Expert Systems with Applications 42 (2015) 3089–3104

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A novel multi criteria decision making model for optimizing time–cost–quality trade-off problems in construction projects

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ARTICLE INFO

Article history:
Available online 23 November 2014

Keywords:
Discrete time–cost–quality trade-off
Shannon's entropy
Multi-criterion decision-making
Evidential reasoning
Multi-objective optimization
NSGA-II
Construction projects

ABSTRACT

The planning phase of every construction project is entangled with multiple and occasionally conflicting criteria which need to be optimized simultaneously. Multi-criterion decision-making (MCDM) approaches can aid decision-makers in selecting the most appropriate solution among numerous potential Pareto optimal solutions. An evidential reasoning (ER) approach was applied for the first time in the context of project scheduling to identify the best Pareto solution for discrete time–cost–quality trade-off problems (DTCQTPs). An exhaustive framework to synthesize the MCDM approaches with multi-objective optimization techniques was also proposed. To identify all global Pareto optimal solutions, a multi-objective genetic algorithm (MOGA) incorporating the NSGA-II procedure was developed and tested in a highway construction project case study. The Shannon's entropy technique served to determine the relative weights of the objectives according to their contributions to the uncertainty of the results obtained. A benchmark case study of DTCQTP was solved using the proposed methodology, and the Pareto optimal solutions obtained were subsequently ranked using the ER approach. By investigating the performance of each scheduling alternative based on multiple criteria (e.g., time, cost, and quality), the proposed approach proved effective in raising the efficiently of construction project scheduling.

1. Introduction

Construction projects are frequently complicated by circumstances in which decision-makers need to narrow down potential alternatives, and decide on an optimal solution, which represents a compromise between various objectives that can often be conflicting. Multi-objective optimization techniques are a convenient and accessible approach that allows for the simultaneous and robust optimization of conflicting and often non-commensurable objectives. In real practice, it is not advisable to arrive at a decision which is grounded on only meeting a single criterion during the decision-making process. This demonstrates the necessity of using multi-criterion assessment approaches to reach a solution that satisfies all the expectations of the decision-makers (DMs) with an acceptable degree of satisfaction. Decisions made during the conceptual design phase of construction engineering projects have an influential role in the overall cost and performance of a project, and in turn this can lead to significant savings if multi-objective optimization is implemented (Mela, Tiainen, Heinisuo, & Baptiste, 2012).

In every construction project, one of the primary challenges is scheduling its execution. Project scheduling problems (PSPs) are therefore a critical part of a project's overall success, especially in terms of managing organizational resources (Tavana, Abtahi, & Khalili-Damghani, 2014). Many operations research studies have focused on PSPs, and a diverse array of optimization techniques have been employed in an attempt to solve these problems (Zhou, Love, Wang, Teo, & Irani, 2013). Discrete time–cost–quality trade-off problems (DTCQTPs) are a branch of PSPs where a project's network of activities is represented on a node network. While being constrained by relations to preceding/succeeding activities, each individual activity in the project network possesses

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http://dx.doi.org/10.1016/j.eswa.2014.11.032
0957-4174/

various execution modes. The correlation between time, cost, and quality for each activity execution mode is expressed via a point by point definition (Sonmez & Bettemir, 2012; Xu, Zheng, Zeng, Wu, & Shen, 2012).

Using exact solution algorithms such as linear programming, integer programming, and others to solve complex project scheduling networks in the DTCQTPs, is both computationally costly and time consuming. Because exact algorithms require very thorough modeling with various equality and inequality constraints, DTCQTPs are known as NP-hard problems (De, Dunne, Ghosh, & Wells, 1997). Three main categories of DTCQTPs-solving procedures can be identified: (a) exact algorithms, e.g., linear programming, integer programming, dynamic programming, and branch and bound algorithms, etc. (Erenguc, Ahn, & Conway, 2001; Moselhi, 1993); (b) heuristic algorithms (Vanhoucke, Debels, & Sched, 2007); and (c) meta-heuristic algorithms (Afruzi, Najafi, Roghanian, & Mazinani, 2014; Afshar, Kaveh, & Shoghli, 2007; Geem, 2010; Mungle, Benyoucef, Son, & Tiwari, 2013; Tavana et al., 2014; Zhang & Xing, 2010). Numerous multi-objective optimization techniques have been used to solve DTCQTPs, and their resultant optimal Pareto solutions have been generated, plotted, and widely reported (El-Rayes & Kandil, 2005). However, no attempt has been made to aid decision-makers in selecting a solution which satisfies the objectives within an acceptable degree. Owing to the multidisciplinary nature of scheduling problems which are closely entwined with various non-commensurable multiple criteria, determining which solution is the best choice to be implemented can be a difficult task. Multiple criteria decision making (MCDM) methods provide an efficient means for supporting the choice of the preferred Pareto optimum (Mela et al., 2012). In this study, an MCDM approach was amalgamated with multi-objective optimization methods to capitalize on the strength of optimization methods in finding Pareto optimal solutions, and the capability of MCDM techniques to rank them.

The aim of MCDM methods is to assist DMs in order to facilitate the process of organizing and synthesizing the required information in an assessment, so that DMs are satisfied and confident with their decision (Løken, 2007). While they differ in terms of their theoretical background, formulation, questions, and types of input and/or output (Hobbs & Meier, 1994), MCDM methods can be classified into three main categories (Belton & Stewart, 2002): (a) value measurement methods; (b) goal, aspiration, and reference level methods; and (c) outranking methods.

In the value measurement method, each alternative is given a numerical value which indicates the solution rank in comparison with the others. Then, in trading off between multiple criteria, different criteria are weighted according to DM-accepted criteria. Multi-attribute utility theory (MAUT), proposed by Keeney and Raiffa (1976), and analytical hierarchy process (AHP), proposed by Saaty (1980), are examples of this category. Other iterative procedures that emphasize solutions closest to a determined goal or aspiration level fall into the second category. These include the technique for order performance by similarity to ideal solution (TOPSIS) and evidential reasoning (ER). In general, these approaches are focused on filtering the most unsuitable alternatives during the first phase of the multi-criterion assessment process (Løken, 2007). In the outranking methods, the alternatives are ranked according to a pairwise comparison, and if enough evidence exists to judge that alternative a is preferable to alternative b, then it is said that alternative a outranks b. *ELECTRE* (Roy, 1991) and *PROMETHEE* (Brans, Vincke, & Mareschal, 1986) are based on this ranking approach.

ER is an MCDM approach developed in the 1990s, which handles ignorance or incomplete assessments as a type of probabilistic uncertainty, fuzziness and vagueness, and qualitative/quantitative attributes within a unified framework. The ER approach uses belief structures, belief matrices, and a rule/utility-based grading technique to aggregate the information. The main advantage of this procedure is that various types of data can be consistently modeled within a unified procedure (Yang, Wang, Xu, & Chin, 2006). Unlike most conventional MCDM approaches, information aggregation of various types of attributes is based on a distributed assessment framework and evidence combination rules drawn from the Dempster–Shafer theory of evidence (Shafer, 1976). Yang and Xu (2002) designed a Windows™-based user-friendly graphical environment intelligent decision system (IDS), which incorporates the ER approach, and is able to model, analyze, and report results in a flexible interface. Thus, in this study, the ER approach was adapted to provide an efficient means of ranking Pareto solutions and determining the most applicable solution.

Since each method draws on different types of inputs and generates equally different outputs, no direct approach can provide a valid comparison of MCDM methods' relative superiority. However, the most suitable approach is one that, while provided with a user-friendly interface, more importantly best satisfies DMs, providing them sufficient confidence to translate their decisions into actions (Løken, 2007). Numerous study reports have enumerated the fundamental dissimilarities between different MCDM methods, and investigated their individual applicability (Løken, 2007; Mela et al., 2012; Opricovic & Tzeng, 2004). In general, most studies have avoided comparing the strength of different approaches in ranking alternatives, and have solved particular case studies using different MCDM approaches without making any comment on the performance of the different methods. This is due to the limitations stemming from limited test problems; any judgment needs rational justification to make such comparisons valid (Mela et al., 2012).

The application of MCDM approaches in optimization techniques falls into two general domains (Chaudhuri & Deb, 2010): (a) the use of MCDM with a given set of Pareto optimal solutions obtained from multi-objective optimization; or (b) the integration of MCDM into multi-objective optimization as a robust parallel searching tool.

The latter application, implemented in the context of hybrid energy systems (HESs), used a fuzzy TOPSIS-based decision support system to analyze the Pareto front and find the best solution (Perera, Attalage, Perera, & Dassanayake, 2013). Tanaka, Watanabe, Furukawa, and Tanino (1995) proposed an interactive genetic-algorithm-based decision support system to apply multi-criterion optimization in selecting the best of many Pareto solutions using a radial basis function network (RBFN). Hapke, Jaszkiewicz, and Słowiński (1998) applied a discrete version of the Light Beam Search (LBS) as an interactive search process seeking the best of the project schedule amongst alternatives. In product delivery scheduling the LBS has also been applied to minimizing the dispersion of unloading and loading in the consignee warehouse (Grajek, Kiciński, Bieńczak, & Zmuda-Trzebiatowski, 2014).

The second type of application has recently drawn the attention of investigators seeking to develop a systematic approach to assist the DM in seeking the most desirable solution within an interactive framework to. An interactive multi-objective optimization technique, NIMBUS, requires the DM to classify objective functions into 5 different classes at the end of each iteration until an aspiration level is met by the DM (Miettinen & Mäkelä, 1995). Kamalian, Takagi, and Agogino (2004) combined an interactive evolutionary computation (IEC) with existing evolutionary synthesis software to design micro-machined resonators, employing human evaluation of the final designs to evaluate the effectiveness of various design alternatives. Chaudhuri and Deb (2010) proposed an interactive multi-objective optimization and decision-making system employing evolutionary methods (I-MODE) to identify regions of interest on the Pareto frontier and further. In this interactive procedure, these regions were investigated until a desired level of satisfaction was attained. However, a review on the literature shows that no attempt has ever been made to use an ER approach to reduce the size of Pareto optimal solutions.

