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An integrated multi-objective model for allocating the limited sources in a multiple multi-stage lean supply chain

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a multiple objective programming model has been presented as a supply chain with the general purpose of adopting an integrated approach such that with making optimal decisions about the optimum allocation of limited sources in the supply chain, selection of the suppliers, production, distribution and supply programming yields, the least cost and the most income and finally maximizes the profit of the chain. The proposed model attempts to regard the integration condition very well with consideration of factors such as the conditions for suppliers, producers, and distributors as well as free relations of producers with each other in the direction of providing products through a process or even the products from each other. The presented model, for more adaptation to reality, is flexible against the dynamism of the demand, and it considers the effect of economic factors on decisions such as inflation. A numerical example is then given to show the applicability of the proposed model. This model covers the operational dimension of the chain very well with appropriate programming for production and controlling the inventories.

1. Introduction

Supply-chain management, applying competitive strategies, integrates suppliers, producers, and distributors with the purpose of promoting responsiveness and flexibility within the organization vis-a-vis the customers. For this purpose, and in order to achieve the competitive benefits for more share of the market, the economic institutions shall manage and supervise the external sources and related elements within the organization as well as the organization itself and its internal source's consideration (Cohen et al., 2006; Garg et al., 2004).

The key problems in a supply chain are managing and control over the supply and demand program, selecting the suppliers, provision of raw materials, production and product programming, product storage, inventory control, distribution, delivery and customer services. Strategic programming of the supply chain results in the customers receiving reliable services, with high quality, fast and with the least-cost (Martin, 2006).

Mathematical programming models are one of the most applied methods for optimizing supply chains. The main purpose of these models is to make optimal decisions for allocating the limited sources in a long term considering various factors (Reiner, 2005; Yu et al., 2005). In general, the presented models for optimizing the activities of the supply chain have been designed in three types. The first type are the models whose objective is to study each part of the chain separately. For example, we can refer to the presented models for selecting the suppliers in a supply chain, the presented models for reducing delivery time in distribution centers, etc. The second type are the models only for reducing the costs, which create relations between both producers and distributors as well as distributors and customers. The third type are those qualitative models for the supply chain presented for either each part of the chain or a combination of the two parts in the chain (Reiner, 2005; Scott et al., 2003). In studying the models of the first and second type, we cannot declare deterministically that the decisions that are made are the most optimum feasible solutions, in that the co-relations between the nods and the sensitivity of affecting a part on another of the models are not a matter of consideration. In the study of the third type, due to non-quantitative decisions, we cannot also refer to them exactly for implementation, and they are only executive for the quality of the performed affairs. In most of these models, the model has become far from the world of reality regarding various hypotheses. However, we have attempted to adopt this model with the real conditions of one supply chain to the extent that it is possible (Ereng et al., 1999).

Considering the facts and the models just presented, the importance of an integrated model of the supply chain with all mentioned properties is clear (Wang et al., 2004).

Many researchers have made tremendous contributions to supply-chain management methods, but the majority of the published articles on supply chain models have focused on the single-period problem with a single objective function, rather than the entire supply chain network in several periods considering multi-objective models (Jang et al., 2002).

For instance, the production/distribution model (PILOT) of Cohen et al. (2006), is a cost function, mixed integer mathematical program with a nonlinear objective function. It is probably one of the earliest successful efforts to model supply chain problems. The authors presented a global supply strategy for manufacturing. This approach seeks to determine the number and locations of plants and distribution centers,

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material flows, plant production volumes, and the allocation of customers to distribution centers. Although this approach has generated a practical application model, it is limited to a single manufacturing stage and considers only one objective function based on supply chain costs. To overcome the limitations of considering only a single manufacturing stage, a number of researchers have considered the integrated production–distribution problem. Cohen and Lee develop a non-linear, stochastic, multi-echelon inventory model to determine the optimal stocking policy for a spare part stocking system, based on achieving an optimal trade-off between holding costs and transportation costs, subject to response time constraints (Cohen and Lee, 1987). This service system has unique characteristics, such as low demand rates, a complex echelon structure, and the existence of emergency shipments to meet unforeseen demand. The solution to this complex model is found using a branch and bound procedure. A mixed-integer programming developed by Robinson et al. considers a cost functional model for a two-echelon incapacitated distribution location problem. The authors provide sensitivity, cost–service tradeoffs, and what-if analyses to clarify all major costs and service trade-offs (Robinson et al., 1993). A fixed-charge network programming technique is used to determine the best shipment routings and shipment size through the distribution system. Vidal and Goetschackx in 1997 presented a critical extensive literature review of strategic production–distribution models. In (Vidal and Goetschackx, 1997), the authors categorize the literature into four groups: previous reviews, optimization models, additional issues for modeling, and case studies. A particular emphasis of their review is on mixed integer programming models. Thus, they identify the main characteristics of the mixed integer programming models, including the terms considered in the objective function, the constraints, and the specific characteristics of the solution methods and computation experiences. However, while there have been a considerable number of papers that consider the supply-chain problems, fewer studies have considered integration of the supply chain. For example, Tzafestas and Kapsiotis propose a mathematical program, with the objective of minimizing the cost of a sub-supplier/supplier/manufacturer in a supply chain (Tzafestas and Kapsiotis, 1994). Three different operational scenarios of optimizing the supply chain are assumed and examined. In the first one, manufacturing optimizes its operational costs without considering the suppliers. In the second scenario, overall cooperation exists between the three levels of the supply chain. In the last scenario, each level of the supply is optimized separately with a partial cooperation among the SC. A numerical example is provided to illustrate the above scenarios, and the simulation technique is used to verify the results. In this example, the differences in the total costs are very small, and the computational times are almost identical to the three scenarios.

To consider the integration of the supply chain, a number of researchers have studied the integrated supply chain problem.

Park (2007) presented a method for integrated production and distribution planning. He investigated the effectiveness of the integration through a computational study with the objective of maximizing the total net profit. It is considered one of the best production–distribution models in the literature, because it is a relatively realistic model considering multiple capacity constraints within a multi-period planning horizon. Moreover, the model involves some fixed costs at different operation stages. Having proposed an MIP (Mix Integer Programming) model, Park then presented alternative solutions and compared them in a computational study. In addition, a sensitivity analysis is carried out on capacities and fixed costs. However, this study assumed that the plants had an unlimited storage capacity, and that the firm could change the fleet size freely without extra cost, but in real operations, these assumptions are not often realized. Additionally, the model did not take changeover cost and production batch size constraints into account at the production stage. Moreover, the solution procedures have some limitations; for example, the decoupled models do not always give feasible solutions, since they ignore the interactions of different operation stages. Although the problem considered a supply-chain network configuration, including multi-plants, multi-retailers, multi-items, and multi-period environment, the key disadvantage is that no raw material procurement activities were considered (Park, 2007). Li et al. (2009) proposed a capacity allocation problem based on a more complex supply chain than has been typically considered in previous quantitative modeling studies. Hsieh and Liu examined a serial supply chain that consists of one supplier and one manufacturer. They investigated the supplier's and the manufacturer's quality investment and inspection strategies in four noncooperative games with different degrees of information revealed (Hsieh and Liu, 2010). Jula and Leachman (2011), propose a mixed integer non-linear programming model for optimizing the supply chains of importers of waterborne containerized goods from Asia to the USA and allocation programming. In their research, they introduce a heuristic algorithm to quickly solve the mathematical model to near optimality. Another problem in this area was configuration of supply chain and some researchers were focused on that. For example, Costantino et al. (2012), propose a model for configuration problem of Manufacturing Supply Chains (MSC) with reference to the supply planning issue. Their results show that the design method provides managers with key answers to issues related to the supply chain strategic configuration and agility, e.g., choosing the right location for distributors and retailers for enhanced MSC flexibility and performance. Another problem in the supply chain management is controlling the costs. Some researchers developed their model regarding to cost. As an example of this field, Pettersson and Segerstedt (2013), express a model to introduce a tool to measure the cost in the supply chain. They applied their model in 30 companies and show how does it works in the real world and they show the applicability of their model by this (Pettersson and Segerstedt, 2013). The remanufacturing process in supply chain management is another problem which some researchers like Giovanni and Zaccour in 2014 are concerned. They consider a two-period closed-loop supply chain (CLSC) game where a remanufacturer appropriates of the returns' residual value and decides whether to exclusively manage the end-of-use product collection or to outsource it to either a retailer or a third-service provider (Giovanni and Zaccour, 2014).

This study analyzes an integrated supply chain operation from raw material purchasing to final product distribution. The aim is to optimize the allocation of capacities among different facilities and product items. In this study, a mixed integer programming model with dynamic characteristics is presented first, and then alternative solution procedures are introduced. The solution procedures include the development of a decomposition heuristic and an integrated heuristic algorithm. A computational study compares the solution procedures and uses sensitivity analysis to show that the heuristics work well. Thus, through adequate modeling, the supply chain problem becomes more realistic sized.

In this paper, an integrated model for the supply chain is presented and an effort is made to define the objective functions so that the decision making in a supply chain is directed towards "lean". Finally, the results of the presented model make optimal decisions against the suppliers, producers, distributors and contractors, considering the relations between the producers such that any producer can supply for other producers.

Moreover, our model is sensitive to some factors such as inflation and the demand variable as well, and it reacts against change in each one of them. Lastly, we have made an effort to present a production programming and inventory control with consideration of its related costs. In general, the proposed model attempts to reduce costs, to increase incomes, and finally to increase the efficiency of the supply chain. In the following section, we propose our model. In Section 3, we illustrate the proposed model using a numerical example, and Section 4 concludes.

2. Proposed model

We assume that there are several suppliers who present their services for several producers who can supply for other producers or send their products for distributors. Some contractors have also been regarded among them. These contractors have an ability for supplying products in process and products and have a direct relation with the producers (for the sale of products in the process) and distributors (for the sale of products). At the end, some distributors have been considered as well, who can be regarded as representatives for the plant sale and a mediate between the producers and the customers (Fig. 1).

