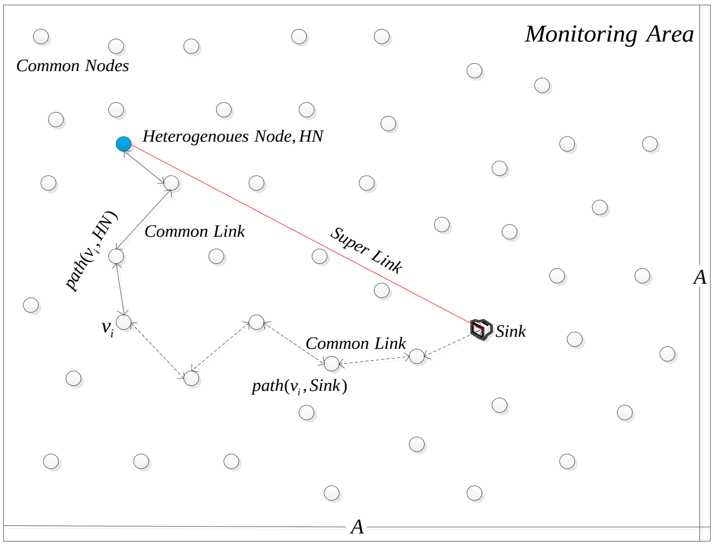

Figure 1 The wireless sensor network with heterogenous nodes

（4）After the placement is completed, all the nodes are still or only slightly moved;

（5）The maximum distance between any two nodes does not exceed their maximum communication radius R;

（6）Each communication link is symmetric.

In Figure 1，common node v_i sends data to Sink, there are two transmission ways: ① Data are sent to Sink by $path(v_i, Sink)$; ② Data are sent to one heterogeneous node by $path(v_i, HN)$, then the heterogeneous node sends it to Sink by supper link. Based on the optimal placement of heterogeneous nodes, the way to transmit data (as shown in Figure 1) is called a heterogeneous routing algorithm for wireless sensor network (HRA).

3.1.2 Heterogeneous node placement

Since each node's position is known, L_{ij}, which denotes the length of path for any two nodes, can be calculated by $Dijkstr$, and then q_{ij}, which denotes the hop number, can be calculated. q_α^{ij} denotes αth node on path L_{ij}, $\alpha = 1, 2, \cdots, q_{ij}$. $d_{l,l+1}^{ij}$ ($l = 1, 2, \cdots, \alpha - 1$) denotes the length of transition path for two adjacent nodes. ht denotes the heterogeneous node. S denotes the Sink node. So L_{ih} denotes the length of the path that common node v_i transmits data to Sink by heterogeneous node ht, L_{iS} denotes the length of the path that common node v_i transmits data to Sink by multi hop among common nodes. $L_i = \min(L_{ih}, L_{iS})$ denotes the minimum length of the path that common node v_i transmits data to Sink.

From paper [11] and [12], it is known that when the distance reduces, the energy consumption will be significantly lower, also the network lifetime will be improved. The energy consumption is reduced by minimizing the distance. The function of total effective transmission distance is

$$f = \sum_{i=1}^{N} L_i \quad (1)$$

Paper [11] has shown it is a NP-hard problem to solve the optimal placement of heterogeneous nodes in the randomly distributed WSN and the optimal solution of the problem cannot be obtained in large-scale network. In this paper, a model for optimal placement of heterogeneous nodes is proposed. A suboptimal solution will be obtained by mix integer programming.

Suppose the number of added heterogeneous nodes is known and the position of heterogeneous nodes can be same to the common nodes' position. The problem of deploying heterogeneous nodes can be translated into the problem of optimizing position selection by mix integer programming.

Suppose that the set of common nodes is $V=\{v_i, 1\le i\le N\}$. The set of position for common nodes is $(X,Y)=\{(x_i,y_i),1\le i\le N\}$. The number of heterogeneous nodes is h and the set of heterogeneous nodes is $H=\{h_k, k=1,2,\cdots,h\}$. $(U,W)=\{(u_k,w_k),k=1,\cdots,h\}$ denotes the set of position for heterogeneous nodes. For simplification, the following assumptions are made.

Sink is denoted by v_0. (x_0,y_0) is its position coordinate. There are $h^{'}$ ($0\le h^{'}\le h$) heterogeneous nodes have the same position as $h^{'}$ common nodes and there are $\hat{h}=h-h^{'}$ heterogeneous nodes have different positions as the common nodes. $V^{'}=\{v_{N+\beta},0\le\beta\le\hat{h}\}$ denotes the set of $\hat{h}$ heterogeneous nodes. $(X^{'},Y^{'})=\{(x_{N+\beta},y_{N+\beta}),0\le\beta\le\hat{h}\}$ denotes the set of the positions for $\hat{h}$ heterogeneous nodes. Let $V^*=V\bigcup\{v_0\}\bigcup V^{'}$, $(X^*,Y^*)=(X,Y)\cup\{(x_0,y_0)\}\cup(X^{'},Y^{'})=\{(x_i,y_i),0\le i\le N+\hat{h}\}$. The Sink node is regarded as a heterogeneous node h_0. There are $h+1$ heterogeneous nodes in the network and the first one is h_0 whose position is $(u_0,v_o)=(x_0,y_0)$. The other nodes will be deployed randomly. Let $H^*=H\bigcup\{h_0\}=\{h_k,0\le k\le h\}$, $(U^*,W^*)=(U,V)\cup\{(u_0,w_0)\}=\{(u_k,w_k),0\le k\le h\}$.

