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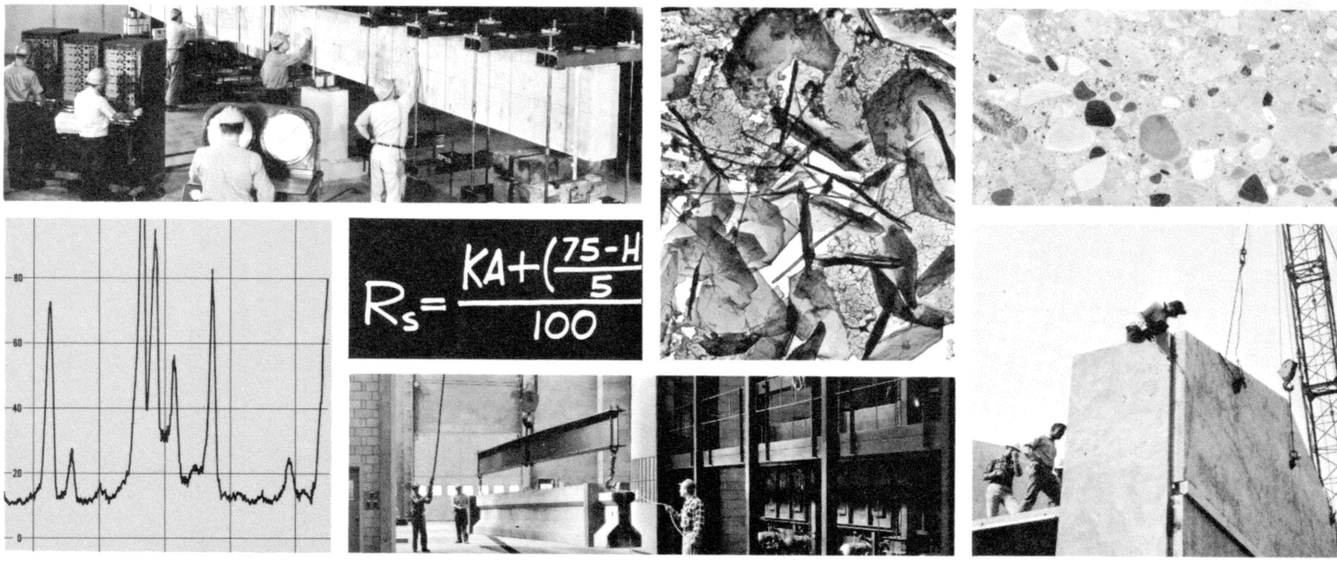

Design Provisions for Shear Walls

by A. E. Cardenas, J. M. Hanson,
W. G. Corley, and E. Hognestad

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PORTLAND CEMENT pca ASSOCIATION

CODE BACKGROUND PAPER

Background material used in preparing ACI 318-71

Design Provisions for Shear Walls

by A. E. Cardenas, J. M. Hanson, W. G. Corley, and E. Hognestad

The background and development of Section 11.16, Special Provisions for Walls, of the ACI Building Code (ACI 318-71) is discussed. These provisions were found to predict satisfactorily the strength of six high-rise and seven low-rise shear walls tested at the laboratories of the Portland Cement Association, as well as the strength of wall specimens tested by other investigators.

The results of the PCA experimental investigations are summarized in an Appendix. Thirteen rectangular shear walls were tested under combinations of lateral and axial loads. One of the specimens was subjected to ten cycles of load reversals.

Keywords: axial loads; **building codes;** cyclic loads; flexural strength; **high-rise buildings;** reinforced concrete; **research;** shear strength; shear stress; **shear walls; structural design.**

■ SHEAR WALLS ARE DEEP, relatively thin, vertically cantilevered reinforced concrete beams. They are commonly used in structures to resist the effects of gravity loads and story shears due to wind or earthquake forces.

This paper summarizes background material for Section 11.16, Special Provisions for Walls, of the 1971 ACI Building Code.[1] The provisions are intended to ensure adequate shear strength. However, other considerations such as flexural strength, energy absorption, lateral stiffness and reinforcement details are equally important to obtain satisfactory structural performance.

There has been relatively little research on the strength and behavior of shear walls. Investigators in Japan[2-4] have been concerned primarily with the strength of low-rise shear walls surrounded by a reinforced concrete or steel frame and subjected to load reversals.

Japanese shear wall design provisions are described in the Standards for Calculation of Reinforced Concrete Structures.[5] They are based on the philosophy that the entire shear force is to be carried by reinforcement, when a certain limiting concrete shear stress is exceeded.

In the early 1950's, Benjamin and Williams,[6-9] at the University of Stanford, conducted extensive static tests on low-rise shear walls surrounded by a reinforced concrete frame. Their proposed design equations[6] had limited practical use due to restrictions in their applicability. An extension of this investigation, dealing with dynamic loads, was conducted by Antebi, Utku and Hansen[10] at the Massachusetts Institute of Technology. Dynamic loads simulated were those due to blast from atomic weapons rather than earthquakes.

Prior to publication of ACI 318-71,[1] the only provisions for design of shear walls in the United States were those contained in Uniform Building Code.[11]

Fig. 1 shows a graphical representation of the provisions for shear walls in Uniform Building Code. Depending on the height-to-depth ratio of the wall, h_w/l_w, the nominal total design shear stress, v_u, is assumed to be resisted either only by

*This paper was prepared as part of the work of ACI-ASCE Committee 426, Shear and Diagonal Tension.

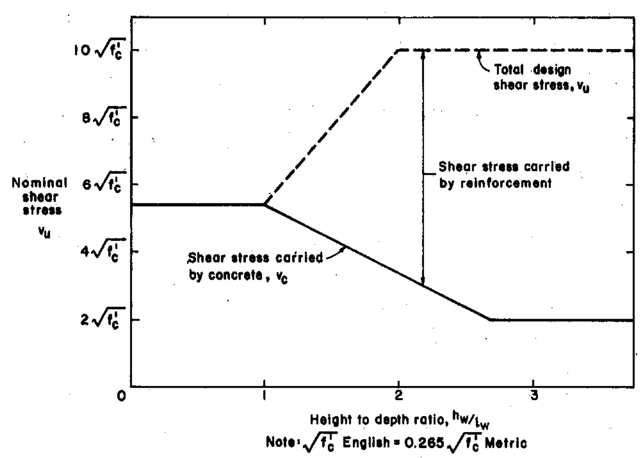

Fig. 1 — Provisions for shear walls in the 1970 Uniform Building Code

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the concrete, or by the concrete and the horizontal reinforcement.