The main purpose of the model is to determine the optimum allocation for the sources in the general supply chain with consideration of the limited capacity of the suppliers, producers, distributors, and transportation and lean conditions of the chain. In this model production capacity is determined considering the available time for each producer in each period, the warehouse capacity for each distributor, and the constraints in dispatch and transportation. The capacity of each producer is independent of other producers. The capacity of the distributors is calculated equal to the maximum access space and with regard to the existing demands in each time period. The supplier's capacity is also determined by considering the maximum raw material that each supplier can provide and the available capacity of the raw materials in each time period. In addition to the above hypothesizes, some other factors have been considered such as changes in the production rate and the production type, production capacity for the products through the process as well as the minimum economic production capacity. Besides the abovementioned, the following preliminary is considered:

- The final product of each phase is not stored in that place, and it is sent to the destination section.
- The demand rates in distribution centers are independent of each other.
- Each product can be produced in one or several production centers, and each production center can produce one or several products.
- The distribution centers supply their shortage from other distribution centers.
- Duration of the time period is determined with regard to the specifications of the problem and by the analyzer.
- The production centers keep some precautionary reservation.
- The cost of waste materials in each place is allocated to the same place.

2.1. Model formulation

2.1.1. Nomenclatures and parameters

Nomenclatures for the proposed model are summarized as follows:

Raw material: The raw material that is acquired in the supply centers.
Products: the product that is produced in the production centers and is sent to the distribution centers.
Products in the process: the products which are produced in the production centers and are sent to other production centers as raw material.

i	Index of the centers for supplying raw materials ($i = 1,2,\ldots,I$)
j,p	Index of the production centers for products in the process and the products ($j = 1,2,\ldots,J$, $p = 1,2,\ldots,J$)
k	Index of the centers for distribution of the products ($k = 1,2,\ldots,K$)
q	Index of the contractors ($q = 1,2,\ldots,Q$)
n	Index of the raw material ($n = 1,2,\ldots,N$)
m	Index of the products during a process ($m = 1,2,\ldots,M$)
l	Index of the products ($l = 1,2,\ldots,L$)
t	Index of the time periods ($t = 1,2,\ldots,T$)

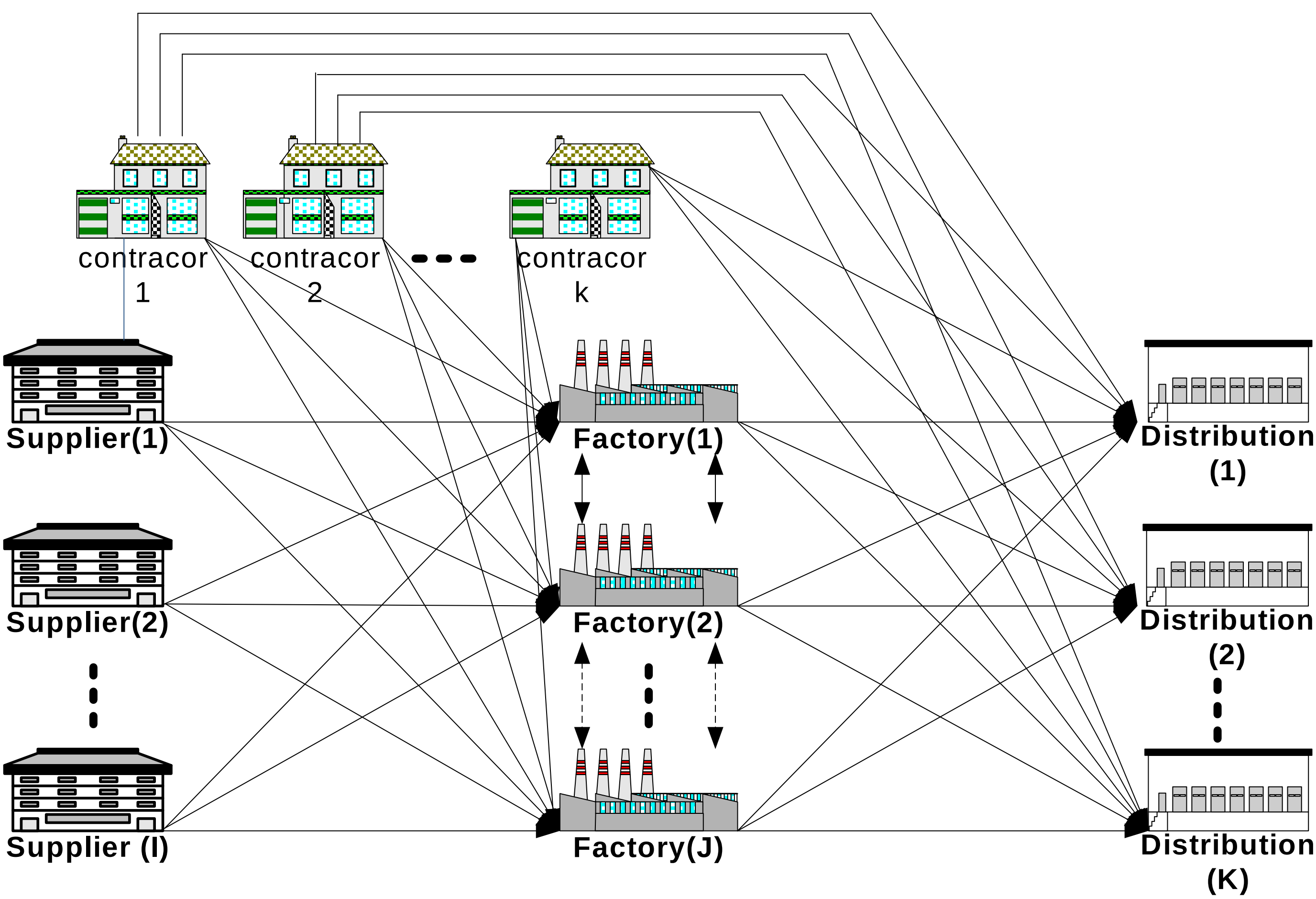


Fig. 1. Supply chain.

r A number that indicates the number of the periods under study ($r \leq T$).
d_{lkt} The rate of demand for the lth type of the product in the kth distribution center during the time period of tth.
pr_{lt} The price of each unit of the lth type product during the time period of tth.
$ctfd_{ljkt}$ The transportation cost for each unit of the product the lth type of the production center jth to the distribution center kth during the time period tth.
$ctsf_{nijt}$ The transportation cost for each raw material of the nth type from supply center ith to the production center jth during the time period tth.
$ctff_{mipt}$ The transportation cost for each product in a process of the mth type of the production center jth to the production center pth during the time period tth.
$ctcf_{lqkt}$ The transportation cost for each unit of the product of the lth type purchased from the contractor qth by the distribution center kth during the time period tth.
$Ctff_{mqjt}$ The transportation cost for each product in a process of the mth type purchased from the contractor qth by the production center jth during the time period tth.
cp_{ljt} The cost of production of each unit of the product of the lth type by the production center jth during the time period tth.
cb_{lqt} The cost of purchase of each unit of the product of the lth type from the contractor qth during the time period tth.
$\overline{c}w_{mqt}$ The cost of purchase of each unit of products in a process of the mth type from the contractor qth during the time period tth.
cdc_{lkt} The cost of distribution of each unit of the product of the lth type in distribution center kth during the time period tth.
ca_{ljt} The cost for setup of the production center jth for producing the product of the lth type during the time period tth.
$\overline{c}a_{mjt}$ The cost for setup of the production center jth for producing the products in a process of the mth type during the time period tth.
$cwhs_{lkt}$ The cost of storage of each unit of a product of the lth type by the distribution center kth during the time period tth.
$cwhf_{mpt}$ The cost of storage of each unit of product in a process of the mth type in the pth type production center during the time period tth.
$cwhrs_{njt}$ The cost of storage of each unit of raw material of the nth type in production center jth during the time period tth.
csh_{lkt} The cost for shortage of each unit of the product of the lth type by the distribution center kth during the time period tth.
csu_{nit} The cost for obtaining each unit of raw material of the nth type in supply center ith during the time period tth.
cw_{mjt} The cost of producing each unit of product in a process of the mth type in the production center jth during the time period tth.
$csvrs_{nit}$ The cost of the wastes in supply center ith for raw material of the nth type during the time period tth.
$csvwf_{mpt}$ The cost of the wastes in production center pth for products in the process of the mth type during the time period tth.
$csvpf_{ljt}$ The cost of the wastes in the production center jth for producing the product of the lth type during the time period tth.
$csvpd_{lkt}$ The cost of the wastes in the distribution center kth for the product of the lth type during the time period tth.
$csvcd_{lqt}$ The cost of the wastes from the contractor qth for the product of the lth type during the time period tth.
$csvcf_{mqt}$ The cost of the wastes from the contractor qth for the products in process of the mth type during the time period tth.
rm_{nl} The rate of raw material type nth that is consumed in producing the product of the lth type.
rmw_{nm} The rate of raw material type nth that is consumed for producing the product of the mth type.
nw_{ml} The rate of products in process the mth type that is consumed for producing the product of lth type.
$svrs_{ni}$ The cost for the wastes in obtaining raw material type nth in supply center ith.
$svpf_{lj}$ The cost for the wastes in producing the products of the lth type in production center jth
$svwf_{mj}$ The percent of the waste in producing the products in process of the mth type in production center jth
$svpd_{lk}$ The percent of the waste from distribution of the products of the lth type in the distribution center kth.
$svlpc_{lq}$ The percent of the waste from the contractor qth for the product of the lth type.
$svmpc_{mq}$ The percent of the waste from the contractor qth for the products in process of the mth type.
tpa_{jt} The access production time of production center jth during the time period tth.
tsp_{lj} The setup time of production center jth for producing the product of the lth type.
tpp_{lj} The required time for producing the product of the lth type in production center jth.
tsw_{mj} The setup time of production center jth for producing the products in the process of the mth type.
twp_{mj} The required time for producing the products in the process of the mth type in production center jth.
$vwhd_{kt}$ The capacity of warehouse for distribution center kth during the time period tth.
$vwhf_{jt}$ The capacity of warehouse for production center jth during the time period tth.
vnp_l The necessary volume for warehousing each unit of the product of the lth type.
vnw_m The necessary volume for warehousing each unit of products in the process of the mth type.
vnr_n Necessary capacity for warehousing each unit of raw material of the nth type.
M^x A large number that is greater than the total amount of the productions.
$\overline{M}^x$ A large number that is greater than the total amount of products in the process.
M^q A large number that is greater than the total amount of the demands for the products.
$\overline{M}^q$ A large number that is greater than the total amount of the products.
b_{ljt} Equal to one in the case that the jth plant can produce the lth product, otherwise it is equal to zero.
c_{mjt} Equal to one in the case that the jth plant can produce the products in the process of the mth type, otherwise it is equal to zero.
e_{lqt} Equal to one in the case that the contractor qth can supply lth product, otherwise it is equal to zero.
f_{mqt} Equal to one in the case that the contractor qth can supply the products in the process of the mth type, otherwise it is equal to zero.

