COUPLED COMMON FIXED POINT THEOREMS FOR φ-CONTRACTIONS IN PROBABILISTIC METRIC SPACES AND APPLICATIONS

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ABSTRACT. In this paper, we give some new coupled common fixed point theorems for probabilistic φ-contractions in Menger probabilistic metric spaces. As applications of the main results, we obtain some coupled common fixed point theorems in usual metric spaces and fuzzy metric spaces. The main results of this paper improve the corresponding results given by some authors. Finally, we give one example to illustrate the main results of this paper.

1. **Introduction and Preliminaries**

In 2006, Bhaskar and Lakshmikantham [1] initially introduced the concept of coupled fixed points and proved the coupled fixed point theorem with application in boundary value problem in metric spaces. Later, Lakshmikantham and Ćirić [16] considered the concept of commuting mappings with the mixed monotone property and proved some coupled fixed point theorems which extends and improves the results of Bhaskar and Lakshmikantham [1]. After the work of Bhaskar and Lakshmikantham [1] and Lakshmikantham and Ćirić [16], many authors study the existence and uniqueness of coupled fixed points for various classes of mappings in metric spaces, cone metric spaces and fuzzy metric spaces (see [21, 4, 14, 23, 17, 12]).

In 1942, Menger [18] introduced the concept of probabilistic metric spaces, which is a generalization of metric spaces. Since then, fixed point theory in probabilistic metric spaces can be considered as a field of probabilistic analysis. Many fixed point theorems for probabilistic contractions are obtained (see [24, 8, 19, 9, 3, 5, 13]).

Recently, some authors also study coupled fixed point theorems for probabilistic contractions (see [7, 6, 25, 20]).

In [25], the authors considered the gauge function φ satisfying the condition that $\varphi(t) < t$ and (or) $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$ and proved some fixed point theorems by using the gauge function φ. In this paper, we prove some coupled fixed point theorems for probabilistic contractions with the gauge function φ in Menger probabilistic metric spaces. Especially, it is worth mentioning that the gauge function φ in our result has the simpler restriction than ones of [25]. Our result improves the corresponding ones given in [25].

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Now, we recall some definitions and results in the theory of probabilistic metric spaces. For more details, the readers refer to [22, 2, 11].

Definition 1.1. A mapping $F : (0, \infty) \to [0, 1]$ is called a *distribution function* if it is non-decreasing and left-continuous with $\inf_{x\in\mathbb{R}} F(x) = 0$. If in addition $F(0) = 0$, then F is called a distance distribution function.

Definition 1.2. A distance distribution function F satisfying $\lim_{t\to\infty} F(t) = 1$ is called a *Menger distance distribution function.*

The set of all Menger distance distribution functions is denoted by D^+. This space D^+ is partially ordered by the usual pointwise ordering of functions, that is, $F \leq G$ if and only if $F(t) \leq G(t)$ for all $t \in [0, \infty)$. The maximal element for D^+ in this order is the distance distribution function ϵ_0 given by

$$\epsilon_0(t) = \begin{cases} 0 & \text{if } t = 0, \\ 1 & \text{if } t > 0. \end{cases}$$

Definition 1.3. A *triangular norm* (shortly, t-norm) is a binary operation $\varDelta$ on [0,1] satisfying the following conditions:
(1) $\varDelta$ is associative and commutative;
(2) $\varDelta$ is continuous;
(3) $\varDelta(a, 1) = a$ for all $a \in [0, 1]$;
(4) $\varDelta(a, b) \leq \varDelta(c, d)$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Two typical examples of the continuous t-norm are $\varDelta_P(a, b) = ab$, $\varDelta_M(a, b) = \min\{a, b\}$ for all $a, b \in [0, 1]$.

Now, the t-norm is recursively defined by $\varDelta^1 = \varDelta$ and

$$\varDelta^n(x_1, \cdots, x_{n+1}) = \varDelta(\varDelta^{n-1}(x_1, \cdots, x_n), x_{n+1})$$

for all $n \geq 2$ and $x_i \in [0, 1]$, $i = 1, 2, \cdots, n + 1$.

A t-norm $\varDelta$ is said to *be of Hadžić-type* if the family $\{\varDelta^n\}$ is equicontinuous at $x = 1$, that is, for any $\epsilon \in (0, 1)$, there exists $\delta \in (0, 1)$ such that

$$a > 1 - \delta \quad \Longrightarrow \quad \varDelta^n(a) > 1 - \epsilon$$

for all $n \geq 1$.

$\varDelta_M$ is a trivial example of a t-norm of Hadžić-type [11].

Definition 1.4. A *Menger probabilistic metric space* (briefly, a Menger PM-space) is a triple $(X, F, \varDelta)$, where X is a nonempty set, $\varDelta$ is a continuous t-norm and F is a mapping from $X \times X \to D^+$ ($F_{x,y}$ denotes the value of F at the pair (x, y)) satisfying the following conditions:
(PM-1) $F_{x,y}(t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(PM-2) $F_{x,y}(t) = F_{y,x}(t)$ for all $x, y \in X$ and $t > 0$;
(PM-3) $F_{x,z}(t + s) \geq \varDelta(F_{x,y}(t), F_{y,z}(s))$ for all $x, y, z \in X$ and $t, s \geq 0$.

