

# Analysis of Steel Plate Shear Walls Using the Modified Strip Model

Jonah J. Shishkin<sup>1</sup>; Robert G. Driver, M.ASCE<sup>2</sup>; and Gilbert Y. Grondin<sup>3</sup>

**Abstract:** Unstiffened steel plate shear walls are an effective and economical method of resisting lateral forces on structures due to wind and earthquakes. Structural engineers require the ability to assess the inelastic structural response of steel plate shear walls using conventional analysis software that is commonly available and is relatively simple and expeditious to use. The strip model, a widely accepted analytical tool for steel plate shear wall analysis, is refined based on phenomena observed during loading of steel plate shear walls in the laboratory. Since the original strip model was proposed as an elastic analysis tool, these refinements are made primarily to achieve an accurate representation of yielding and eventual deterioration of the wall, although moderate improvements in initial stiffness predictions are also made. In assessing each of the proposed refinements, modeling efficiency is evaluated against the accuracy of the solution. A parametric study using the modified strip model examines the effect of varying the angle of inclination of the tension strips on the predicted inelastic behavior of the model. Notably, it was found that the ultimate capacities of steel plate shear wall models with a wide variety of configurations vary little with the variation of the inclination of the strips.

**DOI:** 10.1061/(ASCE)ST.1943-541X.0000066

**CE Database subject headings:** Shear walls; Steel plates; Seismic design; Lateral forces.

## Introduction

A conventional steel plate shear wall (SPSW) consists of thin unstiffened steel plates bounded by steel columns and beams that can be multiple stories high and one or more bays wide, with either simple shear or moment-resisting beam-to-column connections. Numerous research programs have confirmed that this system is an effective method of resisting lateral forces on structures, such as those due to wind and earthquakes. Moreover, they provide an economical solution (Timler et al. 1998) and are increasingly being used in structures. The SPSWs have been shown in large-scale experiments to possess high levels of initial stiffness, strength, ductility, and robustness under cyclic loading (e.g., Timler and Kulak 1983; Driver et al. 1998a; Qu et al. 2008).

Although the design of seismic load resisting systems based on elastic analysis by using loads that have been reduced to account for anticipated ductility and overstrength has been used successfully, modern design codes and standards are increasingly requiring an accurate assessment of the actual inelastic structural response and a primary tool of designers, as part of the overall design process, is the inelastic pushover analysis. Design engineers require the ability to assess inelastic structural response using analysis software that is commonly available and is rela-

tively simple and expeditious to use. This paper proposes refinements to the strip model proposed by Thorburn et al. (1983) to obtain an accurate prediction of the inelastic behavior of SPSWs using a conventional structural analysis software package. Modeling efficiency is evaluated against the accuracy of the solution and a modified version of the strip model is proposed that is shown to be efficient to generate while maintaining a high degree of accuracy. The parameters of the proposed model are generic and can be implemented in any structural analysis program with pushover analysis capabilities. A parametric study is also performed to determine the sensitivity of the predicted nonlinear behavior to variations in the angle of inclination of the infill plate tension strips in the model.

## Strip Model

The strip model was developed for SPSWs by Thorburn et al. (1983), who recognized that the buckling of the infill plate does not represent the ultimate capacity of the system and that the inclined tension field dominates the postbuckling behavior. Fig. 1 shows a typical story of a shear wall. The tension field behavior of each panel was modeled as a series of tension-only strips—10 were shown to be adequate—oriented at the same angle of inclination,  $\alpha$ , as the tension field. The tensile yield strength of the plate material was considered to be the limiting stress and the prebuckling shear resistance of the infill plate was neglected. The boundary beams were assumed to be infinitely stiff in order to reflect the presence of opposing tension fields above and below the modeled panel. Fig. 1 shows the use of hinged connections at the beam ends, although the researchers indicated that frame behavior could also be included. Timler and Kulak (1983) verified the use of the strip model as an accurate analytical tool by comparing predictions from the model with experimental results. From these two research projects, the following equation for  $\alpha$  was developed using the principle of least work

<sup>1</sup>Structural Engineer, WorleyParsons Canada, Suite 120, 5008-86th St., Edmonton, AB, T6E 5S2, Canada.

<sup>2</sup>Professor, Dept. of Civil and Environmental Engineering, Univ. of Alberta, Edmonton, AB, T6G 2W2, Canada (corresponding author).

<sup>3</sup>Professor, Dept. of Civil and Environmental Engineering, Univ. of Alberta, Edmonton, AB, T6G 2W2, Canada.

Note. This manuscript was submitted on August 5, 2008; approved on April 20, 2009; published online on May 2, 2009. Discussion period open until April 1, 2010; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Structural Engineering*, Vol. 135, No. 11, November 1, 2009. ©ASCE, ISSN 0733-9445/2009/11-1357-1366/\$25.00.

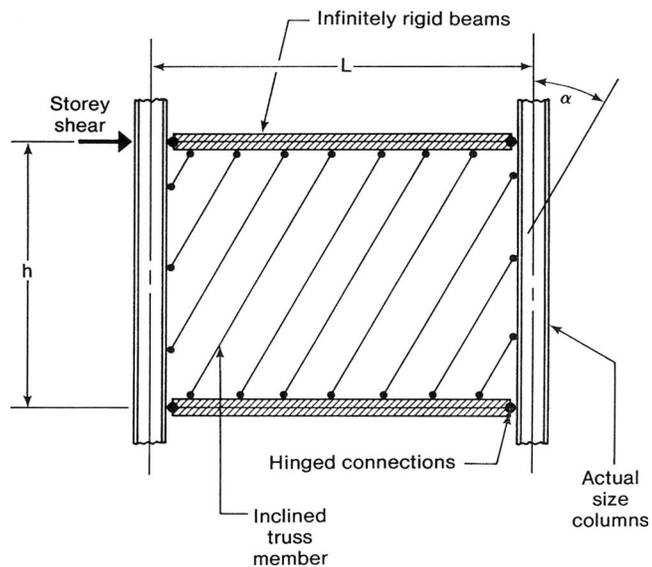


Fig. 1. Strip model (Thorburn et al. 1983)

$$\tan^4 \alpha = \frac{1 + \frac{tL}{2A_c}}{1 + th \left( \frac{1}{A_b} + \frac{h^3}{360I_cL} \right)} \quad (1)$$

where  $t$  represents the thickness of the infill plate,  $I_c$  = moment of inertia of the columns,  $A_b$  and  $A_c$  = cross-sectional areas of the beams and columns, respectively, and  $L$  and  $h$  are defined in Fig. 1.

Since it was originally proposed, the strip model has been used widely in published research projects, as well as by several building designers. Indeed, recent research publications (e.g., Berman and Bruneau 2008) cite nonlinear static pushover analysis using a strip model as providing the “accurate” solution with which other proposed methods of analysis can be compared to assess their efficacy. Several modifications to the original strip model have been proposed over the years. For example, Elgaaly et al. (1993) developed a strip model that incorporated a trilinear stress versus strain relationship for the tension strips. A tension-only strip model was also created by Lubell et al. (2000), where the column and tension strips had trilinear stiffness parameters to account for both yield and postyield strain hardening. Rezaei (1999) proposed another strip model with splayed strips that converge at the beam-to-column joints. The Canadian steel design standard, CSA S16-01 [Canadian Standards Association (CSA) 2001], recommends the strip model as an analytical tool for SPSWs and both FEMA 450 [Building Seismic Safety Council (BSSC) 2003] and the AISC seismic provisions (AISC 2005) recommend its use for analysis in their respective commentaries, but with little guidance as to how it is to be applied for nonlinear analysis. Since the strip model is commonly viewed as an accurate—albeit somewhat time-consuming—method of analyzing SPSWs, the impetus for the investigation presented herein was the need to optimize the implementation of the strip model, to clarify the details of its use, and to highlight any shortcomings in terms of accuracy.