Under above assumptions, the placing problem of heterogeneous node are translated as follows: Selecting $h+1$ nodes from set V^* at whose position the heterogeneous nodes will be deployed to minimize the equation (1). The position of first heterogeneous node h_0 is $(u_0,v_o)=(x_0,y_0)$. The two sets of 0-1 variables are introduced as follows:

（1）The first variable ξ_j is

$$\xi_j=\begin{cases}1, & \textit{one heterogeneous node is deployed on node } v_j\in V^*\\ 0, & \textit{none heterogeneous node is deployed on node } v_j\in V^*\end{cases}$$

where $0\le j\le N+\hat{h}+1$, according to the assumption, there must be a heterogeneous node on v_0, so $\xi_0=1$.

（2）The second variable η_{ij} is

$$\eta_{ij}=\begin{cases}1, & \textit{the data on } v_j\in V \textit{ is transmitted to Sink}\\ & \textit{by the heterogeneous node on } v_j\in V^*\\ 0, & \textit{otherwise}\end{cases}$$

where $1\le i\le N+\hat{h}+1$, $0\le j\le N+\hat{h}+1$.

The placement problem of heterogeneous node can be also expressed as

$$R(MIP):\quad \min\sum_{i=1}^{N^*}\sum_{j=1}^{N^*}L_{ij}\eta_{ij} \tag{2}$$

s. t.

① $\sum_{j=0}^{N^*}\xi_j=h+1$,

② $\sum_{j=0}^{N^*}\eta_{i,j}=1$， $i=1,2,\cdots,N^*$,

③ $\eta_{i,j}\le\xi_j$， $i=1,2,\cdots,N^*$， $j=0,1,2,\cdots,N^*$,

④ $\xi_0=1$， $\xi_j\in\{0,1\}$， $j=1,2,\cdots,N^*$,

⑤ $\eta_{i,j}\in\{0,1\}$， $i=1,2,\cdots,N^*$， $j=0,1,\cdots,N^*$,

⑥ $N^*=N+\hat{h}+1$.

where ① indicates there are $h+1$ heterogeneous nodes will be deployed in the network by regarding Sink node as one heterogeneous node; ② indicates each common node must transmit data by just one heterogeneous node; ③ indicates none heterogeneous node is deployed on $v_j \in V^*$, thus anyone of heterogeneous nodes cannot transmit data from $v_i \in V$ to Sink; ④ and ⑤ indicate the ranges of ξ_j and η_{ij} respectively; ⑥ indicates the order of set V^*. Equation (2) indicates the minimum value of sum of length between each common node and Sink node. The placement problem of heterogeneous node is expressed as the problem of mix integer programming.

With the analysis above all, the problem of the optimal solution for equation (1) has been translated to the problem of the optimal solution for $R(MIP)$. If the solution of the later can be obtained, the solution of the former also can be done. A decomposition method, which consists of Lagrange relaxation and Benders decomposition, is used to solve the problem of mix integer programming.

② in the $R(MIP)$ is regarded as one complex constraint, $R(\pi^0)$ can be defined as follows by Lagrange multiplier $\pi^0 = (\pi_1^0, \pi_2^0, \cdots, \pi_{N^*}^0)$

$$R(\pi^0): \quad \min \sum_{i=1}^{N^*}\sum_{j=1}^{N^*} L_{ij}\eta_{ij} + \sum_{i=1}^{N^*}\sum_{j=1}^{N^*} \pi_i^0 \eta_{ij} - \sum_{i=1}^{N^*} \pi_i^0 \tag{3}$$

s. t.

① $\sum_{j=0}^{N^*} \xi_j = h+1$,

② $\eta_{i,j} \le \xi_j$, $i = 1,2,\cdots,N^*$, $j = 0,1,2,\cdots,N^*$,

③ $\xi_0 = 1$, $\xi_j \in \{0,1\}$, $j = 1,2,\cdots,N^*$,

④ $\eta_{i,j} \in \{0,1\}$, $i = 1,2,\cdots,N^*$, $j = 0,1,\cdots,N^*$,

⑤ $N^* = N + \hat{h} + 1$.

It is shown that the optimal solution of $R(\pi^0)$ satisfies

$$\eta_{ij} = \begin{cases} \xi_j, & L_{ij} + \pi_i^0 \le 0 \\ 0, & otherwise \end{cases} \tag{4}$$

Let $\bar{L}_{ij} = \min\{L_{ij} + \pi_i^0, 0\}$, $\bar{L}_j = \sum_{i=1}^{N^*} \bar{L}_{ij}$, then $R(\pi^0)$ is expressed as by $R'(\pi^0)$

$$R'(\pi^0) \quad \min \sum_j \bar{L}_j \xi_j \tag{5}$$

s. t.

① $\sum_{j=0}^{N^*} \xi_j = h+1$,

② $\xi_0 = 1$, $\xi_j \in \{0,1\}$, $j = 1,2,\cdots,N^*$,

③ $N^* = N + \hat{h} + 1$.

The optimal solution ξ can be obtained by $R'(\pi^0)$. Surely η_{ij} can be obtained by ξ. Furthermore, sorting $\bar{L}_j$: $\bar{L}_0 \le \bar{L}_1 \le \cdots \le \bar{L}_{N^*}$, the optimal solution ξ^0 of $R(\pi^0)$ is $\xi_0^0 = \xi_1^0 = \xi_2^0 = \cdots = \xi_h^0 = 1$ and $\xi_j^0 = 0$, $j = h+1,\cdots,N^*$. Then the optimal solution $\{\xi_j^0, \eta_{ij}^0\}$ of $R(\pi^0)$ can be obtained by equation (4). Taking a vector of 0-1 variables $\xi = (\xi_0, \xi_1, \cdots, \xi_{N^*})$ matching to ①②③ which equation (5) subjects to, $B(\xi)$ is expressed as

$$\eta_0(\xi) = \min \sum_{i=1}^{N^*}\sum_{j=0}^{N^*} L_{ij}\eta_{ij} \tag{6}$$

s. t.