Definition 1.5. Let $(X, F, \varDelta)$ be a Menger PM-space.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to a point $x \in X$ (write $x_n \to x$) if, for any $t > 0$ and $0 < \epsilon < 1$, there exists a positive integer N such that

$$F_{x_n,x}(t) > 1 - \epsilon$$

whenever $n \geq N$;

(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for any $t > 0$ and $0 < \epsilon < 1$, there exists a positive integer N such that

$$F_{x_n,x_m}(t) > 1 - \epsilon$$

whenever $m, n \geq N$.

(3) A Menger PM-space (X, F, Δ) is said to be *complete* if every Cauchy sequence in X is convergent to a point in X.

Definition 1.6. [1] Let X be a nonempty set and $T : X \times X \to X$ be a mapping. An element $(x, y) \in X \times X$ is said to be a *coupled fixed point* of T if

$$T(x, y) = x, \quad T(y, x) = y.$$

Definition 1.7. [16] Let X be a nonempty set and $T : X \times X \to X$, $h : X \to X$ be two mappings.

(1) An element $(x, y) \in X \times X$ is said to be a *coupled coincidence point* of h and T if

$$T(x, y) = h(x), \quad T(y, x) = h(y);$$

(2) An element $(x, y) \in X \times X$ is said to be a *coupled common fixed point* of h and T if

$$T(x, y) = h(x) = x, \quad T(y, x) = h(y) = y.$$

Definition 1.8. Let (X, F, Δ) be a Menger PM-space and $T : X \times X \to X$, $h : X \to X$ be two mappings. The mappings T and h are said to be *weakly compatible* (or *w-compatible*) if they commute at their coupled coincidence points, i.e., if (x, y) is a coupled coincidence point of T and h, then

$$g(F(x, y)) = F(gx, gy).$$

2. Main Results

In this section, let $\mathbb{R}^+ = [0, +\infty)$ and $\mathbb{N}$ denote, the set of all positive integers. Let $\mathbf{\Phi_{w^*}}$ denote, the set of all functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the condition: for each $t_1, t_2 > 0$ there exists $r \geq \max\{t_1, t_2\}$ and $N \in \mathbb{N}$ such that

$$\varphi^n(r) < \min\{t_1, t_2\} \tag{1}$$

for all $n > N$. It is easy to see that (1) implies that, for each $t > 0$, there exists $r \geq t$ and $N \in \mathbb{N}$ such that

$$\varphi^n(r) < t \tag{2}$$

for all $n > N$.

Example 2.1. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ by $\varphi(t) = t/2$ for all $t > 0$. Then $\varphi \in \mathbf{\Phi_{w^*}}$.

Let $\mathbf{\Phi}$ denote the set of all functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the condition:

$$\lim_{n\to\infty} \varphi^n(t) = 0 \tag{3}$$

for each $t > 0$. It is easy to see that, if the function $\varphi \in \mathbf{\Phi}$, then $\varphi \in \mathbf{\Phi_{w^*}}$. But the inverse is not true. See the following example:

Example 2.2. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a function defined by $\varphi(t) = t/2$ for all $t \in [0, 1)$ and $\varphi(t) = \frac{1}{t}$ for all $t \in [1, \infty)$. For each $t_1, t_2 > 0$, there exists $r > \max\{1, t_1, t_2\}$ such that $\frac{1}{r} < \min\{1, t_1, t_2\}$ and hence $\varphi(r) = \frac{1}{2r}$. Further, we have $\varphi^n(r) = \frac{1}{2^n r}$ for each $n \in \mathbb{N}$, which implies that $\varphi \in \mathbf{\Phi_{w^*}}$. However, $\varphi \notin \mathbf{\Phi}$ since $\varphi(1) = 1$.

In [10], Fang introduced a class of functions denoted by $\mathbf{\Phi_w}$. More precisely, let $\mathbf{\Phi_w}$ denote the set of all functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the condition: for each $t > 0$, there exists $r \geq t$ such that

$$\lim_{n\to\infty} \varphi^n(r) = 0.$$

For the property of $\mathbf{\Phi_w}$, the readers can refer to [10]. It is easy to see that, if $\varphi \in \mathbf{\Phi_w}$, then $\varphi \in \mathbf{\Phi_{w^*}}$. But the inverse is not true. See the following example:

Example 2.3. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a function defined by $\varphi(t) = t$ for all $t \in [0, 1]$, $\varphi(t) = \frac{1}{t}$ for all $t \in (1, \infty)$. Then $\varphi \in \mathbf{\Phi_{w^*}}$. In fact, for each $t_1, t_2 > 0$, there exists $r > \max\{1, t_1, t_2\}$ such that $\frac{1}{r} < \min\{1, t_1, t_2\}$. Then we have

$$\varphi^n(r) = \frac{1}{r} < \min\{t_1, t_2\}$$

for all $n \in \mathbb{N}$ and so $\varphi \in \mathbf{\Phi_{w^*}}$. However, for each $t > 0$, it is easy to see that $\lim_{n\to\infty} \varphi^n(t)$ exists and $\lim_{n\to\infty} \varphi^n(t) > 0$ and so $\varphi \notin \mathbf{\Phi_w}$.

For $\mathbf{\Phi}$, $\mathbf{\Phi_w}$ and $\mathbf{\Phi_{w^*}}$, we have $\mathbf{\Phi} \subset \mathbf{\Phi_w} \subset \mathbf{\Phi_{w^*}}$.

