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A new push back design algorithm in open pit mining

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ABSTRACT: In open pit mine design, scheduling is normally based on underlying push backs (i.e. incremental pits). Many different push back design algorithms have been developed in mining industry since 1960. The majority of push back design algorithms use an economic parameter (i.e. metal price) to find series of pits where each of the pits has the highest undiscounted dollar value for the pit size considered. Although these highest value pits may indicate where the next "highest grade ore" phase may be, they may not always indicate where the next "best ore" may be for the schedule that maximizes the net present value (NPV). The "best ore" may be defined as the material that has the highest grade and the least amount of waste stripping. The series of maximum undiscounted value pits which are used to schedule a given open pit mine will not always give a schedule of cash flows that will maximize the NPV. This is because the design of the best value pits does not necessarily take into account the stripping required to get to the "highest grade ore" blocks. In this paper, a new push back design algorithm is presented to develop push backs that indicate where the minimum strip ratio ore will be. This new algorithm finds series of push backs based on minimizing stripping ratio criteria. The minimum strip ratio push backs may be used in conjunction with traditionally designed push backs to develop schedules that result in higher Net Present Value for a given open pit mining project.

1 INTRODUCTION

In a long-term open pit mine design, after defining the ultimate pit limits, an extraction sequence or push backs are generated to be used as a guide during the scheduling process. The sequence of push backs outlined during the planning process are used to determine the yearly extraction schedule of ore and waste production (see Rose, 1985). As such design of a push back sequence plays a key role in defining annual cash flows to be generated from a given open pit mine. There may be many alternative push back sequences that lead to ultimate final pit limits of an open pit mine. Each sequence of push backs results in a different annual cash flow stream that gives different NPV for a project.

Traditionally, a series of pits are generated by using the ultimate pit limit algorithm applied to an economic block model (Dagdelen and Francois-Bongarcon, 1982). In this approach, a series of pits with different sizes are obtained by generating an economic model at different price for the commodity, cutoff grade, or mining and processing costs. An economic block model can be generated by using the following profit equation for each ore block:

$$V=G*R*P-C_P-C_M \quad (1a)$$

For a waste block:

$$V=-C_M \quad (1b)$$

where V = block dollar value ($), G = grade of a block (i.e., oz/ton), P =price ($/ounce of gold), C_m = mining cost ($/ton of material) and C_P= processing cost.

One of the most widely used algorithms in push back design is Whittle's 4-D package (Whittle, 1988). Whittle reduced the number of economic variables affecting the value of ore blocks and the pit shape to one major and one minor factor in his 4D model. Two new variables are obtained by dividing equation (1a) by C_m:

$$V/C_m =G*R*(P/C_m)-(C_p/C_m)-1 \quad (2)$$

P/C_m (mcostm) is the amount of product that should be sold to pay for the mining of a ton of material. This is the only significant variable, because C_p/C_m (cratio) is not expected to change significantly unless there is a significant change in one of the cost component, or a new mining or processing method is introduced (Whittle, 1988). Equation (2) can be written as:

$$V'=G*R*\lambda-\theta \qquad (3)$$

where V' = value generated per unit cost of mining, λ=mcostm, θ=cratio-1.

By using Lerchs and Grosmann's (LG) method, nested pits are generated for different values of λ (Lerchs and Grossmann, 1965). For each possible λ value, push backs are generated for ± 20% of expected θ.

A small 2D example is used to show how multiple push backs can be obtained by changing parameter λ. Grades are written inside each block as ounce (*1000) per ton as in Figure 1. Break-even mine cutoff grade is 52 by equation (4). Each block contains the same tonnage (tonnage contained in a waste and an ore block is assumed to be the same).

30	50	40	40	55	55	30
x	68	68	40	55	62	x
x	x	30	40	30	x	x
x	x	x	65	x	x	x

Figure 1.Cross sectional view of a 2D example grade distribution (1000* ounce per ton).

In calculating dollar values per ton of blocks, it is assumed that gold price is $400/oz, mining cost is $1/ton of material, processing cost is $20/ton of ore and the recovery factor is 100%. It is also assumed that overall pit slope angle is 45 degrees, and ore and waste blocks have equal densities. The economic block dollar values are given in Figure 2.

-1	-1	-1	-1	+1	+1	-1
x	+6	+6	-1	+1	+4	x
x	x	-1	-1	-1	x	x
x	x	x	+5	x	x	x

Figure 2.Cross sectional view of block dollar values of the sample data ($).

