Computers & Geosciences 44 (2012) 86–94

Contents lists available at SciVerse ScienceDirect

Computers & Geosciences

journal homepage: www.elsevier.com/locate/cageo

# Estimating shear wave velocity of soil deposits using polynomial neural networks: Application to liquefaction

Ali Ghorbani [a,*], Yaser Jafarian [b], Mohammad S. Maghsoudi [a]

[a] Faculty of Engineering, Guilan University, PO Box 41635-1477, Rasht, Iran
[b] Department of Civil Engineering, Semnan University, Semnan, Iran

ARTICLE INFO

Article history:
Received 29 September 2011
Received in revised form
3 February 2012
Accepted 2 March 2012
Available online 16 March 2012

Keywords:
Shear wave velocity
In-situ tests
PNN
Standard penetration test
Database
Liquefaction

ABSTRACT

Geophysical and geotechnical field investigations have introduced several techniques to measure in-situ shear wave velocity of soils. However, there are some difficulties for the easy and economical use of these techniques in the routine geotechnical engineering works. For the soil deposits, researchers have developed correlations between shear wave velocity and SPT-N values. In the present study, a new database containing the measured shear wave velocity of soil deposits have been compiled from the previously published studies. Using polynomial neural network (PNN), a new correlation has been subsequently developed for the prediction of shear wave velocity. The developed relationship shows an acceptable performance compared with the available relationships. Three examples are then presented to confirm accuracy and applicability of the proposed equation in the field of liquefaction potential assessment.



## 1. Introduction

Small strain shear modulus ($G_{max}$) and shear wave velocity ($V_s$) are essential parameters for the dynamic analysis of soil systems. Shear wave velocity can be directly used for some geotechnical problems such as evaluating liquefaction potential (e.g., Dobry et al., 1981; Seed et al., 1983; Tokimatsu and Uchida, 1990; Andrus and Stokoe, 2000), soil classification (e.g., Dobry et al., 2000), site response analysis (e.g., Choi and Stewart, 2005), and earthquake-induced ground motions (e.g., Jafarian et al., 2010).

These parameters may be determined via small strain laboratory tests on undisturbed soil samples. Resonant column and bender element tests are the most common devices to estimate small-strain parameters in laboratory; however, cyclic triaxial apparatus with the precise measurement of axial strain has been also used for this purpose. The effect of sample disturbance on small-strain stiffness is significant in laboratory-based measurements since the weak bonds between the soil particles are broken during sampling process. Furthermore, undisturbed sampling in granular deposits is not possible without expensive freezing techniques. Therefore, small-strain shear modulus ($G_{max}$) is more rational to be estimated from shear wave velocity ($V_s$). In-situ shear wave velocitycan be measured by a number of geophysical techniques such as cross-hole test (CHT), down-hole test (DHT), seismic cone penetration tests (SCPTs), refraction micro-tremor (ReMi), multi-station analysis of surface waves (MASW), and spectral analysis of surface waves (SASW). The accuracy of these techniques depends on implementation details, soil conditions, and interpretation methods (Andrus and Stokoe, 1997). Moreover, field measurement techniques involve clear advantages compared with laboratory methods since they examine the soil in its natural condition with minimum disturbance. On the other hand, although in-situ geophysical measurements are the most appropriate methods for this purpose, several limitations such as space constraints, cost considerations, and high noise levels arising from some of these tests, especially in urban areas, make them impractical or restricted in many situations. Hence, empirical equations are essential tools for preliminary assessment of shear wave velocity based on the relevant geotechnical parameters.

Researchers have carried out several studies during the last four decades to develop relationships between shear wave velocity and geotechnical parameters of soils. Majority of these studies have tried to correlate ($V_s$) with SPT blow count, SPT-N, and a few studies have employed other parameters such as effective overburden stress, fine content, depth, cone penetration tip resistance (Kanai et al., 1966; Hamilton, 1976; Ohsaki and Iwasaki, 1973; Campbell and Duke, 1976; Ohta and Goto, 1978; Seed and Idriss, 1981; Jinan, 1987; Lee, 1990; Lodge, 1994; Athanasopoulos, 1995; Sisman, 1995; Iyisan, 1996; Jafari et al., 1997; Kiku et al., 2001; Hanumantharao and Ramana, 2008;

* Corresponding author. Tel.: +98 911 8193170.
E-mail address: ghorbani@guilan.ac.ir (A. Ghorbani).

0098-3004/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cageo.2012.03.002

Hasancebi and Ulusay, 2006; Dikmen, 2009; Brandenberg et al., 2010; Akin et al., 2011).

As discussed above, numerous data were measured in the field condition during the previous studies. These data provide possibility for the development of a predictive correlation for the in-situ shear wave velocity of soils. It is clear that a precise correlation is easier to be used in the routine geotechnical projects compared with the mechanics-based measurement techniques. The current study aims to present such correlation using a large database containing soil parameters and the corresponding values of shear wave velocity. Available correlations of shear wave velocity are reviewed first. Performance of the reviewed correlations is subsequently examined through a comprehensive database of measured shear wave velocity, which was gathered herein. Then, a new correlation is developed via an advanced polynomial neural network algorithm. Reasonable precision and applications of the developed correlation are demonstrated in the final part of the paper.