Lemma 2.4. *Let $\varphi \in \mathbf{\Phi_{w^*}}$. Then, for each $t > 0$, there exists $r \geq t$ such that $\varphi(r) < t$.*

Proof. Suppose that there is $t_0 > 0$ such that $\varphi(r) \geq t_0$ for all $r \geq t_0$. By induction, we obtain that $\varphi^n(r) \geq t_0$ for all $n \in \mathbb{N}$. From (2), it follows that there exists $r \geq t_0$ and $N \in \mathbb{N}$ such that $\varphi^n(r) < t_0$ for all $n > N$, which contradicts $\varphi^n(r) \geq t_0$ for all $n \in \mathbb{N}$. Thus, for each $t > 0$, there exists $r \geq t$ such that $\varphi(r) \leq t$. This completes the proof. □

Lemma 2.5. *Let $\varphi \in \mathbf{\Phi_{w^*}}$ and $F_n, G_n : \mathbb{R} \to [0, 1]$. Assume that, for each $n \in N$, $\sup_{t>0} G_n(t) = 1$ and*

$$F_n(\varphi^n(t)) \geq G_n(t)$$

for all $t > 0$. If each F_n is non-decreasing, then $\lim_{n\to\infty} F_n(t) = 1$ for each $t > 0$.

Proof. Since each $\sup_{t>0} G_n(t) = 1$, for any $\epsilon > 0$, there exists $t_{n,0} > 0$ such that $G(t_{n,0}) > 1 - \epsilon$. Let $t_0 = \sup_{n\geq 1} t_{n,0}$. Then $G_n(t_0) > 1 - \epsilon$ for each $n \in \mathbb{N}$. For any $t > 0$, since $\varphi \in \mathbf{\Phi_{w^*}}$, there exist $r \geq \max\{t, t_0\}$ and $N \in \mathbb{N}$ such that $\varphi^n(r) < \min\{t, t_0\}$ for all $n > N$. Since each F_n is non-decreasing, we have

$$F_n(t) \geq F_n(\varphi^n(r)) \geq G_n(r) \geq G_n(t_0) > 1 - \epsilon$$

for all $n > N$. Thus it follows that $\lim_{n\to\infty} F_n(t) = 1$ for each $t > 0$. This completes the proof. □

Now, we give the main result of this paper.

Theorem 2.6. *Let (X, F, Δ) be a Menger PM-space under a t-norm Δ of Hadžić-type. Let $T : X \times X \to X$ and $h : X \to X$ be two mappings satisfying that*

$$F_{T(x,y),T(u,v)}(\varphi(t)) \geq \Delta(F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)) \tag{4}$$

for all $x, y, u, v \in X$ and $t > 0$, where $\varphi \in \mathbf{\Phi}_{\mathbf{w}^*}$. Suppose that $T(X \times X) \subseteq h(X)$ and $T(X \times X)$ is complete. Then there exists a unique point $(x^*, y^*) \in X \times X$ such that $h(x^*) = h(y^*) = T(x^*, y^*) = T(y^*, x^*)$. Further, if h and T are weakly compatible, then there exists unique $\hat{x} \in X$ such that $\hat{x} = h(\hat{x}) = T(\hat{x}, \hat{x})$.

Proof. Take $x_0, y_0 \in X$ arbitrarily. Since $T(X \times X) \subseteq h(X)$, there exist two sequences $\{x_n\}, \{y_n\} \subseteq X$ such that

$$h(x_{n+1}) = T(x_n, y_n), \quad h(y_{n+1}) = T(y_n, x_n) \tag{5}$$

for all $n \in \mathbb{N} \cup \{0\}$.

Now we prove, by induction, that, for each $n \in \mathbb{N}$,

$$\begin{aligned} &\min\{F_{h(x_{n+1}),h(x_n)}(\varphi^n(t)), F_{h(y_{n+1}),h(y_n)}(\varphi^n(t))\} \\ &\geq \Delta^{2n}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)). \end{aligned} \tag{6}$$

By (4) and (5), for $n = 1$, we have

$$\begin{aligned} F_{h(x_2),h(x_1)}(\varphi(t)) &= F_{T(x_1,y_1),T(x_0,y_0)}(\varphi(t)) \\ &\geq \Delta(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)) \end{aligned}$$

and

$$\begin{aligned} F_{h(y_2),h(y_1)}(\varphi(t)) &= F_{T(y_1,x_1),T(y_0,x_0)}(\varphi(t)) \\ &\geq \Delta(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)). \end{aligned}$$

Hence (6) holds for $n = 1$. Now, assume that (6) holds for some $n \in \mathbb{N}$. Then, by (4)-(6) we have

$$\begin{aligned} &F_{h(x_{n+2}),h(x_{n+1})}(\varphi^{n+1}(t)) \\ &= F_{T(x_{n+1},y_{n+1}),T(x_n,y_n)}(\varphi^{n+1}(t)) \\ &\geq \Delta(F_{h(x_{n+1}),h(x_n)}(\varphi^n(t)), F_{h(y_{n+1}),h(y_n)}(\varphi^n(t))) \\ &\geq \Delta(\Delta^{2n}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)), \Delta^{2n}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t))) \\ &= \Delta^{2n+1}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)). \end{aligned}$$

Similarly, we have

$$F_{h(y_{n+2}),h(y_{n+1})}(\varphi^{n+1}(t)) \geq \Delta^{2n+1}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t))$$

and so

$$\begin{aligned} &\min\{F_{h(x_{n+2}),h(x_{n+1})}(\varphi^{n+1}(t)), F_{h(y_{n+2}),h(y_{n+1})}(\varphi^{n+1}(t))\} \\ &\geq \Delta^{2n+1}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)), \end{aligned}$$

which implies that (6) holds for $n + 1$. Therefore, (6) holds for all $n \in \mathbb{N}$.