Decision-making is very common in various facets of engineering, and PSPs are no exception. In the few studies which have applied MCDM methods to selecting the best Pareto solutions in addressing PSPs, only the Pareto solutions are obtained, plotted and reported. This was one of the issues that motivated the authors of the present paper to apply MCDM methods in solving DTCQTP with the goal of helping DMs select the best project schedule for a given project. Apart from the research of Mungle et al. (2013), in which the best solution was selected based on a fuzzy clustering technique, no other study proposes a comprehensive framework to integrate MCDM methods with multi-objective optimization techniques to more efficiently schedule a construction project. The present paper's novel contributions includes a proposed DTCQTP modeling skeleton encompassing a multi-objective genetic algorithm (MOGA), modified for systematic handling of multi-criterion assessment, along with an ER approach to rank the Pareto solutions. In furtherance of demonstrating the compatibility and ability of the proposed approach, a benchmark case study of DTCQTP is solved, and the best Pareto solution identified.

Following a general description of the methodology with a detailed framework, the present paper details the use of a multi-objective genetic algorithm (MOGA) with NSGA-II procedures in forming the Pareto sets employed in solving DTCQTP. There follows a presentation of the Shannon entropy technique in obtaining the associated relative normalized weights of each objective. Later, an evidential reasoning (ER) approach is described in detail, and the framework to integrate the MCDM approaches with multi-objective optimization techniques, is also proposed in this section. The modeling and formulations to solve DTCQTP are presented in following section. In order to demonstrate the efficiency of the proposed approach, a benchmark case study was solved and the global Pareto solutions identified. The most appropriate solution with respect to the expectations of the DMs, was derived using an ER approach. In the same section, a comparison is made of the efficiency of the results obtained through the proposed methodology and those of Mungle et al. (2013). This comparison showed that using the ER approach was highly effective in finding the optimal solution. Concluding remarks and ideas for future research follow.

2. Methodology

The process of developing the proposed methodology (Fig. 1) begins by setting up and gathering the required information related to form the project scheduling network diagram. Furthermore, the multiple modes of each activity must be determined considering the unavoidable constraints *viz.* the resource limitation and the resource usage plans, technology limitations, various construction methods, etc. Duration (time), as well as the cost and quality of each activity can be quantified. The MOGA with NSGA-II procedure is tailored to guide the algorithm to converge the global Pareto optimal front. Each objective (e.g., time, cost, and quality) will have different relative normalized weights. These can be obtained using the Shannon's entropy technique. In order to relate the aforementioned objectives to the overall performance criterion, a hierarchical structure can be developed with associated computed weights to show how the overall performance can be evaluated. In subsequent steps, the ER approach helps DMs evaluate each alternative (Pareto solutions), assign overall utility scores indicative of their degree of satisfaction with each alternative, while considering all criteria simultaneously. Eventually, the Pareto solution with the highest corresponding utility score is selected as the best solution.

2.1. Genetic algorithm

Various approaches have been implemented to deal with solving DTCQTP (Afshar et al., 2007; El-Rayes & Kandil, 2005; Mungle et al., 2013; Tavana et al., 2014), but no explicit results of Pareto solutions have been reported. Consequently, in this study, the genetic algorithm (GA) was chosen to find the Pareto solutions. The genetic algorithm (GA) is a stochastic search method applicable to optimization problems which is based on biological behavior (Wilson, 1997). In this study, one of the most popular and widely-used metaheuristic algorithms, the multi-objective genetic algorithm (MOGA), was used. El-Rayes and Kandil (2005) also used MOGA to deal with a DTCQTP. Due to space limitations, and since the MOGA procedures are widely known, the steps are only briefly discussed below.

2.1.1. Initial population and chromosome representation

First proposed by Holland (1975), GA is a chromosome-based evolutionary algorithm, which mimicking nature, attempts to seek better offspring from a given evolving population during each subsequent generation. The chromosome consists of cells which are known as genes. In this paper, the position of the gene indicates the number of the activity and the value of each gene represents the option which is assigned for the activity execution mode. Table 1 shows a sample chromosome with 6 activities.

The initial population of the algorithm is generated randomly, allowing the entire range of possible solutions. The population size is set at 300, a value selected based on preliminary model runs, but sufficiently large to ensure convergence to the optimal solutions. Gene values can only take values which do not violate the number of options available for that activity (the upper limit). For example, if activity number 3 has only 4 options, then the gene value of the third position can be an integer in the interval of [1,4]. This representation of the chromosome ensures that no chromosome leads to a non-feasible solution, thus avoiding unnecessary computational efforts and saving time.

2.1.2. Crossover and mutation operator

During each generation, similar to the natural evolutionary process, a pair of 'parent' solutions is selected for breeding and producing a pair of 'children'. The crossover operator attempts to reproduce a pair of 'children' which typically shares many of the characteristics of its 'parents' (Eiben & Smith, 2003). The two point crossover is selected as the crossover operator since it is shown to be efficient in solving DTCQTPs (Mungle et al., 2013).

In order to preserve diversity within the newly generated population one must generate a number of solutions which are entirely different from the previous solutions. Analogous to biological mutation, the mutation operator alters only one gene in a chromosome to generate a non-identical 'child'. In order to alter the chromosomes a swap mutation operator is used to change the values of two randomly selected genes in a chromosome (Eiben & Smith, 2003). The upper limit for the values of the genes is the only constraint which must be checked during the alteration of each chromosome, otherwise non-feasible chromosomes are produced. If the value of a mutated gene violates the upper limit, it is replaced by the maximum allowable value for that specific gene to ensure that no 'child' leads to a non-feasible solution. Obviously, since the lower limit value for all the genes is 1, there is no need to check whether or not there is any value lower than 1.

2.1.3. Selection procedure for next generation

In multi-objective optimization problems, the selection procedure is more complicated than in single-objective optimization problems. Each Pareto-optimal solution represents a compromise considering different objectives, such that the component of the

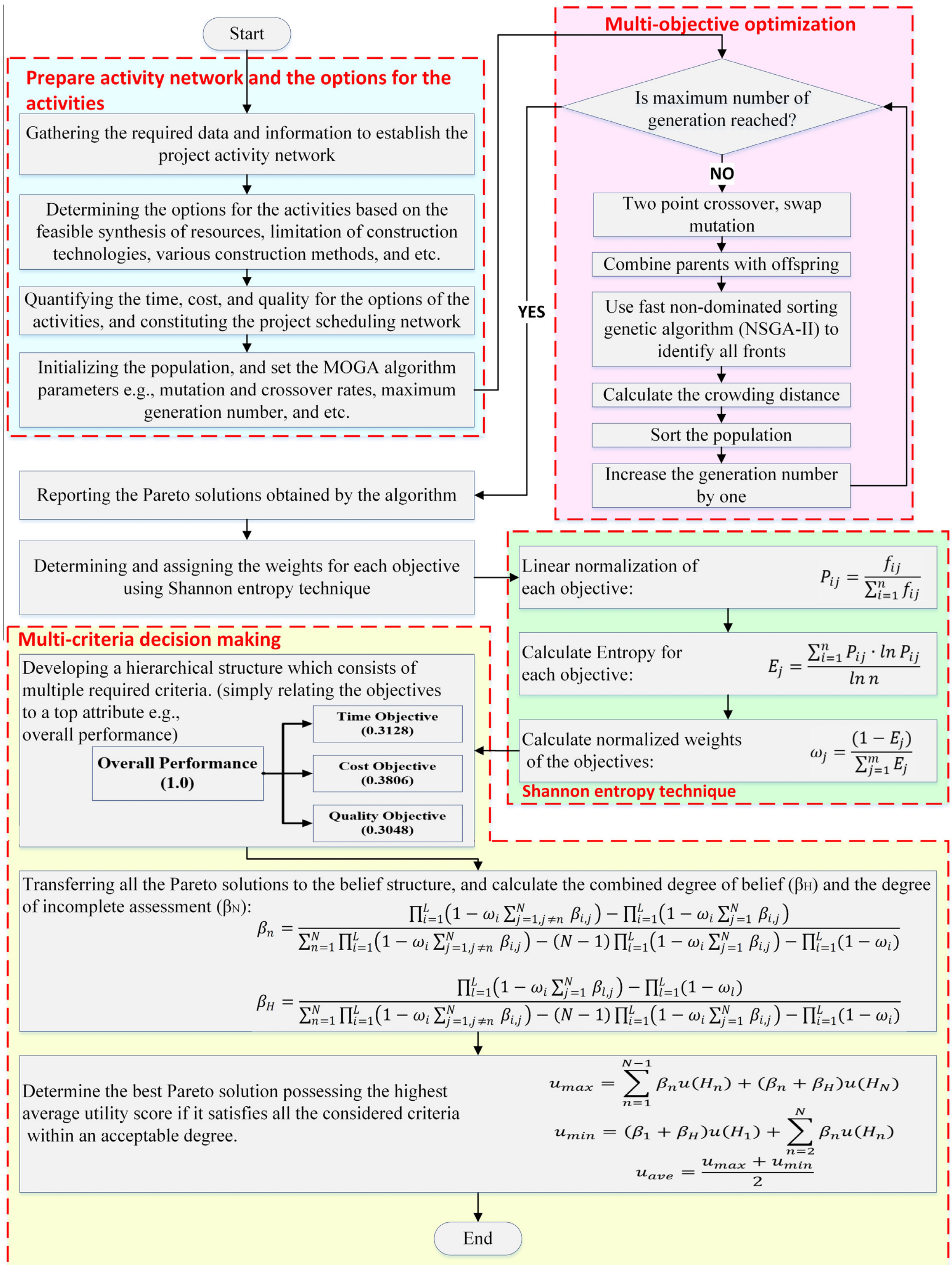


Fig. 1. Flowchart of the proposed methodology.

corresponding vector of objectives cannot simultaneously be improved (Mungle et al., 2013). Any improvement in an objective requires at least one of the other objectives to be scarified, and therefore a set of trade-offs exists. The improved non-dominated sorting genetic algorithm (NSGA-II), proposed by Deb, Pratap, Agarwal, and Meyarivan (2002), is an evolutionary algorithm used

Table 1
Structure of a chromosome.

