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Decision Support

Within-group common weights in DEA: An analysis of power plant efficiency

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Abstract

In many real world applications where DEA is applied, DMUs can often be put into groups, such as those which may be under a single management team. This often means that the multipliers used within a group should be common across that group's members. The case example examined in this regard is one involving a set of power plants, with each containing a set of power units under a common plant management. We develop a goal-programming model for this setting that seeks to derive such a common-multiplier set. The important feature of this multiplier set is that it minimizes the maximum discrepancy among the within-group scores from their ideal levels.

Keywords: DEA; Common weights; Groups; Power plants

1. Introduction

Charnes et al. (1978) presented a methodology for evaluating the relative efficiencies of a set of decision-making units (DMUs). This methodology, data envelopment analysis (DEA), has been applied in numerous settings over the past 25 years. These include the analysis of efficiency of bank branches, hospitals, maintenance crews, etc. The appropriate setting to which the DEA model applies is one wherein the DMUs (e.g., bank branches) are assumed to be comparable, yet with each having its own unique circumstances. Specifically, each DMU is permitted to choose, possibly within bounds, its own set of multipliers for its output/input bundle.

In certain situations, treating each DMU as an independent entity may not be appropriate. It can be argued that if the members of a given subset of the decision-making units are experiencing similar circumstances, then the "pricing" of inputs and outputs should apply uniformly across all members of that subset. An example of this can be found in Cook et al. (1990), where maintenance patrols are evaluated relative to one another. It can be claimed that those patrols within the same "district" experience similar climatic conditions, are subject to similar resource availability, and are managed by the same district engineer. Thus, permitting patrols

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within a district to freely choose input and output multipliers that differ significantly across patrols may not be warranted.

In the current paper we extend the DEA structure to apply to the more general setting where DMUs fall into distinct groups, and where all members of a group are to be treated uniformly in terms of multiplier allocation. The specific problem setting examined is the evaluation of relative efficiencies of a set of power plants. Section 2 describes this problem setting. We demonstrate that the power "units" within the "plants" form natural groupings to which the concept of common input and output multipliers on the members applies. In Section 3 we develop a two-phase optimization model, providing a more realistic assessment of power unit efficiency. Section 4 applies the methodology to the data on eight power plants containing a total of 40 power units. Conclusions are given in Section 5.

2. The problem setting

Ontario Power Generation (formerly, Ontario Hydro) was, in the mid-1900s, a Canadian crown corporation supplying electric power to both Canadian domestic and foreign markets in the northern USA. Two classes of units were managed under the corporation's jurisdiction, namely nuclear and thermal units. While the number of nuclear units is relatively small, a total of 10 thermal plants consisting of 40 thermal units of varying ages, capacities, fuel types are operated by the corporation. Table 1 provides some basic statistics on the 10 power plants.

The power units within any plant (e.g., the eight units comprising plant #1) are very similar in many respects. They are in the same general location, are similar in age and capacity (in megawatt hours), and experience similar maintenance practices. They jointly service the same source of demand (although some units may be down for maintenance at times when others are in full operation), hence are subjected to similar work loadings. Most importantly, they share a common management team. We make this point here to emphasize the fact that performance evaluation should be conducted in a similar manner across all units within a plant. As will be seen in the following section, this will materialize in the form of requiring that common multipliers be applied to all units making up any given plant. We point out here that even though similar conditions may prevail across all DMUs in a group, such as the existence of a common management team, thus necessitating common weights, we do not advocate aggregating all of the units of that group into a single decision-making unit. There is still the need to evaluate the relative efficiency of each member of the group, to discover where gaps exist.