On the other hand, for each $n \in \mathbb{N}$, put

$$F_n(\varphi^n(t)) = \min\{F_{h(x_{n+1}),h(x_n)}(\varphi^n(t)), F_{h(y_{n+1}),h(y_n)}(\varphi^n(t))\}$$

and

$$G_n(t) = \Delta^{2n}(F_{h(x_1),h(x_0)}(t), F_{h(y_1),h(y_0)}(t)).$$

Then, for each $n \in \mathbb{N}$, since Δ is of H-type, we have

$$\sup_{t>0} G_n(t) = 1, \quad F_n(\varphi^n(t)) \geq G_n(t).$$

By Lemma 2.5, we have $\lim_{n\to\infty} F_n(t) = 1$, which implies that

$$\lim_{n\to\infty} F_{h(x_{n+1}),h(x_n)}(t) = 1 \tag{7}$$

for all $t > 0$ and

$$\lim_{n\to\infty} F_{h(y_{n+1}),h(y_n)}(t) = 1 \tag{8}$$

for all $t > 0$.

Now, let $n \in \mathbb{N}$ and $t > 0$. From Lemma 2.4, it follows that there exists $r \geq t$ such that $\varphi(r) < t$. We show, by induction, that, for each $k \in \mathbb{N}$, there exists $n_k \in \mathbb{N}$ such that

$$\begin{aligned}
&\min\{F_{h(x_n),h(x_{n+k})}(t), F_{h(y_n),h(y_{n+k})}(t)\} \\
&\geq \Delta^{n_k}(\min\{F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))\}).
\end{aligned} \tag{9}$$

For $k = 1$, let $n_1 = 1$ such that (9) holds since

$$\begin{aligned}
&\min\{F_{h(x_n),h(x_{n+1})}(t), F_{h(y_n),h(y_{n+1})}(t)\} \\
&\geq \Delta(\min\{F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))\}).
\end{aligned}$$

Assume that (9) holds for some $k \in \mathbb{N}$. Then, by (4), (9) and the monotonicity of Δ, we have

$$\begin{aligned}
&F_{h(x_n),h(x_{n+k+1})}(t) \geq \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(x_{n+1}),h(x_{n+k+1}}(\varphi(r))) \\
&\geq \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \Delta(F_{h(x_n),h(x_{n+k})}(r), F_{h(y_n),h(y_{n+k})}(r))) \\
&\geq \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \Delta(F_{h(x_n),h(x_{n+k})}(t), F_{h(y_n),h(y_{n+k})}(t))) \\
&\geq \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \Delta^{n_k}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \\
&\quad F_{h(y_n),h(y_{n+1})}(t-\varphi(r)))) \\
&= \Delta(1, \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \Delta^{n_k}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \\
&\quad F_{h(y_n),h(y_{n+1})}(t-\varphi(r))))) \\
&\geq \Delta(F_{h(y_n),h(y_{n+1})}(t-\varphi(r)), \Delta(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), \\
&\quad \Delta^{n_k}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))))) \\
&= \Delta(\Delta(F_{h(y_n),h(y_{n+1})}(t-\varphi(r)), F_{h(x_n),h(x_{n+1})}(t-\varphi(r))), \\
&\quad \Delta^{n_k}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r)))) \\
&= \Delta^{n_k+1}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))).
\end{aligned}$$

Similarly, we have

$$F_{h(y_n),h(y_{n+k+1})}(t) \geq \Delta^{b_k+1}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))).$$

Thus one has

$$\begin{aligned}&\min\{F_{h(x_n),h(x_{n+k+1})}(t), F_{h(y_n),h(y_{n+k+1})}(t)\}\\&\geq \Delta^{n_{k+1}}(F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))),\end{aligned}$$

where $n_{k+1} = 2n_k + 2$, which implies that (9) holds for $k+1$. Therefore, (9) holds for all $k \in \mathbb{N}$.

Next, we show that $\{h(x_n)\}$ and $\{h(y_n)\}$ are Cauchy sequences, i.e.,

$$\lim_{m,n\to\infty} F_{h(x_n),h(x_m)}(t) = 1, \quad \lim_{m,n\to\infty} F_{h(y_n),h(y_m)}(t) = 1$$

for any $t > 0$. Let $t > 0$ and $\epsilon > 0$. By hypothesis, $\{\Delta^n : n \in \mathbb{N}\}$ is equicontinuous at 1 and so there exists $\delta > 0$ such that, if $s \in (1-\delta, 1]$, then

$$\Delta^n(s) > 1 - \epsilon \tag{10}$$

for all $n \in \mathbb{N}$. Notice that (7) and (8) imply that

$$\lim_{n\to\infty} F_{h(x_n),h(x_{n+1})}(t-\varphi(r)) = 1, \quad \lim_{n\to\infty} F_{h(y_n),h(y_{n+1})}(t-\varphi(r)) = 1.$$