2.1.2. Decision variables

x_{ljt} The number of producing the lth type product in the jth production center during the time period tth.
$\overline{x}_{mjt}$ The number of producing the products in the process of mth type in the jth production center during the time period tth.
$nlpc_{lqkt}$ The number of products of the lth type from the contractor qth for the kth distribution center during the time period tth.
$nmpc_{mqjt}$ The number of producing the products in the process of mth type from the contractor qth for the jth production center during the time period tth.
nsf_{nijt} The number of delivering the raw material type nth from the supply center ith to the jth production center during the time period tth.

nfd_{ljkt} The number of delivering product type lth from production center jth to the kth distribution center during the time period tth.

nff_{mjpt} The number of delivering the products in the process of mth type from the jth production center to the pth production center during the time period tth.

ndc_{lkt} The number of distribution of product type lth in distribution center kth during the time period tth.

$nwhs_{lkt}$ The remaining amount of product type lth in distribution center kth at the end of the time period tth.

$nwhf_{mpt}$ The remaining amount of products in process type mth in distributing center pth at the end of the time period tth.

nsh_{lkt} The shortage amount of product type lth in distribution center kth at the end of the time period tth.

$nwhrs_{njt}$ The remaining amount of raw material type nth in production center jth at the end of the time period tth.

$nwhcpf_{mjt}$ The warehouse of products in process type mth entered into the production center jth at the end of the time period tth.

$$X_{ljt} = \begin{cases} 1 & \text{if} \quad x_{ljt} > 0 \\ 0 & o.w \end{cases}$$

$$\overline{X}_{mjt} = \begin{cases} 1 & \text{if} \quad \overline{x}_{mjt} > 0 \\ 0 & o.w \end{cases}$$

2.1.3. Objective functions

$$\max Z_1 = \sum_{t=1}^{r}\left(\sum_{l=1}^{L}\left[pr_{lt}\sum_{k=1}^{k}(d_{lkt}-nsh_{lkt})\right]\right) \tag{1}$$

$$\begin{aligned}\min Z_2 = \sum_{t=1}^{r}\Bigg(&\left[\sum_{l=1}^{L}\sum_{j=1}^{J}cp_{ljt}\times x_{ljt}\right]+\left[\sum_{m=1}^{M}\sum_{j=1}^{J}cw_{mjt}\times\overline{x}_{mjt}\right]\\&+\left[\sum_{l=1}^{L}\sum_{k=1}^{K}cdc_{lkt}\times ndc_{lkt}\right]+\left[\sum_{n=1}^{N}\sum_{i=1}^{I}\left(csu_{nit}\times\sum_{j=1}^{J}nsf_{nijt}\right)\right]\\&+\left[\sum_{l=1}^{L}\sum_{q=1}^{Q}\left(cb_{lqt}\times\sum_{k=1}^{K}nlpc_{lqkt}\right)\right]\\&+\left[\sum_{m=1}^{M}\sum_{q=1}^{Q}\left(\overline{cw}_{mqt}\times\sum_{j=1}^{J}nmpc_{mqjt}\right)\right]\Bigg)\end{aligned} \tag{2}$$

$$\begin{aligned}\min Z_3 = \sum_{t=1}^{r}\Bigg(&\left[\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{k=1}^{K}ctfd_{ljkt}\times nfd_{ljkt}\right]+\left[\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J}ctsf_{nijt}\times nsf_{nijt}\right]\\&+\left[\sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{\substack{p=1\\p\neq j}}^{J}ctff_{mjpt}\times nff_{mjpt}\right]\\&+\left[\sum_{m=1}^{M}\sum_{q=1}^{Q}\sum_{j=1}^{J}ctcff_{mqjt}\times nmpc_{mqjt}\right]\\&+\left[\sum_{m=1}^{M}\sum_{q=1}^{Q}\sum_{j=1}^{J}ctcf_{lqkt}\times nlpc_{lqkt}\right]\Bigg)\end{aligned} \tag{3}$$

$$\min Z_4 = \sum_{t=1}^{r}\left(\left[\sum_{l=1}^{L}\sum_{j=1}^{J}X_{ljt}\times ca_{ljt}\right]+\left[\sum_{m=1}^{M}\sum_{j=1}^{J}\overline{X}_{mjt}\times\overline{c}a_{mjt}\right]\right) \tag{4}$$

$$\begin{aligned}\min Z_5 = \sum_{t=1}^{r}\Bigg(&\left[\sum_{l=1}^{L}\sum_{k=1}^{K}cwhs_{lkt}\times nwhs_{lkt}\right]+\left[\sum_{m=1}^{M}\sum_{p=1}^{J}cwhf_{mpt}\times nwhf_{mpt}\right]\\&+\left[\sum_{n=1}^{N}\sum_{j=1}^{J}cwhrs_{njt}\times nwhrs_{njt}\right]\Bigg)\end{aligned} \tag{5}$$

$$\min Z_6 = \sum_{t=1}^{r}\left(\left[\sum_{l=1}^{L}\sum_{k=1}^{K}csh_{lkt}\times nsh_{lkt}\right]\right) \tag{6}$$

$$\begin{aligned}\min Z_7 = \sum_{t=1}^{r}\Bigg(&\left[\sum_{n=1}^{N}\left(\sum_{i=1}^{I}\left(svrs_{ni}\times\sum_{j=1}^{J}nsf_{nijt}\right)\times csvrs_{nit}\right)\right]\\&+\left[\sum_{m=1}^{M}\left(\sum_{j=1}^{J}\left(svwf_{mj}\times\sum_{p=1}^{P}nff_{mjpt}\right)\times csvwf_{mjt}\right)\right]\\&+\left[\sum_{l=1}^{L}\left(\sum_{j=1}^{J}\left(svpf_{lj}\times\sum_{k=1}^{K}nfd_{ljkt}\right)\times csvpf_{ljt}\right)\right]\\&+\left[\sum_{l=1}^{L}\left(\sum_{k=1}^{K}(svpd_{lk}\times ndc_{lkt})\times csvpd_{lkt}\right)\right]\\&+\left[\sum_{l=1}^{L}\left(\sum_{q=1}^{Q}\left(svlpc_{lq}\times\sum_{k=1}^{K}nlpc_{lqkt}\right)\times csvcd_{lqt}\right)\right]\\&+\left[\sum_{m=1}^{M}\left(\sum_{q=1}^{Q}\left(svmpc_{mq}\times\sum_{j=1}^{J}nmpc_{mqjt}\right)\times csvcf_{mqt}\right)\right]\Bigg)\end{aligned} \tag{7}$$

$$\min Z_{8l} = \sum_{t=1}^{r}\left(\sum_{k=1}^{K}nsh_{lkt}\right) \quad \forall l = 1,2,\dots,L \tag{8}$$

$$\min Z_{9n} = \sum_{t=1}^{r}\left(\sum_{j=1}^{J}nwhrs_{njt}\right) \quad \forall n = 1,2,\dots,N \tag{9}$$

$$\min Z_{10m} = \sum_{t=1}^{r}\left(\sum_{p=1}^{P}nwhf_{mpt}\right) \quad \forall m = 1,2,\dots,M \tag{10}$$

$$\min Z_{11l} = \sum_{t=1}^{r}\left(\sum_{k=1}^{K}nwhs_{lkt}\right) \quad \forall l = 1,2,\dots,L \tag{11}$$

$$\min Z_{12n} = \sum_{t=1}^{r}\left(\sum_{i=1}^{I}\left(svrs_{ni}\times\sum_{j=1}^{J}nsf_{nijt}\right)\right)\forall n = 1,2,\dots,N \tag{12}$$

$$\min Z_{13m} = \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svwf_{mj}\times\sum_{p=1}^{P}nff_{mjpt}\right)\right)\forall m = 1,2,\dots,M \tag{13}$$

$$\min Z_{14l} = \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svpf_{lj}\times\sum_{k=1}^{K}nfd_{ljkt}\right)\right)\forall l = 1,2,\dots,L \tag{14}$$

$$\min Z_{15l} = \sum_{t=1}^{r}\left(\sum_{k=1}^{K}(svpd_{lk}\times ndc_{lkt})\right)\forall l = 1,2,\dots,L \tag{15}$$

$$\min Z_{16m} = \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svmpc_{mq}\times\sum_{j=1}^{J}nmpc_{mqjt}\right)\right)\forall m = 1,2,\dots,M \tag{16}$$

$$\min Z_{17l} = \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svlpc_{lq}\times\sum_{k=1}^{K}nlpc_{lqkt}\right)\right) \ \forall l = 1,2,\dots,L. \tag{17}$$