The standard measure of productivity used by management is the ratio of total annual expenditure (operating, maintenance and administration) to total energy produced in megawatt hours per year. While it is the case that the total power production is a principal *output* of the operation, and is certainly the most convenient and readily available indicator of productive capability, management is interested in other, related indicators as well. What may be missing in this simplistic measure of productivity is a consideration of those factors that reflect management's skill. To a great extent, a power unit's efficiency measure should reflect the quality of maintenance that keeps it operating, and the abilities of management in charge of that maintenance. At least two types of other outputs should be considered, namely *outages* and *deratings.*

An outage is a situation in which a unit is shut down; hence it is not generating electric power. Types of outages include

- planned outage, which is scheduled downtime (usually for major overhauls);
- maintenance outage, a form of scheduled down time, for more minor, i.e., routine maintenance;
- forced outage, which is unscheduled and generally caused by equipment failure, environmental requirements, or other unforeseen incidents. There is generally some prior warning for this type of shutdown, and some delay is possible.

Table 1
Thermal plans

Location	# Units	Year built	Fuel utilized	Size (MWH[a])
Plant 1	8	1971–1972	US bit. coal and Western Cdn. coal	500
Plant 2	8	1968	US bit. coal	300
Plant 3	4	1970	US bit. coal	500
Plant 4(1)	1	1964–1966	US bit. coal	100
Plant 4(2)	2	1974–1975	Liquid bit.	150
Plant 5	4	1974	Oil	500
Plant 6	1	1978	Lignite bit. coal	200
Plant 7(1)	4	1956	Gas/coal	100
Plant 7(2)	4	1960	Gas/coal	200
Plant 8	4	1952	US bit. coal	50

[a] Megawatt hours.

- sudden outage, which is a forced outage with no prior warning.

While it can be argued that operating hours essentially capture all forms of outages, it must be recognized that there is a difference between taking a unit out of service on a scheduled basis at non-peak times, versus sudden brownouts or blackouts. The latter ignite public opinion, interrupt business operations, and generally reflect negatively on the organization. Thus, such outages should play a direct role in any measure of efficiency.

A derating is a *reduction* in unit capacity where the operation may, for a number of reasons, operate at only a fraction (e.g., 75% or 50%) of its available (full) capacity. Breakdowns in coal belts, pulverizers or rollers (of which there are several operating in any plant) are a primary cause of such forced deratings. Environmental restrictions, in particular SO_2 emissions, can limit the extent to which a plant can operate a full capacity. Furthermore, such restrictions will often apply to a group of units (e.g., at a given geographical location).

As with outages, there are several forms of deratings, some of which are beyond the control of management and which have nothing to do with maintenance quality (e.g., grid or transmission network load restrictions), while others are a clear reflection of maintenance quality, such as equipment failures.

As with outputs, inputs should include several factors. In addition to expenditures, factors such as plant *age* and *available but not operating time* (ABNOT) should play a role as well. The latter factor (ABNOT) is the time during which the plant is able to operate, but for reasons beyond managements control (such as SO_2 restrictions), the plant is not running.

3. Deriving within-group common weights

3.1. Background

In earlier studies of power plant efficiency, Cook et al. (1998) and Cook and Green (2005) were concerned primarily with examining the hierarchical property of the unit/plant structure. Specifically, that study presented a methodology for evaluating efficiency at two levels. In level 1, the power units within a plant are treated as the "comparable DMUs," and a standard DEA analysis is carried out. In level 2 the units within each plant are aggregated to create a DMU representing the plant itself. Then, the DEA analysis is repeated using the plants as the DMUs. A mechanism is then utilized to adjust the level 1 rating, taking into account the ratings that the various plants received at level 2.

A shortcoming of this earlier approach is that in the level 1 analysis many "efficient" DMUs (power units) result. Two factors contribute to this outcome: (1) the small number of units per plant, and (2) the fact that each power unit is free to choose its own multipliers. One could, of course, restrict multiplier choice by imposing assurance region constraints (see Thompson et al., 1990), but significant differences will still exist between the multiplier vectors of the individual units within a plant. As well, one could argue that different assurance regions may be required for some plants than for others.