Number of activity:	1	2	3	4	5	6
Execution mode:	3	2	4	3	2	1

to generate sets of Pareto-optimal solutions. (Deb et al., 2002) in solving a number of test problems showed the NSGA-II to outperform the Pareto Archive Evolutionary Strategy (PAES) and strength-Pareto evolutionary algorithm (SPEA) in converging the near true Pareto front and in maintaining diversity among the solutions. Low computational requirements, an elitist approach, a parameterless niching approach, and a simple constraint handling procedure, are some of the features of NSGA-II that contribute to its wide application in evolutionary algorithms. In the following section, the NSGA-II procedure is explained in detail (Deb et al., 2002).

NSGA-II uses a fast non-dominated sorting algorithm to form non-dominated fronts $\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_R$, in addition to considering a fixed population size of N during each generation $\bar{t}$, which consists of non-dominated fronts in a combined population of the parents and offspring $P_{\bar{t}}$. The next population $P_{\bar{t}+1}$ is filled starting with solutions in $\bar{F}_1$, then $\bar{F}_2$, and so on. If k is the index of a non-dominated front, $\bar{F}_k$, so that $|\bar{F}_1 \cup \bar{F}_2 \cup \ldots \cup \bar{F}_k| \leqslant N$ and $|\bar{F}_1 \cup \bar{F}_2 \cup \ldots \cup \bar{F}_k \cup \bar{F}_{k+1}| > N$ then solutions from fronts $\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_k$ are placed in $P_{\bar{t}+1}$, and the remaining solutions $(N - |P_{\bar{t}+1}|)$ are selected from the least crowded solutions. Let $p_{\bar{t}}$ and $q_{\bar{t}}$ denote a pair of chromosome in $\bar{t}$th generation, and S_P a set of solutions that $p_{\bar{t}}$ dominates. Then if Q denotes the set of solutions belong to $(k+1)$th front, and n_P and n_q are the domination counters indicating the number of solutions which dominates the solutions $p_{\bar{t}}$, and $q_{\bar{t}}$, respectively; the pseudo-code of the identifying the non-dominated fronts is as follows:

Fast non-dominated sorting procedure:	
For each $p_{\bar{t}} \in P_{\bar{t}}$; set	
$S_P = \emptyset$ and $n_P = 0$;	
For each $q_{\bar{t}} \in P_{\bar{t}}$;	
If $(p_{\bar{t}} \prec q_{\bar{t}})$ then $S_P = S_P \cup \{q_{\bar{t}}\}$	if $p_{\bar{t}}$ dominates $q_{\bar{t}}$, then add $q_{\bar{t}}$ to the set of solutions dominated by $p_{\bar{t}}$.
Else if $(q_{\bar{t}} \succ p_{\bar{t}})$ then $n_P = n_P + 1$	if $q_{\bar{t}}$ dominates $p_{\bar{t}}$, then increment the domination counter of $p_{\bar{t}}$.
If $n_P = 0$ then	$p_{\bar{t}}$ belongs to the first front.
$\bar{F}_1 = \bar{F}_1 \cup \{p_{\bar{t}}\}$	
$k = 1$	initialize the front counter.
While $\bar{F}_i \neq \emptyset$; $Q = \emptyset$	Q is used to store the members of next front.
For each $p_{\bar{t}} \in \bar{F}_i$	
For each $q_{\bar{t}} \in S_P$;	
$n_P = n_P - 1$	
If $n_q = 0$ then	$q_{\bar{t}}$ belongs to the next front.
$Q = Q \cup \{q_{\bar{t}}\}$	
$k = k + 1$; $\bar{F}_i = Q$	

In NSGA-II the crowding distance, $I_{[p_{\bar{t}}]distance}$ is an estimation of the density of solutions surrounding solution $p_{\bar{t}}$ in the population. Finally, the crowded-comparison operator $(\prec_n)$ guides the selection procedure at various stages of the algorithm towards a uniformly spread-out Pareto-optimal solution (Konak, Coit, & Smith, 2006). More specifically, between two solutions that belong to different fronts, the solution within the lower front is preferred, and if they are in the same front then the one with the lower crowding distance is judged to be superior. Let I represents the set of all non-dominated solutions where, $|I| = l$, and f_m^{max} and f_m^{min} denote the maximum and minimum values of the mth objective function, respectively. The pseudo-code of crowding distance calculation is then:

Crowding distance assignment procedure:	
$l = \|I\|$	number of solutions in I.
For each $p_{\bar{t}}, I_{[p_{\bar{t}}]distance} = 0$	initialize the distance.
For each objective m	sort using each objective value and set the distance of first and last points equal to infinity.
$I = sort(I,m); I_{[1]distance} = I_{[l]distance} = \infty$	
For $p_{\bar{t}} = 2$ to $(l-1)$	for all remaining points distance will be calculated.
$I_{[p_{\bar{t}}]distance} = I_{[p_{\bar{t}}]distance} + (I_{[p_{\bar{t}}+1],m} - I_{[p_{\bar{t}}-1],m})/(f_m^{max} - f_m^{min})$	

The NSGA-II procedure after obtaining the non-dominated fronts and crowding distances proceeds as follows:

NSGA-II procedure:	
$\bar{F} = \{\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_R\}$	sort non-dominated fronts in $\bar{F}$.
$P_{\bar{t}+1} = \emptyset$ and $k = 1$	
While $\|P_{\bar{t}+1}\| + \|\bar{F}_k\| \leqslant N$	until the parent population is filled include kth front in the parent population.
$P_{\bar{t}+1} = P_{\bar{t}+1} \cup \bar{F}_k$ and $k = k + 1$	
Sort$(\bar{F}_k, \prec_n)$	sort using crowded-comparison operator.
$P_{\bar{t}+1} = P_{\bar{t}+1} \cup \bar{F}_k[1\ to\ (N - \|P_{\bar{t}+1}\|)]$	choose the first $(N - \|P_{\bar{t}+1}\|)$ elements of $\bar{F}_k$.

2.1.4. Termination criterion

In order to stop the algorithm, a termination criterion representing the maximum number of generations is selected to force the algorithm to continuously seek superior solutions. The higher the maximum number of generations, the more computational effort is required; however, a very low value prevents the algorithm to converge to the optimal solution. Thus, based on preliminary model runs, the maximum number of generations is set to 1000, since higher values did not contribute to improvement of the solutions obtained.

2.2. Determining the normalized weight vector

In order to ascertain the normalized weights of the objectives, the Shannon's entropy technique as first proposed by Shannon (1948) was used in the context of a mathematical model for communication and information (Wang & Lee, 2009). Order preference by similarity to ideal solution (TOPSIS) (Behzadian, Khanmohammadi Otaghsara, Yazdani, & Ignatius, 2012), the simple additive weighted (SAW) approach (Huang, Chang, Li, & Lin, 2013), the ordered weighted averaging (OWA) method

(Yager, 2004), and the analytical hierarchy process (AHP) (Ormerod & Ulrich, 2013) are other representative multi-attribute weighting approaches. In this study, weighting of the attributes was based on the crisp values of the objectives, since, based on the aforementioned techniques, DMs' preferences might not be sufficient and might result in a biased judgment regarding weights. The Shannon's entropy technique expresses the relative intensities of the attribute importance based on the discrimination among data to assess the relative weights (Zhang, Wang, & Wang, 2014). Thus, the Shannon's entropy can provide a more reliable assessment of the relative weights for the objectives in the absence of the DMs' preferences.

Shannon's entropy is a measure of uncertainty associated with the source of information. The information can simply be defined as the values of the objectives. The uncertainty in information is addressed in the Shannon's entropy technique by using probability theory. The underlying assumption is that an event which has a lower probability of happening, is more likely to provide more information by its occurrence. In this respect, an objective which has a sharp distribution also has a lower relative importance with respect to the objective which follows a biased-distribution over an interval (Deng, Yeh, & Willis, 2000; Khorasani et al., 2013).

The Shannon's entropy parameter for the jth objective, denoted by E_j, can be calculated by:

$$E_j = \frac{\sum_{i=1}^{n} P_{ij} \cdot \ln P_{ij}}{\ln n}, \quad \text{where } i \in \{1, 2, \ldots, n\} \text{ and } j \in \{1, 2, \ldots, m\} \tag{1}$$

$$P_{ij} = \frac{f_{ij}}{\sum_{i=1}^{n} f_{ij}}, \quad \text{where } i \in \{1, 2, \ldots, n\} \text{ and } j \in \{1, 2, \ldots, m\} \tag{2}$$

$$\omega_j = \frac{(1 - E_j)}{\sum_{j=1}^{m} E_j}, \quad \text{where } \sum_{j=1}^{m} \omega_j = 1 \tag{3}$$

The jth objective function of ith solution is denoted by f_{ij}, and the linear normalization of jth objective for ith solution, P_{ij}, is then used to calculate the Shannon's entropy value (E_j) of jth objective. The number of solutions and objectives are denoted by n and m, respectively. Eventually, the corresponding relative normalized weight for jth solution indicated by ω_j, is obtained using Eqs. (1)–(3).