The nominal permissible shear stress carried by the concrete, v_c, on shear walls with low h_w/l_w ratios is assumed similar to that in deep beams. It is taken as the straight-line lower bound of results of shear tests on deep beams without web reinforcement reported by dePaiva and Siess.[12] This shear stress is limited to $5.4\phi\sqrt{f_c'}$ for walls with h_w/l_w ratios of 1.0 or less. For h_w/l_w ratios of 2.7 or more, v_c is taken equal to $2\phi\sqrt{f_c'}$, the value recommended for reinforced concrete beams in ACI 318-63.[13]

Shear stress carried by the reinforcement is based on results of shear tests on beams containing web reinforcement reported by Slater, Lord and Zipprodt[14] as well as those reported by dePaiva and Siess.[12] Based on these tests, it is assumed that vertical or horizontal web reinforcement in shear walls with h_w/l_w ratios of 1.0 or less does not appreciably increase the value of v_u above that of v_c attributed to the concrete. Consequently, their total shear stress is limited to $5.4\phi\sqrt{f_c'}$. Shear walls with h_w/l_w ratios of 2.0 or more are considered to behave as beams. Total design shear stress for these walls is taken equal to $10\phi\sqrt{f_c'}$, as recommended in ACI 318-63.[13]

While the UBC provisions represented an advancement in design, additional work, including that by Crist,[15] Leonhardt and Walther,[16] Cardenas and Magura[17] and Cardenas, has led to separate provisions for deep beams and shear walls in Chapter 11 of ACI 318-71.[1] These provisions recognize that there are important differences between deep beams and shear walls. First, deep beams are usually loaded through the extreme fibers in compression. Under these conditions, shear carried by the concrete in a member without web reinforcement is greater than the shear causing diagonal tension cracking. Shear walls, however, are deep members loaded through stubs or diaphragms. This type of member, if it does not contain web reinforcement, may fail at a shear equal to or only slightly greater than the shear causing diagonal cracking.[18] Second, deep beams are not usually subjected to axial loads, whereas the consideration of axial compression or tension may be important in shear walls.

Recognizing the limitations of the existing information on the strength of shear walls, the Portland Cement Association started an experimental investigation in 1968. The highlights of this investigation are described in the Appendix.

DEVELOPMENT OF DESIGN PROVISIONS

Flexural strength

The experimental investigation demonstrated the importance of considering the flexural strength of a shear wall. In many designs of shear walls in high-rise buildings, use of the minimum amount of horizontal shear reinforcement required by the provisions of Section 11.16 of ACI 318-71,[1] 0.0025 times the concrete area, will be adequate to develop the flexural strength of the wall.

Using assumptions that are in accord with those in Section 10.2 of ACI 318-71, the flexural strength of rectangular shear walls containing uniformly distributed vertical reinforcement and subjected to combined axial load, bending and shear, can be calculated as:[17]

$$M_u = A_s f_y l_w \left[\left(1 + \frac{N_u}{A_s f_y}\right)\left(\frac{1}{2} - \frac{\beta_1 c}{2 l_w}\right) - \frac{c^2}{l_w^2}\left(1 + \frac{\beta^2}{3} - \beta_1\right) \right] \quad (1)$$

where

$$\frac{c}{l_w} = \frac{q + \alpha}{2q + 0.85\beta_1}$$

$$q = \frac{A_s f_y}{l_w h f_c'}$$

$$\alpha = \frac{N_u}{l_w h f_c'} \text{ and } \beta = \frac{f_y}{87{,}000}$$

M_u = design resisting moment at section, in. lb
A_s = total area of vertical reinforcement at section, sq in.
f_y = specified yield strength of vertical reinforcement, psi
l_w = horizontal length of shear wall, in.

c = distance from extreme compression fiber to neutral axis, in.
d = distance from extreme compression fiber to resultant of tension force, in.
h = thickness of shear wall, in.
N_u = design axial load, positive if compression, lb
f_c' = specified compressive strength of concrete, psi
β_1 = 0.85 for strength f_c' up to 4000 psi (281.0 kgf/cm²) and reduced continuously to a rate of 0.05 for each 1000 psi (70 kgf/cm²) of strength in excess of 4000 psi (281.0 kgf/cm²)

Eq. (1) can be approximated as:

$$M_u = 0.5\, A_s f_y l_w \left(1 + \frac{N_u}{A_s f_y}\right)\left(1 - \frac{c}{l_w}\right) \quad (2)$$

Based on results of the PCA investigation, Eq. (1) appears to satisfactorily predict the flexural strength of rectangular walls with an h_w/l_w ratio equal to or greater than 1.0.

Fig. 2 shows a comparison of Eq. (1) and (2) for different amounts of Grade 60 uniformly distributed vertical reinforcement for $f_c' = 4000$ psi (281.0 kgf/cm²) and for two ratios of axial compression, $\alpha = 0$ and $\alpha = 0.25$. The comparison shows that for the case of pure bending, $\alpha = 0$, Eq. (2) is in good agreement with the more rigorous Eq. (1). In the case of a rather large axial compression, $\alpha = 0.25$, the greatest difference is about 5 percent. Accordingly, the use of the simplified Eq. (2) appears adequate for practical design.

Shear strength

The distribution of lateral loads on shear walls varies with their height.[19-20] For example, under a lateral wind loading, this distribution may vary from nearly uniform on a wall in a tall building to a single concentrated force on a wall in a low building. Differences in lateral load distribution, geometry, and wall proportions lead to conditions that may make shear strength the controlling criterion in the design of low-rise shear walls.

As pointed out in the report of ACI-ASCE Committee 326 (426), Shear and Diagonal Tension,[21] American design practice is based on the premise that shear capacity of concrete beams is made up of two parts. One part is the shear carried by concrete, and the other part is the shear carried by web reinforcement. Furthermore, these two parts are considered to be independent, so that web reinforcement is required only for that portion of the total shear that exceeds the limit of the shear carried by the concrete.

With the adoption of ACI 318-63, an additional premise became inherent in the shear design provisions. This premise is that the shear carried by the concrete is equal to the shear causing significant inclined cracking. This last assumption underscores the importance of the cracking shear.