To rectify apparent weaknesses in the model of Cook et al. (1998) and Cook and Green (2005) we propose a model for capturing power unit efficiency that accomplishes two goals: First, the model should encompass all power units across all plants simultaneously within the analysis set. This will help to alleviate the problem of the small samples resulting from restricting the analysis set to those power units within a given plant. Second, the model should derive a common set of weights applicable to all power units within the relevant plant.

3.2. Deriving common weights: The ideal point method

To frame the development herein in a general format, consider the situation in which n DMUs are organized into K groups or clusters $\{J_k\}_{k=1}^{K}$. Each DMU_j, $j=1,\dots,n$ is characterized by its own bundles of R outputs $Y_j=(y_{rj})$, and I inputs $X_j=(x_{ij})$. Assume that we adopt as the efficiency measurement technology, the constant returns to scale (CRS) model of Charnes et al. (1978):

$$\begin{aligned} \theta_0 = \max\ & \mu Y_0/\upsilon X_0 \\ & \text{subject to:} \\ & \mu Y_j/\upsilon X_j \leqslant 1, \quad \forall j, \\ & \mu_r, \upsilon_i \geqslant 0, \quad \forall r, i. \end{aligned} \tag{3.1}$$

Let $\{\theta_{j_k}\}$ denote the optimal efficiency ratings arising from (3.1) for members of group k. Note that in general, the optimal multiplier vectors $(\mu^*_{j_k}, \upsilon^*_{j_k})$ yielding the θ_{j_k} can, and generally will be different for the various members $j_k \in J_k$.

We now wish to develop a *common set* of multipliers $(\hat{\mu}, \hat{\upsilon})$ that will be used to derive an efficiency score $(\hat{\mu}Y_{j_k}/\hat{\upsilon}X_{j_k})$ for each member $j_k \in J_k$. A logical property to require of this multiplier vector is that it yields ratings that are as near as possible to the individually optimal ratings θ_{j_k}. Viewed in this manner, deriving such a set of multipliers is a multiple objective problem in which the target is the *ideal point* or vector (θ_{j_k}).

A common approach to ideal point problems, and the one we adopt herein, is to set the θ_{j_k} as *goals* to be achieved. That is, for the set of power units $j_k \in J_{k_0}$,we set the $|J_{k_0}|$ goals:

$$\mu Y_{j_k}/\upsilon X_{j_k} = \theta_{j_k}, \quad j_k \in J_{k_0}. \tag{3.2}$$

Clearly, over achievement of the $|J_{k_0}|$ goals in (3.2) is not possible, since by definition

$$\mu Y_{j_k}/\upsilon X_{j_k} \leqslant \theta_{j_k}$$

for any feasible solution to (3.1). Thus, the only issue becomes the manner in which we choose to capture the extent of under achievement of the $|J_{k_0}|$ goals in (3.2). There would appear to be at least two logical norms for doing this, namely the ℓ^1 and ℓ^∞ norms. Under the ℓ^1 norm, the objective would be to seek a set of multipliers (μ, υ) for which *total* under achievement of the ideal point goals (3.2) is minimized. In that regard, we define a set of $|J_{k_0}|$ goal achievement variables $\{\gamma_{j_k}\}$, and for each k_0 solve the math programming problem:

$$\begin{aligned} &\min \sum_{j_k \in J_{k_0}} \gamma_{j_k} \\ &\text{subject to:} \\ &\mu Y_{j_k}/\upsilon X_{j_k} + \gamma_{j_k} = \theta_{j_k}, \quad j_k \in J_{k_0}, \\ &\mu Y_{j_k} - \upsilon X_{j_k} \leqslant 0, \quad j_k \in J_k, \ \forall k, \\ &\mu_r, \upsilon_i, \gamma_{j_k} \geqslant 0, \quad \forall r, i, j_k. \end{aligned} \tag{3.3}$$

Note that the objective function measures the aggregate of the differences between the ideal efficiency scores θ_{j_k} of the $|J_{k_0}|$ members of group k_0, and those generated by their common multipliers $(\widehat{\mu}, \widehat{\upsilon})$.