① $\sum_{j=0}^{N^*}\eta_{ij}=1$, $i=1,2,\cdots,N^*$

② $0\le\eta_{ij}\le\xi_j$, $i=1,2,\cdots,N^*$, $j=1,2,\cdots,N^*$

Dual function $D(\xi)$ of $B(\xi)$ is defined as

$$\eta_0(\xi)=\max(\sum_{i=1}^{N^*}\pi_i-\sum_{i=1}^{N^*}\sum_{j=0}^{N^*}\pi_{ij}\xi_j) \quad (7)$$

s. t.

① $\pi_i-\pi_{ij}\le L_{ij}$, $i=1,2,\cdots,N^*$, $j=1,2,\cdots,N^*$,

② $\pi_{ij}\ge 0$, $i=1,2,\cdots,N^*$, $j=1,2,\cdots,N^*$.

As the same to solve $R(\pi^0)$, the optimal solution of $B(\xi)$ and $D(\xi)$ can be obtained. The method as follows.

Suppose ξ subjects to $\xi_0=\xi_1=\xi_2=\cdots=\xi_h=1$ and $\xi_j^0=0$, $j=h+1,\cdots,N^*$. Let $L_{ij}^*=\min_{0\le j\le h}L_{ij}$, $i=1,2,\cdots,N^*$. The optimal solution of $B(\xi)$ is

$$\eta_{ij}=\begin{cases}1, & j=\bar{j}\\ 0, & otherwise\end{cases} \quad (8)$$

where $i=1,2,\cdots,N^*$, $j=1,2,\cdots,N^*$. The optimal solution of $D(\xi)$ is $\pi_i=L_{ij}^*$, $i=1,2,\cdots,N^*$, $\pi_{ij}=0$, $i=1,2,\cdots,N^*$, $j=1,2,\cdots,h$, $\pi_{ij}=\max\{0,L_{i\bar{j}}-L_{ij}\}$, $i=1,2,\cdots,N^*$, $j=h+1,h+2,\cdots,N^*$.

3.2 Number of heterogeneous nodes

When the number of common nodes is obtained, the optimal number of heterogeneous nodes can also be obtained. In the wireless sensor network including N common nodes, the average length of the path from common v_i to Sink is $\bar{L}=1/N\sum_{i=1}^{N}L_i$. where L_i denotes the minimum length of the path from v_i to Sink. With different requirements, the minimum number of heterogeneous nodes is different.

Suppose the average minimum length of the path between common nodes and Sink matches with $\bar{L}<r_0$, r_0 is a constant. Let $h_{\min}$ denotes the minimum number of heterogeneous nodes, then,

1) After placing $h_{\min}$ heterogeneous nodes, the average length of the path from common nodes to Sink should subject to $1/N\sum_{i=1}^{N}L_i\le r_0$. That is $1/N^*\min\sum_{i=1}^{N^*}\sum_{j=1}^{N^*}L_{ij}\eta_{ij}\le r_0$.

2) After placing $h_{\min}-1$ heterogeneous nodes, the sum of minimum length of path from common nodes to Sink should not subject to $1/N\sum_{i=1}^{N}L_i\le r_0$. That is $1/N^*\min\sum_{i=1}^{N^*}\sum_{j=1}^{N^*}L_{ij}z_{ij}>r_0$.

Above all, $h_{\min}$, which subjects to $\bar{d}\le r_0$, can be obtained by iteration based on optimal placement of heterogeneous nodes.

3.3 Number of cluster heads

Each common node transmits data to Sink by one heterogeneous node or directly (see Figure 2). In the network, if the heterogeneous nodes are regarded as cluster heads, the clustering can be done with same topological path based on heterogeneous nodes. Static clustering has some weaknesses [13] : cluster heads die quickly, large redundant data and unbalanced load. Before selecting cluster heads, the number of cluster heads should be taken into account.

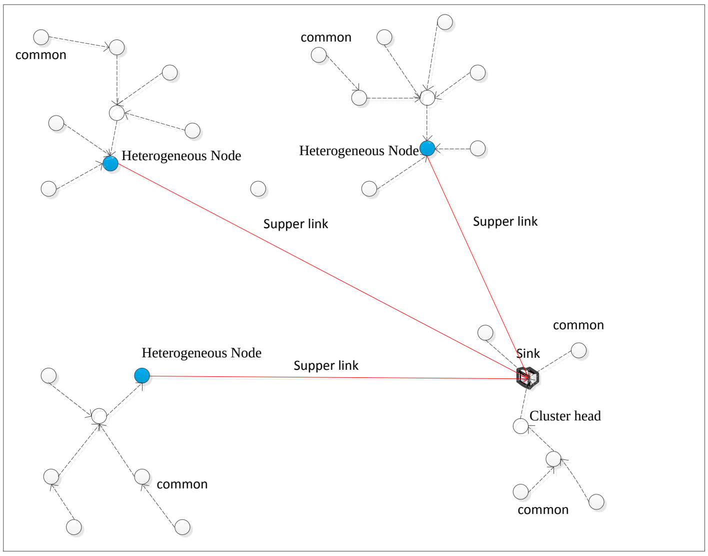

Figure 2 Data transmission of heterogeneous wireless sensor network

The number of cluster heads based on minimum energy consumption is taken into account. n_{CH} denotes the number of cluster heads in the wireless sensor network, N/n_{CH} denotes the number of common nodes around each cluster head. The energy consumption of wireless sensor network includes three parts:

1) The energy consumption of cluster heads E_{CH}, which consists of the receiving energy consumption E_{Rx} from the common nodes into the cluster, the data processing energy consumption E_{FA} and the transmitting energy consumption of processed data E_{Tx};

2) The transmitting energy consumption from common nodes to cluster head E_{com-CH};

3) The energy consumption from one cluster head to another cluster head E_{CH-CH}.