### Study of Potential Modeling Improvements

The typical nonlinear behavior of a properly proportioned SPSW consists of a high initial elastic stiffness followed by tensile yield-

ing of the infill plates, after which the frame develops localized plastic hinges until the ultimate strength of the wall is obtained. This is followed by a gradual deterioration in strength at large displacements. Although hysteresis curves of SPSW behavior capture additional information about cyclic energy dissipation, in general, all of these qualities are reflected in an envelope curve of a cyclically loaded wall. Therefore, it is desirable for the design engineer to be able to model this behavior both accurately and efficiently. Refinements to the strip model, based on a rational approach as well as phenomena observed during laboratory testing, are proposed to provide a model, herein called the “detailed model,” that captures all of the key features of the experimental envelope curve. A four-story SPSW test specimen (Driver et al. 1998a), depicted in Fig. 2(a), was selected for use in developing the detailed model because it is of a large scale and reasonable proportions and comprehensive experimental data were readily available. The geometric arrangement of the specimen using the detailed model is shown in Fig. 2(b). The strip model used as the basis of the development is that of Thorburn et al. (1983).

### Panel Zones

For a moment-resisting connection, the panel zone is the area of the column bounded above and below by the depth of the connecting beam. Driver et al. (1998a) observed that inelastic deformations in the panel zones of their specimen tended to remain small throughout the duration of lateral loading—and these regions remained essentially elastic up to the peak wall capacity—since the primary ductile fuses were the infill plates. Fig. 3 illustrates the frame-joint arrangement used in the detailed model. From the node at the intersection of the beam and column elements called the “connection node,” the panel zone extends to  $d_c/2$  from the connection node for beam elements and  $d_b/2$ , on each side of the connection node, for column elements, where  $d_c$  and  $d_b$  are the depths of the column and beam, respectively. The nodes at the periphery of the panel zones are called “panel nodes.” The panel zone elements were modeled as being effectively rigid to simulate the highly stiff joint region.

### Plastic Hinges

Plastic hinges are required to model the inelastic behavior of SPSWs accurately. Although flexural plastic hinges in frame members have a finite length that is often taken to be approximately equal to the member depth, it is more convenient to model a plastic hinge as occurring at a discrete point. Since the panel zone is assumed to be an effectively rigid area, the hinge nodes (Fig. 2) representing the flexural plastic hinges are located at a distance of one-half the member depth from the boundary of the panel zone. Similarly, a flexural hinge is placed at a distance of  $d_c/2$  from each column base support node. To simulate the yielding of the infill plates, an axial hinge is placed within the length of each pin-ended tension strip.

User-defined moment versus rotation curves describe the behavior of the flexural hinges in the beam and column elements, while force versus elongation curves describe the behavior of the axial hinges in the tension strips. Each hinge is considered to be rigid until yielding commences at that location and up to this point, all deformations occur elastically in the line elements between the hinges. Thereafter, the overall SPSW behavior is influenced by a combination of distributed elastic member deformations and discrete hinge deformations as specified by the hinge behavior definitions. Assuming a linear strain gradient

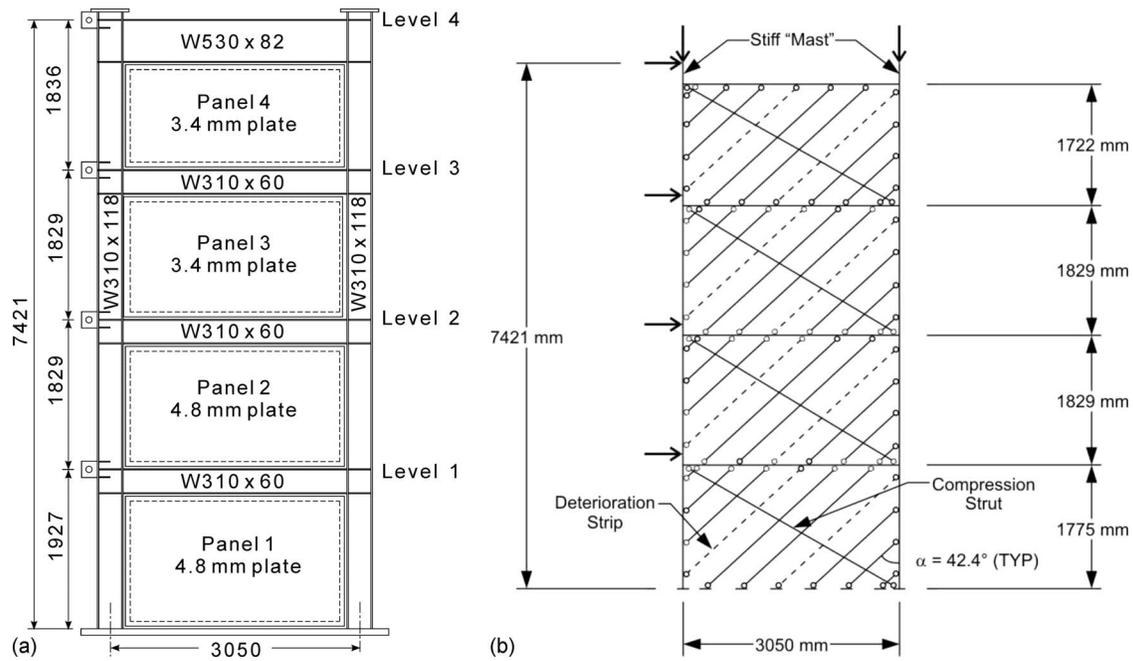


Fig. 2. Driver et al. (1997) SPSW: (a) specimen; (b) detailed model

through the member cross section and neglecting the effects of local buckling (alternative definitions would be required for cross sections with slender elements), the moments used to define the flexural hinges are calculated for the following extreme-fiber strains: yield point, onset of strain hardening, and strain at ultimate stress. Curvatures associated with each of these strain levels are determined and, assuming a hinge length equal to the member depth and constant curvature within the hinge length, the corresponding discrete hinge rotations are established. Using these calculated values, a quadrilinear moment versus rotation hinge curve

is obtained. For the column flexural hinges, the moments specified in the flexural hinge definitions are reduced for the presence of axial force and the detailed model approximates this interaction as follows:

$$M_{pc} = 1.18ZF_y \left( 1 - \frac{P}{A_c F_y} \right) \leq ZF_y \quad (2)$$

where  $M_{pc}$  = plastic moment adjusted for the presence of axial load ( $P$ ),  $Z$  = plastic section modulus, and  $F_y$  = yield stress.

