

# Forward error correction concatenated code in DWDM systems

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**Abstract** The three concatenated coding schemes of the inner-outer type, the parallel type and the consecutive type to improve the current forward error correction (FEC) coding technologies are proposed for dense wavelength-division multiplexing (DWDM) systems, after introducing the development trend of DWDM optical communication systems. The concatenated code is theoretically analyzed. The theoretical analyses and simulation results show that inner-outer concatenated code has a greater redundancy and the decoding of parallel concatenated code is too complex. However, consecutive concatenated code is superior coding scheme with advantages such as better error correction performance, moderate redundancy and easy implementation, therefore it could be better used in high-speed and long-haul DWDM systems.

**Keywords** dense wavelength-division multiplexing (DWDM) system, super forward error correction (Super-FEC), concatenated code, net coding gain (NCG)

## 1 Introduction

In recent years, with the rapid development of dense wavelength-division multiplexing (DWDM) systems, the ITU-T has developed researches on forward error correction (FEC) codes, and relative recommendations such as ITU-T G. 707, G. 975 and G. 709 have been proposed one after the other. All the applied coding code-types in these recommendations are the single linear system cyclic codes, such as the BCH code and RS code [1,2]. The single channel code can produce the moderate net coding gain (NCG), but it may not meet the requirements of high-speed large-capacity DWDM systems. With the increasing development of DWDM systems toward longer distance, larger capacity and higher bit rate, further improvements of the transmission rate and distance are limited due to the accumulated optical effects such as the

dispersion and polarization-mode dispersion (PMD), and the accumulated nonlinear optical effect such as four-wave mixing (FWM), stimulated Raman scattering (SRS), and stimulated Brillouin scattering (SBS) in the DWDM systems [3–6]. As a result, it has become necessary to develop a novel and more powerful super forward error correction (Super-FEC) code-type to gain a higher NCG and better error-correction performance to meet the requirements of the rapid development for DWDM systems. It is also required that this new code-type is able to compensate for serious transmission quality degradation and avoid using dispersion-compensating technologies with expensive and complex devices. In view of this development trend of DWDM systems, it is necessary to do further researches on the novel scheme of the code-type, theoretical analysis and practically verify its application in DWDM systems to improve their communication quality and decrease their cost.

Since the concatenated code has a very powerful outburst-error-correction and random-error-correction performance, it is the main research object of high efficient code-type in DWDM systems.

## 2 Theoretical analysis of concatenated code

### 2.1 Brief introduction of concatenated code

The concatenated code was proposed as a practical method to construct code-type with longer codeword length and better error-correction performance, and it is a special method that a long code can be consisted of some short codes. Essentially, the concatenated code is the special case of the product code, and the most applied concatenated code is constructed by the two codes [4–7].

### 2.2 Inner-outer concatenated code

According to the channel coding theorem, the error probability in the decoding process will tend to zero

exponentially when the codeword length increases [7]. Therefore, a longer codeword should be used to improve the error-correction performance of the FEC code. However, the code rate may decrease with the increase of the codeword length; accordingly the complexity and calculated amount of the decoding devices may increase, which is difficult for its implementation. The coding scheme of inner-outer type concatenated code can solve this contradiction, correspondingly. The encoding process in this coding scheme is divided into a inner-outer serial two stages to operate, with which the requirements on the codeword length for the channel error-correction could be satisfied, the error-correction performance and a high coding gain equal to that of the used longer codeword can be obtained without increasing the complexity of the encoding/decoding process [7]. A theoretical block diagram of the inner-outer type concatenated code is shown in Fig. 1.