Eq. (1) expresses the objective function for optimization of the income. Eq. (2) indicates the objective function for minimization of the costs related to acquiring raw materials from supply centers, production of products in process and products in the production centers, distribution of the products in distribution centers, and purchase of products in process and products from the contractors. Eq. (3) indicates the objective function for minimization related to transportation costs in the whole supply chain. Eq. (4) indicates the objective function for minimization related to the starting up of production centers for producing products in process and products. Eq. (5) indicates the objective function for minimization related to raw materials, products in process and product storage costs. Eq. (6) indicates the objective function for minimization of costs related to the shortage of products in distribution centers. Eq. (7) indicates the objective function for minimization of the costs related to the created wastes in the supply chain. Eq. (8) indicates the objective function for minimization of the shortage number of each type of the products in distribution centers during all of the time periods. Eq. (9) indicates the objective function for minimization of the stored number from each type of raw materials in the production centers, during all the time periods. Eq. (10) indicates the objective function for minimization of the stored number from each type of products during the process in production centers, during all the time periods. Eq. (11) indicates the objective function for minimization of the stored number from each type of product in distribution centers, during all the time periods. Eq. (12) indicates the objective function for minimization of the sent waste rate from each type of raw material from supply centers to producing centers during all the time periods. Eq. (13) indicates the objective function for minimization of the sent waste rate from each type of product through the process from production centers to other production centers during all the time periods. The Eq. (14) indicates the objective function for minimization of the sent waste rate for each type of product from production centers to distribution centers during all the time periods. Eq. (15) indicates the objective function for minimization of the sent waste rate for each type of product from distribution centers to all the customers during all the time periods. Eq. (16) indicates the objective function for minimization of the purchased and sent waste rate for each type of product in the process from the contractors to all the production centers during all the time periods. Eq. (17) indicates the objective function for minimization of the purchased and sent waste rate for each type of purchasing product from the contractors to all the distribution centers during all the time periods.

2.1.4. Constraints

$$\sum_{l=1}^{l}\left(tpp_{lj} \times x_{ljt} + tsp_{lj} \times X_{ljt}\right) + \sum_{m=1}^{m}\left(twp_{mj} \times \bar{x}_{mjt} + tsw_{mj} \times \bar{X}_{mjt}\right) \le tpa_{jt} \quad \forall j = 1,2,\ldots,J \,\&\, \forall t = 1,2,\ldots,T \tag{18}$$

$$\left(\sum_{l=1}^{l} nwhs_{lk(t-1)} \times vnp_l + \sum_{l=1}^{L}\sum_{j=1}^{J} nfd_{ljkt} \times vnp_l \times \left(1-svpf_{lj}\right) + \sum_{l=1}^{L}\sum_{q=1}^{Q} nlpc_{lqkt} \times vnp_l \times \left(1-svlpc_{lq}\right)\right) \le vwhd_{kt} \quad \forall k = 1,2,\ldots,K \,\&\, \forall t = 2,\ldots,T \tag{19}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} nfd_{ljkt} \times vnp_l \times \left(1-svpf_{lj}\right) + \sum_{l=1}^{L}\sum_{q=1}^{Q} nlpc_{lqkt} \times vnp_l \times \left(1-svlpc_{lq}\right)) \le vwhd_{kt} \quad \forall k = 1,2,\ldots,K \,\&\, \forall t = 1 \tag{20}$$

$$\left(\sum_{n=1}^{N}\sum_{i=1}^{I} nsf_{nijt} \times vnr_n \times \left(1-svrs_{ni}\right) + \sum_{m=1}^{M}\sum_{p=1}^{P} nff_{mpjt} \times vnw_m \times \left(1-svwf_{mp}\right) + \sum_{q=1}^{Q}\sum_{m=1}^{M} nmpc_{mqjt} \times vnw_m \times \left(1-svmpc_{mq}\right)\right) \le vwhf_{jt} \quad \forall j = 1,2,\ldots,J \,\&\, \forall t = 1 \tag{21}$$

$$\sum_{n=1}^{N}\sum_{i=1}^{I} nsf_{nijt} \times vnr_n \times \left(1-svrs_{ni}\right) + \sum_{m=1}^{M}\sum_{p=1}^{P} nff_{mpjt} \times vnw_m \times \left(1-svwf_{mp}\right) + \sum_{q=1}^{Q}\sum_{m=1}^{M} nmpc_{mqjt} \times vnw_m \times \left(1-svmpc_{mq}\right) \le vwhf_{jt} \quad \forall j = 1,2,\ldots,J \,\&\, \forall t = 1 \tag{22}$$

$$x_{ljt} \le M^x \times X_{ljt} \times b_{ljt} \quad \forall l = 1,2,\ldots,L \,\&\, \forall j = 1,2,\ldots,J \,\&\, \forall t = 1,2,\ldots,T \tag{23}$$

$$\bar{x}_{mjt} \le \overline{M^x} \times \bar{X}_{mjt} \times c_{mjt} \quad \forall m = 1,2,\ldots,M \,\&\, \forall j = 1,2,\ldots,J \,\&\, \forall t = 1,2,\ldots,T \tag{24}$$

$$\sum_{j=1}^{J} nfd_{ljkt} \times \left(1-svpf_{lj}\right) + \sum_{q=1}^{Q} nlpc_{lqkt} \times \left(1-svlpc_{lq}\right) + nwhs_{lk(t-1)} - \left(\frac{d_{lkt}}{\left(1-svpd_{lk}\right)}\right) = nwhs_{lkt} - nsh_{lkt} \quad \forall l = 1,\ldots,L \,\&\, \forall k = 1,\ldots,K \,\&\, \forall t = 2,\ldots,T \tag{25}$$

$$ndc_{lkt} \times \left(1-svpd_{lk}\right) = d_{lkt} - nsh_{lkt} \quad \forall l = 1,\ldots,L \,\&\, \forall k = 1,\ldots,K \,\&\, \forall t = 1,\ldots,T \tag{26}$$

$$\sum_{j=1}^{J} nfd_{ljkt} \times \left(1-svpf_{lj}\right) + \sum_{q=1}^{Q} nlpc_{lqkt} \times \left(1-svlpc_{lq}\right) + nwhs_{lk(t-1)} \ge nwhs_{lkt} \quad \forall l = 1,\ldots,L \,\&\, \forall k = 1,\ldots,K \,\&\, \forall t = 2,\ldots,T \tag{27}$$

$$\left[\left(\sum_{j=1}^{J} nfd_{ljkt} \times \left(1-svpf_{lj}\right) + \sum_{q=1}^{Q} nlpc_{lqkt} \times \left(1-svlpc_{lq}\right)\right)\right] \ge nwhs_{lkt} \quad \forall l = 1,\ldots,L \,\&\, \forall k = 1,\ldots,K \,\&\, \forall t = 1 \tag{28}$$

$$\sum_{k=1}^{K} nfd_{ljkt} = x_{ljt} \quad \forall l = 1,2,\ldots,L \,\&\, \forall j = 1,2,\ldots,J \,\&\, \forall t = 1,2,\ldots,T \tag{29}$$

$$nwhrs_{njt} = nwhrs_{nj(t-1)} - \sum_{l=1}^{L} rm_{nl} \times x_{ljt} - \sum_{m=1}^{M} rmw_{nm}\bar{x}_{mjt} + \sum_{i=1}^{I} nsf_{nijt}\left(1-svrs_{ni}\right) \quad \forall n = 1,2,\ldots,N \,\&\, \forall j = 1,2,\ldots,J \,\&\, \forall t = 2,\ldots,T \tag{30}$$

$$nwhrs_{njt} = (-1) \times \left(\sum_{l=1}^{L} rm_{nl} \times x_{ljt} + \sum_{m=1}^{M} rmw_{nm}\bar{x}_{mjt}\right) + \sum_{i=1}^{I} nsf_{nijt} \times \left(1-svrs_{ni}\right)) \quad \forall n = 1,2,\ldots,N \,\&\, \forall j = 1,2,\ldots,J \,\&\, \forall t = 1 \tag{31}$$

$$nwhcpf_{mjt} = \sum_{q=1}^{Q} nmpc_{mqjt} \times \left(1 - svmpc_{mq}\right) + \sum_{\substack{p=1 \\ p \neq j}}^{P} nff_{mpjt} \times \left(1 - svwf_{mp}\right)$$
$$\forall m = 1, 2, \ldots, M \& \forall j = 1, 2, \ldots, J \& \forall t = 1, \ldots, T \tag{32}$$

$$\left[\left(nwhf_{mj(t-1)} + nwhcpf_{mj(t-1)} + \overline{x}_{mjt}\right) = \left(nwhf_{mjt} + \sum_{l=1}^{L} nw_{ml} \times x_{ljt} + \sum_{\substack{p=1 \\ p \neq j}}^{P} nff_{mjpt}\right)\right]$$
$$\forall m = 1, 2, \ldots, M \& \forall j = 1, 2, \ldots, J \& \forall t = 2, \ldots, T \tag{33}$$

$$\overline{x}_{mjt} = nwhf_{mjt} + \sum_{l=1}^{L} nw_{ml} \times x_{ljt} + \sum_{\substack{p=1 \\ p \neq j}}^{P} nff_{mjpt}$$
$$\forall m = 1, 2, \ldots, M \& \forall j = 1, 2, \ldots, J \& \forall t = 1 \tag{34}$$

$$\sum_{k=1}^{K} \left(nlpc_{lqkt}\right) \leq M^{q} \times e_{lqt}$$
$$\forall l = 1, 2, \ldots, L \& \forall q = 1, 2, \ldots, Q \& \forall t = 1, 2, \ldots, T \tag{35}$$

$$\sum_{j=1}^{J} \left(nmpc_{mqjt}\right) \leq \overline{M^{q}} \times f_{mqt}$$
$$\forall m = 1, 2, \ldots, M \& \forall q = 1, 2, \ldots, Q \& \forall t = 1, 2, \ldots, T. \tag{36}$$