The axial force versus elongation curve for the hinges in the tension strips is defined to correspond with a multilinear approximation of the stress versus strain curve of the plate material using the same reference strains specified above for the flexural hinges. While it is common practice to model the strip material as being elastoplastic, in the case of the detailed model strain hardening is included. The tension strip axial hinges possess no compressive capacity to simulate the very low buckling capacity of the relatively thin infill plates. That is, for any strip experiencing axial shortening, an axial force of zero is assigned.

### Compression Strut

Driver et al. (1997) discussed the phenomena present in SPSW behavior that are not captured by the strip model that could explain, in part, the tendency of the model to underestimate both the elastic stiffness and the ultimate capacity. The strip model neglects the small contribution to the stiffness and strength of the infill plate from compressive resistance, which may be significant in the corner regions depending on the plate thickness. Moreover, the vertical tension field arising from the overturning moment, which forms in the infill plate near the column on the tension side of the wall, is not taken into account. In the detailed model, the combination of these behaviors—simply called the “compression field” herein for expediency—is modeled using a pin-ended compression strut that extends from corner to corner of each panel and is oriented in the opposite diagonal direction to that of the tension

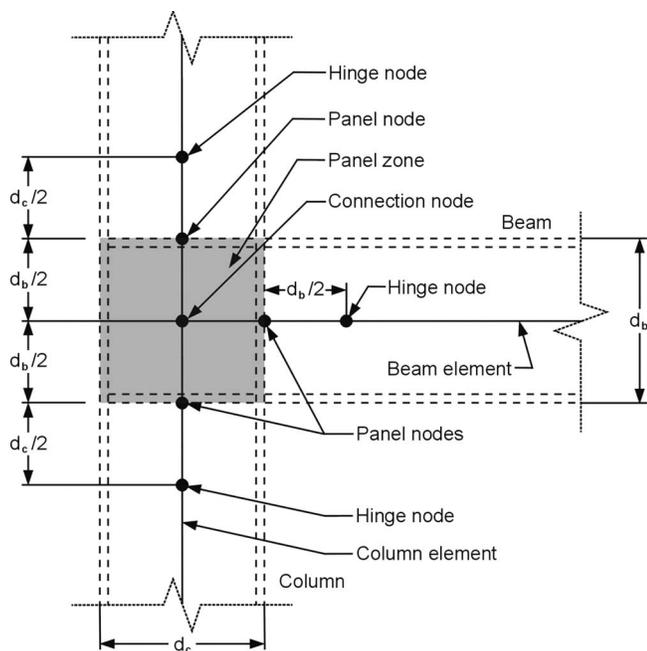


Fig. 3. Typical frame-joint arrangement for moment-resisting connections

strips. Assuming that the whole plate contributes to the compressive resistance, the area of the strut,  $A_{CS}$ , is determined as follows:

$$A_{CS} = \frac{Lt \sin^2 \alpha}{2 \sin \phi \sin 2\phi} \quad (3)$$

where  $\phi$  represents the acute angle of the strut with respect to the column. Based on the equivalent brace method by Thorburn et al. (1983), Eq. (3) implies that a full diagonal compression field forms. It should be emphasized that the compression strut as described above does not physically develop during the lateral loading of a SPSW and that the use of the compression strut is a semiempirical method of modeling the combined effects of the observed additional contributions to the overall strength and stiffness of the SPSW.

A rigid-plastic axial hinge is placed at a discrete point in the compression strut to simulate the sudden “buckling” of the strut once its capacity, or limiting stress, is reached. Based on the empirical observations of Kulak et al. (2001) that considered both the capacity of the wall and energy dissipation characteristics in cyclically loaded models, the value of the limiting stress was set at 8% of the yield strength of the infill plate, which was confirmed to be appropriate in a sensitivity analysis by Shishkin et al. (2005).

### Deterioration Hinge

The tears in the infill plate of the test specimen that arose principally due to the kinking of the stretched plate during load reversals were observed to have formed and propagated primarily in the corners of the infill plates. The tearing of the infill plate contributed to the gradual deterioration in the strength of the specimen. A means of modeling this deterioration is desired so that a maximum strength can be defined, followed by a declining branch of the pushover curve. These features help to characterize the ductility of the SPSW.

Based on empirical observations by Driver et al. (1998a), a discrete axial hinge that includes the effects of deterioration is provided for the tension strips that intersect the frame closest to the opposite corners of the SPSW panel in place of the typical axial hinge described above [see Fig. 2(b)]. The behavior of the deterioration axial hinge is initially identical to that of the typical axial hinge. During the test, significant tearing of the bottom infill plate began when a corresponding strip (of the model) reached an elongation of approximately five times the yield elongation. Therefore, this point was taken as the start of deterioration in the hinge and the deterioration rate was estimated thereafter based on the documented rate of tear propagation observed during the test. By the end of the test, since the tears extended across a width of plate approximately equivalent to the width of one strip in the 10-strip model (Driver et al. 1998a), the capacity of the deterioration hinge is taken linearly to zero at a strip elongation of 10 times the yield elongation (Shishkin et al. 2005). Although this deterioration behavior is recognized to be a function of the cyclic loading for this specific specimen, it is considered to be a severe case and therefore would be expected to be conservative for most applications. As in the case of the typical tension strip hinges, the deterioration axial hinge has no compressive capacity.

### Pushover Analysis

A pushover analysis that included the consideration of  $P$ - $\Delta$  effects was conducted on the detailed model by applying the gravity loads to their full value, followed by the application of the lateral

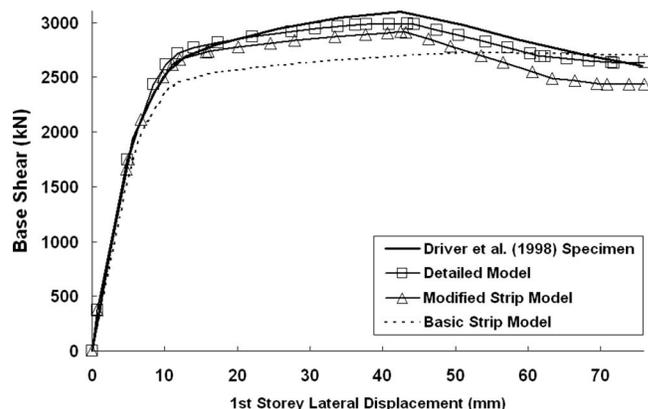


Fig. 4. Response curves for specimen, detailed model, modified strip model, and basic strip model

loads using displacement control, as in the test. The analysis was terminated when a column node at the elevation of the top flange of the beam at Level 1 reached a deflection of 76 mm, which is the maximum deflection at Level 1 recorded during the test. A pushover curve was obtained and is compared to the envelope curve of the test specimen in Fig. 4. The detailed model provides an excellent representation of the elastic portion of the curve. Although there is a very small kink in the curve at a deflection of 0.7 mm, which is due to the buckling of the compression strut in the bottom story, this slight irregularity is considered negligible. The detailed model curve overestimates the measured strength very slightly at a point just beyond the occurrence of initial yielding and predicts a peak strength of 2,990 kN, only 2.9% below the measured ultimate strength (3,080 kN). The peak of the model curve occurred at almost the same deflection as the test specimen (44.3 mm for the detailed model versus 42.5 mm for the specimen). The declining curve of the model descends at approximately the same rate as that of the specimen. Despite slight deviations from the specimen envelope curve, the pushover behavior of the detailed model is in excellent agreement with the test specimen behavior.