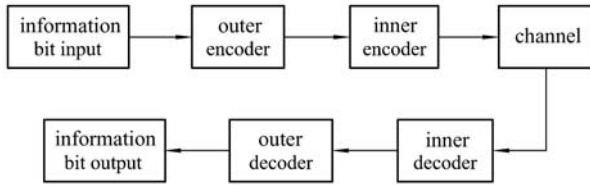


Fig. 1 Theoretical block diagram of inner-outer type concatenated code

When encoding, the  $k_1k_2$  binary information elements are first divided into  $k_2$  segments, each segment includes  $k_1$  information elements and is taken as a symbol. The  $(n_2, k_2)$  outer code is coded according to the nonbinary block code, and the minimum Hamming code distance of the outer code is  $d_2$  with a code rate  $R_2$  of  $k_2/n_2$ . Then, the  $k_1$  information elements in each segment are coded as the  $(n_1, k_1)$  inner code, and the minimum Hamming code distance of the inner code is  $d_1$  with a code rate  $R_1$  of  $k_1/n_1$ , so the  $n_2$  codeword sequence of the  $(n_1, k_1)$  inner code can be obtained. As a result, the  $[n_1n_2, k_1k_2, d_1d_2]$  concatenated code are consisted of the  $n_1n_2$  binary code elements and  $k_1k_2$  information elements, with a rate  $R$  of  $R_1R_2$  and the minimum Hamming code distance of at least  $d_1d_2$ . When decoding, the  $n_2$  symbols consisted of the  $k_1$  code elements can be first obtained according to the decoding rule for the inner code, and then they are sent into the outer decoder and decoded according to the decoding rule of the outer decoder. Finally, the  $k_2$  information symbols can be obtained and there are the  $k_1$  binary information elements in each information symbol. Therefore, the output information bit from the outer decoder is the error-corrected  $k_1k_2$  information elements.

### 2.3 Concatenated code with interleaver

If the inner code and outer code are concatenated directly, the data flow from the inner decoder would enter the outer

decoder directly if there are outburst errors in the data flow, so in this case it seems to be that the outer decoder cannot correct the outburst errors. However, the added interleaver can interleave the data in different blocks to get the new data flow to be sent into the outer decoder, thus the errors are equalized to avoid the occurrence of certain uncorrectable outburst errors. The bit-error rate (BER) can gradually decrease with the increase of the iteration times by applying the iterative decoding. The theoretical block diagram of the improved concatenated code is shown in Fig. 2.

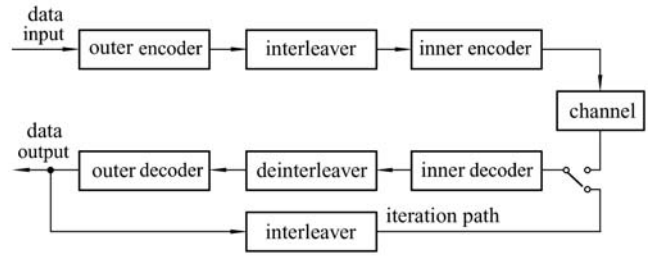


Fig. 2 Theoretical block diagram of concatenated code with interleaver

When encoding, the  $k_1k_2$  information elements are first sent into the outer encoder, and the outer encoder can encode the  $k_1k_2$  information elements into  $k_1n_2$  information elements according to its encoding rule. Then the  $k_1n_2$  information elements are sent into the interleaver, and then sent into the inner encoder after being interleaved. The inner encoder encodes the  $k_1n_2$  information elements according to its encoding rule and exports the  $n_1n_2$  code elements. Thus, in the whole progress of encoding, the encoder of the concatenated code imports the  $k_1k_2$  information elements, exports the  $n_1n_2$  code elements, and produces the codeword of  $[n_1n_2, k_1k_2, d_1d_2]$  concatenated code. Its code rate is  $R = (k_1/n_1) \cdot (k_2/n_2) = R_1R_2$  and the minimum distance is at least  $d_1d_2$ . It is obvious that it is the same as the conventional concatenated code basically, just adding the interleaver in it.

When decoding, the received  $n_1n_2$  code elements are sent into the inner decoder, which can get the  $k_1n_2$  information elements according to the decoding rule for the inner code. Then the  $k_1n_2$  information elements are sent into the deinterleaver to be deinterleaved according to the reverse interleaving rule of the interleaver, thus the  $k_1n_2$  code elements to be sent into the outer decoder can be obtained, and the outer decoder decodes the entered  $k_1n_2$  code elements to get the  $k_1k_2$  information elements of the input encoder for the signal source terminal. If the iterative decoding is applied, it is necessary for the  $n_1n_2$  code elements, corresponding with the  $k_1k_2$  information elements, derived from the previous decoding and obtained by the decoding of both inner and outer decoder, to be sent into the inner and outer decoder over again to implement the decoding operation as the previous decoding

operation after being interleaved through the interleaver. Thus, the iterative decoding process has been achieved for one time.