It can be argued that while (3.3) does provide a set of *collectively best* projections, it may not yield projections that are best in a cooperative or fair sense. To achieve projections that are the most fair in a *cooperative* sense, the goal should be to minimize the penalty imposed on the most disadvantaged unit in a plant; that is, the unit whose final efficiency score is furthest from the idea. To accomplish this, we recommend using a goal programming formulation based on the ℓ^∞ norm. Specifically, let γ be a goal achievement variable, and consider the mathematical programming problem

$$\begin{aligned} &\min \gamma \\ &\text{subject to:} \\ &\mu Y_{j_k}/\upsilon X_{j_k} + \gamma \geqslant \theta_{j_k}, \quad j_k \in J_{k_0}, \\ &\mu Y_{j_k} - \upsilon X_{j_k} \leqslant 0, \quad j_k \in J_k, \ \forall k, \\ &\mu_r, \upsilon_i, \gamma \geqslant 0, \quad \forall r, i. \end{aligned} \tag{3.4}$$

For solution purposes we rewrite (3.4) in the form:

$$\begin{aligned} &\min \gamma \\ &\text{subject to:} \\ &\mu Y_{j_k} - (\theta_{j_k} - \gamma)\upsilon X_{j_k} \geqslant 0, \quad j_k \in J_{k_0}, \\ &\mu Y_{j_k} - \upsilon X_{j_k} \leqslant 0, \quad j_k \in J_k, \ \forall k, \\ &\mu_r, \upsilon_i, \gamma \geqslant 0, \quad \forall r, i. \end{aligned} \tag{3.5}$$

We point out that problem (3.5) is non-linear, by virtue of the product of γ and υ. One might propose that non-linearity could be avoided here by replacing goals (3.2) by the equivalent expression

$$\mu Y_{j_k} - \upsilon(\theta_{j_k} X_{j_k}) = 0, \quad j_k \in J_{k_0},$$

and then introducing the goal achievement variable γ as the difference between the left- and right-hand sides. Specifically, we might consider replacing the first set of constraints in (3.2) by

$$\mu Y_{j_k} - \upsilon(\theta_{j_k} X_{j_k}) + \gamma = 0, \quad j_k \in J_{k_0} \tag{3.6}$$

yielding a linear expression. The problem with (3.6), however, is that scales can come into effect. If output and input values are much larger in scale for some power units than for others, the minimal γ will tend to cater to the large units and ignore the smaller ones. In problem (3.5), however, scale is not an issue since γ captures the difference between the ideal scores θ and ratios that are measured on a unit scale.

In the following section we use model (3.5) to derive efficiency scores for the 40 power units described earlier. While one could apply any nonlinear programming algorithm to approximate the solution of (3.5), we approach it as a parametric linear programming problem, with γ serving as the parameter. Specifically, using the range $[0, \theta]$, we solve (3.5) for a set of values for γ within that range. We note that we are searching for the minimal value of the parameter for which a feasible solution to (3.5) exists.

We demonstrate two approaches.

3.3. Dinkelbach's algorithm

This procedure (see Dinkelbach (1967) and Schaible (1976)), for solving general fractional programming problems, is a parametric linear programming methodology, wherein γ is treated as a parameter. To implement the algorithm, we first note that for any optimal solution (μ^*, υ^*), it is the case that any multiple of this is also an optimal solution. Thus, we may impose a restriction of the form $\sum \mu_r = R$ on problem (3.4). Furthermore, one may view (3.4) as a max min problem, specifically it is equivalent to the problem:

$$\begin{aligned} &\min \max\{\theta_{j_k} - (\mu Y_{j_k}/\upsilon X_{j_k}) | j_k \in J_k\} \\ &\text{subject to:} \\ &\mu Y_j - \upsilon X_j \leqslant 0, \quad j = 1, \ldots, n, \\ &\sum_{r=1}^{R} \mu_r = R, \\ &\mu_r, \upsilon_i \geqslant 0, \quad \forall r, i. \end{aligned} \tag{3.7}$$