The energy consumption model is referred in [14], which indicates the energy consumption of transmitting $k\ bit/s$ data between two nodes.

$$E_{Tx}(k,d)=\begin{cases} k(E_{elec}+\varepsilon_{fs}d^2),\ d<d_0 \\ k(E_{elec}+\varepsilon_{amp}d^4),\ d\geq d_0 \end{cases} \tag{9}$$

The energy consumption of receiving data is $E_{Rx}(k,d)=kE_{elec}$, where $E_{elec}=50nJ/bit$, ε_{fs} and ε_{amp} denotes the energy coefficient of power amplifier with different distance respectively, and $\varepsilon_{fs}=10pJ/bit/m^2$, $\varepsilon_{amp}=0.0013pJ/bit/m^4$. The maximum communication radius is denoted by $d_0=\sqrt{\varepsilon_{fs}/\varepsilon_{amp}}$.

The energy consumption of cluster heads is

$$E_{CH}=E_{Rx}+E_{FA}+E_{Tx}=kE_{elec}(N/n_{CH}-1)+kE_{DA}N/n_{CH}+(kE_{elec}+k\varepsilon d_{CH-CH}^{w}) \tag{10}$$

where, the energy consumption of processing unit data is E_{DA}, the distance from current cluster head to neighbor cluster head is d_{CH-CH}. The maximum communication distance between two cluster heads is lower than the maximum communication radius R, so as to make sure transmitting data from cluster head to Sink without error. For calculating easily, let $d_{CH-CH}=R$. Thus

$$E_{CH}=kE_{elec}(N/n_{CH}-1)+(kE_{DA}N)/n_{CH}+(kE_{elec}+k\varepsilon R^w)=[kN(E_{elec}+E_{DA})]/n_{CH}+k\varepsilon R^w \tag{11}$$

where ε and w is relative to R. When $R<d_0$, then $\varepsilon=\varepsilon_{fs}$, $w=2$; When $R\geq d_0$, the $\varepsilon=\varepsilon_{amp}$, $w=4$.

The energy consumption of common nodes around each cluster head is

$$E_{com-CH} = kE_{elec} + k\varepsilon_{fs} d^2_{com-CH} \quad (12)$$

where d_{com-CH} denotes the communication distance from the common nodes around one cluster head to the cluster head. By the assumptions in [11], the second moment of the distance from the common nodes around each cluster head to the cluster head is

$$E(d^2_{com-CH}) = A^2 / 2\pi n_{CH} \quad (13)$$

where A denotes the area including the common nodes' location. Thus

$$E_{com-CH} = kE_{elec} + k\varepsilon_{fs} A^2 / 2\pi n_{CH} \quad (14)$$

The total energy consumption of each clustering is

$$E_{cluster} = E_{CH} + (N/n_{CH} - 1)E_{com-CH} \approx E_{CH} + (NE_{com-CH})/n_{CH} \quad (15)$$

The total energy consumption between cluster heads is

$$E_{CH-CH} = kE_{elec} + (kE_{elec} + k\varepsilon R^w) = 2kE_{elec} + k\varepsilon R^w \quad (16)$$

Let t denotes the average number of hops from the cluster head nodes to Sink. The average distance from the cluster head nodes to Sink can be obtained. Let $(x_s, y_s) = (rA, qA)$, $0 \le r \le 1$, $0 \le q \le 1$, denotes the location of Sink, the location of someone cluster head is (x, y). Since the cluster head nodes randomly distribute in the area, the probability density function of nodes is $\rho(x, y) = 1/A^2$. Thus

$$E(d^2_{CH-S}) = \int_0^A \int_0^A [(x - rA)^2 + (y - qA)^2] \rho(x, y) dxdy = (2/3 - r - q + r^2 + q^2)A^2 \quad (17)$$

Based on the circular model in [13] and [15], $D(d_{CH-S}) \approx A^2/\pi (1/2 - 1/9\sqrt{\pi})$, the average distance from cluster head nodes to Sink is

$$E(d_{CH-S}) \approx \sqrt{E(d^2_{CH-S}) - D(d_{CH-S})} = A\sqrt{2/3 - r - q + r^2 + q^2 - 1/\pi (1/2 - 1/9\sqrt{\pi})} \quad (18)$$

The average number of hops from cluster head nodes to Sink is $t \approx E(d_{CH-S})/R$. The total energy consumption is as follows

$$\begin{aligned} E_{total} &= n_{CH} E_{cluster} + n_{CH} t_{CH} E_{CH-CH} \\ &= n_{CH} E_{CH} + NE_{com-CH} + n_{CH} t E_{CH-CH} \\ &= kN(E_{elec} + E_{DA}) + kn_{CH} \varepsilon R^w + kN(E_{elec} + \frac{\varepsilon_{fs} A^2}{2\pi n_{CH}}) + n_{CH} t(2kE_{elec} + k\varepsilon R^w) \quad (19) \\ &= kN(2E_{elec} + E_{DA}) + (t+1)kn_{CH} \varepsilon R^w + \frac{kN\varepsilon_{fs} A^2}{2\pi n_{CH}} + 2tkn_{CH} E_{elec} \end{aligned}$$

Let $dE_{total}/dn_{CH} = 0$, the optimal value of n_{CH} is $n_{CH} = A\left(N\varepsilon_{fs} / \{2\pi[(t+1)\varepsilon R^w + 2tE_{elec}]\}\right)^{1/2}$, where ε, w and t can be obtained through above steps.

4 A clustering routing algorithm for heterogeneous wireless sensor network (CHRA)

In this section, a clustering routing algorithm for wireless sensor network (CHRA) is shown as follows, which consists of the heterogeneous routing algorithm for wireless sensor network (HRA) and a cluster heads selecting algorithm by LEACH-C. Firstly, Theorem 1 is given to show the performance of CHRA routing algorithm is better than that of HRA routing algorithm. Secondly, the algorithms are described respectively.

Theorem 1: Performance of CHRA routing algorithm is better than that of HRA routing algorithm.