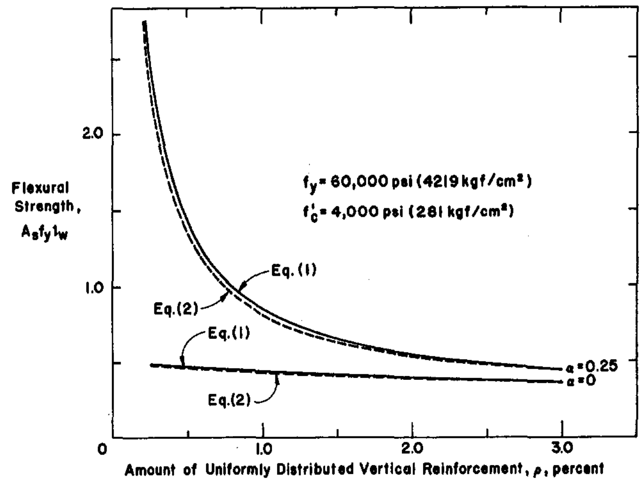

Fig. 2 — Flexural strength of rectangular shear walls

Shear carried by concrete

It is generally recognized that inclined cracking in concrete beams is of two types. In recent years, these types of cracks have been described as either "web-shear" or "flexure-shear." The way in which these cracks develop in reinforced and prestressed concrete beams has been described in detail elsewhere.[22,23]

The provisions of ACI 318-71 use Eq. (11-4) for computing the shear causing flexure-shear cracking in a reinforced concrete member. The limiting value of $3.5\sqrt{f_c'}$ for Eq. (11-4) serves as a measure of the shear causing web-shear cracking. In prestressed concrete beams, the shear causing flexure-shear or web-shear cracking is computed from Eq. (11-11) or (11-12), respectively. Eq. (11-12) predicts web-shear cracking as the shear stress causing a principal tensile stress of approximately $4\sqrt{f_c'}$ at the centroidal axis of the cross-section. Eq. (11-11) as originally developed[23] predicts flexure-shear cracking as the shear stress causing a flexural crack, corresponding to a flexural tensile stress of $6\sqrt{f_c'}$, to form at a section located distance $d/2$ from the section being investigated, plus a small stress, $0.6\sqrt{f_c'}$, intended to represent the shear required to transform the initiating flexural crack into a fully developed flexure-shear crack.

It is important to recognize that Eq. (11-11) for prestressed concrete beams is applicable to reinforced concrete beams subject to axial compression. However, the results would be expected to be conservative, because the shear stress required to transform an initiating flexural crack into a flexure-shear crack will usually be considerably greater than $0.6\sqrt{f_c'}$. Recent work[24] has attempted to take this into account. It follows, therefore, that a similar approach applied to shear walls would be conservative.

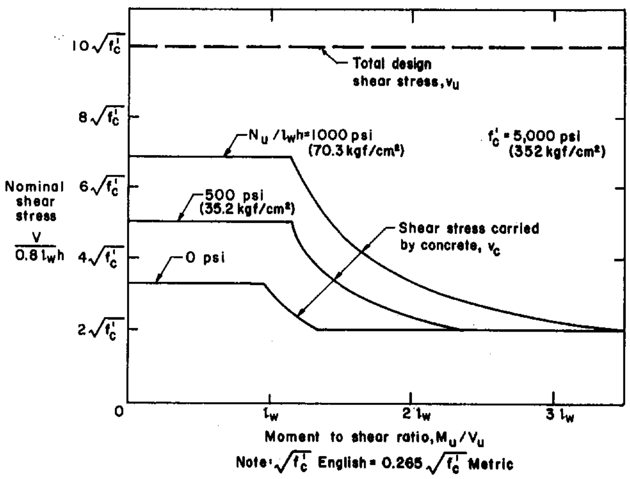

Fig. 3 — Shear carried by concrete in rectangular shear walls

Web-shear cracking would be expected in a shear wall when the principal tensile stress at any interior point exceeds the tensile strength of the concrete. In an uncracked rectangular section, the maximum shear stress due to a shear force, V, is:

$$v_{max} = \frac{3V}{2l_wh} \tag{3}$$

At the occurrence of a principal tensile stress of $4\sqrt{f_c'}$ on a section subjected to combined axial load, N, and shear, Eq. (3) becomes:

$$\frac{3V}{2l_wh} = 4\sqrt{f_c'}\sqrt{1 + \frac{N/l_wh}{4\sqrt{f_c'}}} \tag{4}$$

Eq. (4) can be closely approximated by:[25]

$$\frac{3V}{2l_wh} = 4\sqrt{f_c'} + 0.3\frac{N}{l_wh} \tag{5}$$

Introducing into Eq. (5) the concept of nominal shear stress, $v = V/hd$, and assuming that the effective depth d, is equal to $0.8l_w$, leads to:

$$v_c = 3.3\sqrt{f_c'} + \frac{N_u}{4l_wh} \tag{6}$$

where v_c is the value of nominal shear stress expected to cause web-shear inclined cracking. The subscript u has been added to N to indicate total applied design axial load occurring simultaneously with V_u.

Eq. (6), which is the same as Eq. (11-32) in ACI 318-71, will apply to most low-rise shear walls. In cases where the axial load, N_u, is small, the equation reduces to $v_c = 3.3\sqrt{f_c'}$. Limitations due to the assumption of $d = 0.8l_w$ are discussed later.

Flexure-shear cracking occurs when a flexural crack, because of the presence of shear, turns and becomes inclined in the direction of increasing moment. It is assumed that the flexure-shear cracking strength of a shear wall may be taken equal to the shear from a loading producing a flexural tensile stress of $6\sqrt{f_c'}$ at a section located a distance $l_w/2$ above the section being investigated. For shear walls, an expression for the value of nominal shear stress expected to cause flexure-shear inclined cracking is Eq. (11-33) of ACI 318-71:

$$v_c = 0.6\sqrt{f_c'} + \frac{l_w\ (1.25\sqrt{f_c'} + 0.2N_u/l_wh)}{\dfrac{M_u}{V_u} - \dfrac{l_w}{2}} \tag{7}$$

The shear carried by the concrete therefore corresponds to the least value of v_c computed from Eq. (6) or (7). However, the value of v_c need not be taken less than corresponding values for reinforced concrete beams. Therefore, v_c may be taken at least equal to $2\sqrt{f_c'}$ if N_u is zero or compression, or $2\ (1 + 0.002\,N_u/A_g)\sqrt{f_c'}$ with N_u negative for tension, as given in ACI 318-71.

