking cost in yard c in period t, $\forall c \in \mathcal{C}, t \in \mathcal{T}$.

Binary variables

- $w_{ijr}^t = 1$ if link $\{i, j\}$ is built in standard r *by* time period t (it is available τ_r time periods later), and otherwise, 0, $\forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}, r \in \mathcal{R}$. Notice that w_{ijr}^t is called a step variable, it makes the model stronger than when using the counterpart impulse variable, see e.g., Guignard et al. (1998) for forest harvesting.
- $v_{ij}^t = 1$ if link $\{i, j\}$ is upgraded from dirt to gravel *by* time period t, and otherwise, 0, $\forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}$. Note: The upgrade is available τ_g time periods later, and the link cannot be upgraded in the same time period it is built.
- $e_s^t = 1$, if stand s is harvested in period t, and otherwise, 0, $\forall s \in \mathcal{S}, t \in \mathcal{T}$. Notice that, by construction, it is a so-called impulse variable.

Continuous variables

- x_s^t, area (hectare) of stand s harvested in period t, $\forall s \in \mathcal{S}, t \in \mathcal{T}$.
- y_i^{qt}, volume (cubic meters) of timber of quality q harvested in all stands associated with origin i during period t, $\forall i \in \mathcal{I}^O, q \in \mathcal{Q}, t \in \mathcal{T}$.
- f_{ijr}^{qt}, flow (cubic meters) of timber of quality q transported on link $\{i, j\}$ built in standard r in period t, $\forall \{i, j\} \in \mathcal{L}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T}$. Note: $f_{ijr}^{qt} = 0$ for all existing links in dirt ($\{i, j\} \in \mathcal{L}^E$ and $r = d$) and for all potential links in dirt in winter ($\{i, j\} \in \mathcal{L}^P$, $r = d$ and $t \in \mathcal{T}^W$).
- f_{icr}^{qt}, flow (cubic meters) of timber of quality q transported from node i on link $\{i, c\}$ in standard r to its adjacent c in period t, $\forall i \in \Gamma(c), c \in \mathcal{C}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T}$.
- f_{im}^{qt}, flow (cubic meters) of timber of quality q transported from node i (i.e., a final destination or a stoking yard) to market m at period t, $\forall i \in \mathcal{I}^F \cup \mathcal{C}, q \in \mathcal{Q}, m \in \mathcal{M}_i, t \in \mathcal{T}$.
- z_m^{qt}, amount (cubic meters) of timber delivered as quality q to destination m in period t, $\forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T}$. Note: The timber delivered to the market at the price of quality q can actually be (in part or totally) of a higher quality.
- z_m^{-qt}, unmet timber demand (cubic meters) of quality q requested at destination m in period t, $\forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T}$.

Constraints

1. Road network design. Decisions about new links or upgrades to gravel can only be taken in summer (since is the only period where the work can be done):

$$w_{ijr}^{t-1} \le w_{ijr}^t, \qquad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S, \tag{7a}$$

$$w_{ijr}^{t-1} = w_{ijr}^t, \qquad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^W, \tag{7b}$$

$$v_{ij}^{t-1} \le v_{ij}^t, \qquad \forall \{i, j\} \in \mathcal{L}^P \cup \mathcal{L}^{Ed}, t \in \mathcal{T}^S, \tag{7c}$$

$$v_{ij}^{t-1} = v_{ij}^t, \qquad \forall \{i, j\} \in \mathcal{L}^P, t \cup \mathcal{L}^{Ed} \in \mathcal{T}^W. \tag{7d}$$

A link cannot be upgraded from dirt to gravel if it has not been built two periods earlier (i.e., one year), at least:

$$v_{ij}^t \le w_{ijd}^{t-2}, \qquad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}. \tag{7e}$$

Road incompatibility: A link cannot simultaneously be built in dirt and in gravel:

$$\sum_{r \in \mathcal{R}} w_{ijr}^t \le 1, \qquad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^W. \tag{7f}$$