Hence there exists $n_0 \in \mathbb{N}$ such that, for any $n \geq n_0$,

$$\min\{F_{h(x_n),h(x_{n+1})}(t-\varphi(r)), F_{h(y_n),h(y_{n+1})}(t-\varphi(r))\} \in (1-\delta, 1].$$

Thus, by (9) and (10), we have

$$\min\{F_{h(x_n),h(x_{n+k})}(t), F_{h(y_n),h(y_{n+k})}(t)\} \geq \Delta(F_{h(x_n),h(x_{n+k})}(t), F_{h(y_n),h(y_{n+k})}(t)) > 1-\epsilon$$

for any $k \in \mathbb{N}$. This shows that

$$F_{h(x_n),h(x_{n+k})}(t) > 1-\epsilon, \quad F_{h(y_n),h(y_{n+k})}(t) > 1-\epsilon$$

for all $k \in \mathbb{N}$. This proves that $\{h(x_n)\}$ and $\{h(y_n)\}$ are Cauchy sequences. Since $T(X \times X)$ is complete and $T(X \times X) \subseteq h(X)$, there exist $x^*, y^* \in X$ such that

$$\lim_{n\to\infty} F_{T(x_{n-1},y_{n-1}),h(x^*)}(t) = \lim_{n\to\infty} F_{h(x_n),h(x^*)}(t) = 1 \tag{11}$$

for all $t > 0$ and

$$\lim_{n\to\infty} F_{T(y_{n-1},x_{n-1}),h(y^*)}(t) = \lim_{n\to\infty} F_{h(y_n),h(y^*)}(t) = 1 \tag{12}$$

for all $t > 0$.

Next, we prove that $h(x^*) = T(x^*, y^*)$ and $h(y^*) = T(y^*, x^*)$. Let $t > 0$. By Lemma 2.4, there exists $r \geq t$ such that $\varphi(r) < t$ and so

$$\begin{aligned}&F_{T(x^*,y^*),h(x^*)}(t)\\&\geq \Delta(F_{T(x^*,y^*),T(x_n,y_n)}(\varphi(r)), F_{T(x_n,y_n),h(x^*)}(t-\varphi(r)))\\&\geq \Delta(\Delta(F_{h(x^*),h(x_n)}(r), F_{h(y^*),h(y_n)}(r)), F_{T(x_n,y_n),h(x^*)}(t-\varphi(r))).\end{aligned} \tag{13}$$

Letting $n \to \infty$ in (13), by (11) and (12), we have

$$F_{T(x^*,y^*),h(x^*)}(t) \geq \Delta(\Delta(1,1),1) = 1$$

for all $t > 0$ and so $T(x^*, y^*) = h(x^*)$. Similarly, we can prove that $T(y^*, x^*) = h(y^*)$.

Now, we prove that, if $(x', y') \in X \times X$ is another coupled coincidence point of h and T, then $h(x^*) = h(x')$ and $h(y^*) = h(y')$.

For each $t > 0$, by (4), we have

$$F_{h(x'),h(x^*)}(\varphi(t)) = F_{T(x',y'),T(x^*,y^*)}(\varphi(t)) \geq \Delta(F_{h(x'),h(x^*)}(t), F_{h(y'),h(y^*)}(t))$$

and

$$F_{h(y'),h(y^*)}(\varphi(t)) = F_{T(y',x'),T(y^*,x^*)}(\varphi(t)) \geq \Delta(F_{h(y'),h(y^*)}(t), F_{h(x'),h(x^*)}(t)),$$

which follow that

$$\Delta(F_{h(x'),h(x^*)}(\varphi(t)), F_{h(y'),h(y^*)}(\varphi(t))) \geq \Delta^2\big(F_{h(x'),h(x^*)}(t), F_{h(y'),h(y^*)}(t)\big).$$

By induction, we have

$$\begin{aligned}
&\min\{F_{h(x'),h(x^*)}(\varphi^n(t)), F_{h(y'),h(y^*)}(\varphi^n(t))\} \\
&\geq \Delta(F_{h(x'),h(x^*)}(\varphi^n(t)), F_{h(y'),h(y^*)}(\varphi^n(t))) \\
&\geq \Delta^{2n}\big(F_{h(x'),h(x^*)}(t), F_{h(y'),h(y^*)}(t)\big).
\end{aligned}$$

From Lemma 2.5, it follows that $h(x') = h(x^*)$ and $h(y') = h(y^*)$. This shows that (x^*, y^*) is the unique coupled coincidence point of h and T.