Eq. (18) indicates the restriction related to access time in each plant during each time period. Eqs. (19) and (20) indicate the constraints related to the capacity of warehouse volume in each distribution center during each time period. Eqs. (21) and (22) indicate the constraints related to the capacity of warehouse volume in each production center during each time period. Eq. (23) indicates one logical restriction related to the production or non-production of one product in a production center during a time period. Eq. (24) indicates one logical restriction related to the production or non-production of one product in the process in a production center during a time period. Eq. (25) indicates a logical constraint for calculating the shortage and remaining amount for each product in each distribution center during each time period. Eq. (26) indicates one logical restriction for calculating the sent amounts of each product to the customer in each distribution center at the end of each time period. Eqs. (27) and (28) indicate controlling constraints for optimal performance of the constraints in the Eq. (25). Eq. (29) indicates one logical restriction for determining the production rate and number of sent amounts from each product and from each center to the distribution centers during each time period. Eqs. (30) and (31) indicate logical constraints for calculating the remaining amount from each raw material in each production center at the end of each time period. Eq. (32) indicates one virtual warehouse for each type of product in the process purchased from the contractors and received from other production centers and their maintenance for the next period during all the time periods. Eqs. (33) and (34) indicate logical constraints for calculating the remaining amount from each type of products in the process at each production center at the end of each time period. Eq. (35) indicates the restriction related to the possibility or non possibility for supply of each product by each contractor during each time period for the distribution centers. Eq. (36) indicates the restriction related to the possibility or non possibility for supply of each product in the process by each contractor during each time period for the production centers.

The main aim of the presented model can be summarized as follows:

- Calculation of the optimum allocation of the limited sources in the integrated supply chain.
- Directing the supply chain toward the lean considering Eqs. (3) to (17).
- Obtaining the rate of optimum production from each product in every production center and in each time period.
- Obtaining the rate of optimum production from each product in the process in every production center and in each time period.
- Obtaining the rate of optimum purchase from each product in the process in every contractor center for every production center and in each time period.
- Obtaining the rate of optimum purchase from each product in every contractor center for every distribution center and in each time period.
- Obtaining the rate of optimum delivery from each raw material from every supply center to every production center and in each time period.

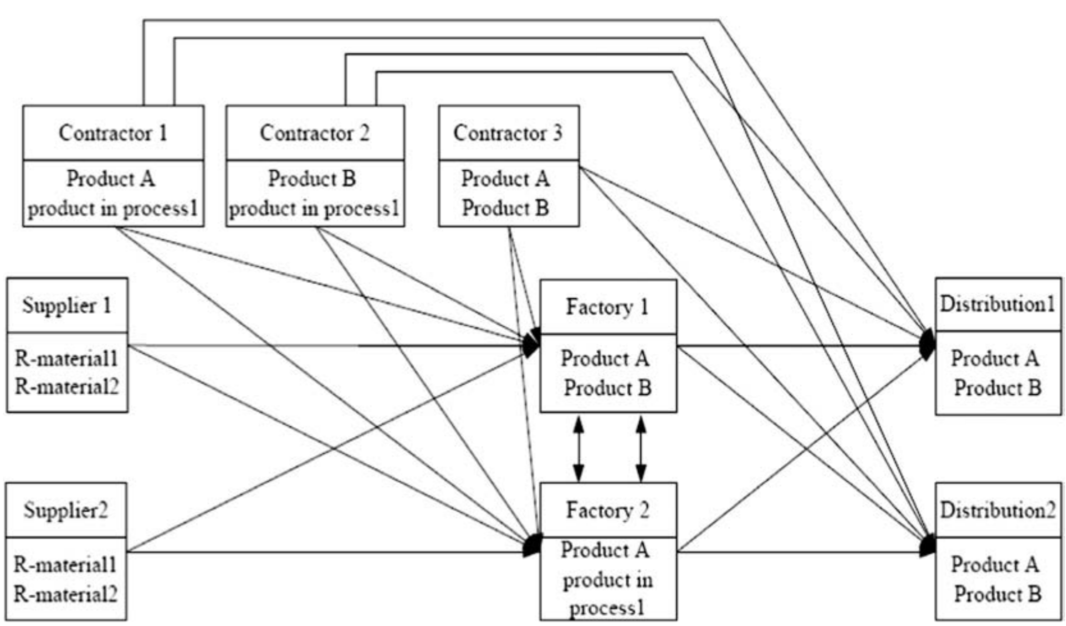


Fig. 2. Numerical example.

- Obtaining the rate of optimum delivery from each product from every production center to every distribution center and in each time period.
- Obtaining the rate of optimum delivery from each work in process material from every production center to every other production center and in each time period.
- Obtaining the rate of optimum distribution and delivery from each product from every distribution center to every customer and in each time period.
- Obtaining the rate of optimum storage for each product in every distribution center and in each time period.
- Obtaining the rate of optimum storage for each product in the process in every production center and at the end of each time period.
- Obtaining the rate of optimum storage for each raw material at every production center and at the end of each time period.
- Obtaining the rate of optimum storage for each product in the process entered into every production center at the end of each time period.
- Programming optimum production
- Controlling the inventory to reduce the costs
- Selecting and evaluating the suppliers, producers and contractors.
- Considering the factor of inflation and increasing the price in the programming.

Finally, very high efficiency for making the optimum decision related to one multi-stage supply chain.

3. Numerical example

In this section, we illustrate our model using a numerical example. To solve the mentioned example, we employ CPLEX Solver.

The related example has been planned in six consecutive time periods (six months) and an annual inflation of 20% in the market prices and costs has been considered. For further study of the model, one sudden fall in sale price was placed in the 4th period. The method of determining entrance parameters is fully random and follows the uniform distribution such that we multiply the selected random number between 0 and 10 by a particular number so that the appropriate parameter becomes a production plant. Available production time for the working centers has been considered in accordance with the hours of work standards in Iran (210 h per month).

The cases that we shall refer to for solving this model are as follows:

- The supply chain consists of two suppliers, two producers, three contractors and two distributors (Fig. 2).
- The considered supply chain consists of two final products 1 & 2, one product in process and two types of raw materials 1 & 2 (Fig. 2).
- Product A is made by combining some raw materials 1 & 2 and also some products in the process the amount of which we obtain in a random form and from uniform distribution in accordance with the described procedure.
- Product B is made of combining some raw materials 1 & 2 and also some products in the process the amount of which we obtain in a random form and from uniform distribution in accordance with the described procedure.
- The product in the process is obtained from composition of a coefficient from raw material 1 with raw material 2.
- Both suppliers have the ability to supplying both raw materials (Fig. 2).
- Producer 1 is able to produce the product A and the product B, and the producer 2 is able to produce the product A and the product in process 1 (Fig. 2).
- Contractor 1 is able to produce the product A and the product in process 1, the contractor 2 is able to produce the product B and the product in process 1, and the contractor 3 is able to produce the product A and the product B (Fig. 2).

The distribution centers 1 and 2 have the ability to supply both products (Fig. 2).

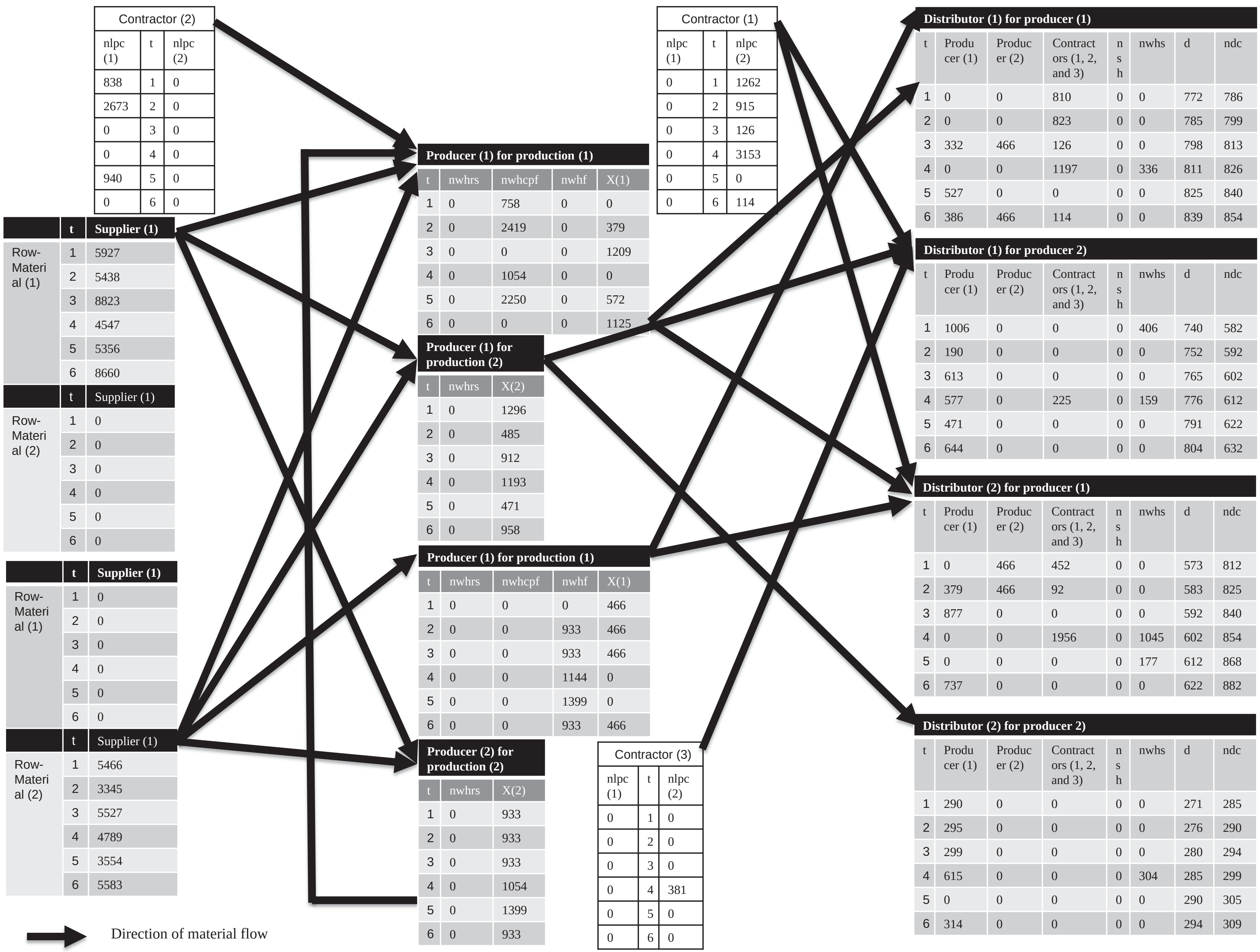
3.1. Solution method

To solve the model, we employ GAMS software (CPLEX Solver). Our approach for solving the proposed model is as follows:

- We have introduced Eq. No.(1) with obj_1.
- We have introduced the minimum of summation of Eqs. (2) to (7) with obj_2.
- Summation of Eqs. (8) to (17) with obj_3 (that obj_3 has been put in the model for obtaining more satisfaction of the customer). Meanwhile when the number of the products and semi-products and raw materials increase, we can substitute the Eq. (37) with obj_3:

$$
\begin{aligned}
obj3 = \min\Bigg(& \max\left[\sum_{t=1}^{r}\left(\sum_{k=1}^{K} nsh_{1kt}\right), \sum_{t=1}^{r}\left(\sum_{k=1}^{K} nsh_{2kt}\right), \dots, \sum_{t=1}^{r}\left(\sum_{k=1}^{K} nsh_{Lkt}\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{j=1}^{J} nwhrs_{1jt}\right) \sum_{t=1}^{r}\left(\sum_{j=1}^{J} nwhrs_{2jt}\right), \dots, \sum_{t=1}^{r}\left(\sum_{j=1}^{J} nwhrs_{Njt}\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{p=1}^{P} nwhf_{1pt}\right), \sum_{t=1}^{r}\left(\sum_{p=1}^{P} nwhf_{2pt}\right), \dots \sum_{t=1}^{r}\left(\sum_{p=1}^{P} nwhf_{Mpt}\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{k=1}^{K} nwhs_{1kt}\right) \sum_{t=1}^{r}\left(\sum_{k=1}^{K} nwhs_{2kt}\right), \dots, \sum_{t=1}^{r}\left(\sum_{k=1}^{K} nwhs_{Lkt}\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{i=1}^{I}\left(svrs_{1i} \times \sum_{j=1}^{J} nsf_{1ijt}\right)\right) \sum_{t=1}^{r}\left(\sum_{i=1}^{I}\left(svrs_{2i} \times \sum_{j=1}^{J} nsf_{2ijt}\right)\right), \right. \\
& \left. \dots, \sum_{t=1}^{r}\left(\sum_{i=1}^{I}\left(svrs_{Ni} \times \sum_{j=1}^{J} nsf_{Nijt}\right)\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svwf_{1j} \times \sum_{p=1}^{P} nff_{1jpt}\right)\right) \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svwf_{2j} \times \sum_{p=1}^{P} nff_{2jpt}\right)\right), \right. \\
& \left. \dots, \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svwf_{Mj} \times \sum_{p=1}^{P} nff_{Mjpt}\right)\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svpf_{1j} \times \sum_{k=1}^{K} nfd_{1jkt}\right)\right) \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svpf_{2j} \times \sum_{k=1}^{K} nfd_{2jkt}\right)\right), \right. \\
& \left. \dots, \sum_{t=1}^{r}\left(\sum_{j=1}^{J}\left(svpf_{Lj} \times \sum_{k=1}^{K} nfd_{Ljkt}\right)\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{k=1}^{K}(svpd_{1k} \times ndc_{1kt})\right) \sum_{t=1}^{r}\left(\sum_{k=1}^{K}(svpd_{2k} \times ndc_{2kt})\right), \right. \\
& \left. \dots, \sum_{t=1}^{r}\left(\sum_{k=1}^{K}(svpd_{Lk} \times ndc_{Lkt})\right)\right] \\
& \max\left[\sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svmpc_{1q} \times \sum_{j=1}^{J} nmpc_{1qjt}\right)\right)\right. \\
& \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svmpc_{2q} \times \sum_{j=1}^{J} nmpc_{2qjt}\right)\right), \dots, \\
& \left. \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svmpc_{Mq} \times \sum_{j=1}^{J} nmpc_{Mqjt}\right)\right)\right] \\
& + \max\left[\sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svlpc_{1q} \times \sum_{k=1}^{K} nlpc_{1qkt}\right)\right)\right. \\
& \left. \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svlpc_{2q} \times \sum_{k=1}^{K} nlpc_{2qkt}\right)\right), \dots, \sum_{t=1}^{r}\left(\sum_{q=1}^{Q}\left(svlpc_{Lq} \times \sum_{k=1}^{K} nlpc_{Lqkt}\right)\right)\right]\Bigg).
\end{aligned} \tag{37}
$$

For adding the functions together, the scale of each function shall be removed. For this reason we perform this action by normalizing the



Contractor (2)

nlpc (1)	t	nlpc (2)
838	1	0
2673	2	0
0	3	0
0	4	0
940	5	0
0	6	0

Contractor (1)

nlpc (1)	t	nlpc (2)
0	1	1262
0	2	915
0	3	126
0	4	3153
0	5	0
0	6	114

Contractor (3)

nlpc (1)	t	nlpc (2)
0	1	0
0	2	0
0	3	0
0	4	381
0	5	0
0	6	0

	t	Supplier (1)
Row-Material (1)	1	5927
	2	5438
	3	8823
	4	4547
	5	5356
	6	8660
	t	Supplier (1)
Row-Material (2)	1	0
	2	0
	3	0
	4	0
	5	0
	6	0

	t	Supplier (1)
Row-Material (1)	1	0
	2	0
	3	0
	4	0
	5	0
	6	0
	t	Supplier (1)
Row-Material (2)	1	5466
	2	3345
	3	5527
	4	4789
	5	3554
	6	5583

Producer (1) for production (1)

t	nwhrs	nwhcpf	nwhf	X(1)
1	0	758	0	0
2	0	2419	0	379
3	0	0	0	1209
4	0	1054	0	0
5	0	2250	0	572
6	0	0	0	1125

Producer (1) for production (2)

t	nwhrs	X(2)
1	0	1296
2	0	485
3	0	912
4	0	1193
5	0	471
6	0	958

Producer (1) for production (1)

t	nwhrs	nwhcpf	nwhf	X(1)
1	0	0	0	466
2	0	0	933	466
3	0	0	933	466
4	0	0	1144	0
5	0	0	1399	0
6	0	0	933	466

Producer (2) for production (2)

t	nwhrs	X(2)
1	0	933
2	0	933
3	0	933
4	0	1054
5	0	1399
6	0	933

Distributor (1) for producer (1)

t	Producer (1)	Producer (2)	Contractors (1, 2, and 3)	nsh	nwhs	d	ndc
1	0	0	810	0	0	772	786
2	0	0	823	0	0	785	799
3	332	466	126	0	0	798	813
4	0	0	1197	0	336	811	826
5	527	0	0	0	0	825	840
6	386	466	114	0	0	839	854

Distributor (1) for producer 2)

t	Producer (1)	Producer (2)	Contractors (1, 2, and 3)	nsh	nwhs	d	ndc
1	1006	0	0	0	406	740	582
2	190	0	0	0	0	752	592
3	613	0	0	0	0	765	602
4	577	0	225	0	159	776	612
5	471	0	0	0	0	791	622
6	644	0	0	0	0	804	632

Distributor (2) for producer (1)

t	Producer (1)	Producer (2)	Contractors (1, 2, and 3)	nsh	nwhs	d	ndc
1	0	466	452	0	0	573	812
2	379	466	92	0	0	583	825
3	877	0	0	0	0	592	840
4	0	0	1956	0	1045	602	854
5	0	0	0	0	177	612	868
6	737	0	0	0	0	622	882

Distributor (2) for producer 2)

t	Producer (1)	Producer (2)	Contractors (1, 2, and 3)	nsh	nwhs	d	ndc
1	290	0	0	0	0	271	285
2	295	0	0	0	0	276	290
3	299	0	0	0	0	280	294
4	615	0	0	0	304	285	299
5	0	0	0	0	0	290	305
6	314	0	0	0	0	294	309

Fig. 3. Supply chain analysis.

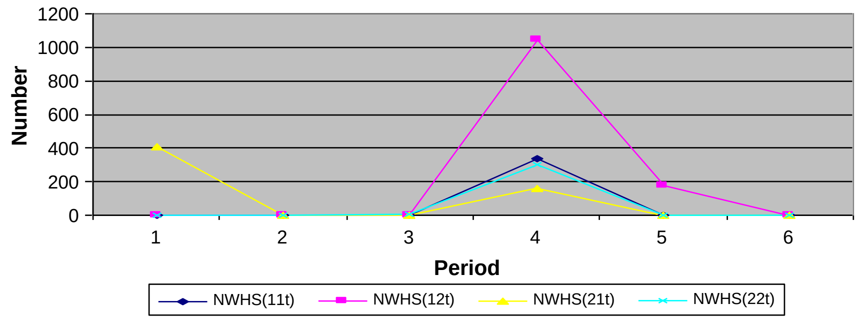


Fig. 4. The amount of the stored product in the warehouse of each distribution center at the end of each time period.