Note: A link built in dirt can be *upgraded* to gravel, but cannot be *built* in gravel.
Origins-to-roads triggers: If a stand is not connected to a existing link at the beginning of the time horizon and it is harvested, then one potential link has to be built, at least. Furthermore, if a stand is not connected to a existing link and it is harvested in winter, then one potential link has to be built in gravel, at least, by this time period, see Andalaft et al. (2003) for more details. For $\forall i \in \mathcal{I}^O$ such that $\left\{\{i, j\} \in \mathcal{L}^E : j \in \Gamma(i)\right\} = \emptyset$:

$$\sum_{t' \in \mathcal{T}: t' \le t} e_s^{t'} \le \min\{t, N_s\} \sum_{\{i,j\} \in \mathcal{L}^P} \sum_{r \in \mathcal{R}} w_{ijr}^{t-\tau_r}, \quad \forall s \in \mathcal{S}_i, t \in \mathcal{T}, \tag{7g}$$

$$\sum_{t' \in \mathcal{T}^W: t' \le t} e_s^{t'} \le \min\{N_t^W, N_s\} \sum_{\{i,j\} \in \mathcal{L}^P} \left(w_{ij,g}^{t-\tau_r} + v_{ij}^{t-\tau_g}\right), \quad \forall s \in \mathcal{S}_i \cup \mathcal{S}_j, t \in \mathcal{T}^W, \tag{7h}$$

where $N_t^W = |\{t' \in \mathcal{T}^W : t' \le t\}|$.
Road-to-road triggers (Andalaft et al., 2003; Guignard et al., 1998): If a potential link $\{i, j\}$ that is not connected to a existing one is built, then, one of the links connecting $\{i, j\}$ must be built, at least:

$$\sum_{r \in \mathcal{R}} w_{ijr}^t \le \sum_{\{i',j'\} \in \mathcal{L}^{\{i,j\}}} \sum_{r \in \mathcal{R}} w_{i'j'r}^t, \quad \forall \{i, j\} \in \mathcal{L}^P : \mathcal{L}^{\{i,j\}} \cap \mathcal{L}^E = \emptyset, t \in \mathcal{T}, \tag{7i}$$

where $\mathcal{L}^{\{i,j\}}$ is the set of links adjacent to $\{i, j\}$.

2. Harvesting decisions:

$$\underline{A}_s e_s^t \le x_s^t \le \overline{A}_s e_s^t, \qquad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{8a}$$

$$\sum_{t \in \mathcal{T}} x_s^t \le \overline{A}_s, \qquad \forall s \in \mathcal{S}, \tag{8b}$$

$$\sum_{t \in \mathcal{T}} e_s^t \le N_s, \qquad \forall s \in \mathcal{S}, \tag{8c}$$

where constraints (8a) bound the harvested area per stand and time period, constraints (8b) bound area harvested per stand, and constraints (8c) bound the number of periods that a stand can be harvested. Notice that (8c) is redundant for the 0–1 model but it is tightening its LP relaxation.

3. Production by origin

$$\sum_{s \in \mathcal{S}_i} B_s^{qt} x_s^t = y_i^{qt}, \qquad \forall i \in \mathcal{I}^O, q \in \mathcal{Q}, t \in \mathcal{T}. \tag{9}$$

4. Flow constraints for origin nodes (10a), intermediate nodes (10b), and final destination nodes (10c):

$$y_i^{qt} + \sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{jir}^{qt} = \sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{ijr}^{qt}, \quad \forall i\in\mathcal{I}^O, q\in\mathcal{Q}, t\in\mathcal{T}, \tag{10a}$$

$$\sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{jir}^{qt} = \sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{ijr}^{qt}, \quad \forall i\in\mathcal{I}^I, q\in\mathcal{Q}, t\in\mathcal{T}, \tag{10b}$$

$$\sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{jir}^{qt} = \sum_{r\in\mathcal{R}}\sum_{j\in\Gamma(i)} f_{ijr}^{qt} + \sum_{m\in\mathcal{M}_i} f_{im}^{qt}, \quad \forall i\in\mathcal{I}^F, q\in\mathcal{Q}, t\in\mathcal{T}. \tag{10c}$$

For stocking yards, arrivals in summer must be equal to dispatches in winter, (10d). On the other hand there are neither arrivals in winter nor dispatches in summer, (10e) and (10f):

$$\sum_{r\in\mathcal{R}, i\in\Gamma(c)} f_{icr}^{qt} = \sum_{m\in\mathcal{M}_c} f_{cm}^{qt+1}, \quad \forall c\in\mathcal{C}, q\in\mathcal{Q}, t\in\mathcal{T}^S, \tag{10d}$$

$$f_{icr}^{qt} = 0, \quad \forall r\in\mathcal{R}, c\in\mathcal{C}, i\in\Gamma(c), q\in\mathcal{Q}, t\in\mathcal{T}^W, \tag{10e}$$

$$f_{cm}^{qt} = 0, \quad \forall q\in\mathcal{Q}, c\in\mathcal{C}, m\in\mathcal{M}_c, t\in\mathcal{T}^S. \tag{10f}$$