For comparison purposes, the test specimen was also analyzed using the strip model recommended in design standard CSA S16-01 [Canadian Standards Association (CSA) 2001]. Since specific details about how to implement the strip model—particularly with respect to the incorporation of inelastic behavior—are not provided in the standard, methods described in recent models presented in the literature (*e.g.*, Driver et al. 1998b) are used. The strip model, called the “basic” strip model here to distinguish it from the modified model, was generated in a similar manner as the detailed model. All hinges were assumed to behave bilinearly. The flexural hinges were located at the connection nodes or base support nodes and the axial hinges were located as they were in the detailed model. Also, the panel zones were not stiffened, no deterioration was modeled, and no compression strut was provided in the basic strip model. The curve from the basic strip model, shown in Fig. 4, underestimates the initial stiffness (taken up to 60% of the peak load) by 10%, after which the discrepancy gradually increases. It also underestimates the ultimate strength of the specimen considerably, although no peak is obtained using this model to permit a direct comparison. Although the basic strip model is conservative for use in design, the detailed model is more accurate in predicting the elastic and inelastic behavior of the SPSW and it characterizes the ductility of the specimen by means of the descending branch.

## Modified Strip Model

Even though the detailed model has been shown to predict the nonlinear behavior of the specimen accurately, if the model could be simplified while retaining good accuracy then the simpler model would be more desirable for use as a design tool. Several parameters of the detailed model are examined more closely to determine their effect on the accuracy of the model. In the following sections, various aspects of the detailed model are simplified, resulting in several distinct simplified models. Each such model undergoes a pushover analysis and the resulting pushover curve is compared to that of the detailed model and the envelope curve of the test specimen itself. Those simplified parameters that result in a more efficient model to generate, while not adversely affecting the accuracy of the model significantly, are retained. This is considered to be the model that demonstrates the best balance between accuracy and modeling efficiency and is termed the “modified strip model.”

## Frame-Joint Arrangement

Modeling the joints as described in the detailed model can be cumbersome and time-consuming. Therefore, an alternative frame-joint arrangement is proposed that relocates the flexural plastic hinges to the panel nodes and the measured or nominal modulus of elasticity is assigned to the members within the panel zone so that the same material model can be used throughout each column or beam. The flexural hinges near the bases of the columns remain at  $d_c/2$  from the base support nodes, as moving them to the base was found to reduce the accuracy of the pushover curve. When the simpler frame-joint arrangement was used in the detailed model, the resulting curve was found to be virtually identical to the detailed model curve. Conversely, placing the hinges right at the connection node resulted in an underestimation of the capacity of the test specimen by 6.7%, with little simplification. Therefore, the frame-joint arrangement with hinges at the panel nodes and actual material properties within the panel zone is adopted.

## Change of Strip Layout

In the strip model proposed by Thorburn et al. (1983), each panel of a SPSW is treated separately when calculating both  $\alpha$  and the resulting tension strip spacing since they are dependent on properties that may vary from story to story. Even when a single average value of  $\alpha$  is used for all panels, this method generally results in a staggered alignment of tension strips in adjacent stories. A simpler method suggested by Timler et al. (1998), hereinafter referred to as “crosshatching,” was investigated in an attempt to reduce the number of required nodes further. The crosshatching method uses the average value of  $\alpha$  and spaces the tension strips at equal intervals so that the strips in panels above and below share common nodes at the beam. The nodes for a SPSW model can be generated rapidly using the crosshatched layout. The detailed model was amended using the crosshatching technique and the tension strips were spaced such that 10 equally spaced strips would represent the bottom panel. Depending on the geometry of the structure and the value of  $\alpha$ , each panel may have slightly more or fewer than 10 tension strips. Cases where some stories have fewer than 10 strips may require a reassessment of the selected strip spacing. When crosshatched tension strips were used in the detailed model, the resulting curve was identical to the detailed model curve up to the peak load and actually provided a

slightly improved representation of the descending curve thereafter. Therefore, due to the savings in modeling effort combined with accurate results, the crosshatching method is adopted.

## Bilinear Hinges

Since flexural hinge rotations may be defined in commercial software in terms of chord rotations over the element length and since the individual element lengths vary in the frame of a SPSW model depending on the locations of the nodes required at the ends of the tension strips, many different hinge definitions may be needed to achieve the desired model. However, if the plastic hinge behavior is modeled as rigid-perfectly plastic, only the plastic moment needs to be specified in the hinge definition, resulting in a single hinge definition per member cross section. In the case of the tension strips, it was found that strain hardening did not occur during the detailed model pushover analysis, making this simplification for the axial hinges logical. It was found that besides being much simpler to implement, the effect of using rigid-plastic hinges throughout the model was small and they were therefore adopted.

## Pushover Analysis

A pushover analysis was performed on the specimen of Driver et al. (1998a) using the modified strip model, utilizing the simplifications from the detailed model as described in the previous sections. The curve for the modified strip model provides an excellent representation of the initial stiffness of the test specimen, predicts the ascending behavior of the specimen near the knee of the curve slightly more accurately than the detailed model, and underestimates the ultimate strength of the specimen by only 5.2% (predicts 2,920 kN), as shown in Fig. 4. The peak of the curve occurs at the same deflection as that of the test specimen envelope curve (42.5 mm) and the declining curve is similar to that of the test specimen, although somewhat steeper. A comparison of the detailed and modified strip models indicates that little accuracy is lost when the simplified parameters are implemented, while at the same time rendering the model significantly more efficient in terms of modeling effort. Moreover, a somewhat more conservative evaluation of the SPSW behavior in terms of both capacity and rate of deterioration is provided when the simplified parameters are used.

## Model Validation

The modified strip model can predict the overall inelastic behavior of the four-story SPSW specimen tested by Driver et al. (1998a) with good accuracy. However, it must be recognized that the model development was influenced by observations during the test itself. Other specimens described in the literature having different geometric properties and configurations can be used to validate the model for general use. (Specimens with shear-type or pinned beam-to-column connections are not presented in this paper since FEMA 450 ([Building Seismic Safety Council (BSSC) 2003] and AISC (2005) require that beam-to-column connections be moment resisting. However, validations for these cases are presented by Shishkin et al. (2005).) The one-story one-bay specimen, SPSW2 [Fig. 5(a)], tested by Lubell et al. (2000) was modeled due to its significant differences from the specimen used for the development of the modified strip model. Of note, the infill plate thickness was 1.5 mm, the  $S75 \times 8$  section does not