When Whittle's Method is applied to this example, no push back is obtained at λ = 338. The first push back is generated at λ=339. θ is kept constant at 21. Figure 3 shows the economic values of blocks for generating the first push back using λ=339. Similarly, the second, third and fourth push backs are generated at λ=355, λ=382, λ=385, respectively. Generated push backs are shown in Figure 4. Push back 1 has an average value of $1.33/ton while the stripping ratio is 2:1. Push backs 2, 3 and 4 have average values of $1.25/ton, $1.0/ton and $0.2/ton while their stripping ratios are 1:3, 0:1 and 4:1 respectively. It is clear that these push backs have the order of decreasing average dollar per ton value, yet the order of random stripping ratios.

-1	-1	-1	-1	-2.36	-2.36	-1
x	+2.1	+2.1	-1	-2.36	+0.0	x
x	x	-1	-1	-1	x	x
x	x	x	+1.0	x	x	x

Figure 3.Cross sectional view of the first push back showing block economic values.

-1	-1 (1)	-1	-1	+1	+1 (2)	-1
x	+6	+6	-1	+1 (3)	+4	x
x	x	-1	-1 (4)	-1	x	x
x	x	x	+5	x	x	x

Figure 4.Cross sectional view of the push backs generated by Whittle's method (The sequence number for each push back is given inside the circle).

If one assumes that a single block is mined in a year with an interest rate of 20%, undiscounted dollar values (UDV) and Net Present Values (NPV) generated from the schedule obtained by Whittle's method can be seen in table 1, columns (2) and (3), respectively.

Table 1.Undiscounted dollar values (UDV) and net present values (NPV) generated from the schedules coming from Whittle's Method, columns (3) and (4), and Minimum Stripping Ratio Method, columns (4) and (5) for 16 year time period, column (1), and 20% rate.

Years (1)	UDV (2)	NPV (3)	UDV (4)	NPV (5)
1	-1	-0.83	1	0.83
2	-1	-0.69	1	0.69
3	-1	-0.58	-1	-0.58
4	-1	-0.48	-1	-0.48
5	6	2.41	4	1.61
6	6	2.01	1	0.34
7	1	0.28	-1	-0.28
8	1	0.23	-1	-0.23
9	-1	-0.19	-1	-0.19
10	4	0.65	6	0.97
11	1	0.13	6	0.81
12	-1	-0.11	-1	-0.11
13	-1	-0.09	-1	-0.09
14	-1	-0.08	-1	-0.08
15	-1	-0.06	-1	-0.07
16	5	0.27	5	0.27
Total	15	2.85	15	3.40

2 MINIMUM STRIPPING RATIO PUSH BACK DESIGN ALGORITHM

It is clear that NPV will be higher for a mining parameter if the ore blocks can be reached during the initial years. Assume that a certain volume of material, V, is to be mined in a year, two alternate strategies may be followed:

The first strategy may be to extract the material in such a way that the next "highest grade ore" is mined during scheduling.

The second strategy may be such that the material that has the grade and provides the least amount of stripping is mined during the scheduling. The second strategy aims of bringing higher positive cash flows forward during the scheduling.

For certain types of mineral deposits, the use of the second technique in conjunction with the first one may generate higher NPV than the first technique by itself.

In this paper, Minimum Stripping Ratio Push Back Design Algorithm that is developed in Ramazan, 1996, is described. The algorithm finds the push backs that each of them has the minimum stripping ratio among all possible push backs with the same size.

2.1 *Definitions*

The following definitions are adapted from Seymour, 1995.

Break-even mine cutoff grade is the grade below which mining and processing a block as an ore block is not economical.

A *node* is the representation of a block.

A *block indicator value* is an indicator of either 1 or 0. It is 1 for an ore block and 0 for a waste block. If densities of an ore and a waste blocks are different, tonnage of an ore block is assigned instead of 1 indicator value.

Mass indicator value is an indicator value of 1 for both ore and waste blocks where densities of ore and waste blocks are the same. If the densities of waste and ore blocks are different, tonnage of a waste block is assigned to a waste block and that of an ore block is assigned to an ore block instead of 1 indicator value.

An *up pointer* connects an underlying node to an overlying node and a *down pointer* connects an overlying node to an underlying node.

The *cumulative indicator mass* at a given node is sum of the masses from the nodes whose pointers directly or indirectly points to the given node.

The *cumulative indicator value* at a given node is the sum of the indicator values from the nodes whose pointers directly or indirectly point to the given node.

The *cumulative indicator strength* at a given node is the cumulative indicator value divided by the cumulative indicator mass.