Now, we show that $h(x^*) = h(y^*)$. In fact, from (4), we have

$$\begin{aligned}
F_{h(x^*),h(y_n)}(\varphi(t)) &= F_{T(x^*,y^*),T(y_{n-1},x_{n-1})}(\varphi(t)) \\
&\geq \Delta(F_{h(x^*),h(y_{n-1})}(t), F_{h(y^*),h(x_{n-1})}(t))
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
F_{h(y^*),h(x_n)}(\varphi(t)) &= F_{T(y^*,x^*),T(x_{n-1},y_{n-1})}(\varphi(t)) \\
&\geq \Delta(F_{h(y^*),h(x_{n-1})}(t), F_{h(x^*),h(y_{n-1})}(t))
\end{aligned} \tag{15}$$

for all $t > 0$. Let $M_n(t) = \Delta(F_{h(y^*),h(x_n)}(t), F_{h(x^*),h(y_n)}(t))$ for all $t > 0$. From (14) and (15), it follows that

$$M_n(\varphi^n(t)) \geq \Delta^2\big(M_{n-1}(\varphi^{n-1}(t))\big) \geq \cdots \geq \Delta^{2n}\big(M_0(t)\big)$$

for all $t > 0$. By Lemma 2.5, we have $\lim_{n\to\infty} M_n(t) = 1$, which implies that

$$\lim_{n\to\infty} F_{h(y^*),h(x_n)}(t) = \lim_{n\to\infty} F_{h(x^*),h(y_n)}(t) = 1$$

for all $t > 0$. Hence $h(x_n) \to h(y^*)$ as $n \to \infty$. Since the limit point of $\{h(x_n)\}$ is unique, $h(x^*) = h(y^*)$.

Suppose that h and T are, in addition, weakly compatible. Let $\hat{x} = h(x^*)$. Then $\hat{x} = h(y^*)$ since $h(x^*) = h(y^*)$. Further, we have

$$h(\hat{x}) = h(h(x^*)) = h(T(x^*, y^*)) = T(h(x^*), h(y^*)) = T(\hat{x}, \hat{x}),$$

which implies that $(\hat{x}, \hat{x})$ is a coupled coincidence point of h and T. Since g and F have a unique coupled point of coincidence, we can conclude that $h(\hat{x}) = h(x^*)$, i.e., $h(\hat{x}) = \hat{x}$. Therefore, we have $\hat{x} = h(\hat{x}) = T(\hat{x}, \hat{x})$, that is, $\hat{x}$ is a common fixed point of h and T.

Finally, we prove the uniqueness of common fixed point of h and T. Let $v \in X$ such that $v = h(v) = T(v, v)$. By (4), we have

$$F_{\hat{x},v}(\varphi(t)) = F_{T(\hat{x},\hat{x}),T(v,v)}(\varphi(t)) \geq \Delta(F_{h(\hat{x}),h(v)}(t), F_{h(\hat{x}),h(v)}(t)) = \Delta^2\big(F_{\hat{x},v}(t)\big),$$

which implies that

$$F_{\hat{x},v}(\varphi^n(t)) \geq \Delta^{2n}\big(F_{\hat{x},v}(t)\big).$$

By Lemma 2.5, it follows that $F_{\hat{x},v}(t) = 1$ for all $t > 0$. Hence $\hat{x} = v$. This completes the proof. □

Let $\mathbf{\Psi}$ denote the set of functions $\psi : [0,1] \to [0,1]$ satisfying the conditions $\psi^{-1}(0) = \{0\}$, $\psi^{-1}(1) = \{1\}$ and $\psi(t) > t$ for all $t \in (0,1)$. Then, by Theorem 2.6, we get the following:

Corollary 2.7. *Let (X, F, Δ) be a Menger PM-space under a t-norm Δ of Hadžić-type. Let $T : X \times X \to X$ and $h : X \to X$ be two mappings satisfying that*

$$F_{T(x,y),T(u,v)}(\varphi(t)) \geq \psi(\Delta(F_{h(x),h(u)}(t), F_{h(y),h(v)}(t))) \tag{16}$$

for all $x, y, u, v \in X$ and $t > 0$, where $\varphi \in \mathbf{\Phi}_{\mathbf{w}^*}$ and $\psi \in \mathbf{\Psi}$. Suppose that $T(X \times X) \subseteq h(X)$ and $T(X \times X)$ is complete. Then there exists a unique point $(x^*, y^*) \in X \times X$ such that $h(x^*) = h(y^*) = T(x^*, y^*) = T(y^*, x^*)$. Further, if h and T are weakly compatible, then there exists a unique point $\hat{x} \in X$ such that $\hat{x} = h(\hat{x}) = T(\hat{x}, \hat{x})$.

Proof. Since $\psi \in \mathbf{\Psi}$, $\psi(t) \geq t$ for all $t \in [0,1]$, by (16),

$$\begin{aligned} F_{T(x,y),T(u,v)}(\varphi(t)) &\geq \psi(\Delta(F_{h(x),h(u)}(t), F_{h(y),h(v)}(t))) \\ &\geq \Delta(F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)) \end{aligned}$$

for all $x, y, u, v \in X$ and $t > 0$. From Theorem 2.6, it follows that the conclusion holds. This completes the proof. □

In Corollary 2.7, if $\psi(t) = \sqrt{t}$ for all $t \in [0,1]$, then we have the following:

Corollary 2.8. *Let (X, F, Δ) be a Menger PM-space under the t-norm Δ of Hadžić-type. Let $T : X \times X \to X$ and $h : X \to X$ be two mappings satisfying that*

$$F_{T(x,y),T(u,v)}(\varphi(t)) \geq \sqrt{\Delta(F_{h(x),h(u)}(t), F_{h(y),h(v)}(t))}$$

, for all $x, y, u, v \in X$ and $t > 0$, where $\varphi \in \mathbf{\Phi}_{\mathbf{w}^*}$. Suppose that $T(X \times X) \subseteq h(X)$ and $T(X \times X)$ is complete. Then there exists a unique point $(x^*, y^*) \in X \times X$ such that $h(x^*) = h(y^*) = T(x^*, y^*) = T(y^*, x^*)$. Further, if h and T are weakly compatible, then there exists a unique point $\hat{x} \in X$ such that $\hat{x} = h(\hat{x}) = T(\hat{x}, \hat{x})$.