2.3. Evidential reasoning

Decision making is very common in various facets of engineering, and PSPs are no exception. Many methods have been developed and used to deal with MCDM problems: additive utility (value) function methods (Keeney & Raiffa, 1976), outranking methods (Guitouni, Martel, Bélanger, & Hunter, 2008), and Evidential Reasoning (ER) (Bazargan-Lari, 2014), amongst others. MCDM is an inherently intricate bi-level dynamic process that facilitates efficient cooperation among managers and engineers. Using MCDM methods, DMs are able to handle their criteria and preferences more efficiently, and furthermore the data can be easily transferred to the engineers, thus eliminating the time-consuming and costly iterative procedure of reviews and feedback during the planning phase of construction. The DMs at the managerial level form the preference structure, while scrutinizing the concomitants of selecting each solution constitute the engineering level (Opricovic & Tzeng, 2004). The framework of amalgamating the MCDM approaches with the multi-objective optimization techniques is shown in Fig. 2. The first two stages are undertaken at the managerial level, and the remainder at the engineering level.

The ER approach is a general approach for analyzing MCDM problems under uncertainty. The ER approach encompasses all the steps in the MCDM framework by using belief structures and belief matrices. The ER methodology for information aggregation comprises a rule-and-utility-based information transformation technique for dealing with various types of information of both a quantitative and qualitative nature under the necessary conditions of utility and value equality. The ER approach for MCDM problem analysis involves the following steps (Xu, 2012):

1. Identifying and analyzing a decision problem having multiple and occasionally conflicting criteria.
2. Transforming various belief structures into a unified belief structure using the rule or utility based information transformation techniques.
3. Aggregating information using the ER algorithm.
4. Generating the distributed assessment outcomes, utility scores, or utility intervals if some information is missing. In this step, the solution with higher utility score is preferred over the solutions with lower corresponding values.

In multi-objective optimization where there exists conflict among the objectives, the Pareto solutions can be very numerous, and it might be tedious for DMs to finally reach a single compromise solution. The output of any multi-objective optimization algorithm is a collection of non-dominated solutions. Each non-dominated solution satisfies the project's objectives to a degree, which necessitates the implementation of the MCDM approach to select the best non-dominated solution. The MCDM problems deal with how to rank the solutions based on multiple criteria. Mungle et al. (2013) used a fuzzy clustering technique in order to find a non-dominated solution representing the best scheduling alternative. However, it has some limitations since it cannot impart multiple criteria (e.g., qualitative and uncertain quantitative attributes). Hence, it is believed that by using the ER approach the non-dominated solutions can be ranked according to multiple attributes which can provide more practical and efficient schedule alternatives for PSPs.

In finding the best solution, a wide range of various quantitative and qualitative criteria should be identified. These criteria can be gathered and obtained via project stakeholders' and managers' expectations. One of the approaches to deal with MCDM is the ER approach, which is able to consider various types of uncertainties such as ignorance and fuzziness. In conducting surveys, some participants might not give any answer to a specific attribute due to not having any or little knowledge of the matter, which is termed 'ignorance' in the ER approach. Different human judgments *viz.* crisp/vague and complete/incomplete can be imparted into the ER approach utilizing the belief structure. The ER approach is based on the Dempster–Shafer theory of evidence (Shafer, 1976), and decision theory for dealing with various types of criteria of both a quantitative and qualitative nature in decision-making (Bazargan-Lari, 2014). The ER approach has also been applied in research areas such as regional hospital solid waste assessment (Abed-Elmdoust & Kerachian, 2012), and determining the best layout of water quality monitoring stations (Bazargan-Lari, 2014). In order to implement an ER approach, the following steps must be followed:

1. Identify and analyze the multiple assessment criteria of the MCDM problem through exhaustive investigation from expert negotiation, identifying stakeholders' and DMs' expectations and requirements, as well as understanding DMs' preferences in terms of the weight associated with each criterion. Various types of contributing attributes (*e.g.*, quantitative, qualitative, precise numbers, fuzziness uncertainty, belief structures and comparison numbers) are gathered in this step. For example, the cost of construction equipment might be precise, while the cost of excavation might be expressed within a range. The technical ability of a subcontractor might be expressed as a belief structure for which it might be 'Good' to a degree of belief

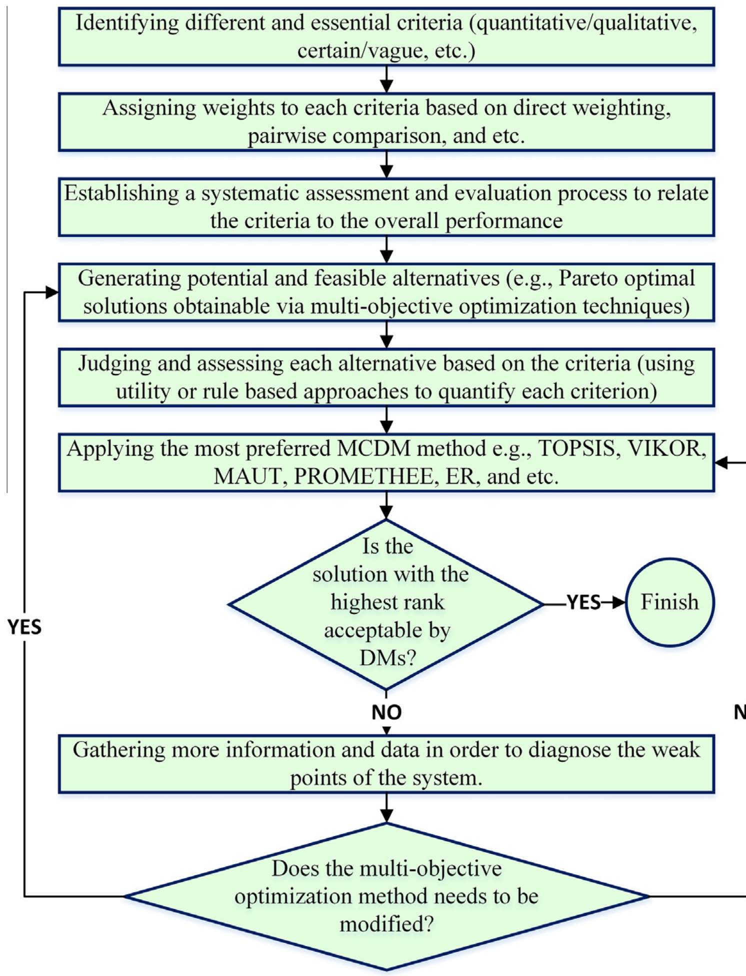


Fig. 2. Framework outlining the amalgamation of MCDM with multi-objective optimization methods.

of 34%, and simultaneously be 'Very Good' with a degree of belief of 63%, which is expressed as {(Good, 0.34), (Very Good, 0.63)}. The summation of both degrees of belief is equal to 97% (0.34% + 0.63%) with 3% as ignorance. Given the lack of knowledge regarding the technical capability of a subcontractor, some might prefer to not assess that criterion termed lack of evidence and therefore the summation of probabilities does not reach one.

2. Different types of belief structures should be transformed into a unified belief structure using the rule or utility-based information transformation technique. In this step, the belief structures are translated; for example, the 'Very Good' and 'Very Bad' are assigned one and zero respectively, and the other grades may/may not be evenly distributed (Fig. 3).
3. The ER algorithm and formulation can serve to aggregate the assessment information of the agglomeration of various types of criteria to obtain the overall assessment of each alternative.
4. Develop the utility scores or utility intervals in the case of missing information. The utility-based ranking can judge each alternative overall performance considering every single criterion simultaneously through a systematic and rational prioritizing methodology which provides the best alternative for the schedule project, and one which is favored by all the DMs since the final acceptable solution is a trade-off between the preferences of the DMs.

The weights of each criterion can be determined through different approaches (e.g., pairwise comparison, directly by the DMs, Shannon's entropy, etc.). Since the weights express the relative importance, the normalized values are more beneficial than the absolute values

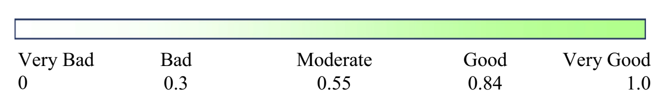


Fig. 3. Utility based representation of a belief structure.

$$\omega_i = \frac{W_i}{\sum_{i=1}^{L} W_i}, \quad i \in \{1, 2, \ldots, L\} \tag{4}$$

$$\text{S.T. } 0 \leqslant \omega_i \leqslant 1, \quad \sum_{i=1}^{L} \omega_i = 1$$

The weight assigned to the ith basic criterion indicating the reflectiveness to the general criterion is denoted by W_i and ω_i is the normalized weight of ith basic criterion which is calculated using Eq. (4). The linguistic terms such as 'worst', 'good', and so on are called grades where the whole set of grades is $H = \{H_n, n = 1, 2, \ldots, N\}$. As demonstrated by Guo, Yang, Chin, and Wang (2007) and Wang, Yang, and Xu (2006), the analytical format of the ER algorithm can calculate the combined degree of beliefs β_n of the nth grade, where $n \in \{1, 2, \ldots, N\}$ and β_H represents the incompleteness assessment of the whole set of H. Different from the recursive format of the ER approach, the analytical format is preferred since it does not require any iteration to evaluate the multiple attributes,