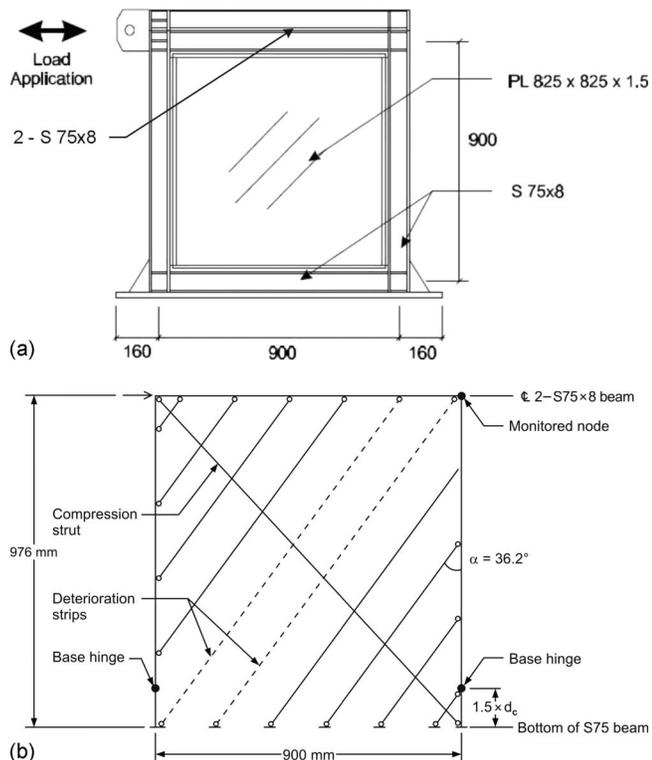


Fig. 5. Lubell et al. (2000) SPSW2: (a) specimen; (b) modified strip model

meet the column flexibility limit requirements prescribed in Clause 20.4.2 of CSA S16-01 [Canadian Standards Association (CSA) 2001] and no vertical loads were applied.

Figure 5(b) shows the geometric arrangement of the modified strip model for the SPSW2 specimen. The base of the model was located at the bottom of the lowest  $S75 \times 8$  beam and a fixed base support condition was imposed. The base hinges of the columns were located at  $d_c/2$  above the top flange of the bottom beam, which was omitted from the model since in the analysis the region of the column that would be bounded by the beam remains elastic and deforms a negligible amount compared to the hinge. The height of the model extended to the centerline of the top beam. The lateral load was applied from one side of the wall at the level of the centerline of the top beam, as in the test. A node on the other column at that level was monitored and the analysis was terminated when the node reached the maximum lateral deflection imposed during the test (50 mm).

The peak capacity of the model pushover curve occurs at a considerably smaller deflection than observed in the test, as shown in Fig. 6, which suggests that the deterioration behavior of the modified strip model (MSM) does not accurately reflect the deterioration of SPSW2. This is likely due to the very thin infill plate that would be less susceptible to localized kinking, leading to the formation of tears, and supports the notion that the deterioration hinge proposed for the model is conservative. To assess the wall response without the plate tearing, the deterioration behavior of the strips was omitted and bilinear tension-only axial hinges were used for all of the inclined tension strips. The pushover curve from this model provides an excellent representation of the latter part of the curve, as seen in Fig. 6. A small kink forms in the model curve early in the analysis due to the buckling of the compression strut. After the formation of the kink, the stiffness of the model is equal to that of the specimen but the model curve lies

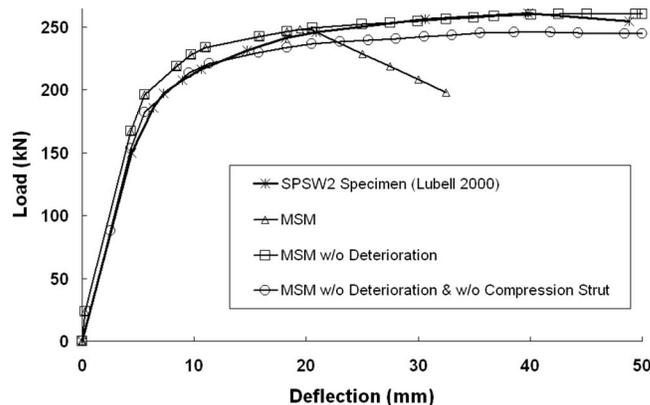


Fig. 6. Response curves for SPSW2 specimen and models

slightly above that of the specimen. Despite this deviation, the model without the deterioration accurately predicts the ultimate strength of the specimen, underestimating it by only 0.4% at the same deflection (260 kN for the model versus 261 kN for the specimen).

The model of the SPSW2 specimen without the deterioration hinges was also analyzed with the compression strut removed. Fig. 6 demonstrates that the initial stiffness of this model agrees well with the initial stiffness of the test specimen, while the predicted ultimate strength of the specimen (245 kN) is underestimated by 6.1%. The pushover curves of the models with and without the compression strut suggest that early in the loading stage, the compression field does not form in the very thin infill plate. However, at a point approximately halfway through the initial yield portion of the envelope curve, the full strength of the compression field starts to be developed. A design engineer can readily create two models for shear walls with very thin infill plates—one with a bilinear compression strut and one without—and obtain a good estimate of the complete response.

## Frame Force Results

Modern seismic design provisions follow capacity design principles whereby the frame design forces would generally be determined based on the expected capacity of the infill plates, including overstrength. However, for wind applications, frame forces are taken directly from the analysis of the model under the design loads. Therefore, moments and axial forces for the first-story frame members were extracted from the pushover analysis of the modified strip model and were compared to those determined from strain measurements during the test of Driver et al. (1997). Moments and axial forces were evaluated for the first-story columns and beam at a total base shear of 2,020 kN since these values were available for the test specimen.

Fig. 7 shows that the first-story frame force effects obtained from the modified strip model are generally conservative when compared to those of the test specimen, with the exception of the east column axial forces (the specimen was pushed to the west), where the model underestimated the test results. Since in most cases compression forces govern for column design, the underestimated tensile forces may not cause the column sections to be inadequately designed but would influence splice design and base anchorage for uplift. The model overestimates by a considerable margin the first-story column moment near the base of the east column but provides a good estimate near the top of that column