Dinkelbach's approach involves replacing the max min problem by the equivalent formulation:

$$\begin{aligned} &\min s \\ &\text{subject to:} \\ &(\theta^*_{j_k} - \gamma)\upsilon X_{j_k} - \mu Y_{j_k} \leqslant s, \quad j_k \in J_k, \\ &\mu Y_j - \upsilon X_j \leqslant 0, \quad j = 1, \ldots, n, \\ &\sum_{r=1}^{R} \mu_r = R, \\ &\mu_r, \upsilon_i \geqslant 0, \quad \forall r, i. \end{aligned} \tag{3.8}$$

Treating γ as a parameter, the LP model (3.8) is solve for s, and this is continued until $s = 0$. Precise details on the selection of γ at each iteration can be found in Dinkelbach (1967). Table 3 displays the results from the application. We discuss this in the next section.

3.4. Consecutive interval search

A convenient and simple search procedure is to start at the lower end of the γ range ($\gamma = 0$), and increment the parameter until a feasible solution is found. Clearly, the smaller the increments, the more accurate will be the solutions, but as well the more iterations needed. Here, we chose an increment of 0.0001. Possibly, more efficient search tactics could be applied, such as the half-interval method, but our approach appears to converge relatively quickly, and suffices for the purpose at hand. Table 4 provides similar results to those in Table 3.

3.5. Alternate optima

In the solution of (3.5), or its equivalents, alternate optima may arise. Specifically, for any given (approximate) optimal value of $\gamma = \gamma^*$, more that one pair of optimal solution vectors (μ^*, υ^*) and (μ^0, υ^0), may exist. This is common in the general area of "compromise programming", of which this problem is an example. Hence, different alternate optima can yield different relative rankings of the members of any group. While in some respects, non-uniqueness of the solution may be seen as undesirable, choices for optima can have its advantages. While some optimal vectors (μ, υ) can yield efficiency scores that are radically, (and possibly undesirably) different from one another, others produce scores that can be more clustered, and less controversial for management to defend.

It is important to point out that it is possible that none of the DMUs will be 100% efficient. The fact that the ℓ^∞ norm is applied here, means that the choice of multipliers (μ, υ) is designed to make the distances of the actual efficiencies from their ideal values (θ_{j_k}) as near equal as possible. Fig. 1 illustrates this phenomenon. Hyperplane #1 represents the optimal solution for DMUs A1–A7, while hyperplane #2 is optimal for the remainder of the DMUs. The only frontier unit on hyperplane #1,

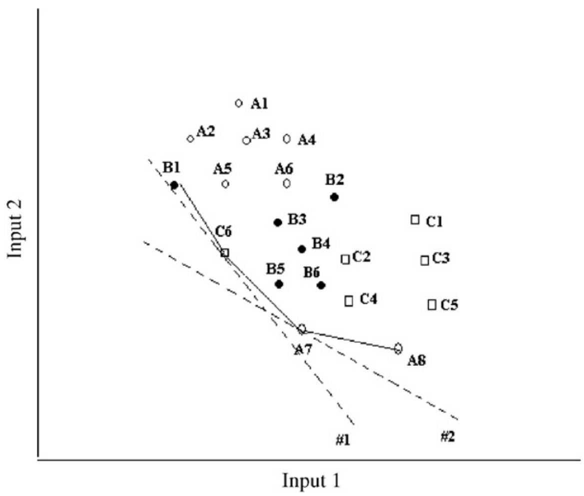

Fig. 1. Three groups of DMUs and supporting hyperplanes.

however, is C6 (not an 'A' unit), and the only one on hyperplane #2 is A7 (not a 'B' or 'C' unit). Thus, in this illustration, since DMUs are forced to apply group-common multipliers, all final efficiency scores would be strictly less than unity. We emphasize that this phenomenon may be rare, and is purely a function of the numbers involved, not the particular application setting.

