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Search for a market equilibrium in the oligopoly heat market

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Abstract

The paper considers a single-buyer model of the oligopoly heat market. A mathematical model of the heat market is developed. It allows us to take into consideration economic and technical aspects of heating system's operation. The problem of the determination of the Cournot-Nash equilibrium in the heat market at a linear demand function and nonlinear costs of heat sources is investigated and solved. To search for the Cournot-Nash market equilibrium we suggest an iterative algorithm based on step-by-step optimization of heat volumes produced by heat sources. The developed model was applied to perform practical studies on a heating system with two heat sources.

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Keywords: heating system, heat market, single-buyer model, Cournot-Nash equilibrium.

1. Introduction

District heating in Russia is the main way of providing consumers with heat. The emergence of a great number of owners in this sector of the economy, related to the process of energy sector liberalization, resulted in the new economic relations among heat producers, suppliers and consumers, and the creation of heat market. There are many heat supply organizations in the heat market, which belong to different owners, for example, municipal heat energy enterprises, heat supply enterprises of territorial generating companies, energy companies of industrial enterprises and others. Each of the owners has their interests and seeks to obtain the greatest benefit from their activity. Therefore, to solve the problem of control of the expansion of heating systems in the existing market conditions it is necessary to apply new

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approaches to their modeling, calculation and optimization that should be aimed at establishing the incentives for heat supply organizations to meet the given heat demand.

2. An organizational model of a competitive heat market

The heat market model is normally designed on the basis of existing heating systems. As in many other fields of economic activity one of the possible forms of market arrangement in heat supply can be a gradual establishment of competitive relations. This model corresponds to the situation which emerges in the cities of the country where heat is supplied from several heat sources (HS) that belong to different owners. In terms of microeconomics such a form of heat market arrangement relates to oligopoly [1]. Currently quite many studies are devoted to the analysis and modeling of oligopoly markets [2-4]. In literature, however, there are virtually no examples of the oligopoly model application in solving the problems of heating system operation and expansion in a market environment, although they are successfully applied in the electric power industry [5-7 et al.], in the oil and gas industry [8, 9].

The model of competition among HSs in the heat market can be implemented when the following requirements are met: there are two and more HSs belonging to different owners; integration of heat networks with different types of ownership into a unified heat network company; HSs have a surplus heat output.

The relations among market participants are built according to a certain pattern and are as follows. Heat network company is a regulated natural monopoly in the heat market. Based on the forecasts of heat demand it supplies heat to consumers at a tariff determined as a sum of heat production and heat transportation from HSs to consumers tariffs. The purchase price of heat is not regulated because the heat network company can select a supplier in the market. Each HS produces the amount of heat which could maximize their profit provided that the HSs in total should cover the demand for heat set by consumers and meet the desire of consumers to pay for this demand, whereas heat network company seeks to minimize network costs, considering physical and technical constraints and optimal heat carrier flows in heat networks, and determines the tariff of heat transportation at the level of average total costs of the heat networks. The organizational structure of control of such market is as a rule called the single-buyer model. In Russia and in the world this model is widely applied in modeling of the electricity market [5, 10]. Currently among the most popular approaches to modeling and forecasting of possible situations in solving the problems of system expansion in the conditions of oligopoly markets, we can accentuate the Cournot model.

3. A mathematical model of heat sources

The heat market, with a great number of HSs and heat networks, is simulated by hydraulic circuit, which presents the design diagram of the actual heat supply system composed of m nodes and n branches. The hydraulic circuit presents the aggregate of ordered sets: nodes $J=\{j:j=1,\dots,m\}$, composed of subsets J_{HS} (heat sources), J_{CON} (consumers); and branches $I=\{i:i=1,\dots,n\}$ representing the given pairwise links between nodes. Simulation uses the time interval, which is determined by some initial time τ_0 and its final (computational) value T.