Proof: Supposing there are h heterogeneous nodes in HRA routing algorithm. There is a part of common nodes around the heterogeneous nodes. The common nodes transmit data to the heterogeneous nodes and the heterogeneous nodes transmit the disposed data to Sink. In fact, the h heterogeneous nodes, whose energy and bandwidth are unlimited, are equivalent to h Sink nodes. In Figure 3, each circle includes one heterogeneous node and some common nodes. Each circle can be regarded as one sub homogeneous network with single Sink. If the sub-network is clustered for finding the cluster head nodes, the performance of clustered sub-network is improved [16; 17]. In Figure 4, the clustered sub network is shown. The algorithm of all sub-network is clustered by LEACH-C in HRA is defined as CHRA.

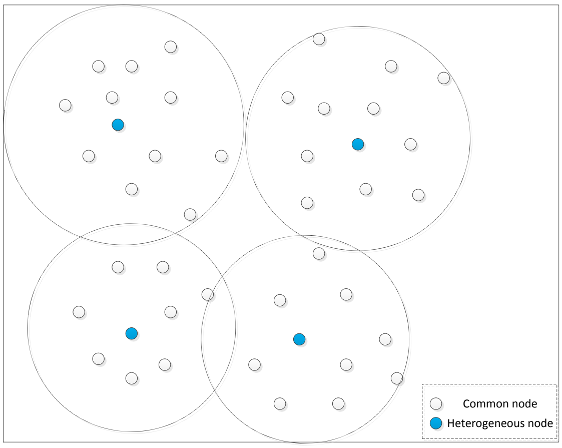

Figure 3 Clustered network with one heterogeneous node as

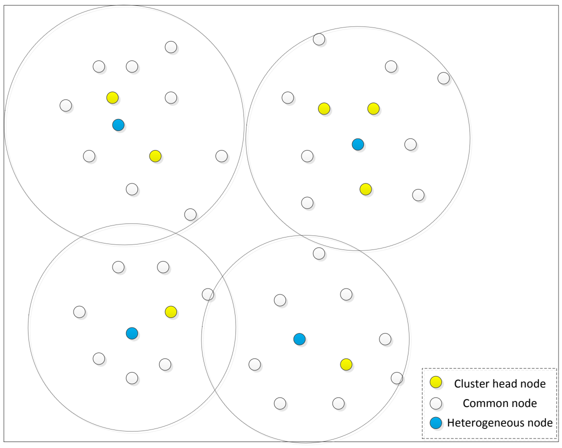

Figure 4 Clustered sub-network with one heterogeneous node as center

Obviously, the performance of CHRA routing algorithm is better than that of HRA routing algorithm. In HRA routing algorithm, all common nodes are divided into two kinds. The first kind includes the nodes which transmit data to Sink without heterogeneous node and the others belong to the second kind. However, in CHRA routing algorithm, not only all common nodes are divided in two kinds, but each circle in Figure 4 has the cluster head nodes which can balance the energy consumption and improve the life. So, the performance of CHRA is better than HRA.

4.1 The heterogeneous routing algorithm for wireless sensor network (HRA)

Algorithm 1 represents the heterogeneous routing algorithm for wireless sensor network (HRA). Firstly, the initial value of ξ need be got; Secondly, calculate the $h+1$ optimal value $(\eta^{h'},\xi^{h'})$ of $R'(\pi^0)$, $B(\xi)$ and $D(\xi)$ for each h' (line 2-35); Thirdly, find $(\eta^{h'},\xi^{h'})$ to minimize $R'(\pi^0)$ which is the optimal placement (η,ξ) of heterogeneous nodes (line 36-37).

Algorithm 1: The heterogeneous routing algorithm for wireless sensor network (HRA)

Input: The initial heterogeneous nodes location vector ξ

Output: Optimal heterogeneous nodes placement result $<\eta,\xi>$

$Q=\{1\}$, N , h // Initialization
Calculate (π_i^1,π_{ij}^1)
for $h^{'}=0:h$
  for $j=0:N+h-h^{'}+1$
    for $i=1:N+h-h^{'}+1$
      Calculate (η_{ij}^*,ξ_j^*) // The optimal solution of $R(\pi^j)$
      Calculate $(\overline{\eta}_{ij},\xi_j)$ // The optimal solution of $B(\xi^*)$
      Calculate (π_i^*,π_{ij}^*) // The optimal solution of $D(\xi^*)$
      Calculate $\eta_0(\xi^*)$ // The object function of $D(\xi^*)$
      if $\pi_i^*=\pi_i^j$
        $\left\langle\eta^{h^{'}},\xi^{h^{'}}\right\rangle\leftarrow\left\langle\overline{\eta}_{ij},\overline{\xi}_j\right\rangle$ // Store $(\overline{\eta}_{ij},\overline{\xi}_j)$
        break
      else
        Calculate $(\overline{\eta}_{ij}^*,\overline{\xi}_j^*)$ // The optimal solution of $R(\pi_i^*)$
        if $\overline{p}_j^*=p_j^*$
          $\left\langle\eta^{h^{'}},\xi^{h^{'}}\right\rangle\leftarrow\left\langle\overline{\eta}_{ij},\overline{\xi}_j^*\right\rangle$ // Store $(\overline{\eta}_{ij},\overline{\xi}_j)$
          break
        else
          $\pi_i^{j+1}=\pi_i^*$
          Calculate $(\eta_{ij}^{'},\overline{p}_j)$ // The optimal solution of $B(\overline{\xi})$
          Calculate $(\overline{\pi}_i,\overline{\pi}_{ij})$ // The optimal solution of $D(\overline{\xi})$
          Calculate $\eta_0(\overline{\xi})$ // The object function of $D(\overline{\xi})$
        end
        if $\eta_0(\overline{\xi})=\eta_0(Q)$
          $\left\langle\eta^{h^{'}},\xi^{h^{'}}\right\rangle\leftarrow\left\langle\eta_{ij}^{'},\overline{p}_j\right\rangle$ // Store $(\eta_{ij}^{'},\overline{p}_j)$
          break
        else
          $\pi_i^{j+1}=\overline{\pi}_i$
        end
      end
    end
    $Q=Q\bigcup\{j+1\}$
    $j=j+1$
  end
end
$\left\langle\eta,\xi\right\rangle\leftarrow\left\langle\eta_{optimal}^{h^{'}},\xi_{optimal}^{h^{'}}\right\rangle$ // Find $\left\langle\eta^{h^{'}},\xi^{h^{'}}\right\rangle$ subject to $\min R^{'}(\pi^0)$

```
    return ⟨η, ξ⟩
```