5. Demand constraints. The amount of timber delivered to a market as quality q is bounded by the flow arriving to the nodes that can serve that market. Notice that the timber's quality can be higher than requested at a lost, as state above:

$$\sum_{q'\in\mathcal{Q}:q'\le q} z_m^{q't} \le \sum_{q'\in\mathcal{Q}:q'\le q}\sum_{i\in\mathcal{I}^F\cup\mathcal{C}} f_{im}^{q't}, \quad \forall q\in\mathcal{Q}, m\in\mathcal{M}, t\in\mathcal{T}, \tag{11a}$$

$$\underline{Z}_m^{qt} \le z_m^{-qt} + z_m^{qt} \le \overline{Z}_m^{qt}, \quad \forall q\in\mathcal{Q}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{11b}$$

6. Capacity constraints
Capacity at stocking yards (in summer):

$$\sum_{q\in\mathcal{Q}}\sum_{r\in\mathcal{R}}\sum_{i\in\Gamma(c)} f_{icr}^{qt} \le C_c, \quad \forall c\in\mathcal{C}, t\in\mathcal{T}^S. \tag{12a}$$

Flow through a potential link in dirt (i.e., $r = d$) is only allowed in any period if the link has been previously built in dirt and not upgraded to gravel:

$$\sum_{q\in\mathcal{Q}}\left(f_{ijd}^{qt'} + f_{ijd}^{qt'}\right) \le U_{ijd}^t\left(w_{ijd}^{t-\tau_d} - v_{ij}^{t-\tau_g}\right), \quad \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}^S. \tag{12b}$$

If there is any flow on dirt through a potential link in dirt between the beginning of the time horizon and period t, the link must have been built *by* time period t:

$$\sum_{t'\in\mathcal{T}:t'\le t}\sum_{q\in\mathcal{Q}}\left(f_{ijd}^{qt'} + f_{ijd}^{qt'}\right) \le U_{ijd}^t w_{ijd}^{t-\tau_d}, \quad \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}^S,$$
$$r = d \text{ (dirt)}. \tag{12c}$$

Even more, if there is some flow on dirt through a potential link in dirt during or after time period t, the link cannot have been upgraded by time period t:

$$\sum_{t'\in\mathcal{T}:t\le t'}\sum_{q\in\mathcal{Q}}\left(f_{ijd}^{qt'} + f_{ijd}^{qt'}\right) \le U_{ijd}^t\left(1 - v_{ij}^{t-\tau_g}\right), \quad \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}^S. \tag{12d}$$

The dirt links are only available in summer, so, no flow is allowed through them in winter:

$$f_{ijd}^{qt} + f_{jid}^{qt} = 0, \quad \forall q\in\mathcal{Q}, \{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}^W. \tag{12e}$$

Flow through a potential link in gravel is only allowed at any time period if the link has been built in gravel or upgraded from dirt to gravel *by* that period:

$$\sum_{t'\in\mathcal{T}:t'\le t}\sum_{q\in\mathcal{Q}}\left(f_{ijg}^{qt'} + f_{jig}^{qt'}\right) \le U_{ijg}^t\left(w_{ijg}^{t-\tau_g} + v_{ij}^{t-\tau_g}\right), \quad \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}. \tag{12f}$$

For an existing link in dirt, flow is only allowed provided they have not been upgraded to gravel:

$$\sum_{t'\in\mathcal{T}:t\le t'}\sum_{q\in\mathcal{Q}}\left(f_{ijd}^{qt'} + f_{jid}^{qt'}\right) \le U_{ijd}^t\left(1 - v_{ij}^{t-\tau_g}\right), \quad \forall\{i,j\}\in\mathcal{L}^{Ed}, t\in\mathcal{T}^S. \tag{12g}$$

For such links, flow on gravel is allowed only if it has been upgraded:

$$\sum_{t'\in\mathcal{T}:t'\le t}\sum_{q\in\mathcal{Q}}\left(f_{ijg}^{qt} + f_{jig}^{qt}\right) \le U_{ijg}^t v_{ij}^{t-\tau_g}, \quad \forall\{i,j\}\in\mathcal{L}^{Ed}, t\in\mathcal{T}^S. \tag{12h}$$

For existing links in gravel, flow is upper bounded:

$$\sum_{q\in\mathcal{Q}}\left(f_{ijg}^{qt} + f_{jig}^{qt}\right) \le U_{ijg}^t, \quad \forall\{i,j\}\in\mathcal{L}^{Eg}, t\in\mathcal{T}. \tag{12i}$$