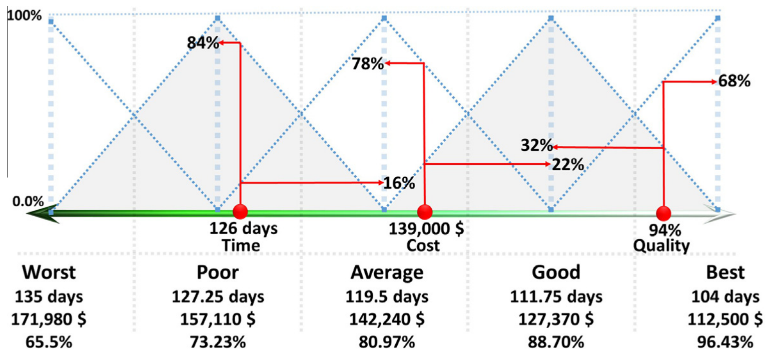


Fig. 4. Transferring each attribute to belief structure.

thus providing greater flexibility for optimization and evaluation (Yang et al., 2006). β_n and β_H can be calculated using Eqs. (5) and (6), respectively:

$$\beta_n = \frac{\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1,j\neq n}^{N}\beta_{i,j}\right)-\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1}^{N}\beta_{i,j}\right)}{\sum_{n=1}^{N}\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1,j\neq n}^{N}\beta_{i,j}\right)-(N-1)\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1}^{N}\beta_{i,j}\right)-\prod_{i=1}^{L}(1-\omega_i)} \quad (5)$$

$$\beta_H = \frac{\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1}^{N}\beta_{i,j}\right)-\prod_{i=1}^{L}(1-\omega_i)}{\sum_{n=1}^{N}\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1,j\neq n}^{N}\beta_{i,j}\right)-(N-1)\prod_{i=1}^{L}\left(1-\omega_i\sum_{j=1}^{N}\beta_{i,j}\right)-\prod_{i=1}^{L}(1-\omega_i)} \quad (6)$$

The degree of belief of ith basic criterion for the jth grade is denoted by $\beta_{i,j}$ and N is the number of grades of set H. In order to rank the alternatives, one must translate the combined degrees of belief and the incomplete assessment (β_n and β_H) into one single utility score. Hence, it is necessary to generate numerical values equivalent to the belief structure:

$$u_{max} = \sum_{n=1}^{N-1}\beta_n u(H_n) + (\beta_n + \beta_H)u(H_N) \quad (7)$$

$$u_{min} = (\beta_1 + \beta_H)u(H_1) + \sum_{n=2}^{N}\beta_n u(H_n) \quad (8)$$

$$u_{ave} = \frac{u_{max} + u_{min}}{2} \quad (9)$$

where u_{max}, u_{min}, and u_{ave} are the maximum, minimum, and the average utility score, such that $u(H_n)$ is a function showing the utility score of the nth grade. For example, if $n = 5$, and all the grades are spaced equally in the interval of [0,1], then $u(H_n)$ = {0,0.25,0.5,0.75,1}. From Eqs. (7) and (8) it is clear that if there is no incomplete assessment ($\beta_H = 0$), all three cases of maximum, minimum, and average utility scores are exactly the same and can be determined as:

$$u_{max} = u_{min} = u_{ave} = \sum_{n=1}^{N}\beta_n \cdot u(H_n) \quad (10)$$

In Table 2, a stepwise procedure to evaluate a given solution is solved in order to illustrate the underlying idea of the ER in more detail.

Table 2
A stepwise example showing the procedures in ER evaluation.

The ER approach evaluation of a solution with corresponding values for the attributes as follows:

	Attributes:	Time (day)	Cost ($)	Quality (%)
	Values:	125	139,000	70.40
Step 1.	Determine the worst and best possible values for the attributes (e.g., time, cost, and quality)			
		Time (day)	Cost ($)	Quality (%)
	Best:	104	112,500	96.43
	Worst:	135	171,980	65.50
Step 2.	Assign normalized weights for each attribute.			
	Attributes:	Time	Cost	Quality
	ω_i:	0.3128	0.3806	0.3048

Step 3. Calculate the belief structure for each attribute as shown in Fig. 4. The x-axis indicates the grades e.g., 'Worst', 'Poor', 'Average', 'Good', 'Best' and the corresponding values for each attribute. The quality of 94% lies between 'Good' and 'Best' grades with the quality values of 88.7% and 96.43%, respectively. This attribute belongs to 'Best' grade with 68% belief, and with 32% degree of belief belongs to grade 'Good'. The same procedure is done for the time and cost attributes. The belief structures for each attribute are as below:

Belief structures of the attributes

Grades:	Worst	Poor	Average	Good	Best
Time	0	84	16	0	0
Cost	0	0	78	22	0
Quality	0	0	0	32	68

Step 4. Calculate the combined degree of belief β_n based on Eq. (5). Since there is no incomplete assessment $\beta_H = 0$

Grades:	Worst	Poor	Average	Good	Best
β_n:	0.0	0.244	0.377	0.186	0.193

Step 5. Calculate the utility scores according to Eq. (5)

$$u_{max} = u_{min} = u_{ave} = 0.402$$

The average utility score indicates that the solution satisfies DMs to an extent of 40.2% when considering all the attributes simultaneously

3. Modeling formulation and interpretation

The cost component for each activity can be an agglomeration of various factors which are required to complete the activities successfully. Generally, direct and indirect costs are the two main elements that constitute the overall cost of each activity. The direct cost is the overall cost spent directly in order to successfully accomplish the activities, and is directly related to the execution phase. The direct cost of jth option of ith activity is denoted by $\tilde{c}_{ij}$. The cost might also consist of indirect costs ($\tilde{C}_d$), which originate from the managerial cost of a construction organization and any other indirect costs which can be measured in cost per day. In this study, the indirect cost is assumed to be a fixed amount, and its amount varies with project duration. Different types of construction contracting methods may also impose other types of costs, namely, tardiness penalty ($\tilde{C}_p$) and incentive cost ($\tilde{C}_{in}$), both of which can be measured in cost per day. For any delay occurring in total project time in comparison with the DMs' desired time ($\tilde{T}_d$), the main contractor(s) might be charged a tardiness fine on a daily basis, usually at a fixed price per day. In contrast, for any early completion, they might be rewarded for each day of this early completion period.

A thorough model to solve the DTCQTP can be expressed as:

$$\text{Minimize } f_1 = \max_{\forall p \in P}\{\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_n\} \tag{11}$$

$$\text{Minimize } f_2 = \sum_{i=1}^{N} \tilde{c}_{ij} + f_1 \cdot \tilde{C}_d + \bar{u}(f_1 - \tilde{T}_d) \cdot (\tilde{T}_d - f_1) \cdot \tilde{C}_p + \bar{u}(\tilde{T}_d - f_1) \cdot (\tilde{T}_d - f_1) . \tilde{C}_{in} \tag{12}$$

$$\text{Maximize } f_3 = \alpha Q_{min} + (1 - \alpha) Q_{ave} \tag{13}$$

$$Q_{min} = min\{\tilde{q}_{ij} : x_{ij} = 1\} \tag{14}$$

$$Q_{ave} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{m} q_{ij} \cdot x_{ij}}{N} \tag{15}$$

If all paths of the network is the set of $P = \{p|p = 1, 2, \ldots, n\}$, and i_p is the ith activity on path p, and n_p is the number of activities on path p, then the total implementation time of pth path ($\tilde{T}_p$) is the summation of the durations of all the activities on path p, which can be mathematically calculated as $\tilde{T}_p = \sum_{i_p}^{n_p} \tilde{t}_{ij}$. The first objective function (f_1) refers to the total project duration which is obtained by considering the maximum implementation time, $\tilde{T} = \{\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_n\}$, where $\tilde{T}$ represents all paths of the project network (Eq. (11)). The second objective function (f_2) represents the total project cost which is the summation of each activity cost ($\tilde{c}_{ij}$) denoting the direct cost. It is later added to the indirect cost calculated by multiplying the fixed cost of the indirect cost ($\tilde{C}_d$) with the project duration (f_1). Other cost components such as the project tardiness penalty and incentive costs are also considered. The unit step function, $\bar{u}(x)$, is either one or zero for non-negative and negative values of x, respectively. Then, the total project cost can be mathematically computed as shown in Eq. (12). x_{ij} is the index variable of ith activity when performed in jth option. If $x_{ij} = 1$ then the jth option for ith activity is selected and when $x_{ij} = 0$ it means that the jth option of ith activity is not selected. The next objective function (f_3) estimates the project quality through Eq. (13). If the quality of the ith activity of jth option is shown by $\tilde{q}_{ij}$ the estimation of the project overall quality is a linear relationship between the minimum quality of all the selected alternatives (Q_{min}), which is calculated according to Eq. (14), and the average quality of all the chosen alternatives (Q_{ave}) which is calculated using Eq. (15). A higher value of α means a greater emphasis on the fact that the quality of no activity in the schedule is too low, while a lower value ensures that the overall project quality is aimed at not lying too far away from the average quality (Q_{ave}). Using the α parameter ensures that the third objective (f_3) represents a close estimation of the overall project quality since only the average value might not be a good measurement of the total obtainable project quality. Therefore, if an activity with a very low quality is selected, it lowers f_3 more significantly than in the case where only average value is considered, thus automatically, throughout the optimization algorithm, an attempt is made so that not only the average quality is at a high standard, but also no activity with a very poor quality is to be selected. Using the step function one ensures that either the tardiness penalty or the incentive cost is added to the total cost. It must be noted that the total cost is from the viewpoint of the project's main contractor and that of the owner, meaning that incentive and tardiness costs are summed negatively and positively with the total cost, respectively. If we consider the cost from the owner's viewpoint, then the incentive cost would be negative but the tardiness cost would be positive. To take into account all the expenditures in relation to the project the first case is considered (*i.e.*, the contractors' viewpoint), which is the more common approach in DTCQTPs.