4.2 The clustering routing algorithm for heterogeneous wireless sensor network (CHRA)

Algorithm 2 represents the clustering routing algorithm for heterogeneous wireless sensor network (CHRA). Firstly, deploy h heterogeneous nodes in the wireless sensor network by algorithm 1; Secondly, select cluster heads by LEACH-C; Thirdly, the network select cluster heads per round until the number of common nodes is lower than the threshold. (line 7-12).

Algorithm 2: The clustering routing algorithm for heterogeneous wireless sensor network (CHRA)

Input: The initial heterogeneous nodes location vector ξ
The upper bound of average length from common nodes to Sink r_0
The process time threshold T_0
The node number threshold N_0
Output: The residue nodes number $N_{residue}$

1: Deploy the heterogeneous nodes in the wireless sensor network by algorithm 1 and 2
2: Calculate n_{CH} by (3-6) and select the cluster nodes uniformly
3:   **for** $N > N_0$ **do**
4:     **if** $q_{\alpha}^{ij} == 0$ **do** // The common node number between node v_i to Sink
5:       $L = L_{iS}$
6:     **end if**
7:     **if** $T \geq T_0$ **do**
8:       Calculate n_{CH} by (3-6) and select the cluster nodes uniformly
9:       Calculate N_T // The common nodes number at T
10:       $T = 0$
11:     **end if**
12:   **end for each**
13: $N_{residue} \leftarrow N_T \in N_T \leq N_0$ // Find the residue common node number
14: **return** $N_{residue}$

Then, the clustering routing algorithm for wireless sensor network is analyzed. The clustering routing algorithm for wireless sensor network mainly includes the heterogeneous routing algorithm for wireless sensor network and the selecting cluster heads by LEACH-C. The time complex of the heterogeneous routing algorithm for wireless sensor network is $O((h+1)(n+rm)\ln(\mu_0/\varepsilon))$, where h is the number of heterogeneous nodes, μ is the control variable in the decomposed master problem, x is the control variable in the decomposed subproblem, r and n are the dimension of μ and x respectively, m is the number of constraints in the subproblem, $\varepsilon > 0$ is the stopping criterion. The time complex of calculating and selecting cluster heads by LEACH-C is $O(k)$, where k is number of hops. Hence, the time complex of the clustering routing algorithm for heterogeneous wireless sensor network is $O(kN_0(h+1)(n+rm)\ln(\mu_0/\varepsilon))$.

5 Analysis of Simulation Results

In this section, the performance of LEACH-C [13], LEACH-Sin [18], ccWSN [19], HRA and CHRA is compared by simulation in Omnet++. In simulation, 50, 100, 150, 200, 250 and 300 nodes are respectively and randomly distributed in 6 different square areas whose edge-length are respectively 160m, 226m, 277m, 320m, 358m and 392m. There is only one Sink node in wireless sensor network and the setting of simulation parameters is shown in table 1.

In the verification experiments, the average energy consumption of nodes, stable period, lifetime and average delivery delay are analyzed.

1) Average energy consumption of nodes;

2) Stable period: the number of running rounds when the first node dies is called the stable period of network;

3) Lifetime: the number of running rounds when 10% of nodes die is called the lifetime of network;

4) Average delivery delay.

To eliminate the effect of the randomness of experiment, it's applied that taking average from 30 times experiments in each square area.

Table 1 Parameters of simulation experiments

parameter	value	parameter	value
Initial energy	0.5J	Frames in one round	10 frames
E_{elec}	50nJ/bit	Size of broadcast packet	20B
E_{fs}	10pJ/bit/ m^2	Maximum communication radius	30m
E_{amp}	0.0013pJ/bit/ m^4	Delay time of processing	5~10ms
E_{DA}	5pJ/bit/signal	Frame size	9800bit
Distance threshold d_0	87.7m	Number of heterogeneous nodes	4

Figure 5 shows the position of nodes and Sink, Figure 6 shows the dynamic cluster formation during one round by LEACH-C and Figure 7 shows the dynamic cluster formation during next round with LEACH-C within the 160m×160m square region, in which 50 nodes are randomly distributed. In Figure 5-7, "*" denotes the Sink node, "+" denotes the common node, "○" denotes the heterogeneous node and "△" denotes the cluster head. After one round ends, the cluster heads are selected by LEACH-C again. Figure 8 depicts the average energy consumption in different square areas. Figure 9 and 10 show the stable period and lifetime in different square areas. Figure 11 depicts the average delivery delay in different square areas. At last, the performance of wireless sensor network between static and mobile Sink is discussed. Figure 12 and 13 show the average energy consumption and average delivery delay comparing static with mobile Sink in different square areas respectively.