#### 2.4 Error-correction of concatenated code

Using the concatenated code, a small quantity of random errors in the channel can be corrected by the inner code. When the outburst error or the random error is too much that it is beyond the error correction capability of the inner code, the inner decoder may produce wrong decoding, and the output codeword will contain the error code, which however merely corresponds to a few error code elements in the outer code and can be easily corrected by the outer decoder. Thus, the concatenated code is effective for the correction of the combined channel error and the relatively long outburst error. Moreover, the implementation of its encoding/decoding circuit is simple with less cost. Therefore, it is more suitable for the concatenated code to be applied in DWDM systems.

### 3 Improved schemes of FEC code in DWDM systems

#### 3.1 Improved scheme of inner-outer concatenated code

Because of the limited performances of existing FEC codes, it is considered that the concatenated codes with better error and outburst-error correction performances, compared to general cyclic codes, can be used in long-haul optical communications. The general concatenated code is to apply inner code and outer code in series, and its principle is shown in Fig. 1. The inner code may adopt the binary BCH code, and the outer code may adopt multiple system RS code. The Super-FEC code using this serial concatenated coding scheme with the inner and outer code can theoretically provide more superior than single RS code or single BCH code.

Here, the RS (127, 119) or RS (127, 111) is concatenated with BCH (15, 7), and the outer code RS (127, 119) and inner code BCH (15, 7) are selected for research. By choosing different EbNos, namely signal-to-noise ratio (SNR), to simulate the RS (127, 119)+BCH (15, 7) concatenated code, different BERs can be acquired and the plots of the simulated results compared with the RS (255, 239) code are shown in Fig. 3.

It can be seen from Fig. 3 that these two RS+BCH serial concatenated codes have a superior error correction performance, the NCGs are respectively up to 8.0 and 8.9 dB, which are solely 3.1 and 4.0 dB more than that of the RS (255, 239) code at the BER of  $10^{-8}$ ; the NCGs of these two concatenated codes can be up to about 9.0 and 10.1 dB at the BER of  $10^{-13}$ , which are 4.1 and 5.2 dB

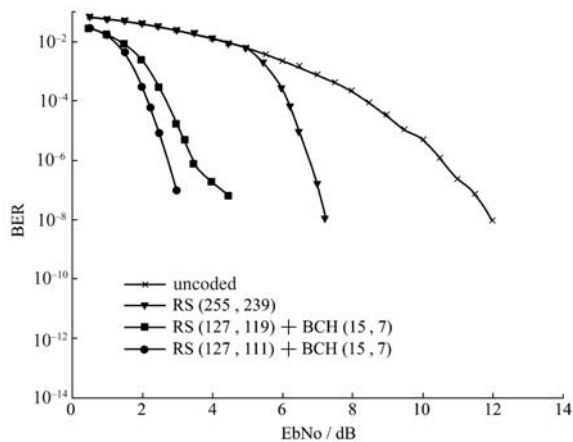


Fig. 3 Simulated results of inner-outer concatenated code

more than that of the RS (255, 239) code. Therefore, they are of the Super-FEC coding scheme with superior error correction performances. It is still known from Fig. 3 that this concatenated coding scheme has the better error correction performance at the low noise condition, because one of its subcode is the BCH (15, 7) with greater redundancy. However, these two concatenated codes of Super-FEC coding scheme with the inner and outer code have an obvious defect, that is, it has greater redundancies of 128.7% and 145.2%, respectively. Thus, it is still necessary to study their practical applications in the future, which is also a direction to improve the FEC coding scheme.

#### 3.2 Improved scheme of parallel-concatenated code

Since the RS+BCH serial concatenated codes have excessive redundancy, it is considerable to abandon this coding scheme and apply the novel coding scheme of the parallel-concatenated codes [5,6]. Furthermore, the interleaved code could be added into the concatenated codes. The structure of this novel coding scheme may be different from that of the RS+BCH serial concatenated code, but it is also named as the concatenated codes because they have the same characters: that is the novel error correction code structure consists of a few different codes or same codes, they can both have better error correction performances, and both are a direction of Super-FEC codes.

The parallel-concatenated coding scheme is shown in Fig. 4.