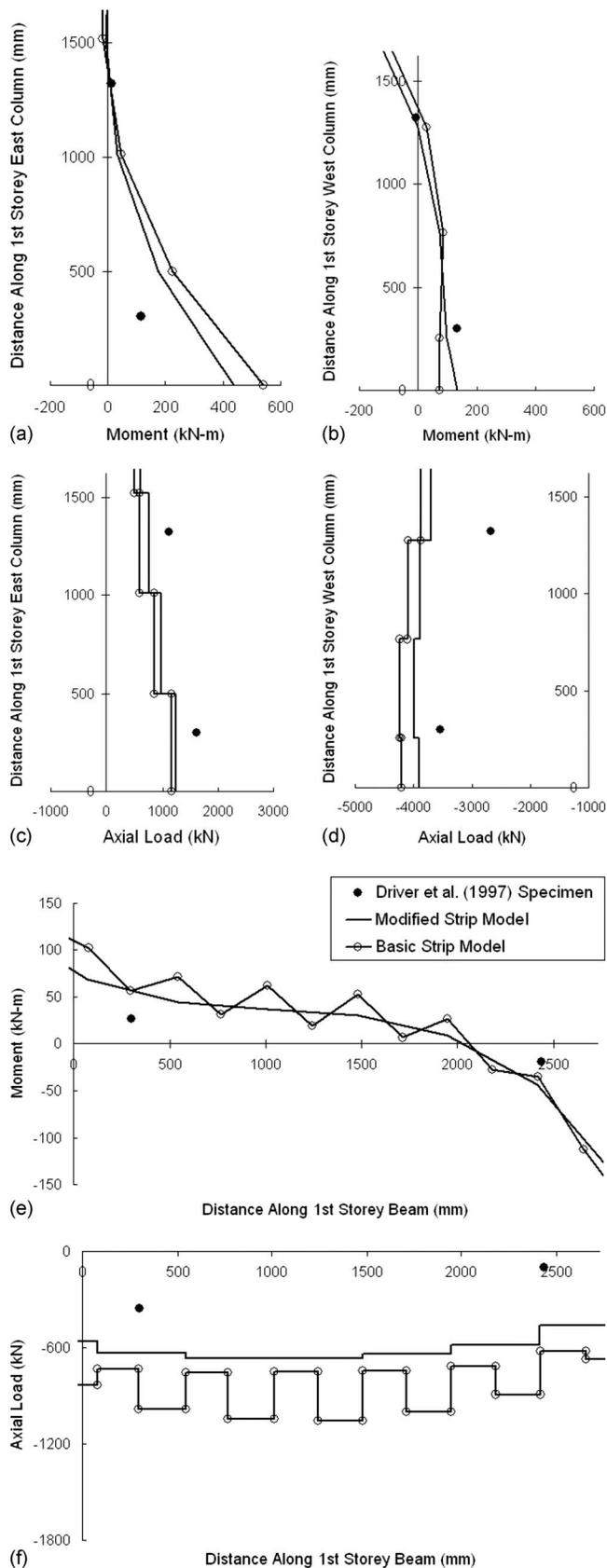


Fig. 7. Frame force comparisons

and in the west column throughout, although there is a slight underestimation of the moment near the base of the west column. It is evident from the results presented in Fig. 7 that more research is needed to provide a simple model that can accurately predict both the overall inelastic pushover response and the internal frame forces of a SPSW. Nevertheless, the modified strip model produces an excellent representation of the pushover behavior and provides generally conservative frame forces. The frame forces from the basic strip model described previously are included in Fig. 7 for comparison with the results of the modified strip model. In all cases, the modified strip model provides either equivalent or improved estimates of the frame forces and significant improvements are revealed in several locations. Similar observations have been made for the second-story columns and beam (Shishkin et al. 2005). Therefore, while still giving generally conservative results, the modified strip model is found to be a better tool for predicting frame forces than the basic strip model.

## Parametric Study

The design and analysis of SPSWs can be an iterative and time-consuming process due in part to the dependence of the angle of the tension strips,  $\alpha$ , on the cross-sectional properties of the boundary members and the infill plate thickness, according to Eq. (1). As the design evolves, the calculated angle of inclination changes, which appears to require a revision to the strip model geometry each time. A preliminary study by Driver et al. (1998b) showed that varying  $\alpha$  significantly had little effect on the predicted pushover curve of their four-story specimen as represented by the basic strip model. Therefore, in an effort to simplify the analysis process further, a parametric study was conducted to investigate the influence of varying the value of  $\alpha$  on the inelastic behavior of SPSWs as predicted by the modified strip model.

## Design and Grouping of Models

The values and distribution of seismic loads for the design and analysis of the SPSWs of a “regular” office building in Vancouver, British Columbia, were determined using the equivalent static force procedure, as described in the 2005 National Building Code (NBC) of Canada [National Research Council of Canada (NRCC) 2005]. Although the Canadian code is used, the conclusions of this study are considered to be generally applicable. In order to apply the equivalent static force procedure in high earthquake zones, the NBC limits regular buildings to a maximum height of 60 m and a fundamental period of less than 2 s [National Research Council of Canada (NRCC) 2005]. Thus, by selecting a constant story height,  $h$ , of 3,800 mm, the structure can have a maximum of 15 stories. Structures of one, four, and 15 stories, with design base shears per wall of 119, 1,140, and 1,800 kN, respectively, were included in the parametric study. All parametric study models have fixed column bases. The factored gravity loads, consisting of dead, live, and snow loads, were applied to the columns at each story as point loads. Dead loads were arbitrarily but reasonably selected, while live and snow loads were taken from the NBC. The frame members were designed according to standard CSA S16-01 [Canadian Standards Association (CSA) 2001]. A minimum practical infill plate thickness,  $t_{\min}$ , of 3.0 mm was assumed based on handling and welding considerations and this value governed in all cases. The column sections were kept the same throughout the height of the one- and four-story models, whereas three-story column tiers of a single section

**Table 1.** Summary of 1-, 4-, and 15-Story Models

Group	$L/h$	$\omega_h$	% difference <sup>a</sup>
1-A	0.75	2.5	+1.6
	1.0	2.5	+0.8
	1.5	2.5	+2.0
	2.0	2.5	+1.4
1-B	1.5	1.7	+3.0
	1.5	2.5	+2.0
	1.5	3.1	-3.7
4-A	0.75	2.5	+0.4
	1.0	2.5	-0.6
	1.5	2.2	0.0
	2.0	2.0	+0.8
4-B	1.5	1.6	+4.2
	1.5	2.2	0.0
15	0.75	1.7	+0.5
	1.5	1.4	-0.2
	2.0	1.3	+1.3

<sup>a</sup>Difference in ultimate strength of 50° model as compared to 38° model.

were assumed for the design of each 15-story SPSW. Further details regarding the assumed structure, the design loads, and the design of the members can be found in Shishkin et al. (2005).

The parameters investigated in the study were the aspect ratio of the panel,  $L/h$ , and the column flexibility parameter,  $\omega_h$ , defined in CSA S16-01 [Canadian Standards Association (CSA) 2001] as

$$\omega_h = 0.7h \sqrt[4]{\frac{t}{2LI_c}} \leq 2.5 \quad (4)$$

The limit of 2.5 for this parameter is also specified indirectly in the AISC seismic provisions (AISC 2005). The aspect ratio was varied by changing  $L$  and keeping  $h$  constant (3,800 mm). The parameter  $\omega_h$  was varied by keeping  $t$  and the aspect ratio constant while using different column cross sections to reflect column flexibility at the design limit (2.5) as well as flexibilities below and above the limit. To facilitate comparisons, the models are arranged in groups, as seen in Table 1. Group 1-A contains one-story models with  $\omega_h$  at the CSA S16-01 limit and with various aspect ratios, while Group 1-B contains one-story models with an aspect ratio of 1.5 and with various values of  $\omega_h$ . Groups 4-A and 4-B are arranged in a similar manner for the four-story models. Group 15 contains the 15-story models. For each set of parameters listed in Table 1, two models with values of  $\alpha$  at the limits prescribed by CSA S16-01 (38 and 50°) are analyzed. The average values of  $\alpha$  calculated according to Eq. (1) all fell within the range of 38–50°.

For the case of the four-story models with aspect ratios of 1.5 and 2.0 in Group 4-A, the corresponding values of  $\omega_h$  used for the one-story models in Group 1-A could not be used without requiring unrealistically thick infill plates. The reason for this is that a column section selected to obtain a value of  $\omega_h$  equal to 2.5 would not have adequate strength to resist the design forces. Therefore, keeping the infill plate at  $t_{\min}=3.0$  mm, the column sections for the models with aspect ratios of 1.5 and 2.0 were designed so that  $\omega_h$  would be as close to 2.5 as the design column forces would allow. Due to this restriction, the values of  $\omega_h$  in Group 4-B do not exactly match those of Group 1-B and a reasonable design for a flexible column ( $\omega_h > 2.5$ ) could not be obtained and thus was omitted from Group 4-B. Due to the high column loads, the low-

est columns for the 15-story models could not be designed with cross sections resulting in  $\omega_h \geq 2.5$  with the selected value of  $t$ . Therefore, the column sections at the bottom of each model are sized to resist the design column forces, resulting in low values of the parameter  $\omega_h$ . Near the top of the SPSWs, the size of the column cross section is governed by the  $\omega_h \leq 2.5$  requirement rather than the column design forces.