When modeling the behavior of heat sources, the single-buyer model assumes that at each time they operate their heat production volumes and make their decisions simultaneously based on the heat production price formed in the market, constraints on the heat production volumes and heat production costs. The heat production costs of the heat sources have the form of quadratic dependence [11]:

$$Z_{j\tau}(Q_{j\tau})=\alpha_j\cdot Q_{j\tau}^2+\beta\cdot Q_{j\tau}+\gamma_j, \quad (1)$$

where $Z_{j\tau}(Q_{j\tau})$ – heat production costs of the heat sources, EUR; $Q_{j\tau}$ – heat production volume, GJ/h; α_j, β_j, γ_j– coefficients of approximation of heat source cost characteristic.

In the market conditions the behavior of the j-th heat source is determined by the profit gained from the heat production at each time instant τ, in turn the production volume of the heat source maximizing profit at $Q_{j\tau}>0$, depends on the expected heat volume to be produced by the other producers. In the case the expected production volumes coincide with the real ones, this state can be called equilibrium. This equilibrium is often called the Cournot- Nash equilibrium [12].

Thus, the optimal values of heat production volumes are determined when solving the problem of obtaining the total maximum profit over the entire time interval [τ_0, T], for each j-th HS, considering the constraints on its heat production volumes:

$$\sum_{\tau=\tau_0}^{T} P_{j\tau}(Q_{j\tau}) = \sum_{\tau=\tau_0}^{T} w_\tau^{HS} \cdot Q_{j\tau} - \sum_{\tau=\tau_0}^{T} Z_{j\tau}(Q_{j\tau}) \to \max, \quad (2)$$

$$Q_{j\_\min} \le Q_{j\tau} \le Q_{j\_\max}, \quad (3)$$

where $P_{j\tau}(Q_{j\tau})$ –profit of the j-th HS, EUR; w_τ^{HS} – heat production price, EUR/GJ; $Q_{j\_\min}$ and $Q_{j\_\max}$ – minimum and maximum levels of the j-th HS productive capacity, respectively, GJ/h.

4. A mathematical model of heat network

To mathematically describe the problem of search for the minimum costs of heat network company we use the following extremal problem in a continuous statement [13]:

$$Z_\tau^{NET} = \sum_{i=1}^{n} Z_{i\tau}^{NET}(x_{i\tau}) = F_1 + F_2 \cdot \sum_{i=1}^{n} x_{i\tau}^2 \cdot \left|x_{i\tau}\right| \cdot s_i \to \min , \quad (4)$$

$$\mathbf{A}\mathbf{x}_\tau = \mathbf{G}_\tau, \quad (5)$$

where Z_τ^{NET} – heat network costs, EUR; F_1 – semi-fixed costs, EUR; F_2 – coefficient at semi-variable costs of heat network; **A** – incidence matrix of linearly independent branches and nodes; $x_{i\tau}$ – heat carrier flow rate in the i–th section of the heat network at time instant τ, t/h; s_i – coefficient of hydraulic resistance of the i–th branche, mh^2/t^2; $\mathbf{x}_\tau$– vector of flow rates in the network sections; $\mathbf{G}_\tau$ – vector of mass flow rates at nodes at time instant τ, t/h.

It is known from [13] that the minimum total costs of heat network are achieved at optimal flow distribution determined by solving the following system of equations:

$$\mathbf{A}\mathbf{x}_\tau = \mathbf{G}_\tau, \quad (6)$$

$$\overline{\mathbf{A}}^{T} \overline{\mathbf{P}}_\tau = \mathbf{h}_\tau - \mathbf{H}_\tau, \quad (7)$$

$$\mathbf{h}_\tau = \mathrm{S}\mathbf{X}_\tau \, \mathbf{x}_\tau \quad (8)$$

where $\overline{\mathbf{A}}^{T}$ – transposed incidence matrix of heat network; $\overline{\mathbf{P}}_\tau$ – vector of nodal pressures, mwc; $\mathbf{h}_\tau$ – differential pressure vector, mwc; $\mathbf{H}_\tau$– vector of effective heads, mwc; **S** and $\mathbf{X}_\tau$ – diagonal matrices made up of the values of coefficients of hydraulic resistances of branches s (mh^2/t^2) and absolute values of flow rates in them $|x_\tau|$ (t/h).