Fig. 3 shows a diagram of Eq. (6) and (7) as a function of the moment to shear ratio, M_u/V_u, for selected values of axial compression, expressed as N_u/l_wh. The upper horizontal portion represents the web-shear cracking strength, as given by Eq. (6). The transition to the suggested minimum of $2\sqrt{f_c'}$ represents the flexure-shear cracking strength, as given by Eq. (7).

Shear carried by reinforcement

The contribution of reinforcement to shear strength of concrete beams has traditionally been based on the "truss analogy." This concept is discussed in the report of ACI-ASCE Committee 326 (426), Shear and Diagonal Tension.[21] Applied to shear walls, this contribution, expressed in terms of nominal shear stress, is:

$$v_s = \rho_h f_y \tag{8}$$

where

$\rho_h = \dfrac{A_v}{sh} =$ ratio of horizontal shear reinforcement

Shear reinforcement restrains the growth of inclined cracking, increases ductility, and provides a warning in situations where the sudden formation of inclined cracking may lead directly to distress. Accordingly, minimum shear reinforcement is highly desirable in any main load-carrying member. In shear walls, the specified minimum reinforcement area of 0.0025 times the gross area of the shear wall, provides a shear stress contribution of about $2\sqrt{f_c'}$ to the strength of the wall.

For low walls, it is reasonable to expect that the horizontal shear reinforcement is less effective than indicated by Eq. (8). However, the vertical reinforcement in the wall will contribute to its shear strength, in accord with the concept of shear-friction.[26] Because of insufficient test data to develop recommendations for walls with low

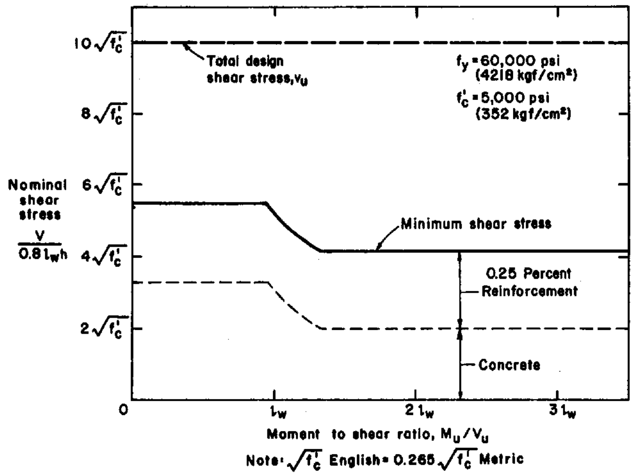

Fig. 4 — Minimum shear strength of rectangular shear walls

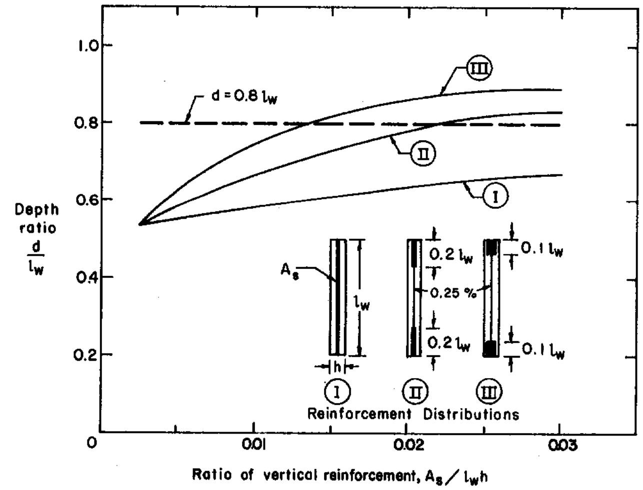

Fig. 5 — Variation of effective depth in rectangular shear walls

height to depth ratios, the amount of vertical reinforcement required is equal to the amount of horizontal reinforcement when h_w/l_w is less than 0.5. When h_w/l_w is greater than 2.5, the required minimum vertical reinforcement area is 0.0025. Between h_w/l_w ratios of 0.5 and 2.5, the required minimum is determined by linear interpolation, as expressed by Eq. (11-34) of ACI 318-71.

The shear capacity of rectangular shear walls containing minimum shear reinforcement is plotted in Fig. 4 as a function of the moment to shear ratio. The curves have been plotted for a concrete strength, f_c', of 5000 psi (350 kgf/cm²), and a yield stress of the horizontal reinforcement, f_y, of 60,000 psi (4200 kgf/cm²). The diagram shows that for these conditions, the minimum shear strength of low-rise walls is of the order of $5.4\sqrt{f_c'}$, and that of high-rise walls is of the order of $4.1\sqrt{f_c'}$.

Definition of nominal shear stress

In the design provisions, nominal shear stress is used as a measure of shear strength. Nominal shear stress, as defined by Eq. (11-31) of ACI 318-71, is given by:

$$v_u = \frac{V_u}{\phi h d} \tag{9}$$

where

V_u = total applied design shear force at section
ϕ = capacity reduction factor (Section 9.2, ACI 318-71)
h = thickness of shear wall
d = distance from extreme compression fiber to resultant of tension force

In shear walls, the effective depth, d, depends mainly on the amount and distribution of vertical reinforcement. Fig. 5 shows the variation of the effective depth with these variables. The value of $d = 0.8\,l_w$ is also shown in Fig. 5. This value is not necessarily conservative or unconservative, because the equations for shear attributed to the concrete have been modified to account for the proposed value of d. The equation for shear attributed to the reinforcement depends on the ability to effectively reinforce for shear over the vertical projection of the assumed inclined crack.

Limitation on ultimate shear stress

A limitation on ultimate shear stress is generally considered to represent failure due to crushing of concrete "struts" in beam webs. For reinforced concrete beams, ACI 318-63[18] limited the nominal ultimate shear stress to $10\sqrt{f_c'}$.* There is some indication[27] that the shear strength of a beam without web reinforcement may decrease with increasing depth. Other tests[16] on beams with low a/d ratios indicate that the limiting shear stress may be less than $10\sqrt{f_c'}$. However, the tests reported in this paper indicate that shear stresses up to $10\sqrt{f_c'}$ can be attained in walls with web reinforcement, even under load reversals. Attainment of shear stresses of this magnitude requires careful reinforcement detailing.