Remark 2.9. In [25, Theorem 2.1], the t-norm Δ is required to satisfy that $\Delta \geq \Delta_P$. However, Corollary 2.8 has no this restriction. Also, the function φ in [25, Theorem 2.1] needs to satisfy the condition that $\sum_{n=1}^{\infty} \varphi^n(t) < +\infty$ for all $t > 0$. Obviously, this condition is stronger than the one on φ in Corollary 2.8. Thus Corollary 2.8 improves Theorem 2.1 in [25].

3. Applications

As applications of Theorem 2.6, we first give a coupled fixed point theorem in usual metric spaces.

Lemma 3.1. [10] *Let (X, d) be a metric space. Define a mapping $F : X \times X \to \mathcal{D}^+$ by*

$$F_{x,y}(t) = \begin{cases} 0, & t \leq 0 \text{ or } d(x, y) > t > 0, \\ 1, & d(x, y) \leq t \ (t > 0) \end{cases} \tag{17}$$

for all $x, y \in X$. Then (X, F, Δ_M) is a Menger PM-space. It is called the induced Menger PM-space by (X, d) and it is complete if and only if (X, d) is complete.

Theorem 3.2. *Let (X, d) be a complete metric space and $\varphi \in \mathbf{\Phi}_{\mathbf{w}^*}$ be a non-decreasing function. Let $T : X \times X \to X$ and $h : X \to X$ be two mappings satisfying*

$$d(T(x, y), T(u, v)) \leq \varphi(\min\{d(x, u), d(y, v)\}) \tag{18}$$

for all $x, y, u, v \in X$. Suppose that $T(X \times X) \subseteq h(X)$ and $T(X \times X)$ is complete. Then there exists a unique point $(x^*, y^*) \in X \times X$ such that $h(x^*) = T(x^*, y^*)$ and $h(y^*) = T(y^*, x^*)$. Further, if h and T are weakly compatible, then there exists unique $\hat{x} \in X$ such that $\hat{x} = h(\hat{x}) = T(\hat{x}, \hat{x})$.

Proof. For any $t > 0$, if $t < d(h(x), h(u))$ or $t < d(h(y), h(v))$, then we have

$$\min\{F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)\} = 0$$

and hence (4) holds. If $t \geq d(h(x), h(u))$ and $t \geq d(h(y), h(v))$, then we have

$$\min\{F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)\} = 1.$$

Since φ is non-decreasing, from (18), it follows that

$$d(T(x, y), T(u, v)) \leq \varphi(\min\{d(h(x), h(u)), d(h(y), h(v))\}) \leq \varphi(t).$$

From (17), we have $F_{T(x,y),T(u,v)}(\varphi(t)) = 1$ and so (4) holds. Therefore, T and h satisfy the condition (4) for all $x, y, u, v \in X$. Therefore, from Theorem 2.6, it follows that Theorem 3.2 holds. This completes the proof. □

Before giving applications in fuzzy metric spaces, we first recall the concept of fuzzy metric spaces in the sense of Kramosil and Michálek as follows:

Definition 3.3. [15] A *fuzzy metric space* in the sense of Kramosil and Michálek is a triple (X, M, Δ), where X is a nonempty set, Δ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$,

(KM-1) $M(x, y, 0) = 0$;

(KM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

(KM-3) $M(x, y, t) = M(y, x, t)$;

(KM-4) $M(x, z, t + s) \geq \Delta(M(x, y, t), M(y, z, s))$;

(KM-5) $M(x, y, \cdot) : \mathbb{R}^+ \to [0, 1]$ is left continuous.

Let (X, M, Δ) be a KM-fuzzy metric space. It is known that, if $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$, then (X, F, Δ) is a Menger PM-space (see [10, Lemma 2.2]), where

$$F_{x,y}(t) = \begin{cases} M(x, y, t), & t \geq 0, \\ 0, & t < 0. \end{cases}$$

Thus the conclusion of Theorem 2.6 holds in a KM-fuzzy metric space and so we can state the conclusion as follows:

Theorem 3.4. *Let (X, M, Δ) be a KM-fuzzy space under the t-norm Δ of Hadžić-type. Let $T : X \times X \to X$ and $h : X \to X$ be two mappings satisfying*

$$M(T(x, y), T(u, v), \varphi(t)) \geq \Delta(M(h(x), h(u), t), M(h(y), h(v), t))$$

for all $x, y, u, v \in X$ and $t > 0$, where $\varphi \in \mathbf{\Phi_{w^*}}$. Suppose that $T(X \times X) \subseteq h(X)$, $T(X \times X)$ is complete, h and T are weakly compatible. Then there exists unique $u \in X$ such that $u = h(u) = T(u, u)$.