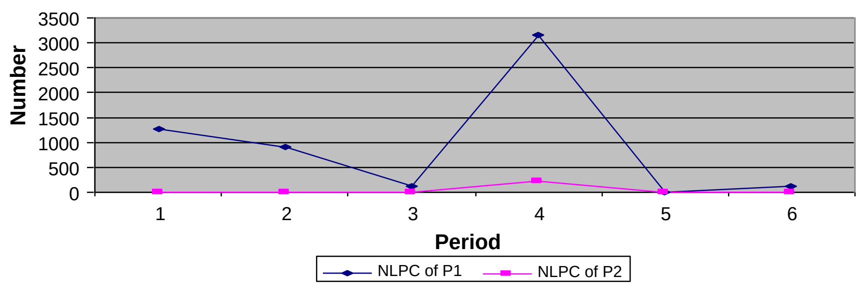


Fig. 5. The amount of the purchased products from the contractors at the end of each time period.

functions. Operations for normalizing the functions have been performed in the following form:

I. At first we solve the model with the objective function of obj_1 (income function), and we call the resulting optimum response F_1^* e II.
II. Then we solve the model with the objective function of obj_2 (the cost functions are added at first, and finally we call their minimum obj_2), and we call the resulting optimum response F_2^* and pass to the stage III.
III. Then we solve the model with obj_3 (first number functions are added together (or we use Eq. (37)) and finally we called their minimum mod as obj_3) as an objective function.

We solved the model with the following normalized objective function (Eq. (38)):

$$\max Z = \left(w_1 \times \frac{obj_1}{F_1^*} + w_2 \times \frac{F_2^*}{obj_2} + w_3 \times \frac{F_3^*}{obj_3}\right). \tag{38}$$

In Eq. (38), w_1, w_2, and w_3 are selected by the manager using the model practically, and $w_1 + w_2 + w_3 = 1$. In this section we supposed:

$w_1 = 0.4, w_2 = 0.4$ and $w_3 = 0.2$

3.2. The results obtained from GAMS (CPLEX Solver)

$F_1^* = 4058107,\ \ F_2^* = 2032125,\ \ F_3^* = 839.5476$

Z^*: 0.7965629.

3.3. Analysis of the results

Fig. 3 shows the schematic form of analyzing the results:

The following points obtained results in the previous subsection.

The inputs of the example have been selected in a completely random form, but for indicating and studying several affecting factors, the 20% inflation has been considered for the costs in the periods and also a 30% reduction in demand has been considered for the 4th period.

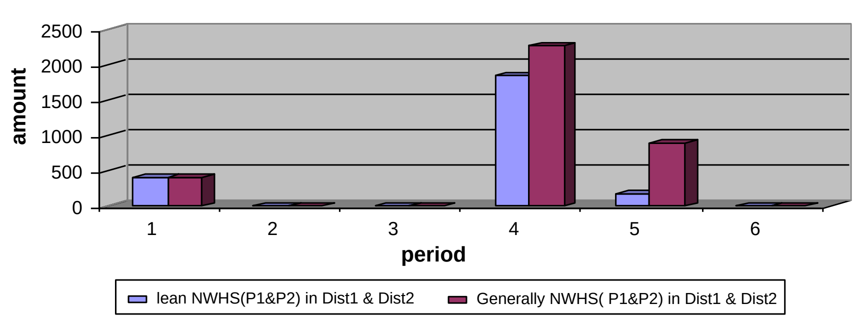


Fig. 6. Comparison of the stored amount of product in the lean state (with obj_3) and in the ordinary state (without obj_3).

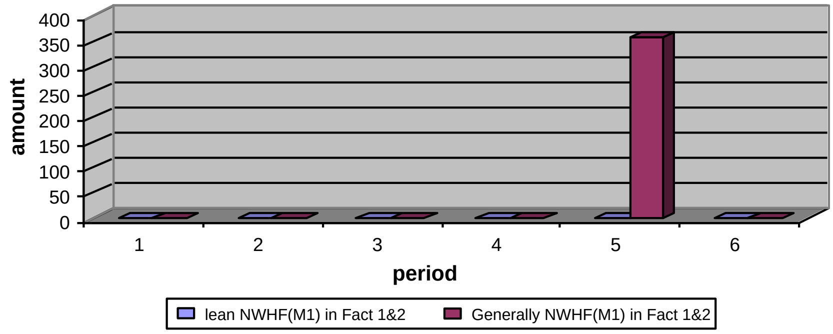


Fig. 7. Comparison of the work in process amount of product stored in the lean state (withobj_3) and in the ordinary state (without obj_3).

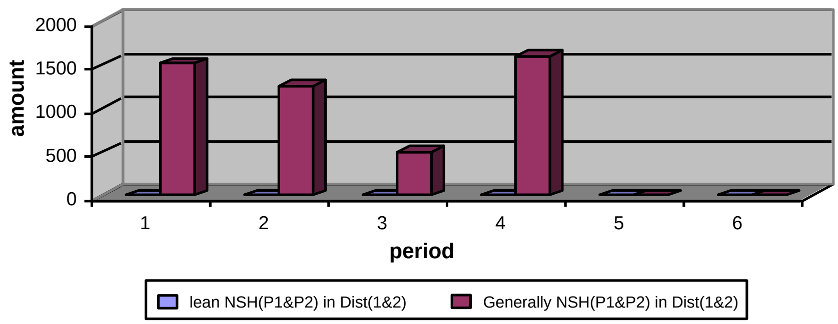


Fig. 8. Comparison of the shortage amount of product in the lean state (with obj_3) and in the ordinary state (without obj_3).

Meanwhile, in the 4th period, due to a reduction in demand, a reduction for the product sale price in a market (in the section related to the clients' market and also in the section related to purchasing from the contractors) has as well been considered. Now we study the results.

Regarding the sudden fall for the sale and purchase of products 1 and 2 and also sudden fall for the demand we expect to create a particular procedure in the 4th period. For this reason, we study the obtained results.

The obtained results from Figs. 4 and 5 are as follows.

Considering the reduction of the demand in the period of 4 and following it reduction of the price in the market with the policy of increasing the clients and a policy for reduction of the mutual price of the contractors in the said example, the model decided to purchase more than the necessity from contractors (Fig. 5) and following this purchase the amount of stored products in the distribution centers increases (Fig. 4) in order that the profit in the supply chain reaches the maximum.

We study the effect of obj_3 in the direction of reaching the model toward lean (Figs. 6–8).

From the above figures, we can understand that the rate of resulted shortage and the rate of stored inventory has been reduced at the end of each period for the products and products in the process at each distribution and production center considerable with increasing and it was attempted to reduce the rate of stored inventory for each product in process and product considering the costs and incomes (in this example the shortage rate has become zero). The result from the obtained production plan indicates the movement of the chain towards lean (the rate of stored raw materials in both cases is zero in this example)

- The obtained results in Tables 1 to 13 and Fig. 3 are a very appropriate programming for purchase, production and controlling the inventory in the direction of increasing incomes, reducing the costs and lean of the supply chain.
- From Table (2) we can derive the rate of optimum production for each product in the process in each production center and at each time period.
- From Table (3) we can derive the rate of optimum delivery of the products in process from every production center to another production center and at each time period.
- From Table (4) we can derive the rate of optimum storage for each raw material in each production center and at the end of each time period.
- From Table (5) we can derive the rate of optimum storage of the product in the process in each production center and at the end of each time period.
- From Table (6) we can derive the rate of optimum purchase for each product in the process in each contracting center for every production center and at each time period.
- From Table (7) we can derive the rate of optimum storage of the product in the process entered into each production center at the end of each time period.
- From Table (8) we can derive the rate of optimum production for each product in each production center and at each time period.
- From Table (9) we can derive the rate of optimum delivery of the product from each production center to each distribution center and at each time period.
- From Table (10) we can derive the rate of optimum purchase for each product in each contracting center for every distribution center and at each time period.

Table 1
The number of sent raw materials from each supplier to each producer in each time period.

t	1	2	3	4	5	6
nfs_{111t}	2623	2134	5519	2414	2554	5356
nfs_{112t}	3304	3304	3304	2133	2832	3304
nfs_{121t}	0	0	0	0	0	0
nfs_{122t}	0	0	0	0	0	0
nfs_{211t}	0	0	0	0	0	0
nfs_{212t}	0	0	0	0	0	0
nfs_{221t}	4021	1898	4080	3700	2007	4136
nfs_{222t}	1447	1447	1447	1089	1447	1447

Table 2
The number of sent products in process produced by each producer in each time period.

t	1	2	3	4	5	6
$\bar{x}_{11t}$	0	0	0	0	0	0
$\bar{X}_{11t}$	0	0	0	0	0	0
$\bar{x}_{12t}$	933	933	933	1053	1399	933
$\bar{X}_{12t}$	0	1	1	1	1	1

Table 3
The number of sent products in process from the producers to other producers in each time period.

t	1	2	3	4	5	6
nff_{111t}	0	0	0	0	0	0
nff_{112t}	0	0	0	0	0	0
nff_{121t}	0	0	0	1054	1399	0
nff_{122t}	0	0	0	0	0	0

Table 4
The number of stored raw materials in the store of each producer at the end of each time period.

t	1	2	3	4	5	6
$nwhrs_{11t}$	0	0	0	0	0	0
$nwhrs_{21t}$	0	0	0	0	0	0
$nwhrs_{12t}$	0	0	0	0	0	0
$nwhrs_{22t}$	0	0	0	0	0	0

Table 5
The number of stored products in process in the store of each producer at the end of each time period.

t	1	2	3	4	5	6
$nwhf_{11t}$	0	0	0	0	0	0
$nwhf_{12t}$	0	0	0	0	0	0

Table 6
The number of products in process purchased by each producer from each contractor in each time period.