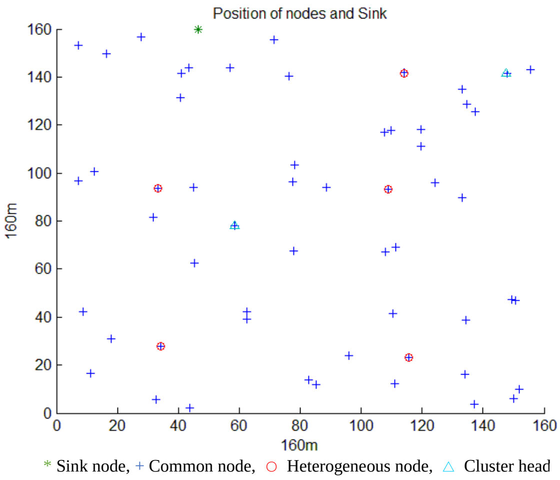

Figure 5 Position of nodes and Sink

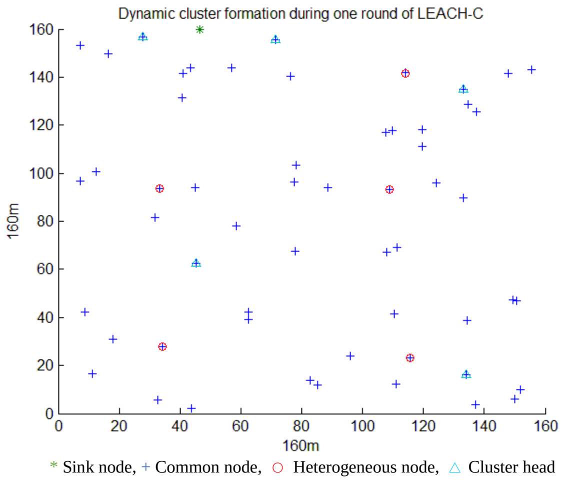

Figure 6 Dynamic cluster formation during one round with LEACH-C

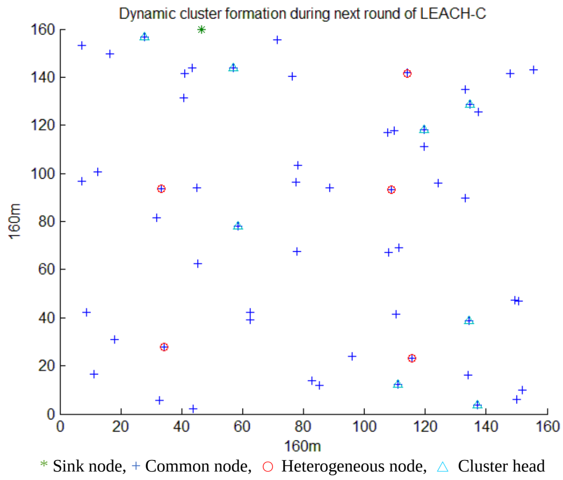

Figure 7 Dynamic cluster formation during next round by LEACH-C

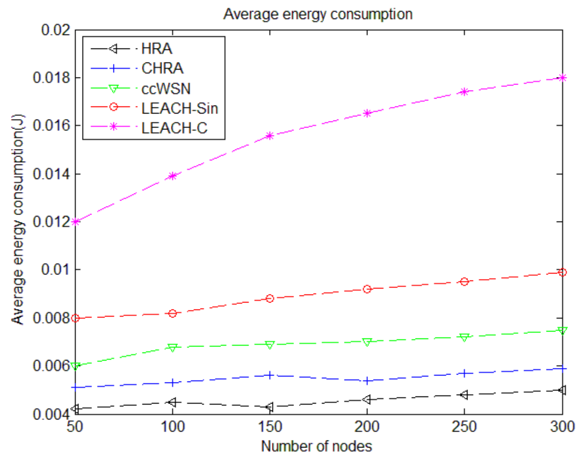

Figure 8 Average energy consumption

Figure 8 depicts average energy consumption in different square areas. Because LEACH-C routing algorithm has purposeless than the others, the average energy consumption of LEACH-C is largest than the others. Because HRA routing algorithm transmits data on the shortest path, the average energy consumption of HRA is less than those of LEACH-C, LEACH-Sin and ccWSN. Based on the principle of HRA routing algorithm, CHRA routing algorithm selects cluster heads dynamically to balance the energy consumption of nodes. CHRA routing algorithm may not select the shortest path to transmit data, the average energy consumption of CHRA is higher than that of HRA. The simulations indicate that HRA reduces average energy consumption up to 14% than CHRA, up to 29% than ccWSN and up to 46% than LEACH-Sin.

Figure 9 shows the stable period and Figure 10 shows the lifetime in different square areas. Although HRA routing algorithm is on the shortest path, the HRA routing algorithm doesn't select common nodes as cluster heads, which will cause common nodes to die quickly and reduce the lifetime and stable period. Based on the principle of HRA routing algorithm, CHRA dynamically selects cluster heads from common nodes. In CHRA routing algorithm, some common nodes are clustered by regarding heterogeneous nodes as cluster heads. In one cluster with heterogeneous cluster-head node, common nodes are clustered again and some ones are selected as cluster heads. CHRA reduces the average length of transmission paths in network. The simulations indicate that LEACH-Sin improves stable period up to 14% and lifetime up to

9.3% comparing with HRA, ccWSN improves stable period up to 20% and lifetime up to 16% comparing with HRA, CHRA improves stable period up to 27% and lifetime up to 22% comparing with HRA.