7. Variables' domain definition:

$$
\begin{aligned}
w_{ijr}^t &\in \{0,1\} && \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T}, r\in\mathcal{R},\\
e_s^t &\in \{0,1\} && \forall s\in\mathcal{S}, t\in\mathcal{T},\\
v_{ij}^t &\in \{0,1\} && \forall\{i,j\}\in\mathcal{L}^P, t\in\mathcal{T},\\
x_s^t &\ge 0 && \forall s\in\mathcal{S}, t\in\mathcal{T},\\
f_{im}^{qt} &\ge 0 && \forall q\in\mathcal{Q}, i\in\mathcal{I}^F\cup\mathcal{C}, m\in\mathcal{M}_i, t\in\mathcal{T},\\
f_{ijr}^{qt} &\ge 0 && \forall\{i,j\}\in\mathcal{L}, r\in\mathcal{R}, q\in\mathcal{Q}, t\in\mathcal{T},\\
y_i^{qt} &\ge 0 && \forall i\in\mathcal{I}^O, q\in\mathcal{Q}, t\in\mathcal{T},\\
z_m^{qt}, z_m^{-qt} &\ge 0 && \forall q\in\mathcal{Q}, m\in\mathcal{M}, t\in\mathcal{T}.
\end{aligned}
$$

Objective function

The objective function under consideration consists of maximizing the profit NPV, i.e., income minus cost. It can be expressed as follows:

$$\max \sum_{t\in\mathcal{T}}\Bigg\{\sum_{m\in\mathcal{M}}\sum_{q\in\mathcal{Q}} R_m^{qt} z_m^{qt} - \Bigg[\sum_{m\in\mathcal{M}}\sum_{q\in\mathcal{Q}} S_m^{qt} z_m^{-qt} + \sum_{s\in\mathcal{S}} P_s^t x_s^t - \sum_{i\in\mathcal{I}^O}\sum_{q\in\mathcal{Q}} \overline{P}_i^{qt} y_i^{qt}$$

Table 8
TCVaR: Results for model (5).

Inst.	ρ_1^3	ρ_1^6	$\overline{Z}_{MIP}$	Z_{MIP}	GAP	t_0 (seconds)
i1	$\frac{1}{4}$	$\frac{1}{4}$	2716.35	2663.12	1.96	18664.18
i1	$\frac{1}{4}$	5	9594.78	9388.87	2.16	> 24 hour[a]
i1	5	$\frac{1}{4}$	8966.35	8798.92	1.87	2945.52
i1	5	5	14994.94	14723.37	1.81	15856.19
i2	$\frac{1}{4}$	$\frac{1}{4}$	3428.40	3362.39	1.92	45987.24
i2	$\frac{1}{4}$	5	12627.90	12432.10	1.98	13841.28
i2	5	$\frac{1}{4}$	11347.15	11129.96	1.91	7012.70
i2	5	5	20308.53	19999.71	1.52	2536.65
i3	$\frac{1}{4}$	$\frac{1}{4}$	2813.85	2743.07	2.52	> 24 hour[a]
i3	$\frac{1}{4}$	5	9993.54	9679.57	3.14	> 24 hour[a]
i3	5	$\frac{1}{4}$	8634.14	8464.95	1.96	20850.28
i3	5	5	1552.34	1521.90	1.96	72990.54

[a] Elapsed time limit reached (24 hour).

$$+\sum_{\{i,j\}\in\mathcal{L}^P}\sum_{r\in\mathcal{R}}H^t_{ijr}w^t_{ijr}+\sum_{\{i,j\}\in\mathcal{L}^P\cup\mathcal{L}^{Ed}}\overline{H}^t_{ij}\nu^t_{ij}$$
$$+\sum_{\{i,j\}\in\mathcal{L}}\sum_{r\in\mathcal{R}}\sum_{q\in\mathcal{Q}}D^{qt}_{ijr}f^{qt}_{ijr}+\sum_{c\in\mathcal{C}}\hat{H}^t_c\sum_{i\in\Gamma(c)}\sum_{r\in\mathcal{R}}\sum_{q\in\mathcal{Q}}f^{qt}_{icr}$$
$$+\sum_{i\in\mathcal{I}^F\cup\mathcal{C}}\sum_{m\in\mathcal{M}_i}\sum_{q\in\mathcal{Q}}\overline{D}^{qt}_{im}f^{qt}_{im}\Bigg]\Bigg\}. \tag{A.13}$$