t	1	2	3	4	5	6
$nmpc_{111t}$	0	0	0	0	0	0
$nmpc_{112t}$	0	0	0	0	0	0
$nmpc_{121t}$	838	2673	0	0	940	0
$nmpc_{122t}$	0	0	0	0	0	0
$nmpc_{131t}$	0	0	0	0	0	0
$nmpc_{132t}$	0	0	0	0	0	0

Table 7
The number of purchased and received products in process that have been stored in each production center at the end of each period.

t	1	2	3	4	5	6
$nwhcpf_{11t}$	758	2419	0	1054	2250	0
$nwhcpf_{12t}$	0	0	0	0	0	0

Table 8
The number of produced product by each producer in each time period.

t	1	2	3	4	5	6
x_{11t}	0	397	1209	0	572	1125
X_{11t}	0	1	1	0	1	1
x_{12t}	466	466	466	0	0	466
X_{12t}	1	1	1	0	0	1
x_{21t}	1296	485	912	1193	471	958
X_{21t}	1	1	1	1	1	1
x_{22t}	0	0	0	0	0	0
X_{22t}	0	0	0	0	0	0

Table 9
The number of sent products from each producer to each distributor in each time period.

t	1	2	3	4	5	6
nfd_{111t}	0	0	332	0	527	388
nfd_{112t}	0	379	877	0	0	737
nfd_{121t}	0	0	466	0	0	466
nfd_{122t}	466	466	0	0	0	0
nfd_{211t}	1006	190	613	577	471	644
nfd_{212t}	290	295	299	615	0	314
nfd_{221t}	0	0	0	0	0	0
nfd_{222t}	0	0	0	0	0	0

Table 10
The number of purchased product from each contractor by each distributor in each time period.

t	1	2	3	4	5	6
$nlpc_{111t}$	810	823	126	1197	0	114
$nlpc_{112t}$	452	92	0	1956	0	0
$nlpc_{121t}$	0	0	0	0	0	0
$nlpc_{122t}$	0	0	0	0	0	0
$nlpc_{131t}$	0	0	0	0	0	0
$nlpc_{132t}$	0	0	0	0	0	0
$nlpc_{211t}$	0	0	0	0	0	0
$nlpc_{212t}$	0	0	0	0	0	0
$nlpc_{221t}$	0	0	0	0	0	0
$nlpc_{222t}$	0	0	0	0	0	0
$nlpc_{231t}$	0	0	0	225	0	0
$nlpc_{232t}$	0	0	0	0	0	0

Table 11
The number of stored products in the warehouse of each distributor at the end of each time period.

t	1	2	3	4	5	6
$nwhs_{11t}$	0	0	0	336	0	0
$nwhs_{12t}$	0	0	0	1045	177	0
$nwhs_{21t}$	406	0	0	159	0	0
$nwhs_{22t}$	0	0	5	304	0	0

Table 12
The number of resulted shortage for each product from each distributor in each time period.

t	1	2	3	4	5	6
nsh_{11t}	0	0	0	0	0	0
nsh_{12t}	0	0	0	0	0	0
nsh_{21t}	0	0	0	0	0	0
nsh_{22t}	0	0	0	0	0	0

Table 13
The number of sent products from each distributor to customer in each time period.

′t	1	2	3	4	5	6
ndc_{11t}	786	799	813	826	840	854
ndc_{12t}	812	825	840	854	868	882
ndc_{21t}	582	592	602	612	622	632
ndc_{22t}	285	290	294	299	305	309

- From Table (11) we can derive the rate of optimum delivery for every raw material at each supply center to each production center and at each time period.
- From Table (12) we can derive the rate of optimum shortage for each product in each distribution center and at each time period.
- From Table (13) we can derive the rate of optimum distribution and delivery of the product from each production center to each distribution center to the client and at each time period.
- Subsequently, we can also select and evaluate the suppliers, producers, distributers and contractors in the next subsections.

3.3.1. Ranking the suppliers in production plant 1 & 2 for supplying raw materials 1 & 2 in each time period

In the above table, the column 1 indicates the rate of planning for purchase of raw material 1 from the supplier 1, column 2 expresses the rate of planning for purchase of raw material 1 from the supplier 1, column 3 expresses the percent of planned purchase of raw material 1 from the supplier 1, column 4 expresses the percent of planned purchase of raw material 1 from the supplier 2, column 5 expresses the 1st rank for selecting the supplier in each period for extracting raw material 1 and column 6 expresses the second rank for selecting the supplier in each period for extracting raw material 1.

In the above table, column 1 indicates the rate of planning for purchase of raw material 2 from the supplier 1, column 2 expresses the rate of planning for purchase of raw material 2 from the supplier 1, column 3 expresses the percent of planned purchase of raw material 2 from the supplier 1, column 4 expresses the percent of planned purchase of raw material 2 from the supplier 2, column 5 expresses the 1st rank for selecting the supplier in each period for extracting raw material 2 and column 6 expresses the second rank for selecting the supplier in each period for extracting raw material 2.

Tables 14 and 15 indicate model efficiency for selecting the suppliers. Due to the reason that the costs in this numerical example are increased in a linear form and no sudden fall or growth of the cost is considered in this section, regarding the input numbers and costs since in period 1 the costs for supplying raw material 1 from supplier 1 and the costs for supplying raw material 2 from supplier 2 are at a lower level than the competitor, to the end of period 6 these two suppliers were selected with no change (Tables 14 and 15).

Table 14
Selection and evaluation of the suppliers for extracting the raw material 1.

Period	1	2	3	4	Rank 1*	Rank 2*
1	5927	0	100%	0%	S1	S2
2	5438	0	100%	0%	S1	S2
3	8823	0	100%	0%	S1	S2
4	4547	0	100%	0%	S1	S2
5	5356	0	100%	0%	S1	S2
6	8660	0	100%	0%	S1	S2

* S1: Supplier 1, S2: Supplier 2.

Table 15
Selection and evaluation of the suppliers for extracting the raw material.

Period	1	2	3	4	Rank 1	Rank 2
1	0	5468	0%	100%	S2	S1
2	0	3345	0%	100%	S2	S1
3	0	5527	0%	100%	S2	S1
4	0	4789	0%	100%	S2	S1
5	0	3554	0%	100%	S2	S1
6	0	5583	0%	100%	S2	S1

Table 16
Selection and evaluation of the producers and contractors for receiving the product 1.

Period	1	2	3	4	5	6	7	8	Rank 1	Rank 2	Rank 3	Rank 4
1	0	466	1262	0	0%	27%	73%	0%	C1	S2	S1	C3
2	379	466	915	0	22%	26%	52%	0%	C1	S2	S1	C3
3	1209	466	126	0	67%	26%	7%	0%	S1	S2	C1	C3
4	0	0	3153	0	0%	0%	100%	0%	C1	C3 & S2 & S1		
5	527	0	0	0	100%	0%	0%	0%	S1	C3 & C1 & S2		
6	1125	466	114	0	66%	27%	7%	0%	S1	S2	C1	C3

C1: Contractor 1 & C3: Contractor 3.

3.3.2. Ranking the producers and contractors for the provision and producing of product 1 in each time period

In Table 16 column no. 1 indicates the rate of planning for receiving product 1 from producer 1, column no. 2 indicates the rate of planning for receiving product 1 from producer 2, column no. 3 indicates the rate of planning for receiving product 1 from contractor 1, column no. 4 indicates the rate of delivery for planning of receiving product 1 from contractor 3, column no. 5 indicates the planned percent for delivery of product 1 from producer 1, column no. 6 indicates the planned percent for delivery of product 1 from producer 2, column no. 7 indicates the planned percent for delivery of product 1 from contractor 1, column no. 8 indicates the planned percent for delivery of product 1 from contractor 3, column no. 9 indicates the rank of 1 for supplying product 1, column no. 10 indicates the rank of 2 for supplying product 1, column no. 11 indicates the rank of 3 for supplying product 1, and column no. 12 indicates the rank of 4 for supplying product 1.

Considering the obtained results from Table 16 a full evaluation is obtained from the suppliers of product 1.

In Table 17 column no. 1 indicates the rate of planning for receiving product 2 from producer 1, column no. 2 indicates the rate of delivery for planning of receiving product 2 from contractor 2, column no. 3 indicates the rate of delivery for planning of receiving product 2 from contractor 3, column no. 5 indicates the planned percent for delivery of product 2 from contractor 2, column no. 6 indicates the planned percent for delivery of product 2 from contractor 3, column no. 8 indicates the rank 2 for supplying product 2, and column no. 9 indicates the rank 3 for supplying product 2. We can also perform this action for selecting and evaluating the suppliers in the case of semi-made goods.

4. Conclusion

In this paper, an integrated model for programming and allocation of the limited sources in a multiple multi-stage lean supply chain was presented. The main purpose of the model is to adopt the optimum strategies in the supply chain with regard to the objective functions of

Table 17
Selection and evaluation of the producers and contractors for receiving the product 2.

Period	1	2	3	4	5	6	Rank 1	Rank 2	Rank 3
1	1296	0	0	100%	0	0	S1	C3 & C2	
2	485	0	0	100%	0	0	S1	C3 & C2	
3	912	0	0	100%	0	0	S1	C3 & C2	
4	1192	0	225	84%	0	16%	S1	C3	C2
5	471	0	0	100%	0	0	S1	C3 & C2	
6	958	0	0	100%	0	0	S1	C3 & C2	

C2: Contractor 2

minimization of the raw materials' costs, production, transportation, distribution, optimization of incomes, and minimization of the shortage number and waste. A considerable point in this model is the definition of objective functions in a manner that the supply chain is directed toward lean. The integrated approach of the model for supplying the whole chain and its effort for making a decision in all the problems related to one multiple multi-stage lean supply chain are the main privileges of the presented method compared to other methods.

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