Dastfan and Driver (2008) derived a flexibility parameter,  $\omega_L$ , analogous to  $\omega_h$ , that applies at the top of the SPSW where one end of the tension field is anchored by the top beam. They defined the associated boundary member flexibility limit as follows:

$$\omega_L = 0.7 \sqrt[4]{\left(\frac{h^4}{I_c} + \frac{L^4}{I_b}\right) \frac{t}{4L}} \leq 2.5 \quad (5)$$

where  $I_b$ =top beam strong-axis moment of inertia and the remaining parameters have been defined previously. With the exception of the case in Group 1-B that was designed with flexible boundary elements (i.e.,  $\omega_h > 2.5$ ), all of the models meet the  $\omega_L$  criterion expressed in Eq. (5).

### Analysis Results

A pushover analysis, including  $P$ - $\Delta$  effects, was conducted for each parametric study model. Gravity loads were first applied to their full value, followed by the application of the distributed seismic loads using displacement control. The base shear and first-story lateral deflection were used to quantify the model responses.

Fig. 8(a) shows the response curves for Group 1-A. For each aspect ratio, the ultimate strengths of each model ( $\alpha=38$  and 50°) are nearly identical (see Table 1). For panel aspect ratios of 0.75 and 1.0, the initial stiffnesses are approximately equal for  $\alpha=38$  and 50°. As the aspect ratio increases, the predicted initial stiffnesses of models with  $\alpha=38^\circ$  become somewhat lower than those with  $\alpha=50^\circ$ . Fig. 8(b) shows the response curves for Group 1-B. In all cases, the models with  $\alpha=38^\circ$  provide a slightly lower estimate of the initial stiffness than those with  $\alpha=50^\circ$  and the predicted ultimate strengths are again in good agreement with one another (see Table 1). The response curves for groups 4-A and 4-B display similar results to those of groups 1-A and 1-B, respectively, as shown in Figs. 8(c and d). The peak loads for  $\alpha=38$  and 50° are all within 4.2%, with five out of six cases being within 1.0% (see Table 1). For some of the 15-story models, full pushover curves could not be obtained (see Fig. 9) due to computational difficulties. However, these “partial” pushover curves define the initial stiffness, initial yield, and postyield portions of the predicted SPSW inelastic response right up to the ultimate capacity, but they do not predict the deterioration of strength. Because the ultimate load was achieved, a comparison between the 38 and 50° 15-story models is still possible. Fig. 9 shows that varying the angle of the tension strips from 38 to 50° has a negligible effect on the pushover curves. Although not discussed in this paper, SPSWs with pinned beam-to-column connections have also been found to be insensitive to the selected value of  $\alpha$  (Shishkin et al. 2005).

Varying the angle of inclination of the tension strips within the range permitted by standard CSA S16-01 [Canadian Standards Association (CSA) 2001] does not affect the predicted ultimate strength of a SPSW significantly. To eliminate the need to revise the angle repeatedly during the design development, a suitable constant value can be used. Since the 38° models tend to exhibit slightly less stiff behavior than the 50° models, a value of

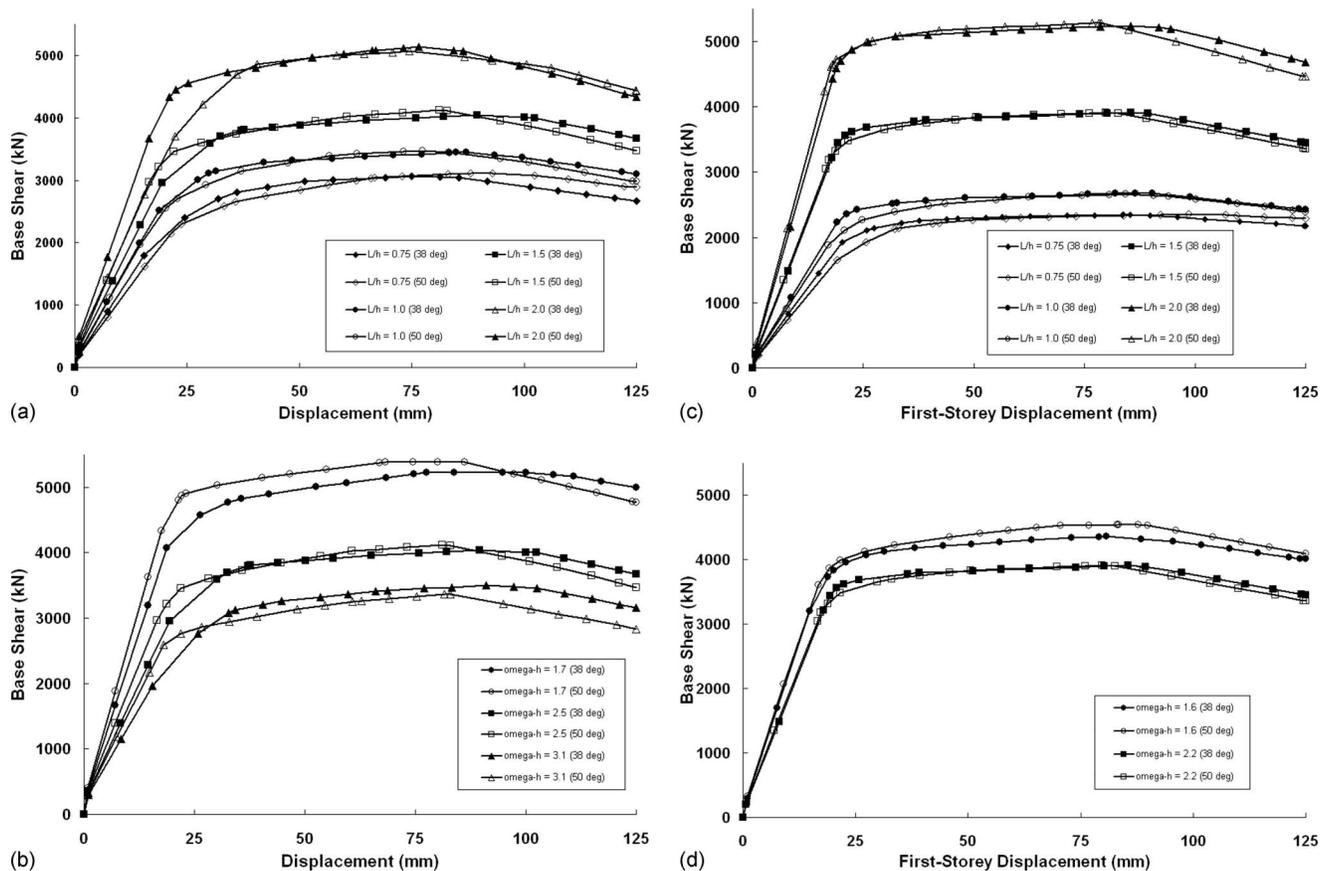


Fig. 8. Response curves for one- and four-story models: (a) Group 1-A; (b) Group 1-B; (c) Group 4-A; and (d) Group 4-B

$\alpha = 40^\circ$  is recommended throughout the design process to achieve accurate but generally slightly conservative results. The observations regarding the insensitivity of the pushover behavior to the selected value of  $\alpha$  apply to both the basic and the modified strip models.