Appendix B. Additional computational results

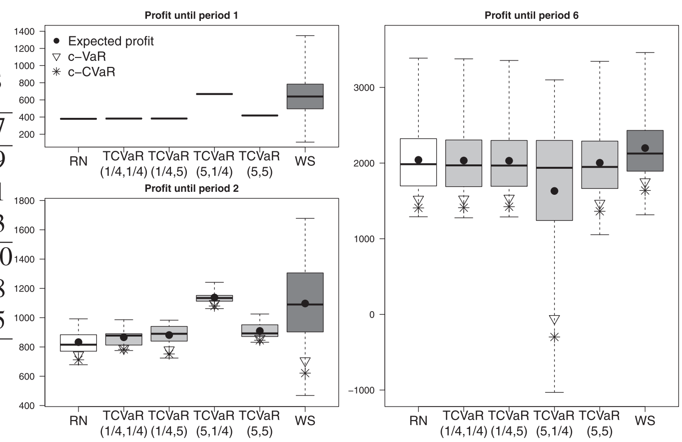
B1. Computational comparison between the TCVaR and RN models

We have tested the impact of managing the risk in two points, namely, an intermediate period and the end of the time horizon, so, say, $\tilde{\mathcal{T}}_1=\{3,6\}$, and, in order to ascertain better the impact of the risk-averse term on the objective function, different combinations of the weight parameter ρ^t_1 have been tested, for $t\in\tilde{\mathcal{T}}_1$.

Table 9
TCVaR: Results and profit distribution.

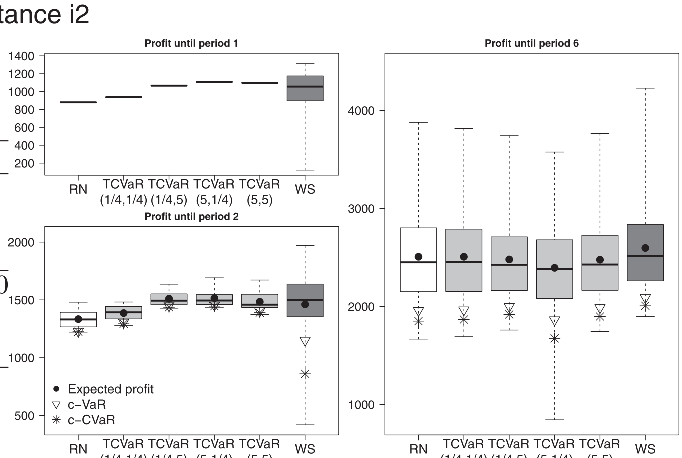
Instance i1

t	Stat.	RN	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ 5$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ 5$	WS
1	Expected	17.3	17.4	17.4	30.4	19.0	28.7
2	Expected	37.9	39.4	40.1	51.7	41.4	49.9
	c-VaR	33.9	36.0	35.5	49.9	39.1	32.1
	c-CVaR	32.4	35.6	34.2	49.1	38.5	28.3
6	Expected	92.9	92.5	92.4	74.2	91.1	100.0
	c-VaR	69.3	69.4	69.9	-2.4	66.8	79.8
	c-CVaR	64.0	64.1	64.8	-13.6	62.0	74.5



Instance i2

t	Stat.	RN	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ 5$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ 5$	WS
1	Expected	33.9	36.1	41.1	42.7	42.3	38.5
2	Expected	51.4	53.4	58.2	58.3	57.1	56.3
	c-VaR	47.3	50.3	55.8	56.1	54.1	44.3
	c-CVaR	47.2	49.8	55.3	55.7	53.5	33.1
3	Expected	96.6	96.6	95.5	92.2	95.4	100.0
	c-VaR	75.5	75.7	77.1	71.8	76.6	80.5
	c-CVaR	71.3	71.9	73.9	64.5	73.1	77.3



Instance i3

t	Stat.	RN	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $\frac{1}{4}\ 5$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ \frac{1}{4}$	TCVaR $\rho_1^3\ \rho_1^6$ $5\ 5$	WS
1	Expected	27.2	31.0	33.3	36.1	34.0	35.2
2	Expected	42.9	45.8	46.3	48.8	46.8	51.2
	c-VaR	36.6	42.2	44.8	47.2	45.1	40.2
	c-CVaR	36.5	42.0	43.9	47.1	45.0	35.4
3	Expected	94.5	96.0	94.4	89.9	92.6	100.0
	c-VaR	69.5	71.7	72.0	65.1	70.9	76.8
	c-CVaR	62.1	67.0	67.6	57.3	66.9	73.4