After the optimal heat network costs are determined, the price of heat transportation from heat source to consumers is calculated by the following equation:

$$w_{\tau}^{\mathrm{NET}} = Z_{\tau}^{\mathrm{NET}} / \sum_{j \in J_{\mathrm{HS}}} Q_{j\tau}. \tag{9}$$

where w_{τ}^{NET} – heat production price, EUR/GJ.

5. A mathematical model of heat consumers

Denote by J_{CON} a set of consumers in the heat market, which can be represented in the form of two subsets: household and industrial consumers. The demand of household consumers for heat is determined by the heat load duration curve:

$$Q_{j\tau}^{\mathrm{HC}}=[1-(1-r)\cdot(\tau/\tau_{\mathrm{HP}})^{(g-r)/(1-g)}]\cdot Q_j^{\mathrm{HL}}+Q_j^{\mathrm{HW}}, \tag{10}$$

where $Q_{j\tau}^{\mathrm{HC}}$ – demand of household consumers for heat, GJ/h; $Q_{j\tau}^{\mathrm{HL}}$ – design heating load, GJ/h; Q_j^{HW}– design hot water supply load, GJ/h; r and g – coefficients of heat load curve non-uniformity; τ_{HP}– duration of heating period, h.

The demand of industrial consumers for heat is modeled by the demand characteristic, which is obtained from real calculations and can be represented by linear dependence [14]:

$$Q_{j\tau}^{\mathrm{IC}}=\xi_j - \vartheta_j w_{\tau}, \tag{11}$$

where $Q_{j\tau}^{\mathrm{IC}}$ – demand of the j-th industrial consumer, GJ/h; ξ_j,>0, $\vartheta_j>0$ – coefficients obtained from the approximation of the factual data on the heat volume purchased by an industrial consumer, depending on price; w_{τ} – purchase price, EUR/GJ.

6. A mathematical model of heat market

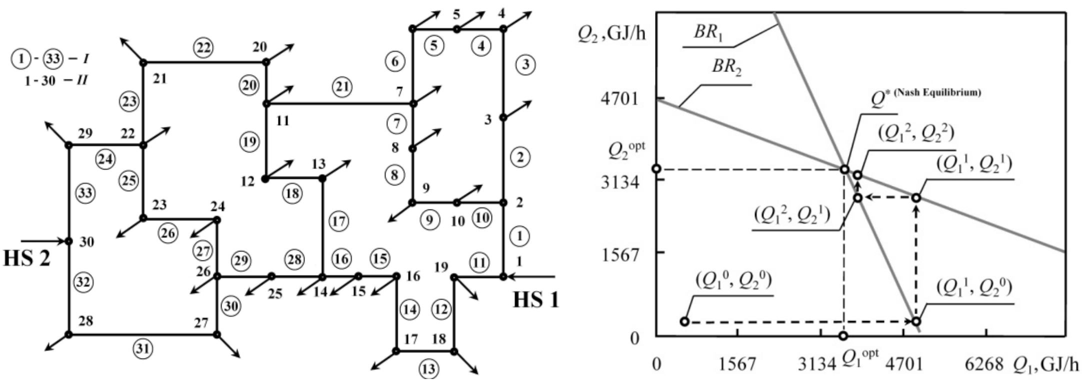
One of the indices determining a tradeoff among the interests of heat market participants is an equilibrium price of heat production w_{τ}^{HS} at time instant τ. Heat generation price for the HS is:

$$w_{\tau}^{\mathrm{HS}} = \left(\sum_{j \in J_{\mathrm{CON}}} \vartheta_j\right)^{-1} \cdot \Big(\sum_{j \in J_{\mathrm{CON}}} Q_{j\tau}^{\mathrm{HC}} + \sum_{j \in J_{\mathrm{CON}}} \xi_j - \sum_{j \in J_{\mathrm{HS}}} Q_{j\tau}\Big) - \sum_{i=1}^{n} Z_{i\tau}^{\mathrm{NET}}(x_{i\tau}) \Big/ \sum_{j \in J_{\mathrm{HS}}} Q_{j\tau}. \tag{12}$$