4. Implementation of the proposed model

To verify and demonstrate the efficacy of the proposed model to integrate the ER approach into DTCQTPs, a highway construction project activity network consisting of 18 activities, first proposed by Feng, Liu, and Burns (1997), was adapted. The activity on the node network diagram of the case study is illustrated in Fig. 5. Mungle et al. (2013) modified the data to account for the quality associated with each of the options of the activities. The corresponding time, cost, and quality for each mode of the activities are listed in Table 3. The indirect cost is assumed to be 50$ per day with the due date being taken as 121 days. The incentive reward and the tardiness penalty are $120/day and $200/day, respectively. The relative importance, α, between Q_{min} and Q_{ave} is taken as 0.4 which ensures that no activity in the schedule is preferred that has a quality lower than the average quality of all the selected options for the activities.

In DTCQTPs where each activity can be executed in several modes, the solution space increases exponentially for medium and large size problems (Tavana et al., 2014). In this case example, each activity possesses approximately 3.4 alternatives, leading to 3.6 billion possible activity schedules for the entire project. Hence, a tailored MOGA was developed in order to have the capacity to converge on near true optimal solutions among the large number of potential solutions for the project schedule. Furthermore, a proposed model was enabled to select the best solution among numerous Pareto solutions via the ER approach. The novelty of employing an ER approach in a MCDM problem of DTCQTP also provides more practical solutions.

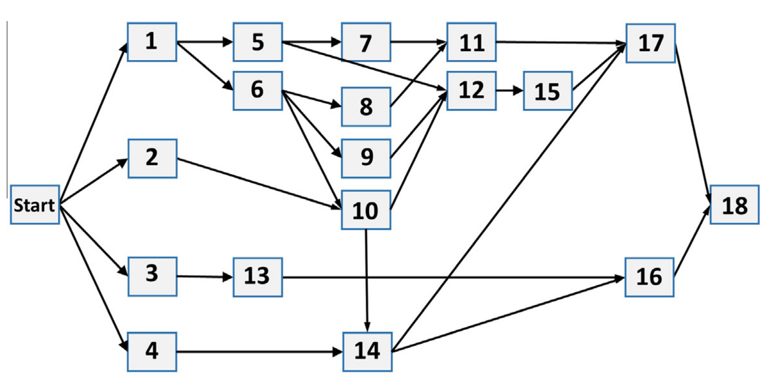


Fig. 5. Eighteen activities on node network (ANN) representation of the case example.

Table 3
Data of the 18 activities of the network case example.

Activity number	Preceding activities	Options-(time (day), cost ($))				
		1	2	3	4	5
1	–	(14,2400)	(15,2150)	(16,1900)	(21,1500)	(24,1200)
2	–	(15,3000)	(18,2400)	(20,1800)	(23,1500)	(25,1000)
3	–	(15,4500)	(22,4000)	(33,3200)	–	–
4	–	(12,45,000)	(16,35,000)	(20,30,000)	–	–
5	1	(22,20,000)	(24,17,500)	(28,15,000)	(30,10,000)	–
6	1	(14,40,000)	(18,32,000)	(24,18,000)	–	–
7	5	(9,30,000)	(15,24,000)	(18,22,000)	–	–
8	6	(14,220)	(15,215)	(16,200)	(21,208)	(24,120)
9	6	(15,300)	(18,240)	(20,180)	(23,150)	(25,100)
10	2,6	(15,450)	(220,400)	(33,320)	–	–
11	7,8	(12,450)	(16,350)	(20,300)	–	–
12	5,9,10	(22,2000)	(24,1750)	(28,1500)	(30,1000)	–
13	3	(14,4000)	(18,3200)	(24,1800)	–	–
14	4,10	(9,3000)	(15,2400)	(18,2200)	–	–
15	12	(16,3500)	–	–	–	–
16	13,14	(20,3000)	(22,2000)	(24,1750)	(28,1500)	(30,1000)
17	11,14,15	(14,4000)	(18,3200)	(24,1800)	–	–
18	16,17	(9,3000)	(15,2400)	(18,2200)	–	–

The proposed MOGA algorithm with a fuzzy based clustering technique to solve DTCQTPs was coded in MATLAB R2013a. The algorithm parameters are set as follows: $n_{Pop} = 300$ with *Generation* = 1000, $P_c = 0.9$, $P_m = 0.1$. The mean program running time, without any attempt to improve the computational time, was 15.3 min on a personal computer (Intel Core i5-3230 M with CPU 2.6 GHz with 4 GB memory), which is an acceptable time in comparison with the solution space consisting of almost 3.6 billion possible scenarios, and only searching 0.0081% of the total number of potential solutions to obtain the Pareto solutions.

The proposed algorithm was able to find the same Pareto solutions in 16 optimization trials out of the total 20 that were performed; this implies that the proposed approach is able to attain the same global Pareto optimal solutions with 80% accuracy. In the remaining four trials, the Pareto solutions were in a maximum of three points which were not global optimum points. This arose due to the stochastic nature of the proposed approach. However, in comparison with the low percentage of search space and significant solution space, the accuracy of the algorithm is noteworthy.

In every 16 runs of the algorithm, exactly 105 Pareto solutions were identified; due to space limitations they are plotted in two sub-figures (a and b) of Fig. 6. To simplify reading the data from the figures the Pareto solutions are initially sorted according to the time objective, and the solutions with identical time objectives are then sorted with respect to the cost objective. Fig. 6(a) shows the cost objective *vs*. time of the project, while in Fig. 6(b) the same Pareto solutions are shown in terms of quality *vs*. objective. The Pareto solutions obtained for the 18 activity network benchmark case study of DTCQTP are provided in Table A.1 (Appendix A), and the utility scores of each solution for each objective, are also listed there. This can be used for future research studies.

The corresponding weights of each objective were calculated using Shannon's entropy technique (see Section 2.2). The fact that the cost objective has the highest normalized weight with 38.06%, and time and quality have almost the same weights with 31.28% and 30.48%, respectively (Table 4), indicates that the cost objective has a greater contribution to the uncertainty associated with the obtained results and as a result a higher weight is assigned to this attribute.

The Pareto solutions obtained can now be ranked according to their overall performance, denoting the degree to which each alternative is acceptable when considering all the criteria simultaneously. In order to assess the overall performance of each solution, one must provide a hierarchical structure to relate the time, cost, and quality attributes with their associated normalized weights to the overall performance criterion (Fig. 7). The solutions are then ranked according to the utility scores obtained for the overall performance. The 23rd Pareto solution possesses the highest utility score (64.67%; Fig. 8), which denotes that this alternative for scheduling the project satisfies the overall performance considering all the objectives simultaneously within 64.67%. However, this may not be sufficient for a final decision to select the best solution, since each solution needs to be investigated to identify its weak and strong points regarding each objective. In fact, the ER approach has the capacity to provide DMs with a transparent view of the performance of each objective in each criterion. Therefore, in the following steps of selecting the best solution, one must more deeply investigate the solutions.

Among the Pareto solutions, the 2nd, 23rd, 37th, and 71st solutions were selected to show how the overall assessment is done. The corresponding utility scores with respect to each objective in addition to the overall performance of each solution are plotted in Fig. 9, where it can be seen that the 23rd solution has the highest utility score in both time and quality objectives with 100% and 97.96%, respectively. However, the cost objective of this solution has the lowest performance in being only 4.2% cost objective. Hence, the 23rd solution might not be desirable to be implemented since its performance with respect to the cost objective is extremely low. On the other hand, the 71st solution has an overall utility score of 40.11%, which is also quite low to be selected, and the 37th solution does not have an acceptable performance in terms of the cost objective (with only 10.34% for the utility score) either. With a thorough investigation, the DMs can select the most appropriate solution with a similar approach in an iterative attempt to obtain the solution that fits well with the DMs' expectations (e.g., additional data can be gathered to modify the weights of the objectives). This procedure can be continued in order to arrive at a consensus on a Pareto solution to be selected.

The ER approach facilitates the procedure of investigating the overall performance of each solution by providing more details of the solution performance with respect to each objective. The preconception of DMs about the performance of each solution gives more confidence to the DMs to implement their chosen project schedule, and thus more efficiently manage organizational resources.

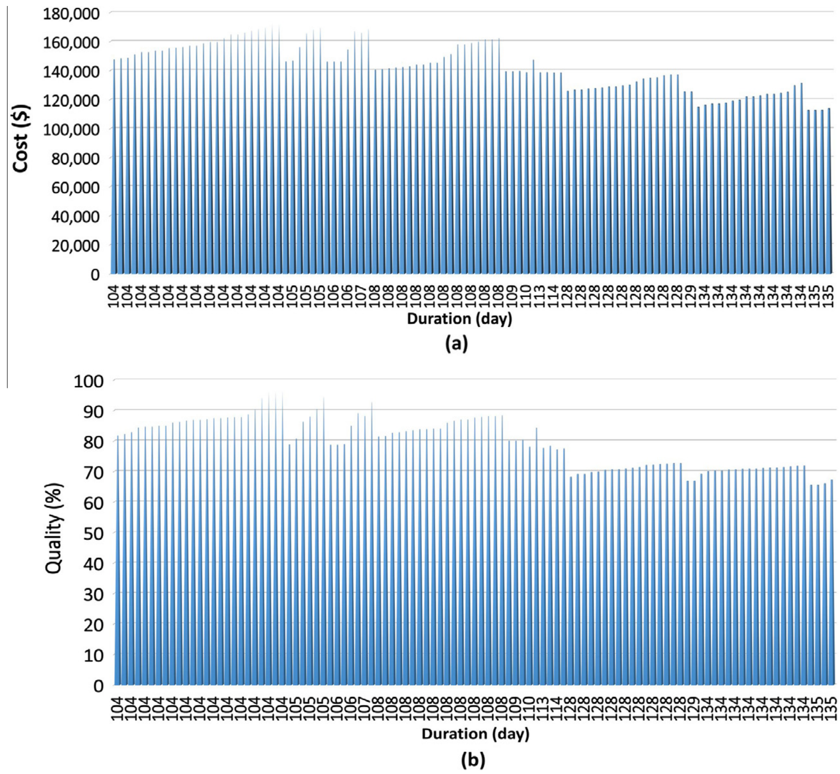


Fig. 6. Pareto solutions of the benchmark example.