Based on the input bit stream 0/1, first perform a transverse encoding and a system code is acquired by applying the RS code or BCH code; then, perform an interleaved encoding for the original information by applying the matrix interleaved code, namely perform a novel encoding for all the rows in Fig. 4 by applying the RS code or BCH code again. Thus, the two-group check code can be obtained, and every information bit is protected, checked and corrected by these two independent check codes. In this way, the possible errors can be better checked out.

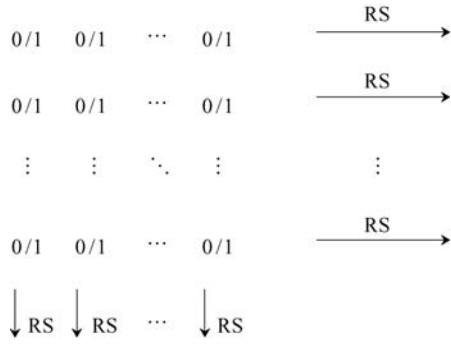


Fig. 4 Parallel-concatenated coding scheme

There is no doubt that the parallel-concatenated coding scheme has good performance in error correction. Although this capability is theoretically better, the codes are involved with the problem of error judgment when decoding. Since the line-code and row-code both can check out the error, it is difficult to accept or reject the checked error, or even no error is judged into the error. Therefore, it is difficult to select the final decoding information.

A feasible judgment method is the application of soft-judgment algorithm, with which the results are recorded down after the line- and row-code are separately corrected. The best results are adopted as the output codes. The concrete way is to compute the Euclidean distance for the corrected vectors and received vectors, which can be expressed as

$$d^2 = \sum_{i=1}^n (\rho_i - s_i)^2,$$

where,  $\rho_i$  is the possible transmitted code word, namely the corrected results, and  $s_i$  is the actually received sequence level.

However, consecutive  $s_i$  cannot be processed and the quantified problem should be taken into account in practice. It is very complex to apply the parallel-concatenated coding scheme in practice, as the considerably complex decoding circuits and soft-judgment algorithms are needed, which makes its real-time performance worse. At this point, it cannot meet the requirement for long-haul optical communications. Therefore, it is not fit to apply this parallel-concatenated coding scheme at present. However, the parallel-concatenated coding scheme is absolutely one of the Super-FEC coding schemes with bright prospect, therefore, it is necessary to research further the parallel-concatenated code in the future.

### 3.3 Improved scheme of consecutive concatenated code with interleaver

Because of the particularities of high-speed optical communications of both superior error correction capabilities

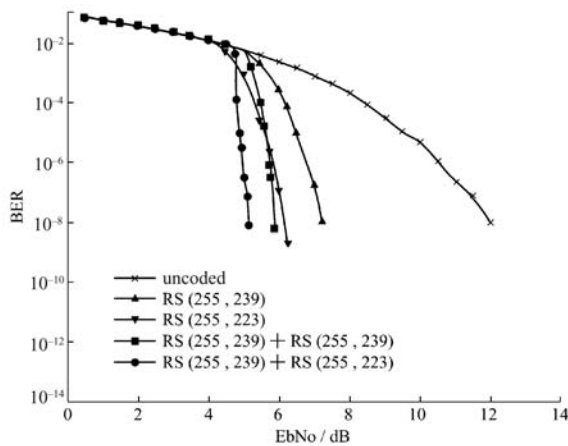
and complex device arising from the complex encoding-decoding circuits, it is considered that the qualities of optical communications can be further improved by combining the concatenated code and interleaved code with the classic RS code or BCH code. Besides, more practical and real-time encoding/decoding can be applied in long-haul optical communications by means of designing novel concatenated coding schemes. Since there are considerably complex encoding/decoding and implementation in the parallel-concatenated coding scheme, it can be assumed to perform twice encoding for the information directly, instead of applying the parallel-concatenated coding scheme. The way to perform twice encoding for the information directly is that the data are encoded again after performing the encoding once, thus there is a larger probability to correct the error that arises. However, it is not obvious to improve the performances for directly performing encoding again based on the data after encoding. Therefore, the data is encoded again after the encoded data is interleaved by the use of the interleaved code scheme, so in this way, there are both further improvements on the error correction performance of the system and considerable enhancement of the capability of outburst-error correction. The considered coding scheme is shown in Fig. 2.