It is of note that the design base shear resistances that are specified in standard S16-01 [Canadian Standards Association (CSA) 2001] and the AISC seismic provisions (AISC 2005) are also insensitive to the angle of inclination of the tensions field,  $\alpha$ , over a practical range of such values. Therefore, it is reasonable to replace the term “ $\sin 2\alpha$ ” with  $\sin(2 \times 40^\circ) = 0.985 \approx 1.0$ .

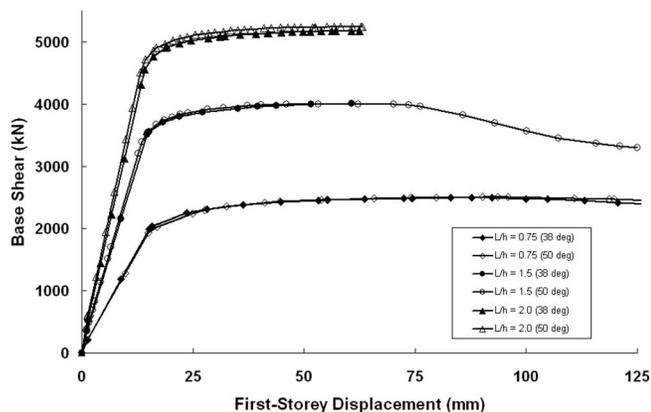


Fig. 9. Response curves for 15-story models

## Summary and Conclusions

The modified strip model, based on the original strip model by Thorburn et al. (1983), incorporates bilinear flexural hinges positioned at the edges of the frame panel zones, bilinear axial hinges in the tension strips, a simplified method of spacing the strips, a diagonal compression strut with a bilinear axial hinge to represent phenomena present in a continuous plate that are not captured by discrete strips, and a conservative deterioration behavior that simulates experimentally observed tearing of the infill plate under extreme cyclic loading. This model has been shown to be an accurate tool for predicting the inelastic pushover response of SPSWs, as well as being relatively efficient to implement. Although the modified strip model provides more accurate frame forces than the basic strip model, further improvement of the prediction of frame forces is an area where additional research on the strip model is needed. For cases with very thin infill plates, the compression strut can be omitted from the model to obtain the initial stiffness and retained to obtain the ultimate capacity.

Pushover curves obtained using the modified strip model were found to be relatively insensitive to variations in  $\alpha$ , particularly for multistory SPSWs. It is proposed that a single value of  $\alpha = 40^\circ$  can be used throughout the SPSW design process to achieve accurate, but generally slightly conservative, results. If only the ultimate capacity is required, values between  $38$  to  $50^\circ$  have been shown to give similar results. These conclusions apply whether or not the compression strut is used in the model.

## Acknowledgments

Funding for this research was provided by the Steel Structures Education Foundation and the Natural Sciences and Engineering Research Council of Canada. The writers are grateful to the research team of Professor Carlos Ventura at the University of British Columbia for providing some of the test data used to validate the modified strip model. The first writer would like to gratefully acknowledge the financial support from the Alberta Region of the Canadian Institute of Steel Construction through the G. L. Kulak scholarship.

## References

- AISC. (2005). "Seismic provisions for structural steel buildings." *ANSI/AISC 34105*, Chicago, Ill.
- Berman, J. W., and Bruneau, M. (2008). "Capacity design of vertical boundary elements in steel plate shear walls." *Eng. J.*, 45(1), 57–71.
- Building Seismic Safety Council (BSSC). (2003). "NEHRP recommended provisions for seismic regulations for new buildings and other structures." FEMA 450, Washington, D.C.
- Canadian Standards Association (CSA). (2001). "Limit states design of steel structures." *CAN/CSA S16-01*, Toronto.
- Dashtfard, M., and Driver, R. G. (2008). "Flexural stiffness limits for frame members of steel plate shear wall systems." *Proc., Annual Stability Conf.*, Structural Stability Research Council, Rolla, Mo.
- Driver, R. G., Kulak, G. L., Elwi, A. E., and Kennedy, D. J. L. (1998a). "Cyclic test of a four-storey steel plate shear wall." *J. Struct. Eng.*, 124(2), 112–120.
- Driver, R. G., Kulak, G. L., Elwi, A. E., and Kennedy, D. J. L. (1998b). "FE and simplified models of steel plate shear wall." *J. Struct. Eng.*, 124(2), 121–130.
- Driver, R. G., Kulak, G. L., Kennedy, D. J. L., and Elwi, A. E. (1997). "Seismic behaviour of steel plate shear walls." *Structural Engineering Rep. No. 215*, Dept. of Civil and Environmental Engineering, Univ. of Alberta, Edmonton, Alta.
- Elgaaly, M., Cacesse, V., and Du, C. (1993). "Postbuckling behavior of steelplate shear walls under cyclic loads." *J. Struct. Eng.*, 119(2), 588–605.
- Kulak, G. L., Kennedy, D. J. L., Driver, R. G., and Medhekar, M. (2001). "Steel plate shear walls—An overview." *Eng. J.*, 38(1), 50–62.
- Lubell, A. S., Prion, H. G. L., Ventura, C. E., and Rezai, M. (2000). "Unstiffened steel plate shear wall performance under cyclic loading." *J. Struct. Eng.*, 126(4), 453–460.
- National Research Council of Canada (NRCC). (2005). *National building code of Canada*, Ottawa.
- Qu, B., Bruneau, M., Lin, C. H., and Tsai, K. C. (2008). "Testing of full scale two-story steel plate shear wall with reduced beam section connections and composite floors." *J. Struct. Eng.*, 134(3), 364–373.
- Rezai, M. (1999). "Seismic behaviour of steel plate shear walls by shake table testing." Ph.D. dissertation, Univ. of British Columbia, Vancouver, B.C.
- Shishkin, J. J., Driver, R. G., and Grondin, G. Y. (2005). "Analysis of steel plate shear walls using the modified strip model." *Structural Engineering Rep. No. 261*, Dept. of Civil and Environmental Engineering, Univ. of Alberta, Edmonton, Alta.
- Thorburn, L. J., Kulak, G. L., and Montgomery, C. J. (1983). "Analysis of steel plate shear walls." *Structural Engineering Rep. No. 107*, Dept. of Civil Engineering, Univ. of Alberta, Edmonton, Alta.
- Timler, P. A., and Kulak, G. L. (1983). "Experimental study of steel plate shear walls." *Structural Engineering Rep. No. 114*, Dept. of Civil Engineering, Univ. of Alberta, Edmonton, Alta.
- Timler, P. A., Ventura, C. E., Prion, H., and Anjam, R. (1998). "Experimental and analytical studies of steel plate shear walls as applied to the design of tall buildings." *Struct. Des. Tall Build.*, 7, 233–249.