COMPARISON OF DESIGN PROVISIONS WITH TEST RESULTS

The proposed design provisions for shear strength of shear walls have been compared with experimental results reported by Muto and Kokusho,[2] Ogura, Kokusho and Matsoura,[3] Benjamin and Williams,[6,7] Antebi, Utku and Hansen,[10] and the PCA Laboratories.[17] In the computation of nominal shear stress, the effective depth, d, was taken equal to the distance from the extreme compression force to the resultant of the tension

*ϕ was not included here, so that the value is comparable to stresses in ACI 318-71.

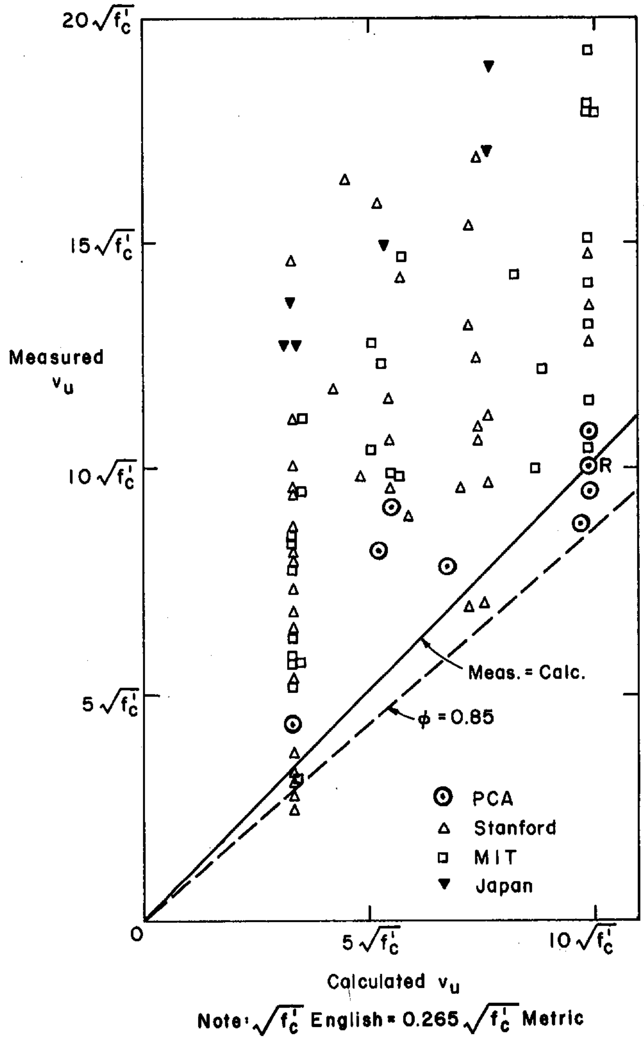

Fig. 6 — Comparison of measured and calculated strengths

force, or $0.8\, l_w$, whichever was greater. Results of the tests carried out by PCA are summarized in Tables A1 and A2 in the Appendix.

Fig. 6 compares calculated and measured shear strength for these test results. The solid line represents equality between calculated and measured shear stresses, and the dashed line represents consideration of the ACI capacity reduction factor, ϕ, equal to 0.85.

The two PCA test results plotted under the solid line are for specimens where the shear failure was observed to have been precipitated by loss of anchorage of the flexural reinforcement. The PCA test result marked with an *R* corresponds to the specimen subject to load reversals. Comparison of measured and calculated strengths in Fig. 6 indicates that the design provisions are satisfactory.

OTHER CONSIDERATIONS

In the development of design provisions for shear walls, the main emphasis was on evaluation of flexural and shear strength under static loadings. However, considerations of energy absorption, where earthquake resistance is required, and lateral stiffness are also important factors influencing the behavior of walls. Properly detailed reinforcement is also essential to obtain satisfactory performance.

Based on results of a recent investigation,[28,29] Paulay has indicated that energy absorption and stiffness characteristics of a wall may be significantly improved if the shear reinforcement does not yield when the wall reaches its flexural capacity. The apparent reason for this is that the widths of the inclined cracks are restrained, thereby maintaining aggregate interlock across the crack, and doweling action of the main reinforcement. Paulay has suggested that the total shear in a wall subject to load reversals should be taken by shear reinforcement. This requirement appears reasonable where great energy absorption is required.

In cases where high ductility is essential, as may be the case in spandrels or piers, it may be desirable to physically divide the wall into two or more parts as suggested by Muto.[30] This would have the effect of substantially increasing the *M/V* ratio of the wall elements thereby making flexure the predominant consideration. In any case, it is desirable to provide shear strength capacity in excess of the flexural strength.

The importance of careful detailing of shear walls must be emphasized. From experience, many researchers have found it is sometimes difficult to apply very large concentrated loads to walls, without experiencing local failures.

The possibility of tension in unexpected locations should also be given careful consideration. When beam action begins to break down due to the formation and growth of inclined cracks, particularly in deep members, the steel stress at the intersection of the inclined cracking and the flexural reinforcement tends to be controlled by the moment at a section through the apex of the inclined cracking. These stresses can be quite different from those calculated on the basis of the moment at a section through the lower extremity of the crack. Consequently, adequate anchorage of main reinforcement at force application points is essential.

CONCLUSIONS

Results of tests summarized in this paper indicate that flexural strength, as well as shear strength, must be considered in an evaluation of the load-carrying capacity of a shear wall. For use in design, the flexural strength of shear walls with height to depth ratios, h_w/l_w, of 1.0 or more can be satisfactorily predicted using Section

10.2, Assumptions, of ACI 318-71. Equations for determining the design flexural capacity of rectangular walls with uniformly distributed vertical reinforcement are presented in this paper.

For use in design, the shear strength of walls can be satisfactorily predicted using Section 11.16, Special Provisions for Walls, ACI 318-71.

In the design of shear walls, considerations such as energy absorption, lateral stiffness, and detailing of reinforcement need special attention.

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Mr. D. D. Magura, former PCA Research Engineer, initiated the experimental investigation. Laboratory technicians B. J. Doepp, B. W. Fullhart, W. H. Graves, W. Hummerich, Jr., and O. A. Kurvits performed the laboratory work.

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APPENDIX

PCA investigation

In this investigation, thirteen large rectangular shear wall specimens have been tested under static combinations of axial load, bending, and shear. Six of the specimens, SW-1 through SW-6, represented walls in high-rise buildings.[18] The remaining seven, SW-7 through SW-13, represented walls in low-rise buildings. One of the low-rise shear walls, SW-13, was subjected to ten cycles of load reversals.