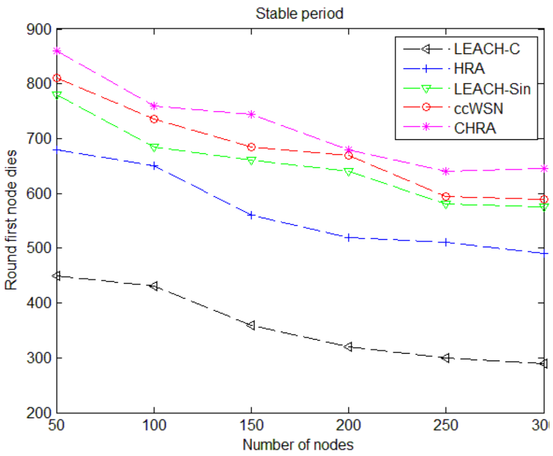

Figure 9 Stable period

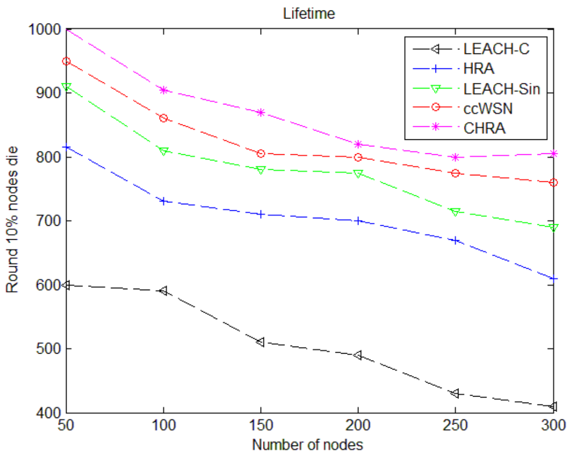

Figure 10 Lifetime

Figure 11 depicts the average delivery delay in different square areas. By transmitting data on the shortest path, the average delivery delay of HRA and CHRA is less than LEACH-C, LEACH-Sin and ccWSN. Firstly, CHRA routing algorithm needs select common nodes to be cluster heads. Secondly, CHRA routing algorithm optimizes the path to transmit data. So, the average delivery delay of CHRA is more than that of HRA. The simulations indicate that HRA reduces average delivery delay up to 19% comparing with CHRA, up to 37% comparing with ccWSN, up to 38% comparing with LEACH-Sin.

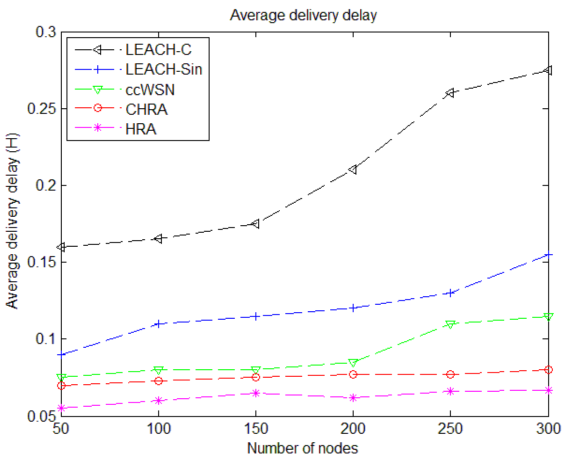

Figure 11 Average delivery delay

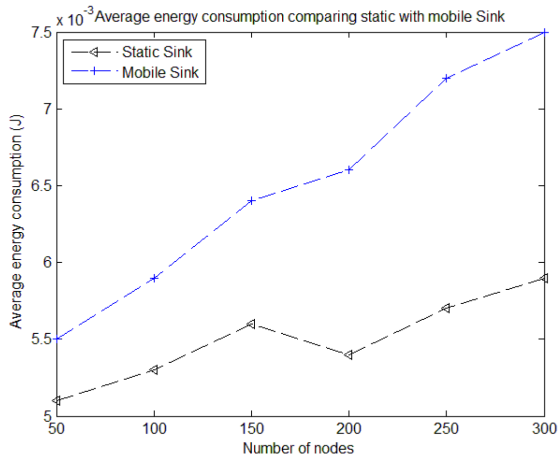

Figure 12 Average energy consumption comparing static with mobile

At last, the performance of wireless sensor network between static and mobile Sink is discussed. The mobile model is random waypoint and the Sink speed is 5 m/s. Figure 12 shows average energy consumption with static and mobile Sink in different square areas. Figure 13 shows average delayed time with static and mobile Sink in different square areas. Figure 12 indicates that the average energy consumption in the wireless sensor network with static is lower than the one with mobile Sink. Figure 13 indicates that average delayed time in the wireless sensor network with static is lower than the one with mobile Sink. The reason of the tendency in Figure 12 and 13 are same. When Sink node is mobile, it would send its location to each node. And then, the nodes take energy to receive the information sent by Sink node. With the number of nodes is bigger, the energy took to receive the information sent by Sink node is more. So, the average energy consumption in the wireless sensor network with static Sink is lower than the one with mobile Sink, the average delayed time in the wireless sensor network with static is also lower than the one with mobile Sink.

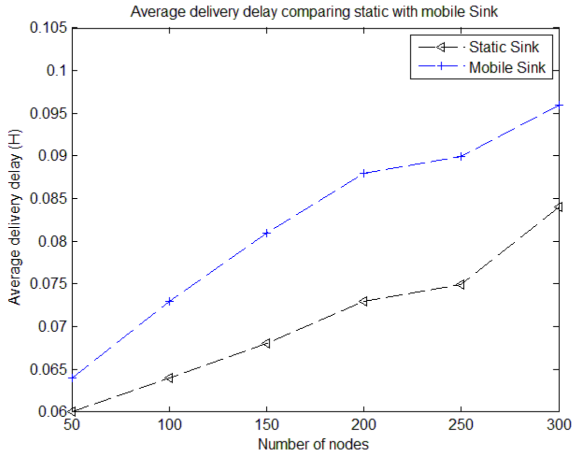

Figure 13 Average delivery delay comparing static with mobile Sink

6 Conclusion and future work

In this paper, the clustering routing algorithm for wireless sensor network (CHRA) is presented to balance energy and prolong network lifetime. We consider the combination of the heterogeneous nodes and cluster heads. Firstly, the model of optimal placement of heterogeneous nodes is built. Secondly, a clustering routing algorithm for wireless sensor network (CHRA) is presented, which consists of the heterogeneous routing algorithm for wireless sensor network (HRA) and a cluster heads selecting algorithm by LEACH-C. Thirdly, the performance of our proposed routing algorithm and some previous typical routings is evaluated via extensive experiments. The results indicate that our proposed routing algorithm can availably prolong network lifetime and stable period. And it can also balance the energy consumption significantly. In the future work, it is worth studying the optimal placement of heterogeneous nodes with limited energy and cluster heads of mobile Sink or multi-Sinks in wireless sensor network.

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