Remark 3.5. In [12], the function φ is required to be non-decreasing, upper semi-continuous from the right and satisfy

$$\sum_{n=0}^{\infty} \varphi^n(t) < +\infty$$

for all $t > 0$. Obviously, if φ satisfies the condition

$$\sum_{n=0}^{\infty} \varphi^n(t) < +\infty$$

for all $t > 0$, then $\varphi \in \mathbf{\Phi_{w^*}}$. The converse is not true (see Example 2.2). Thus the condition on φ is simpler than one in [12]. Thus Theorem 3.4 improves the corresponding ones in [12].

Finally, we give an example to illustrate Theorem 2.6.

Example 3.6. Let $X = \{2^n : n \in \mathbb{N}\} \cup \{0\}$ and define the mapping $F : X \times X \to \mathcal{D}^+$ by $F_{x,y}(0) = 0$ for all $x, y \in X$, $F_{x,x}(t) = 1$ for all $x \in X$ and $t > 0$,

$$F_{x,y}(t) = F_{y,x}(t) = \begin{cases} \dfrac{3}{5}, & 0 < t \leq |x - y|, \\ 1, & t > |x - y| \end{cases}$$

for all $x, y \in X$ with $x \neq y$. It is easy to see that (X, F, Δ_M) is a complete Menger PM-space.

Let $T : X \times X \to X$ and $h : X \to X$ be two mappings defined by

$$T(x, y) = 0$$

for all $x, y \in X$ with $xy = 0$,

$$T(2, y) = 0$$

for all $y \in X$,

$$T(x, y) = x$$

for all $x, y \in X$ with $xy \neq 0$ and $x \neq 2$ and

$$h(0) = 0, \quad h(2^n) = 2^{n+1}$$

for each $n \in \mathbb{N}$, respectively. It is easy to see that $T(X \times X) = h(X) = \{2^{n+1} : n \in \mathbb{N}\} \cup \{0\}$ and so $h(X)$ is complete. We also see that T and h are weakly compatible.

Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a function defined by

$$\varphi(t) = \begin{cases} t, & 0 \leq t \leq 1, \\ t - 1, & t > 1. \end{cases}$$

Then $\varphi \in \mathbf{\Phi_{w^*}}$. In fact, for each $t_1, t_2 \in (0, \infty)$, there exists $r = N + \epsilon$, where $N \in \mathbb{N}$ with $N > \max\{t_1, t_2\}$ and $\epsilon < \min\{t_1, t_2\}$ such that $\varphi^n(r) = \epsilon < \min\{t_1, t_2\}$ for all $n \in \mathbb{N}$ with $n \geq N$. Thus $\varphi \in \mathbf{\Phi_{w^*}}$. However, $\varphi \notin \mathbf{\Phi_w}$ since $\lim_{n\to\infty} \varphi^n(t) \in (0, 1)$ for all $t > 0$.

Now, we show that T and h satisfy the condition (4). For all $x, y, u, v \in X$, if $xy = 0$ and $uv = 0$, then T and h satisfy (4) since $T(x, y) = T(u, v) = 0$. For all $x, y, u, v \in X$ with $xy \neq 0$ or $uv \neq 0$ and $t > 0$, if $\varphi(t) > |T(x, y) - T(u, v)|$, then we have

$$F_{T(x,y),T(u,v)}(\varphi(t)) = 1 \geq \min\{F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)\}.$$

Next, assume that $\varphi(t) \leq |T(x, y) - T(u, v)|$. We show the condition (4) by the following cases:

(A) $xy = 0$, $u = 2^n$, $v = 2^m$: For all $t > 0$, $\varphi(t) < |T(x, y) - T(u, v)| = 2^n$ implies that $t < 2^n + 1 < 2^{n+1} = h(u)$ and so

$$F_{T(x,y),T(u,v)}(\varphi(t)) = \frac{3}{5} = \min\{F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)\} = \frac{3}{5}.$$

(B) $xy \neq 0$ and $uv \neq 0$: Let $x = 2^s$, $y = 2^l$, $u = 2^m$ and $v = 2^n$ for each $l, s, m, n \in \mathbb{N}$. For all $t > 0$, $\varphi(t) < |T(x, y) - T(u, v)| = |2^l - 2^m|$ implies that $t < |2^l - 2^m| + 1 < |2^{l+1} - 2^{m+1}| = 2|2^l - 2^n| = |h(x) - h(u)|$ and so

$$F_{T(x,y),T(u,v)}(\varphi(t)) = \frac{3}{5} = \min\{F_{h(x),h(u)}(t), F_{h(y),h(v)}(t)\}.$$

By the cases above, (4) holds for all $x, y, u, v \in X$ and $t > 0$. Therefore, by Theorem 2.6, there exists $x^* \in X$ such that $x^* = T(x^*, y^*) = h(x^*)$. In fact, $x^* = 0$.

4. Conclusion

In this paper, we have proved some new coupled fixed point theorems for φ-contractions in Menger PM-spaces and fuzzy metric spaces with the t-norm of H-type. In the results, the gauge function φ only needs to satisfy the condition (1), i.e., for each $t_1, t_2 > 0$, there exists $r \geq \max\{t_1, t_2\}$ and $N \in \mathbb{N}$ such that

$$\varphi^n(r) < \min\{t_1, t_2\}$$

for all $n > N$. In fact, it is the weakest condition in the similar results given in some papers. Therefore, the results in this paper improve some theorems in the papers [12, 25]. Can The condition (1) be weakened further? This question is an interesting and worthy question for further investigation.

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