All test specimens were rectangular reinforced concrete members with a thickness $h = 3$ in. (7.92 cm) and a depth $l_w = 6$ ft 3 in. (1.90 m). For convenience, the specimens were tested as horizontal cantilevered beams. However, in describing the specimens, reference is always made to the position of a wall in an actual building rather than its position during testing. Fig. A1 shows the test setup for one of the high-rise walls. Loading rods extending through the test floor were used to apply the simulated static lateral forces. Post-tensioning rods, running horizontally in the photo, were used to apply the simulated gravity loads. The portion of the specimen to the right of the support represents a foundation providing full restraint to the base of the wall.

Shear wall specimens SW-1 through SW-6 represent the lower portion of a shear wall in a frame-shear wall structural system.[20,21] The height of the specimen corresponds to the distance between the base of the wall and its point of contraflexure. It was assumed that 50 percent of the total shear force at the base of the wall would be applied at the point of contraflexure. The remaining 50 percent was uniformly distributed between the point of contraflexure and the base of the wall.

Four of the six high-rise shear wall specimens SW-1, SW-2, SW-3 and SW-6 had a height of 21 ft (6.40 m), the other two, SW-4 and SW-5, were 12 ft (4.09 m) high. An axial compressive stress of about 420 psi (29.5 kgf/cm^2) was applied. The main variable was the amount and distribution of the vertical reinforcement. Horizontal shear reinforcement equal to 0.27 percent of the concrete cross-sectional area was provided in each of the six specimens.

Six of the seven specimens representing low-rise shear walls, SW-7 through SW-12, were subjected to a