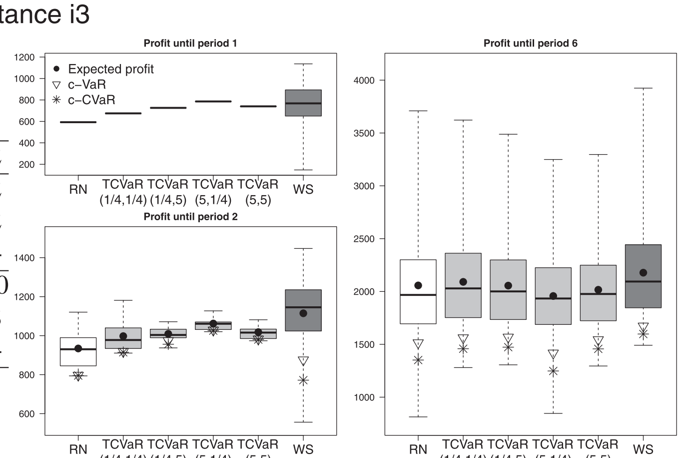


Table 10
ECVaR: Results for model (6).

Inst.	ρ_1^1	ρ_1^2	$\bar{Z}_{MIP}$	Z_{MIP}	GAP	t_0 (seconds)
i1	$\frac{1}{4}$	0	2444.42	2396.36	1.96	35033.26
i1	$\frac{1}{4}$	$\frac{1}{4}$	2875.61	2819.26	1.96	1111.53
i1	$\frac{1}{4}$	5	1114.98	10934.02	1.93	14771.75
i1	5	0	9321.70	9139.42	1.95	80076.30
i1	5	$\frac{1}{4}$	9754.30	9579.14	1.96	54083.62
i1	5	5	18000.40	17651.02	1.99	1179.69
i2	$\frac{1}{4}$	0	1419.94	1383.30	1.96	11392.24
i2	$\frac{1}{4}$	$\frac{1}{4}$	3025.71	2966.38	1.96	18374.89
i2	$\frac{1}{4}$	5	3576.17	3506.06	1.96	17933.73
i2	5	0	12259.96	12025.93	1.91	16544.01
i2	5	$\frac{1}{4}$	12814.18	1256.99	1.94	37515.63
i2	5	5	23263.51	22859.61	1.73	2654.12
i3	$\frac{1}{4}$	0	2501.44	2452.40	1.96	66026.98
i3	$\frac{1}{4}$	$\frac{1}{4}$	2967.05	2882.59	2.85	> 24 hour[a]
i3	$\frac{1}{4}$	5	11367.54	11185.94	1.60	38454.60
i3	5	0	9655.31	9471.58	1.90	19253.64
i3	5	$\frac{1}{4}$	10141.07	9932.20	2.06	> 24 hour[a]
i3	5	5	18462.22	18113.08	1.89	43353.94

[a] Elapsed time limit reached (24 hour).

The solution obtained by the TCVaR model (5) is shown in Table 8. The headings are as follows: ρ_1^3 and ρ_1^6, weight factors in the objective function for the risk management in $t = 3$ and 6, respectively; $\bar{Z}_{MIP}$, best upper bound for the optimal solution provided by the MIP solver at the time at which the solver's execution is stopped; Z_{MIP}, solution value of the incumbent solution; *GAP*, related optimality gap defined as $100\frac{\bar{Z}_{MIP}-Z_{MIP}}{Z_{MIP}}$; and *nn*, number of nodes in the B&C tree that have been explored up to the stopping time, say t_0 (seconds), of the solver's execution.

Table 9 shows the expected profit, computed VaR and computed CVaR obtained from the profit up to the end of periods $t = 1,\ 2,\ 6$ for the instances i1, i2, and i3. As a reference, the last column shows the results for the WS approach. Different results are presented for TCVaR, depending on the pairs (ρ_1^3, ρ_1^6) of the weight parameters that have been used for $t = 3, 6$.

As can be observed, all stochastic models practically provide the same expected profit at the end of the planning horizon ($t = 6$), while c-CVaR and c-VaR are slightly higher. However, TCVaR provides an expected profit for $t = 1, 2$ (especially, for the former) much higher than RN as well as a significant improvement in c-VaR and c-CVaR.

It is worth analyzing the results related to the combination $(\rho_1^3 = 5,\ \rho_1^6 = 0.25)$ as shown in Table 9. Notice that period $t = 6$ is the last one in the time horizon considered in the experiment. It is a simulation where the decision-maker is assumed to give a higher weight to CVaR for $t = 3$ than to CVaR for $t = 6$. As a result, it can be observed that the expected profit and the profit distribution up to $t = 6$ (i.e., the profit that considers the whole time horizon) in the whole set of scenarios are very poor with respect to the same results up to $t = 3$; in fact, they are very good for $t = 1$ and 2, as shown in the tables. Notice also that the profit results are very different for a simulation with opposite priorities. It is worth paying attention to this point, since the profit results could be very poor up to $t = 6$ in the irreversible situation where the decision-maker adopts a shortsighted strategy. Note that this strategy is based on a shorter time horizon (say, up to $t = 3$), whose results in the simulation are very good.