A node is a *root node* if there is no pointer assigned from the node. Initially, each node is a root node of itself.

2.2 *Steps of the algorithm*

The steps of the minimum strip ratio push back design algorithm are same as the one generated in Seymour, 1995 except the steps 1 and 2.

Step 1: Find the break-even mine cutoff grade.

$$\text{Mcutoff} = (C_p+C_m)/[(P-S)*R] \quad (4)$$

Where Mcutoff =break-even mine cutoff grade (ounce/ton), C_p =processing cost ($/ton of ore), C_m =mining cost ($/ton of material), P =price of gold ($/ounce), S =selling cost ($/ounce of gold), R =recovery factor (%).

Step 2: If the average grade of a block is greater than the break-even mine cutoff grade assign indicator value 1 to the block. If it is less or equal to the mine cutoff, assign indicator value 0 to the block (See Definitions). Assign cumulative block indicator value (initially same as indicator value), cumulative indicator mass (initially tonnage of a block), and indicator strength (cumulative indicator value over cumulative indicator mass).

Step 3: If block A has an overlying block B whose root strength is lower than the root strength of A, A and B are connected by assigning a pointer from A to B. Cumulative indicator value, mass and strength of the block B is updated as follows:

$$\text{CIV (B)=CIV (B)+CIV (A),} \quad (5)$$

$$\text{CM (B)=CIM (B)+CIM (A),} \quad (6)$$

$$\text{CIS (B)=CIV (B)/CIM (B)} \quad (7)$$

Where CIS (X) = strength of the node X, CIV (X) = the Cumulative Indicator Value of node X and CIM (X) = the Cumulative Indicator Mass of node X.

All the values of A is kept constant and B becomes root node if there is not any pointer assigned from B to another block in the previous steps.

Step 4: If node (A), which has higher root strength than that of an overlying node (B), has already a pointer connecting to another node, this pointer should be reversed towards A (making A root), before making another connection from node A. Note that there can be only one pointer going out from a node, but there can be more than one pointer pointing to a node. If a pointer is reversed, go to Step 3. Otherwise, go to the next step.

Step 5: If there is a node that is connected to another node with a down pointer, and if its strength is higher than the strength of the root node of its own branch, this pointer is pruned. The values are updated as follows:

CIV (B)=CIV (B)-CIV (A) (8)

CM (B)=CIM (B)-CIM (A) (9)

CIS (B)=CIV (B)/CIM (B) (10)

If a pointer is pruned, go to Step 3. Otherwise, go to the next step.

Step 6: Repeat Step 3 through Step 5 for all the nodes in the model.

Step 7: After all the nodes are searched for connections, all the nodes should be converted into root node by reversing pointers (a pass is completed). Reversing starts from a root node. After each reversing, pointers are checked for pruning as in step 4.

Step 8: If a pointer is pruned in step 7, go to step 3. Otherwise, go to the next step.

Step 9: When no pointer is pruned or set in a pass, the process is stopped. All the blocks that are connected to a root block, which has a positive strength, make a push back. Blocks are ordered starting from the highest root strength node to the lowest one.

These steps are based on the algorithm given by Vallet (1976) and Seymour (1995). For theoretical proofs one should refer to these publications.

2.3 *Illustration of steps of the algorithm*

A small 2D block model configuration with the grades is given in Figure 5 taken from Figure 1.

A(50)	B(40)	C(40)	D(68)	E(68
	F(68)	G(40)	H(68)	

Figure 5.Cross sectional view of grade distribution of a small example 2D data (1000*ounce/ton).

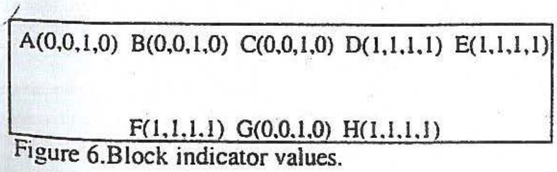

Figure 6.Block indicator values.

From equation 1, Break-even mine cutoff = (20+1) /400 = 0.052 ounce /ton. Therefore, nodes D, E, F and H are ore blocks, and A, B, C and G are waste blocks (Step 1).

Initial values are assigned to the blocks as shown in Figure 6. The numbers inside a node from left to right are block indicator value, cumulative indicator value, cumulative indicator mass and block indicator strength (Step 2).