Table 4
Normalized weights of the objectives obtained from Shannon's entropy technique.

Objectives:	Time	Cost	Quality
Normalized weights:	0.3128	0.3806	0.3048

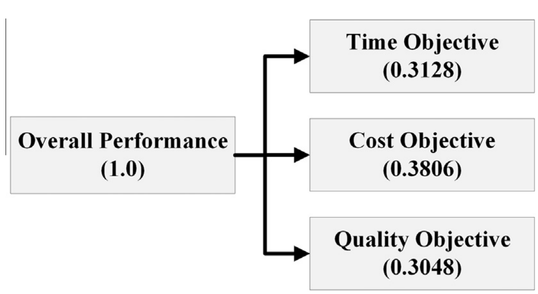


Fig. 7. One level hierarchical structure of overall performance assessment criteria.

As mentioned earlier, the ER approach is able to provide DMs with informative output data indicating the weak points of each alternative at any desired level. In this study, we divided the overall performance into five grades viz 'worst', 'poor', 'average', 'good', and 'best', which are equally spaced in the interval [0,1]. These five grades can be used to reflect the combined belief degree β_n (Fig. 10). On this basis, the overall performance of the 23rd solution is believed to belong to the 'best' grade with a degree of 61.56%. On the other hand, the 23rd solution has the highest degree of belief of the 'worst' grade with 30.22%, in comparison with the 2nd, 37th, and 71st solutions with degrees of belief for the 'worst' grade of 0.0%, 22.81% and 15.17%, respectively. As a result, the DMs might decide not to select the 23rd solution as the best solution although it has the highest utility score. In this case, the DMs can apply improvement strategies to better the performance of the 23rd solution or start investigating another solution. Figs. 9 and 10 show it to be more advisable to implement the 2nd solution since it has an acceptable utility

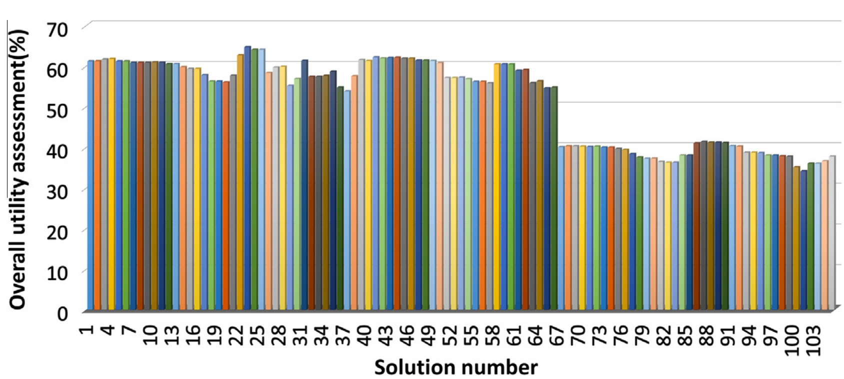
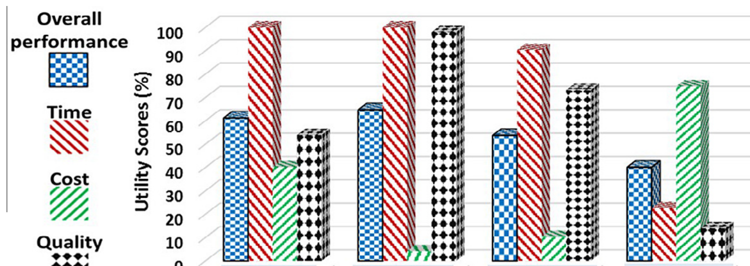
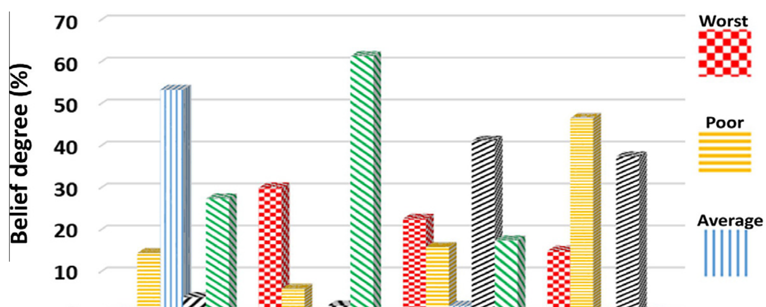


Fig. 8. Overall performance utility assessment of each solution.



	Solution 2	Solution 23	Solution 37	Solution 71
Overall Performance	61.19	64.67	53.75	40.11
Time	100.00	100.00	90.32	22.58
Cost	40.18	4.20	10.34	74.77
Quality	53.90	97.96	73.17	14.00

Fig. 9. Utility scores for 2nd, 23rd, 37th, and 71st Pareto solutions with respect to each objective.



	Solution 2	Solution 23	Solution 37	Solution 71
Worst	0.00	30.22	22.81	15.17
Poor	14.66	6.11	16.09	46.84
Average	53.55	0.00	2.04	0.34
Good	4.15	2.11	41.41	37.64
Best	27.64	61.56	17.66	0.00

Fig. 10. Combined degrees of belief (β_n) for 2nd, 23rd,37th, and 71st Pareto solutions with respect to the overall performance.

score with respect to the overall performance (61.19%), and its combined degrees of belief are high in terms of the 'average' and 'best' grades (53.55% and 27.64%, respectively) with a zero value for the 'worst' grade. Since there is no incomplete assessment, $\beta_H = 0$.

To summarize, the ER approach is highly efficient in identifying the performance of each alternative, and it enables the DMs to form a transparent and rational judgment about the best alternative. Using the ER approach in construction project scheduling provides more efficient management strategies, and the DMs can be more confident about their selected alternative since the DMs have a clear understanding of the performance of each alternative. In the present case study, we observed that although the 23rd solution had the highest utility score, it was not advisable to select it as the best solution since it has a very high degree of belief in the 'worst' grade. With further investigation, the 2nd solution was chosen to be more rational and practicable for the DMs.

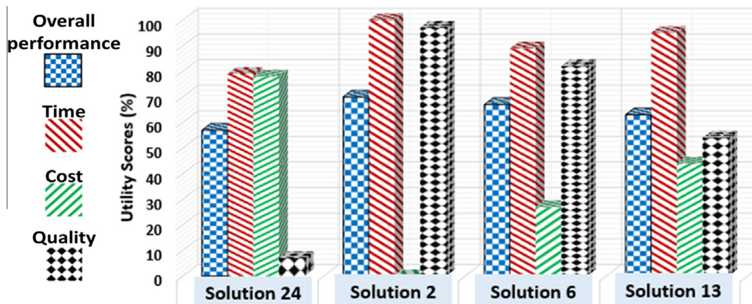
4.1. Comparing the results with the literature

Mungle et al. (2013) identified 25 solutions for the same benchmark problem using fuzzy clustering-based optimization using MOGA, and the best compromise solution was also identified. In order to demonstrate that the currently proposed methodology can generate more efficient schedules for construction projects, the reported results from Mungle et al. (2013) were ranked based on the ER approach. The weights of the attributes were assumed to be equal in order to make the comparison valid. The best solution identified by Mungle et al. (2013), the three solutions with highest utility score using the ER approach, along with their respective attribute values are presented in Table 5. The worst

Table 5
Best solutions according to Mungle et al. (2013) and the ER approach.

Solution number	Attributes		
	Time (days)	Cost ($)	Quality (%)
24[a]	108	124,110	70.734
2	104	168,480	85.667
6	106	153,120	83.233
13	105	143,345	78.634
Worst value	123	168,480	69.600
Best value	104	111,355	86.234

[a] Best compromise solution according to Mungle et al. (2013).



	Solution 24	Solution 2	Solution 6	Solution 13
Overall Perfomance	57	70	67	63
Time	79	100	89	95
Cost	78	0	27	44
Quality	7	97	82	54

Fig. 11. Performance comparison of the selected solutions from Mungle et al. (2013).

and best possible values are from the data reported in Mungle et al. (2013).

Using the ER approach the performance of the solutions listed in Table 5 are illustrated in Fig. 11. The 24th solution has the worst overall performance with only 57%, in comparison with 2nd, 6th, 13th solutions with corresponding overall performance of 70%, 67%, and 63% respectively. It must be noted that with almost 7% performance in the quality attribute, the 24th solution is too low to be accepted as the best solution: although the objectives of time and cost are satisfied within 79% and 78%, respectively, the low quality of the solution would result in inefficient planning. With respect to the aim of trade-off, a solution which establishes a compromise between time, cost, and quality is more preferable. Thus, the 2nd solution with zero performance in cost attribute, since it has the highest cost, is rejected as a best solution for the same reason as the 24th solution. Comparing the remaining solutions, the 13th solution is a better trade-off between the objectives, since the objectives are simultaneously satisfied within an acceptable degree, whereas the 6th solution has very low performance in terms of cost objective, with only 27%. As shown the ER approach provides a transparent and panoramic view of the performances of the alternatives, and the DMs are able to find the weak and strong points of each alternative. Consequently, the 13th solution is a better and more efficient alternative based on the assessments that were provided through the ER approach.