B2. Computational comparison between the ECVaR and RN models

ECVaR performs the risk management for the instances we have experimented with in each of the given subtrees in the scenarios tree, that is, for scenario groups in $\mathcal{N}^t,\ t = 1, 2$. For a better assessment of the impact of the risk averse term in the ob-

Table 11
ECVaR: Results for the three instances.

t	Stat.	RN	ECVaR ρ_1^1	ρ_1^2	ρ_1^1	ρ_1^2	ρ_1^1	ρ_1^2	ρ_1^1	ρ_1^2	ρ_1^1	ρ_1^2	ρ_1^1	ρ_1^2	WS
Instance i1															
			$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	5	5	0	5	$\frac{1}{4}$	5	5	
1	Expected	17.3	16.4		16.4		11.4		16.6		16.9		16.6		28.7
2	Expected	37.9	38.0		38.1		43.3		40.4		40.9		42.0		49.9
	c-VaR	33.9	33.0		33.0		34.7		35.2		35.8		37.3		32.1
	c-CVaR	32.4	32.8		32.8		32.7		33.6		34.8		35.5		28.3
6	Expected	92.9	93.0		93.0		92.0		92.5		92.8		92.3		100.0
	c-VaR	69.3	69.8		69.8		68.8		69.9		70.3		70.0		79.8
	c-CVaR	64.0	64.4		64.4		63.9		64.9		64.9		64.9		74.5
Instance i2															
			$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	5	5	0	5	$\frac{1}{4}$	5	5	
1	Expected	33.9	41.7		41.7		34.8		41.8		41.1		41.4		38.5
2	Expected	51.4	56.6		56.7		56.9		58.5		58.5		59.2		56.3
	c-VaR	47.3	53.5		53.5		51.6		55.7		55.9		55.5		44.3
	c-CVaR	47.2	53.0		53.0		51.4		55.7		55.5		55.3		33.1
3	Expected	96.6	96.0		96.0		95.7		95.6		95.5		95.9		100.0
	c-VaR	75.5	76.8		76.9		75.9		77.4		77.5		76.9		80.5
	c-CVaR	71.3	73.1		73.2		72.5		73.9		73.7		73.6		77.3
Instance i3															
			$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	5	5	0	5	$\frac{1}{4}$	5	5	
1	Expected	27.2	29.4		28.3		29.8		34.4		33.2		32.2		35.2
2	Expected	42.9	46.2		46.8		48.9		46.9		48.7		47.5		51.2
	c-VaR	36.6	41.7		42.0		43.6		45.2		45.0		43.8		40.2
	c-CVaR	36.5	41.5		41.8		42.5		43.5		44.8		43.0		35.4
3	Expected	94.5	96.1		95.9		96.1		94.4		95.0		94.2		100.0
	c-VaR	69.5	70.8		71.4		71.6		72.6		73.2		72.9		76.8
	c-CVaR	62.1	66.4		66.5		66.9		68.3		68.7		68.1		73.4