4. An analysis of power unit efficiency

Earlier a description was given of a problem setting involving thermo-generating plants, wherein it was argued that efficiency should be viewed in terms of a set of outputs and inputs. Table 1 shows the number of thermal units operating at each of eight locations. Given also are the construction dates, fuel

Table 2
Data for power plants

Group	Unit	Outputs			MAINT	OCCUP
		OPER	OUT	EQDER		
Plant 1	1	573	95	110	538	895
	2	560	138	120	290	770
	3	637	151	150	386	886
	4	685	139	160	290	760
	5	542	157	130	343	721
	6	520	100	120	470	810
	7	531	122	60	439	820
	8	511	135	160	293	888
Plant 2	1	521	102	93	440	771
	2	634	93	102	324	780
	3	610	86	75	378	825
	4	538	95	106	380	815
	5	591	116	119	241	880
	6	650	123	105	141	766
	7	621	107	91	355	823
	8	686	125	110	270	750
Plant 3	1	620	120	130	350	750
	2	550	81	95	630	770
	3	525	105	125	495	860
	4	580	125	106	345	800
Plant 4(1)	1	430	105	140	190	810
Plant 4(2)	1	560	110	105	280	770
	2	510	125	95	180	820
Plant 5	1	650	170	140	300	7000
	2	550	120	120	275	800
	3	580	160	110	447	650
	4	640	110	130	370	720
Plant 6	1	480	95	125	228	880
Plant 7(1)	1	320	70	110	230	790
	2	250	60	110	220	790
	3	370	100	140	320	840
	4	280	90	100	280	810
Plant 7(2)	1	520	120	100	281	750
	2	430	100	140	302	850
	3	470	110	150	227	770
	4	410	80	110	254	825
Plant 8	1	475	100	120	179	750
	2	560	150	120	143	800
	3	510	120	110	114	750
	4	425	140	90	172	820

types and capacities in megawatt hours. We point out that two of the plants (#4 and #7) are each broken down into two groups for a total of 10 groupings. This breakdown is imposed due to the difference in construction dates for the different units at the two locations (e.g., units 7(1) were constructed in 1956 versus those at 7(2) which were built in 1960).

Table 2 displays the raw data for the 40 plants under analysis. Shown are three outputs and two inputs. These outputs and inputs are defined as follows:

4.1. Outputs

- OPER—a function of equivalent full capacity operating hours. This factor accounts for the fact that when operating at less than 100% capacity (e.g., if the unit is derated to 50% capacity), the operating hours during this period are prorated.