As a result, the mathematical model for a search of equilibrium between demand and supply in the oligopoly heat market is reduced to a search for the maximum profit of HSs (2) subject to (3), (4), (6)-(12).

7. Practical implementation of the developed model

An aggregate calculated scheme of the heat supply system is presented in Fig. 1a. Nodes with numbers 1 and 30 correspond to the nodes of HSs, nodes 2-28, 31-33 correspond to the household consumers and node 29 – to the industrial consumer.



a) (b)

Fig 1. (a) calculated scheme of heat supply to consumers (*I* – numbers of heat network sections; *II* – numbers of heat network nodes); (b) equilibrium search procedure.

Table 1 presents technical and economic indices obtained from the calculations for HSs, heat network and consumers, average annual prices of heat production and average annual prices of heat transportation.

Table 1

Calculated indices		
Heat sources	HS 1	HS 2
Heat production volume, million GJ	28.7	20.11
Heat production costs, million EUR	95.2	61.6
Heat production price cost, EUR/GJ	3.31	3.06
Average annual heat price, EUR/GJ	4.80	
Profit, million EUR	42.6	2.63
Heat networks		
Total network costs, million EUR	46.8	
Average annual heat transportation cost, EUR/GJ	0.95	
Consumers		
Heat consumption by household consumers, million GJ	24.68	
Heat consumption by industrial consumers, million GJ	24.13	
Final heat price, EUR/GJ	5.75	

The price for consumers is determined by a sum of average annual prices of heat production and heat transportation by heat networks. Table 1 shows that the average annual price for HS 1 and HS 2 exceeds their price costs by 31 percent and 36 percent, respectively. The profit gained from the heat production makes up 31 percent and 36 percent of the total revenue for the first and second HSs, respectively. The share of covering the total annual load of consumer for the first and second HSs made up 58 percent and 42 percent, respectively.

The process of searching for equilibrium between demand and supply, using the Cournot model can be presented graphically (Fig.1b). To illustrate the search for the Cournot equilibrium we consider the time interval τ=1. X-axis in Fig.1a corresponds to the productive capacity of HS 1 and Y-axis – to the productive capacity of HS 2. Lines BR_1 and BR_2 (response curves of the first heat source and second heat source, respectively) represent a set of values of the heat production volumes that correspond to the maximum profit which could be gained by one of the heat sources at a set heat production volume by the other heat source.

The equilibrium search procedure starts at any point (Q_1^0, Q_2^0), which for example can be represented by a zero point. Here each heat source successively optimizes their volume of heat production considering the possibility of obtaining the maximum profit at a specified volume of heat production by the other heat source. The heat production volumes obtained in the iterative process (according to the direction of arrow in Fig. 1b) correspond to the Cournot-Nash equilibrium which is located at point Q^* on the crossing of curves of HS responses.

Conclusions

We formulated one of the possible approaches based on the Cournot model to solve the problem of the equilibrium search in the oligopoly heat market

A distinctive feature of the approach lies in the fact that along with the problems traditionally solved for HSs, it makes it possible to consider physical-technical characteristics of heat network and economic factors, related to the costs of heat production and transportation. This approach allows to determine the best HS loading levels to meet a set demand for heat and to get the maximum profit, and also the conditions of minimum heat network costs to be met in the considered time period. Its graphical interpretation reflecting the search for the optimal heat production volumes is presented.

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