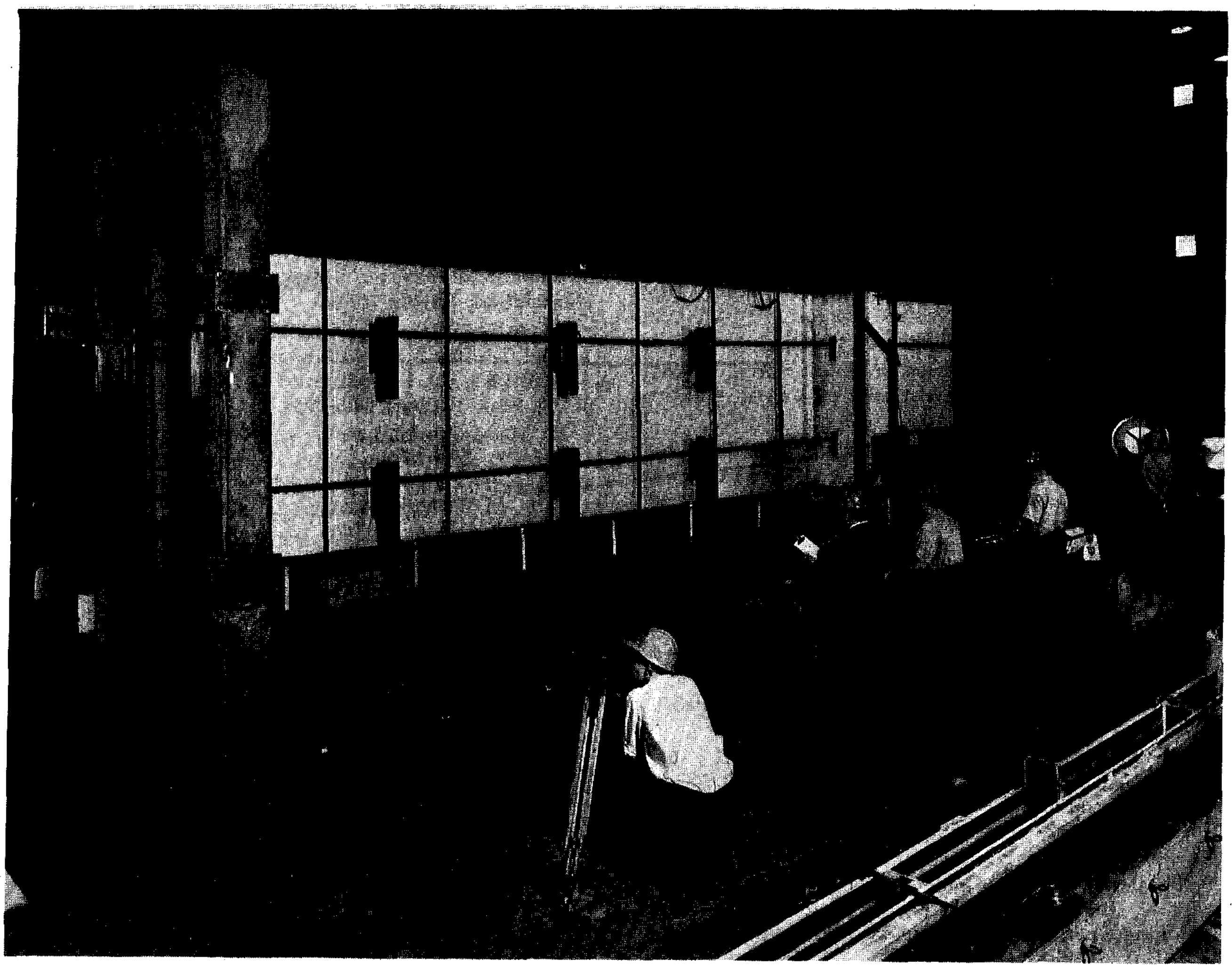


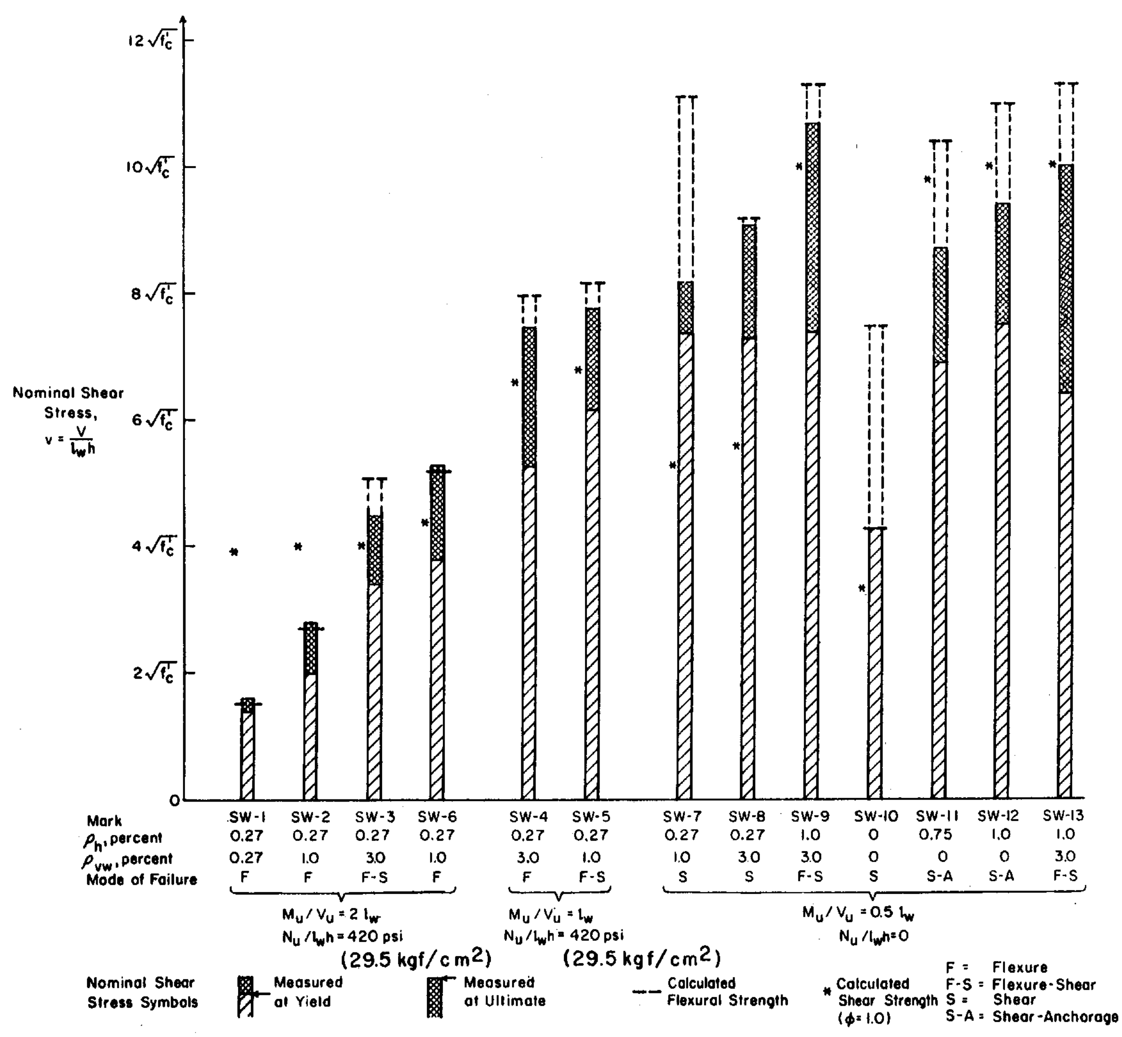
Fig. A1 — Test setup for shear wall investigation single static lateral force applied at the top of the wall. These specimens had a height, h_w, equal to their depth, l_w, of 6 ft 3 in. (1.90 m). At the top of these specimens, the thickness of the wall was enlarged to simulate the effect of floor slabs framing into the shear wall. The enlarged section distributes the applied shear force along the top of the specimen. No axial compression was applied to these specimens. Variables investigated were the amount and distribution of vertical reinforcement and the amount of horizontal shear reinforcement.

The seventh of the low-rise shear wall specimens, SW-13, was subjected to ten cycles of load reversals. All of the characteristics of this specimen were similar to those of specimen SW-9 previously tested under static loads. The objective of the test was to evaluate the effect of the cyclic loading on the strength and behavior of low-rise shear walls. Tables A1 and A2 summarize material properties, variables investigated and test results for all 13 specimens.

A summary of the results of the PCA investigation is presented in Fig. A2 in the form of bar graphs. Comparison of test results for specimens representing high-rise shear walls SW-1, SW-2, SW-3 and SW-6 shows that 0.27 percent of horizontal reinforcement, an amount considered to be nearly a practical minimum, is sufficient to develop the flexural strength of walls with varying amounts and distribution of vertical reinforcement.

Specimens SW-4 and SW-5 were designed to have the same flexural capacity as that of specimens SW-3 and SW-6. However, their height, h_w, was less. For the applied loads, the moment to shear ratio, M_u/V_u, at section $l_w/2$ from the base of the wall, was l_w for SW-4 and SW-5, and $2l_w$ for SW-3 and SW-6. Minimum horizontal reinforcement was again sufficient to develop nearly the caleulated flexural strength, even though the shear stresses were substantially higher. This implies that a greater proportion of the shear was carried by the concrete at the lower M_u/V_u ratio.

In the group representing low-rise shear walls, specimens SW-7 and SW-8 also indicate that walls with minimum horizontal shear reinforcement have a high load-carrying capacity. Comparisons of specimens SW-11 and SW-12 with SW-7, and also SW-9 with SW-8,



Note: $\sqrt{f'_c}$ English = $0.265\sqrt{f'_c}$ Metric

Fig. A2 — Results of PCA investigation

TABLE A1 — DIMENSIONS AND MATERIAL PROPERTIES OF TEST SPECIMENS

Mark	Height	Concrete		Reinforcement				Axial stress
				Vertical		Horizontal		
	h_w ft	Compressive strength f_c' psi	Tensile splitting strength f'_{sp} psi	Ratio ρv*	Yield stress f_y psi	Ratio ρ_h	Yield stress f_y psi	$N_u/l_w h$ psi
SW-1	21.0	7420	660	0.0027	60,200	0.0027	61,300	415
SW-2	21.0	6880	650	0.0100	65,400	0.0027	61,000	430
SW-3	21.0	6780	615	0.0300	66,000	0.0027	60,000	420
SW-4	12.0	6740	585	0.0300	60,000	0.0027	60,000	430
SW-5	12.0	5900	565	0.0230†	60,000	0.0027	60,000	425
SW-6	21.0	5950	590	0.0230†	63,000	0.0027	70,000	430
SW-7	6.25	6240	630	0.0230†	65,000	0.0027	60,000	None
SW-8	6.25	6160	565	0.0300	65,000	0.0027	67,500	None
SW-9	6.25	6240	630	0.0300	65,000	0.0100	60,000	None
SW-10	6.25	5850	565	0.0165‡	65,000	None	None	None
SW-11	6.25	5540	535	0.0230‡	65,000	0.0075	65,000	None
SW-12	6.25	5570	530	0.0230‡	65,000	0.0100	65,000	None
SW-13	6.25	6300	630	0.0300	64,500	0.0100	66,000	None

*$\rho_v = \frac{A_s}{l_w h}$, where A_s = total area of vertical reinforcement, $l_w = 75$ in. and $h = 3$ in.