Table 3
Group-common multipliers and efficiency scores: Dinkelbach's method

DMU	Original efficiency	OPER	OUT	EQDER	MAINT	OCCUP	New efficiency	γ
Plant 1-1	0.70443	0.27507	1.82284	1.50070	0.16615	0.83385	0.59336	0.11107
Plant 1-2	0.89133						0.84850	
Plant 1-3	0.85654						0.84139	
Plant 1-4	1						1	
Plant 1-5	1						0.95772	
Plant 1-6	0.71187						0.67073	
Plant 1-7	0.71698						0.60591	
Plant 1-8	0.90530						0.79423	
Plant 2-1	0.74290	0.87815	0.44585	0.00000	0.19137	0.80863	0.71079	0.03211
Plant 2-2	0.88875						0.86355	
Plant 2-3	0.80838						0.77626	
Plant 2-4	0.72706						0.70352	
Plant 2-5	0.78366						0.75319	
Plant 2-6	1						0.96789	
Plant 2-7	0.82495						0.80857	
Plant 2-8	1						1	
Plant 3-1	0.91281	1.04763	0.00000	0.26485	0.00000	1.00000	0.91195	0.01237
Plant 3-2	0.78288						0.78098	
Plant 3-3	0.69041						0.67803	
Plant 3-4	0.80699						0.79462	
Plant 4(1)	0.99049	0.00000	0.00000	2.31203	0.77937	0.22063	0.99049	0
Plant 4(2)-1	0.80654	0.44083	1.92555	0.09786	0.37174	0.62826	0.79775	0.00880
Plant 4(2)-2	0.82451						0.81572	
Plant 5-1	0.53833	0.23036	1.04398	0.16284	0.75321	0.24679	0.17917	0.35916
Plant 5-2	0.80450						0.67114	
Plant 5-3	1						0.64084	
Plant 5-4	0.98023						0.62106	
Plant 6	0.78841	0.09158	0.00000	2.25557	0.71569	0.28431	0.78841	0
Plant 7(1)-1	0.71933	0.00000	0.01132	3.08370	0.56391	0.43609	0.71697	0.01018
Plant 7(1)-2	0.73555						0.72536	
Plant 7(1)-3	0.79209						0.79165	
Plant 7(1)-4	0.61549						0.60530	
Plant 7(2)-1	0.81846	0.00000	1.61419	1.84129	0.51281	0.48719	0.74159	0.07688
Plant 7(2)-2	0.80046						0.73676	
Plant 7(2)-3	1						0.92312	
Plant 7(2)-4	0.68613						0.62324	
Plant 8-1	0.92040	0.00000	1.67951	1.81337	0.50300	0.49700	0.83311	0.08729
Plant 8-2	1						1	
Plant 8-3	1						0.93238	
Plant 8-4	0.89354						0.80625	

To bring the scale of values for the units of measurement within the range of the scales used for other factors, we apply a scaling factor of 1/10, i.e., $\mathrm{OPER} = 1/10 \times \text{full capacity operating hours}$.

- OUT—a function of the number of forced and sudden outages.
- $\mathrm{OUT} = N - K$(# forced outages + # sudden outages). Sudden and forced outages, as unscheduled shutdowns of operations, are often consequences of equipment failure. Again, to bring scales into line we arbitrarily choose $N = 200$, $K = 10$.
- EQDER—a function of forced deratings caused by equipment failure.

Table 4
Group-common multipliers and efficiency scores: consecutive interval search method

DMU	Original efficiency	OPER	OUT	EQDER	MAINT	OCCUP	New efficiency	γ
Plant 1-1	0.70443	0.00042	0.00277	0.00228	0.00025	0.00127	0.59333	0.11110
Plant 1-2	0.89133						0.84856	
Plant 1-3	0.85654						0.84139	
Plant 1-4	1						1	
Plant 1-5	1						0.95773	
Plant 1-6	0.71187						0.67068	
Plant 1-7	0.71698						0.60602	
Plant 1-8	0.90530						0.79420	
Plant 2-1	0.74290	0.00136	0.00042	0.00028	0.00030	0.00125	0.71070	0.03220
Plant 2-2	0.88875						0.86913	
Plant 2-3	0.80838						0.77626	
Plant 2-4	0.72706						0.70877	
Plant 2-5	0.78366						0.75743	
Plant 2-6	1						0.96780	
Plant 2-7	0.82495						0.80762	
Plant 2-8	1						1	
Plant 3-1	0.91281	0.00140	0.00000	0.00034	0.00000	0.00133	0.91215	0.01240
Plant 3-2	0.78288						0.78142	
Plant 3-3	0.69041						0.67801	
Plant 3-4	0.80699						0.79500	
Plant 4(1)	0.99049	0.00000	0.00000	0.00707	0.00238	0.00068	0.99049	0
Plant 4(2)-1	0.80654	0.00076	0.00331	0.00017	0.00064	0.00108	0.79774	0.00880
Plant 4(2)-2	0.82451						0.81571	
Plant 5-1	0.53833	0.00057	0.00258	0.00040	0.00186	0.00061	0.17913	0.35920
Plant 5-2	0.80450						0.67104	
Plant 5-3	1						0.64080	
Plant 5-4	0.98023						0.62103	
Plant 6	0.78841	0.00022	0.00000	0.00546	0.00173	0.00069	0.78841	0
Plant 7(1)-1	0.71933	0.00000	0.00002	0.00658	0.00120	0.00093	0.71700	0.01020
Plant 7(1)-2	0.73555						0.72540	
Plant 7(1)-3	0.79209						0.79169	
Plant 7(1)-4	0.61549						0.60529	
Plant 7(2)-1	0.81846	0.00000	0.00328	0.00375	0.00104	0.00099	0.74156	0.07690
Plant 7(2)-2	0.80046						0.73672	
Plant 7(2)-3	1						0.92310	
Plant 7(2)-4	0.68613						0.62325	
Plant 8-1	0.92040	0.00000	0.00391	0.00422	0.00117	0.00116	0.83310	0.08730
Plant 8-2	1						1	
Plant 8-3	1						0.93237	
Plant 8-4	0.89354						0.80626	