In fact, the consecutive concatenated coding scheme is slightly different from the parallel-concatenated coding scheme. Simply speaking, based on the parallel-concatenated coding scheme shown in Fig. 4, the line-code is assumed as the row-code, which is repeatedly encoded. Namely, as shown in Fig. 4, first the information is encoded once, then a matrix interleaving is performed and encoding is applied once again after interleaving the system code that has been encoded once. Finally, the decoding is implemented twice according to the sequence of the data when decoding. Thus, this scheme can not only enhance chances of checking more error theoretically, but also can solve the outburst-error in optical communication channels more effectively, because the RS code itself can correct a small quantity of the outburst-error. Moreover, the consecutive concatenated code after interleaving the RS code can have more effective outburst-error correction. The most important point is that the encoding/decoding structures of this consecutive concatenated coding scheme are very simple, the structures of the existing RS code, BCH code can be applied, and the soft encoding/decoding scheme is not necessary. Thus, it is possible to implement the consecutive concatenated code in practice, which is good for the special optical communication transmission media. In theory, the consecutive concatenated codes can provide more effective error correction performances because the interleaved codes are introduced, and it can have an improved capability of the outburst-error correction. Therefore, the consecutive concatenated codes are theoretically a more effective



and feasible code type structure as well as a good Super-FEC coding scheme.

To operate the consecutive concatenated code, the two RS (255, 239) codes or one RS (255, 239) and one RS (255, 223) code are concatenated together as Fig. 2. After the different  $E_b/N_0$  (SNR) parameters are set in the additive white Gaussian noise (AWGN) channel of the system structure for the consecutive concatenated code, the BER results of the RS (255, 239)+RS (255, 239) code and the RS (255, 239)+RS (255, 223) code are separately tested under the condition of different SNR. The plots of the simulated results compared with the RS (255, 239) code are shown in Fig. 5.



**Fig. 5** Simulation results of consecutive concatenated code with interleaver

It can be known from Fig. 5 that these novel RS+RS consecutive concatenated codes can have the NCGs of 6.2 and 6.8 dB respectively, which are respectively 1.4 and 2.0 dB more than that of the RS (255, 239) code at the BER of  $10^{-8}$ ; they can respectively have about 6.3 and 7.1 dB NCGs, which are respectively 2–3 dB more than that of the RS (255, 239) code at the BER of  $10^{-13}$ . Obviously, the NCGs of the novel consecutive concatenated code are less than that of the inner-outer serial concatenated code, but their redundancies are 13.8% and 22% respectively, which is more moderate. Therefore, this novel consecutive concatenated code is very suitable to be used in high-speed and long-haul DWDM systems.

In addition, all the consecutive concatenated codes have an additional interleaved code structure, with an interleaved depth of 239 byte (1912 bit) for the RS (255, 239)+RS (255, 239) code and 223 byte (1784 bit) for the RS (255, 239)+RS (255, 223). Therefore, the consecutive concatenated codes have a superior outburst-error correction performance. For

example, the RS (255, 239) code can correct 8 byte errors, and it can correct 15296 bit outburst-errors after interleaving 239 bytes; the RS (255, 223) code can correct 16 byte errors, and it can correct 28544 bit outburst-errors after interleaving 223 bytes. Thus, it can be seen that all the consecutive concatenated codes have a superior error correction performance and may correct the outburst-error possibly from non-linear effects effectively in high-speed and long-haul DWDM systems.

## 4 Conclusions

The three improved concatenated coding schemes for the existing FEC coding technologies in DWDM systems are proposed, and the concatenated codes are theoretically analyzed in this paper. The theoretical analyses and simulation results show that the inner-outer concatenated code has a greater redundancy and the decoding of the parallel-concatenated code is too complex. However, the consecutive concatenated code with the interleaver is a superior concatenated code with advantages such as better error correction performance, moderate redundancy and easy implementation, and is more suitable to be used in high-speed and long-haul DWDM systems.

**Acknowledgements** This work was supported by the Key Special Foundation of the National High Technology Development Program (863) of China (No. 2005AA123730) and the Doctoral Initiation Foundation of Chongqing University of Posts and Telecommunications, China (No. A2007-47).

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