5. Conclusion

In this paper, and for the first time, in addition to solving the DTCQTP, an exhaustive framework to rank the obtained Pareto solutions was proposed and tested. A multi-objective genetic algorithm (MOGA) with NSGA-II procedure was tailored to solve DTCQTP, and it was then utilized to solve an 18 activity network benchmark case study from the literature. The global optimal Pareto solutions were ranked using the ER approach using the weights obtained from Shannon's entropy technique. The results indicated that the ER approach is more efficient in ranking the Pareto solutions when compared to the results of Mungle et al. (2013). The proposed approach of this research study enables the DMs to know the performance of each solution with a transparent view with respect to each attribute, and consequently decide on an optimal solution with more confidence. A detailed framework to integrate MCDM methods into multi-objective optimization techniques was proposed in order to consider all the influential criteria which facilitate the process of reaching a consensus regarding a chosen solution. The authors believe that the proposed methodology in applying the MCDM approach (*i.e.*, the ER approach) can generate more practical solutions in terms of project scheduling. It is expected that the proposed approach can assist industry, project managers, and researchers in the planning phase of construction projects.

The ER approach has the ability to deal with various types of criteria (e.g., quantitative/qualitative, certain/ambiguity/randomness, and complete/incomplete). Furthermore, the ER approach provides a transparent and panoramic view of the performances of the alternatives, and the DMs are able to find the weak points of each alternative, and if necessary improvement strategies can then be applied to better an alternative performance. Given these reasons, the ER approach is more powerful than other existing MCDMs method in providing informative outcomes. In this respect, some further applications of ER should be made beyond DTCQTPs, such as in various decision analysis applications, assessment and evaluation of multiple alternatives in the field (e.g., environmental impact assessment), organizational self-assessment, efficient subcontracting plans in bidding and evaluation processes, etc.

In terms of future studies based on this research, the authors plan to expand the usage of the proposed method by designing a knowledge-based system to help automate the process of ranking, analyzing, and searching for a best solution. The highly efficient performance in solving DTCQTPs obtained using the ER approach provided the authors of this paper with the aspiration of developing a computer system to emulate the decision-making ability of DMs using the ER approach. Another idea is to establish approaches that can help construction contractors and decision-makers to develop more efficient subcontracting plans during the bidding process via the development of multiple criteria assessment procedures. This would be useful because the bidding and evaluation process of a construction project can be a tedious and time-consuming process with no well-established criteria and approaches.

Appendix A

See Tabel A.1.

Table A.1
All the Pareto solutions obtained for the 18 activity network benchmark case example of DTCQTP.

#	Objectives			Utility scores in terms of			
	Time (day)	Cost ($)	Quality (%)	Overall performance (%)	Time (%)	Cost (%)	Quality (%)
1	104	147,230	81.70	61.16	100.00	41.61	52.38
2	104	148,080	82.17	61.19	100.00	40.18	53.90
3	104	148,525	82.83	61.62	100.00	39.43	56.03
4	104	150,980	84.33	61.77	100.00	35.31	60.88
5	104	152,480	84.60	61.16	100.00	32.78	61.75
6	104	152,480	84.60	61.16	100.00	32.78	61.75
7	104	153,480	84.87	60.82	100.00	31.10	62.63
8	104	153,480	84.87	60.82	100.00	31.10	62.63
9	104	155,130	85.93	60.83	100.00	28.33	66.05
10	104	155,380	86.20	60.93	100.00	27.91	66.93
11	104	155,980	86.57	60.90	100.00	26.90	68.12
12	104	156,980	86.83	60.50	100.00	25.22	68.96
13	104	156,980	86.83	60.50	100.00	25.22	68.96
14	104	158,480	87.10	59.73	100.00	22.70	69.84
15	104	159,480	87.37	59.30	100.00	21.02	70.71
16	104	159,480	87.37	59.30	100.00	21.02	70.71
17	104	161,980	87.60	57.80	100.00	16.81	71.45
18	104	164,530	87.77	56.21	100.00	12.53	72.00
19	104	164,530	87.77	56.21	100.00	12.53	72.00
20	104	165,980	88.60	56.00	100.00	10.09	74.68
21	104	167,280	90.17	57.65	100.00	7.90	79.76
22	104	168,480	93.93	62.65	100.00	5.88	91.92
23	104	169,480	95.80	64.67	100.00	4.20	97.96
24	104	171,980	96.43	64.02	100.00	0.00	100.00
25	104	171,980	96.43	64.02	100.00	0.00	100.00
26	105	145,880	78.77	58.34	96.77	43.88	42.90
27	105	146,550	80.67	59.64	96.77	42.75	49.05
28	105	155,900	86.23	59.84	96.77	27.03	67.02
29	105	165,300	87.90	55.14	96.77	11.23	72.42
30	105	167,795	90.43	56.86	96.77	7.04	80.60
31	105	169,400	94.27	61.30	96.77	4.34	93.02
32	106	145,800	78.63	57.39	93.55	44.01	42.45
33	106	145,800	78.63	57.39	93.55	44.01	42.45
34	106	145,815	78.90	57.62	93.55	43.99	43.32
35	106	154,365	84.87	58.65	93.55	29.61	62.63
36	106	166,865	89.03	54.71	93.55	8.60	76.08
37	107	165,830	88.13	53.75	90.32	10.34	73.17
38	107	168,330	92.67	57.51	90.32	6.14	87.84
39	108	140,160	81.33	61.52	87.10	53.50	51.18
40	108	140,760	81.53	61.28	87.10	52.49	51.83
41	108	141,210	82.60	62.15	87.10	51.73	55.29
42	108	141,810	82.80	61.90	87.10	50.72	55.93
43	108	142,060	83.07	62.02	87.10	50.30	56.81
44	108	142,660	83.43	62.07	87.10	49.29	57.97
45	108	143,660	83.70	61.87	87.10	47.61	58.84
46	108	143,660	83.70	61.87	87.10	47.61	58.84
47	108	145,160	83.97	61.38	87.10	45.09	59.72
48	108	145,160	83.97	61.38	87.10	45.09	59.72
49	108	149,060	85.83	61.28	87.10	38.53	65.73
50	108	151,160	86.47	60.73	87.10	35.00	67.80
51	108	157,810	86.93	57.08	87.10	23.82	69.29
52	108	157,810	86.93	57.08	87.10	23.82	69.29
53	108	158,660	87.57	57.19	87.10	22.39	71.35
54	108	159,660	87.83	56.82	87.10	20.71	72.20
55	108	161,160	88.10	56.13	87.10	18.19	73.07
56	108	161,160	88.10	56.13	87.10	18.19	73.07
57	108	162,160	88.37	55.77	87.10	16.51	73.94
58	109	139,230	80.03	60.45	83.87	55.06	46.98
59	109	139,230	80.03	60.45	83.87	55.06	46.98
60	109	139,480	80.30	60.42	83.87	54.64	47.85
61	110	138,480	78.00	58.89	80.65	56.32	40.41
62	110	147,045	84.23	59.08	80.65	41.92	60.56
63	113	138,455	77.57	55.82	70.97	56.36	39.02
64	113	138,555	78.37	56.30	70.97	56.20	41.61
65	114	138,360	77.17	54.53	67.74	56.52	37.73
66	114	138,375	77.43	54.71	67.74	56.50	38.57
67	128	125,690	68.13	40.05	22.58	77.82	8.50
68	128	126,405	69.10	40.29	22.58	76.62	11.64
69	128	126,405	69.10	40.29	22.58	76.62	11.64
70	128	127,205	69.70	40.21	22.58	75.28	13.58
71	128	127,05	69.83	40.11	22.58	74.77	14.00
72	128	128,005	70.40	40.21	22.58	73.93	15.84
73	128	128,605	70.60	39.97	22.58	72.92	16.49

Table A.1 (*continued*)

#	Objectives			Utility scores in terms of			
	Time (day)	Cost ($)	Quality (%)	Overall performance (%)	Time (%)	Cost (%)	Quality (%)
74	128	128,605	70.60	39.97	22.58	72.92	16.49
75	128	129,405	70.83	39.63	22.58	71.58	17.23
76	128	130,005	71.03	39.39	22.58	70.57	17.88
77	128	132,005	71.33	38.35	22.58	67.21	18.85
78	128	134,105	72.00	37.53	22.58	63.68	21.02
79	128	134,755	72.07	37.18	22.58	62.58	21.24
80	128	135,005	72.33	37.22	22.58	62.16	22.08
81	128	136,405	72.43	36.43	22.58	59.81	22.41
82	128	137,005	72.63	36.20	22.58	58.80	23.05
83	128	137,005	72.63	36.20	22.58	58.80	23.05
84	129	125,148	66.83	38.01	19.35	78.74	4.30
85	129	125,148	66.83	38.01	19.35	78.74	4.30
86	134	114,705	69.13	40.99	3.23	96.29	11.74
87	134	116,105	70.03	41.37	3.23	93.94	14.65
88	134	116,905	70.27	41.20	3.23	92.59	15.42
89	134	116,905	70.27	41.20	3.23	92.59	15.42
90	134	117,505	70.47	41.09	3.23	91.59	16.07
91	134	118,905	70.57	40.33	3.23	89.23	16.39
92	134	119,505	70.77	40.22	3.23	88.22	17.04
93	134	121,905	70.80	38.70	3.23	84.19	17.14
94	134	121,905	70.80	38.70	3.23	84.19	17.14
95	134	122,505	71.00	38.59	3.23	83.18	17.78
96	134	123,605	71.10	38.01	3.23	81.33	18.11
97	134	123,605	71.10	38.01	3.23	81.33	18.11
98	134	124,405	71.33	37.81	3.23	79.98	18.85
99	134	125,005	71.53	37.70	3.23	78.98	19.50
100	134	129,505	71.80	35.10	3.23	71.41	20.37
101	134	131,155	71.87	34.11	3.23	68.64	20.59
102	135	112,500	65.50	35.98	0.00	100.00	0.00
103	135	112,500	65.50	35.98	0.00	100.00	0.00
104	135	112,580	65.97	36.58	0.00	99.87	1.52
105	135	113,640	67.23	37.71	0.00	98.08	5.59

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