Table 5
Computational efficiency of the two algorithms

Group	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10
t^c	1111	322	124	0	88	3592	0	102	769	873
t^d	4	5	2	0	2	11	0	4	4	4

EQDER = $N - K$ (# equipment related deratings), with $N = 200$ and $K = 10$ as above.

Since on the output side, any measure used must be such that "bigger is better," one cannot *directly* take outages as an output. To achieve the bigger is better condition, we subtract outages from some constant to create a proper scale measure. The value 200 has been chosen arbitrarily, but at the same time to yield "OUT" values that are in line with the scales used for other factors. Some sensitivity analyses were done relative to this parameter (200), and the particular value chosen was found to have very little effect on the final relative efficiency outcomes.

4.2. Inputs

- MAINT—the total labor and materials expenditures in thousands of dollars.
  Clearly, we could separate this into monetary inputs, but for purposes here we aggregate the two amounts into one figure.
- OCCUP—a function of total occupied hours, that is
  OCCUP = 1/10 (total hours available – available but not operating hours).

4.3. Analysis

A DEA analysis of the 40 power units was conducted, and the resulting efficiency scores are displayed in column 2 of Tables 3 and 4. We then solved model (3.5) using the two heuristics discussed earlier, deriving common multipliers and corresponding within-group efficiency scores. Shown as well are the values for γ that measure the maximum distances from the ideal scores.

There is reason to prefer the Dinkelbach solution methodology in two respects: First, one gets greater accuracy with this approach in terms of the values of γ; see the last column in each of the two tables. In order to have arrived at equally acceptable solutions with the consecutive search method, one would have to choose an increment of 0.00001 for the parameter. The second argument in favor of Dinkelbach's algorithm, is the generally smaller number of iterations required before a solution is reached. Table 5 provides the number of iterations in the 10 problems solved. Here, t^c and t^d represent respectively the numbers of iterations in the consecutive search and Dinkelbach algorithms. Clearly, since the computational complexity of the feasibility checking in the two methods is approximately the same, Dinkelbach's algorithm is the more efficient approach.

5. Conclusions

In many real world applications where DEA is applied, DMUs can often be put into groups, the members of which may be under a single management team, or should be evaluated under the same assumptions. This often means that the multipliers used within a group should be common across that group's members. The case example examined in this regard is one involving a set of power plants, where each contains a set of power units under a common plant management. We develop a goal-programming model for this setting that seeks to derive such a common-multiplier set. The important feature of the derived multiplier set is that it minimizes the maximum discrepancy among the within-group scores from their ideal levels. In this manner, the model seeks to minimize the detrimental impact on the most disadvantaged member of each group. We believe this model structure is an important addition to the DEA methodology, and is one deserving of further research.

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