The nodes A, B, C, D and E are said to be overlying nodes and node F, G and H are underlying nodes. Initially all the nodes are root nodes. Since indicator strength of node A, which is 0, is less than that of node F, which is 1, F is connected to A as in Figure 7 and the values of node A are updated (Step 3).

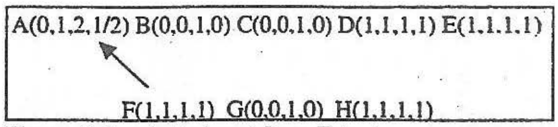

Figure 7.A pointer is set from F to A.

After connection, A is the root of F. Since the indicator strength of B, which is 0, is less than the indicator strength of the root of F (node A), which is 1/2, a pointer should be set from F to B. Before a pointer is set from F to B, the pointer from F to A should be reversed to make F the root (Step 4). The block values after reversing the pointer are shown in Figure 8.

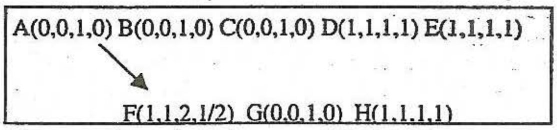

Figure 8.The pointer is reversed from A to F making F root.

After pointer reversal process, it should be checked if there is a down pointer with strength higher than that of its root (Step 5). In Figure 8, node A has a down pointer, but its strength is less than that of the strength of the root F. So the pointer is not pruned and node connecting process will continue.

The indicator strength of B, which is 0, is less than that of F, which is 1/2, which means that a pointer is set from F to B as in Figure 9.

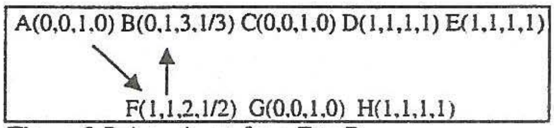

Figure 9.Pointer is set from F to B.

Indicator strength of C, which is 0, is lower than that of the root of F (node B), which is 1/3, hence the pointer from F to B is reversed and a pointer is assigned from F to C (see Figure 10).

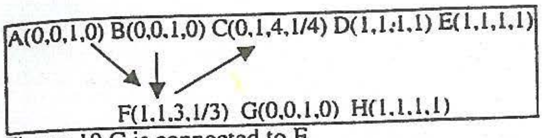

Figure 10.C is connected to F.

All the overlying nodes are connected to node F. Therefore, the search process for the node connection should continue with the next node. Since G is a 0 strength block, node H is searched for the connection. As it can be seen from Figure 10, the strength of C, which is 1/4, is less than that of H, which is 1. Therefore, a pointer should be set from F to C as shown in Figure 11.

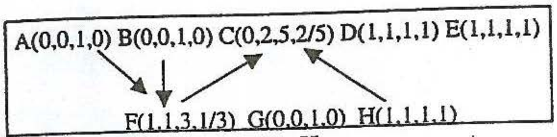

Figure 11.C is connected to H.

The indicator strengths of both nodes D and E, which are 1, are greater than that of the root node of H (node C), which is 2/5. Therefore, no more connection can be made between any nodes. The next step is to reverse the pointers starting from the root node to convert each node to a root node (Step 7). The pointer from C to F is reversed and the values are updated as in Figure 12.

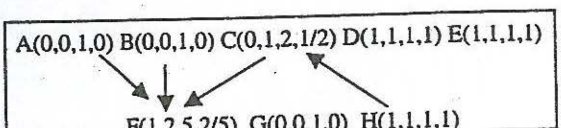

Figure 12.C is connected to H.

It is shown in Figure 1 that node C has a down pointer and its strength, which is 1/2, is greater than that of the root (node F), which is 2/5. Therefore, the pointer from C to F is pruned (Step 5). The blocks after pruning process are given in Figure 13.

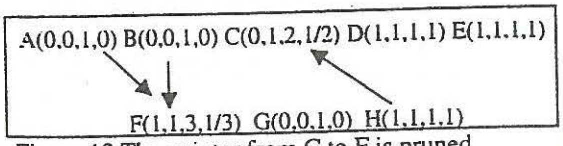

Figure 13.The pointer from C to F is pruned.

There is no more pointer that can be pruned or assigned. Therefore, the algorithm is stopped at this point. All the blocks are ordered from the higher to lower indicator strength of their root node. The blocks that are connected together with higher indicator root strength are scheduled to be mined earlier than those connected with lower root strength. Blocks D and E are scheduled to be mined at the earliest period of production with 0:2 stripping ratio. Then, C and H are to be mined in the next production period with 1:1 stripping ratio. A, B and F are scheduled to be mined in the last production period with 2:1 stripping ratio. It is clear that block G is not economical to mine in this example.