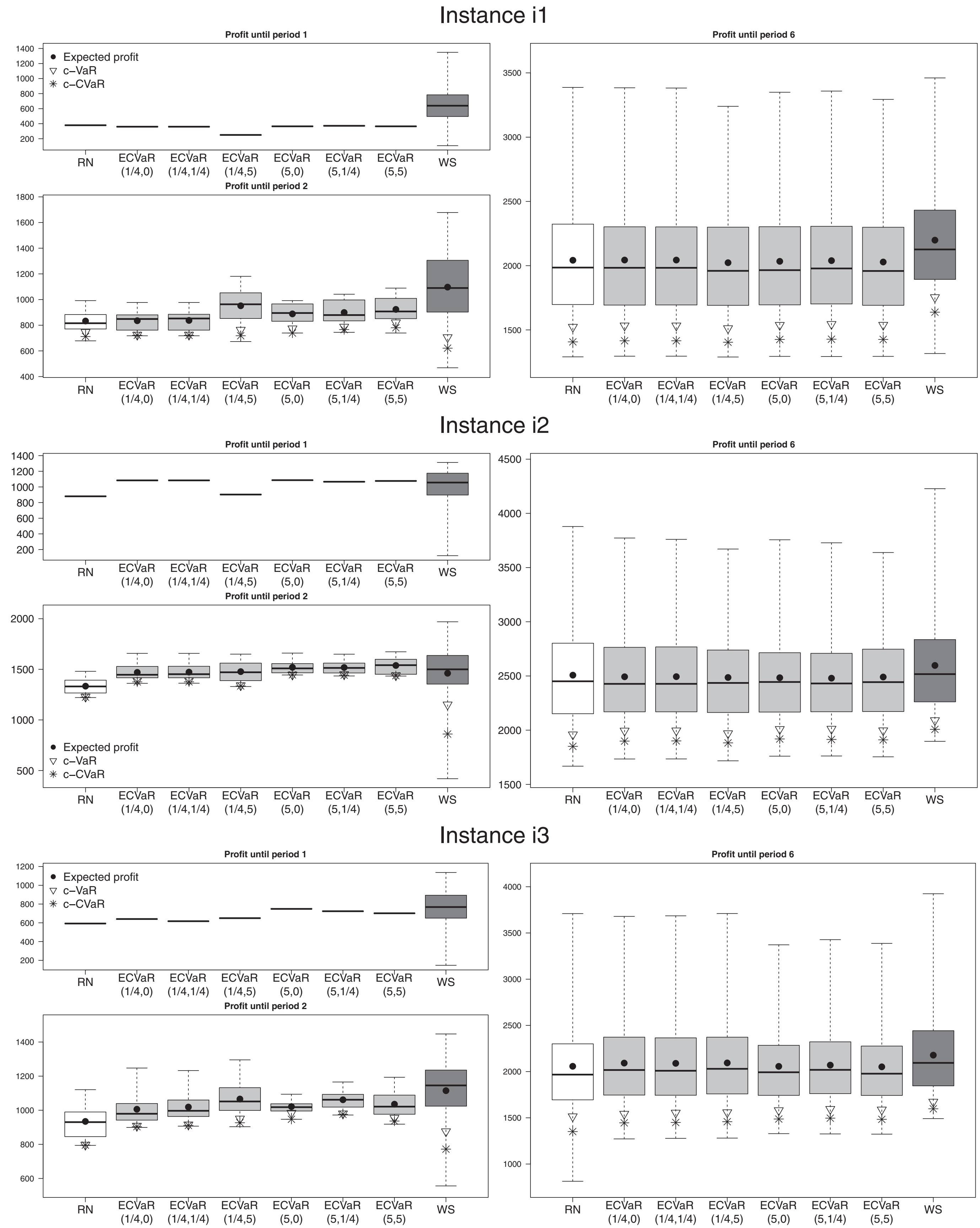


Fig. B1. ECVAR: Profit distribution.

jective function of the ECVaR model (6), several combinations of the weight parameters ρ_1^t for $t = 1, 2$ have been considered; in particular, the following ones have been tested: $\rho_1^1 \in \{0.25, 5\}$ and $\rho_1^2 \in \{0, 0.25, 5\}$. Notice that $\rho_1^2 = 0$ represents the traditional CVaR measure, then the risk management is only performed for the total profit along the whole time horizon and, so, no control is performed at intermediate periods.

The main results are shown in Table 10. The headings are as in Table 8. It can be observed that the elapsed time required for solving instance i1 is very high compared with the time required by the TCVaR model (5). So, it is unacceptable; see in Section 5 an outline of our future research plans on the issue.

The expected profit, computed VaR and computed CVaR up to the periods $t = 1, 2, 6$ obtained by the ECVaR model (6) are shown in Table 11. As in the previous tables, the last column shows the results for the WS approach, the values being normalized. Different results are presented, depending on the values of the pair (ρ_1^1, ρ_1^2) of the weight parameters that have been used. Again, it can be observed that all the stochastic models practically provide the same expected profit at the end of the time horizon ($t = 6$), but CVaR and VaR are slightly higher in ECVaR than in RN. However, ECVaR tends to provide an expected profit higher than the RN expected profit at the end of periods $t = 1, 2$ (10% at $t = 1$ and 20% at $t = 2$, for instance i2 and 4–8% at $t = 1$ and 10–15% at $t = 2$ for instance i3).

Fig. B.9 shows the profit distribution at the end of time periods $t = 1, 2, 6$ for the different combinations tested.

Also in this case, it can be concluded that ECVaR moves the profits to the early periods. Observe in the boxplot shown for instance i2 in Fig. B.9 that the conclusion is even stronger, since the profit distributions in ECVaR for periods $t = 1, 2$ are much better than in RN, being almost equal at the end of the time horizon.

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