†One-third of total vertical reinforcement concentrated within a distance $l_w/10$ from either extremity of cross section (amount of reinforcement in interior region $\rho_{vw} = 0.01$).

‡One-half of total vertical reinforcement concentrated within a distance $l_w/10$ from either extremity of cross section ($\rho_{vw} = 0$).

To convert to SI equivalents: 1 ft = 0.305 m; 1 psi = 0.0703 kgf/cm².

TABLE A2 — TEST RESULTS

Mark	Calculated parameters		Flexural strength		Shear strength			Measured/Calculated		Observed mode of failure
					Measured		Calculated†			
	Moment to shear ratio M_u/V_u at $l_w/2$ from base	Ratio d/l_w, at ultimate	Measured moment, M_u, at base kip-ft	Calculated* moment, M_u, at base kip-ft	Shear, V_u, at $l_w/2$, kips	$\frac{V_u‡}{hd\sqrt{f_c'}}$	$\frac{v_c\ v_s}{\sqrt{f_c'}}$	Moment at the base	Shear at $l_w/2$ from base	
SW-1	$2.0l_w$	0.58	406	379	26.5	1.7	3.9	1.07	0.44	Flexure
SW-2	$2.0l_w$	0.62	675	650	41.4	2.8	4.0	1.04	0.70	Flexure
SW-3	$2.0l_w$	0.71	1073	1200	66.0	4.5	4.0	0.90	1.13	Flexure-Shear
SW-4	$1.0l_w$	0.71	1077	1139	108.6	7.4	6.6	0.95	1.12	Flexure
SW-5	$1.0l_w$	0.78	1078	1121	108.6	7.8	6.8	0.96	1.15	Flexure-Shear
SW-6	$2.0l_w$	0.78	1179	1154	72.5	5.3	4.4	1.02	1.20	Flexure
SW-7	$0.5l_w$	0.74	729	980	116.7	8.2	5.3	0.74	1.55	Shear
SW-8	$0.5l_w$	0.65	801	1009	128.1	9.1	5.6	0.79	1.63	Shear
SW-9	$0.5l_w$	0.65	954	1000	152.7	10.7	10.0	0.95	1.07	Flexure-Shear
SW-10	$0.5l_w$	0.94	429	700	68.7	4.3	3.3	0.61	1.30	Shear
SW-11	$0.5l_w$	0.94	856	1000	137.0	8.7	9.8	0.86	0.89	Shear-Anchorage
SW-12	$0.5l_w$	0.94	925	1000	148.0	9.4	10.0	0.93	0.94	Shear-Anchorage
SW-13§	$0.5l_w$	0.65	888	1000	142.1	10.0	10.0	0.89	1.00	Flexure-Shear

*Based on compressive concrete limiting strain of 0.003, strain compatibility and measured material properties.
†Calculated from proposed shear strength equations.
‡d used is $0.8l_w$ or greater.
§SW-13 was subjected to 10 cycles of load reversals.
To convert to SI equivalents: 1 kip = 453.6 kgf; $\sqrt{f_c'}$, U.S. = 0.265 $\sqrt{f_c'}$ metric.

indicate that additional horizontal shear reinforcement will further increase capacity.

Comparisons of SW-8 with SW-7, and SW-9 with SW-12, show that the lateral load carrying capacity also increases with vertical web reinforcement. However, these observations are qualified somewhat by the observation that specimens SW-9, SW-11 and SW-12 did not fail in shear. In addition, at failure there was yielding of the vertical reinforcement in all of these specimens.

The ultimate shear stress of SW-10, a specimen with no horizontal or vertical web reinforcement, was $4.3 \sqrt{f_c'}$.

Specimen SW-13 was subjected to a total of ten cycles of increasing levels of load reversals. Comparison of this specimen with SW-9, a physically similar specimen that was subjected to one-directional loading, shows no significant decrease in strength. Both of these specimens developed shear stresses of the order of $10 \sqrt{f_c'}$.

Notation

a = shear span, distance between concentrated load and face of support, in.

A_g = gross area of section, sq in.

A_s = total area of vertical reinforcement at section, sq in.

A_v = area of horizontal shear reinforcement within a distance, s, sq in.

c = distance from extreme compression fiber to neutral axis, in.

d = distance from extreme compression fiber to resultant of tension force, in.

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_c' = specified compressive strength of concrete, psi

f_y = specified yield strength of reinforcement, psi

h = thickness of shear wall, in.

h_w = total height of wall from its base to its top, in.

l_w = depth or horizontal length of shear wall, in.

M_u = design resisting moment at section, in./lb

N_u = design axial load at section, positive if compression, lb

q = $A_s f_y / l_w h f_c'$

s = vertical spacing of horizontal shear reinforcement, in.

v_c = nominal permissible shear stress carried by concrete, psi

v_u = nominal total design shear stress, psi

V = shear force at a section, lb

V_u = total applied design shear force at section, lb

α = $N_u / l_w h f_c'$

β = $f_y / 87{,}000$

β_1 = 0.85 for strength f_c' up to 4000 psi (281.0 kgf/cm^2) and reduced continuously to a rate of 0.05 for each 1000 psi (70.3 kgf/cm^2) of strength in excess of 4000 psi (281.0 kgf/cm^2).

ϕ = capacity reduction factor (Section 9.2 ACI 318-71)

ρ_h = A_v / sh

ρ_v = $A_s / l_w h$

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This publication is based on the facts, tests, and authorities stated herein. It is intended for the use of professional personnel competent to evaluate the significance and limitations of the reported findings and who will accept responsibility for the application of the material it contains. Obviously, the Portland Cement Association disclaims any and all responsibility for application of the stated principles or for the accuracy of any of the sources other than work performed or information developed by the Association.

KEYWORDS: axial loads, building codes, cyclic loads, flexural strength, high-rise buildings, reinforced concrete, research, shear strength, shear stress, shear walls, structural design.

ABSTRACT: Discusses background and development of Sec. 11.16, Special Provisions for Walls, of the ACI Building Code (ACI 318-71). These provisions were found to predict satisfactorily the strength of six high-rise and seven low-rise shear walls tested at the PCA laboratories, as well as the strength of wall specimens tested by other investigators. Results of the PCA investigations are summarized in the Appendix.

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