2.4 *Comparison of the algorithm with Whittle's method on a 2D example data*

Minimum Stripping Ratio Method is also applied to the same example data set after finding the ultimate final pit limits. Initially four variables are assigned to each block: indicator value, cumulative indicator value, cumulative mass and strength (see step 2). The initial values of variables are as follows from the most left block to the right:
First Bench: (0, 0, 1, 0), (0, 0, 1, 0), (0, 0, 1, 0), (0,0,1,0), (1, 1, 1, 1), (1, 1, 1, 1), (0, 0, 1, 0).
Second Bench: (1, 1, 1, 1), (1, 1, 1, 1), (0, 0, 1, 0), (1, 1, 1, 1), (1, 1, 1, 1).
Third Bench: (0, 0, 1, 0), (0, 0, 1, 0), (0, 0, 1, 0).
Fourth Bench: (1, 1, 1, 1).

The block indicator values of the example data is given in Figure 14. By applying the steps given in *Steps of the algorithm* section, 4 push backs are generated as shown in Figure 15. UDV's (undiscounted dollar values) and NPV's (net present values) coming from the schedule of push backs from Minimum Stripping Ratio Method are also shown in Table 1, columns (4) and (5), respectively. The same assumptions are applied as in Whittle's method (each block is mined in a year with 20% interest rate).

0	0	0	0	1	1	0
x	1	1	0	1	1	x
x	x	0	0	0	x	x
x	x	x	1	x	x	x

Figure 14.Cross sectional view of the block indicator values of the example.

It should be noticed from Figure 15 that the stripping ratio increases as the push back sequence increases. The first push back has 0.0 stripping ratio. The second, third and fourth push backs have 1:1, 1.5:1 and 4.0:1 stripping ratios, respectively. Therefore, it is clear that the schedule obtained by this algorithm will reach the ore blocks faster than any other conventional method.

-1	-1	-1	-1	+1 (1)	+1	-1
x	+6 (3)	+6	-1	+1 (2)	+4	x
x	x	-1	-1 (4)	-1	x	x
x	x	x	+5	x	x	x

Figure 15.Cross sectional view of the push backs generated by Minimum Stripping Ratio method (The sequence number for each push back is given inside the circle).

3. MINIMUM STRIPPING RATIO PUSH BACK DESIGN BY USING MODIFIED WHITTLE ALGORITHM

Minimum stripping ratio push backs can also be obtained by using Whittle's Method. If the same grade value (maximum grade in the block model) is assigned to all the ore blocks, the push backs generated by Whittle's method will give the same result as the minimum stripping ratio method. Assigned grades are given in Figure 16.

30	50	40	40	68	68	30
x	68	68	40	68	68	x
x	x	30	40	30	x	x
x	x	x	68	x	x	x

Figure 16.Cross sectional view of the assigned grades (1000* ounce per ton).

When Whittle's Method is applied, no push back is obtained at $\lambda = 308$. The first push back is generated at $\lambda=309$. As in the application of the method to the original grades, θ is kept constant at 21. The second push back is generated at $\lambda=324$. The third and fourth push backs are generated at $\lambda=331$, $\lambda=368$, respectively. The generated push backs are shown in Figure 17. As one can notice, the push backs in Figure 15 and Figure 17 are exactly the same.

-1	-1	-1	-1	+1 (1)	+1	-1
x	+6 (3)	+6	-1	+1 (2)	+4	x
x	x	-1	-1 (4)	-1	x	x
x	x	x	+5	x	x	x

Figure 17.Cross sectional view of the push backs generated by Modified Whittle's method (The sequence number for each push back is given inside the circle).

It is shown that Minimum Stripping Ratio algorithm can produce up to 6% better NPV than the conventional Whittle's method at 15% interest rate and 16% more NPV at 20% rate. The reason to develop a new algorithm instead of using Whittle's method in generating minimum stripping ratio push backs is that Whittle's method does not guarantee the optimality in generating push backs where initial stripping may be required. The new algorithm gives the optimum result in generating push backs which has the minimum stripping ratio.

4 CONCLUSIONS

The algorithm presented here will generate push backs that show progression of pits from low stripping areas of the deposit towards high strip ratio areas.

During the production scheduling exercise, it is not only important to know where the highest incremental value pits are but also to know where the low strip ratio material is.

Maximization of net present value of a given project can only be attained if the planning engineer considers push backs generated by these two different methods together during the scheduling exercise.

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