## 2. Review of available relationships

The most frequent functional form of the previous equations is $V_s = A \cdot N^B$, where the constants $A$ and $B$ are calculated using statistical regression of a given data set. A summary of the available equations are given in Table 1.

Kanai et al. (1966) presented an equation based on 70 field case histories gathered from Japan for all soil types. Campbell and Duke (1976) employed 63 field measurements and proposed two equations which only depend on the depth of soil layer ($D$) for two types of recent and older alluvium sediments. Seed and Idriss (1981) developed another equation to correlate shear wave velocity to the values of SPT blow counts which were corrected for depth (i.e., $N_{1,60}$). Using statistical analysis Lee (1990), examined different combinations of the multiple regression models and introduced equations for predicting shear wave velocity in SM, CL, and ML soils. For a seismic region located in the eastern Turkey Iyisan (1996), proposed several equations and evaluated effects of various parameters on the values of shear wave velocity such as SPT blow counts ($N$), vertical overburden stress ($\sigma_v$), rains mean diameter ($D_{50}$), corrected SPT-N value($N_{1,60}$), tip resistance of cone penetration test ($q_c$), and depth of the sediment ($H$). Furthermore Jafari et al. (1997), used numerous field data gathered from southwest of Tehran and developed some relationships predicting shear wave velocity of all soil types. Hasancebi and Ulusay (2006) studied correlations using 97 case histories collected from an area in the north-western part of Turkey and proposed their equations for sands, clays, and for all soils. Recently Brandenberg et al. (2010), employed regression analysis and proposed a predictive equation of shear wave velocity for Caltrans bridges sites. They used a total data of 79 boring logs from 21 bridges and predicted $\ln(V_s)$ for sands, silts, and clays as a function of SPT resistance and effective overburden pressure.

## 3. Database development and influential parameters

Majority of the previous studies, which are cited in Table 1, employed limited number of data from one particular site. Therefore, these models cannot guarantee reasonable performance for various site conditions. In this study, a large database was compiled from 10 different sites, which can potentially lead to a generalized equation. The compiled database contains 80 boreholes with totally 394 data measurements as summarized in Table 2.

The database includes some geotechnical parameters such as number of standard penetration blow counts, SPT-N, vertical overburden stress, ($\sigma_v$), effective overburden stress, ($\sigma'_v$), and the corresponding values of shear wave velocity. The values of shear wave velocity were measured using seismic cone penetration test (SCPT) and spectral analysis of surface waves test (SASW). Table 3 shows that input variables as well as the output vary in a wide range. Accordingly, $N_{1,60}$, ($\sigma_v$), ($\sigma'_v$) and ($V_s$) are distributed within 0–75, 12.1–408.9 (kPa), 7.5–233.7 (kPa), and 66–363 (m/s) ranges, respectively.

Fig. 1 is demonstrated to evaluate performance of the previous models (cited in Table 1) for the new compiled database. The figure

**Table 1**
Some empirical relations for predicting shear wave velocity based on SPT blow count.

| References | | Soil type | | | | Remark |
|---|---|---|---|---|---|---|
| | | All | Sand | Silt | Clay | |
| 1 | Kanai et al., (1966) | $V_s = 19N^{0.6}$ | – | – | – | |
| 2 | Hamilton (1976) | $V_s = 128D^{0.28}$ | – | – | – | |
| 3 | Ohsaki and Iwasaki (1973) | $V_s = 81.4N^{0.39}$ | – | – | – | |
| 4 | Campbell and Duke (1976) | $V_s = 319D^{0.386}$ | – | – | – | $D$ in feet |
| 5 | Ohta and Goto (1978) | $V_s = 85.3N^{0.3481}$ | – | – | – | |
| | | $V_s = 92.1D^{0.339}$ | – | – | – | |
| 6 | Seed and Idriss (1981) | $V_s = 61.4N^{0.5}$ | – | – | – | |
| 7 | Jinan (1987) | $V_s = 116(N+0.318)^{0.202}$ | – | – | – | $D$ in feet ; $Vs$ in $m/s$ |
| | | $V_s = 90.9(D+0.62)^{0.212}$ | – | – | – | |
| 8 | Lee (1990) | – | $V_s = 57.4N^{0.49}$<br>$V_s = 57.4D^{0.46}$ | $V_s = 105.6D^{0.32}$ | $V_s = 114.4D^{0.31}$ | |
| 9 | Athanasopoulos (1995) | $V_s = 107.6N^{0.36}$ | – | – | – | |
| 10 | Sisman (1995) | $V_s = 32.8N^{0.51}$ | – | – | – | |
| 11 | Iyisan (1996) | $V_s = 51.5N^{0.516}$ | – | – | – | |
| 12 | Jafari et al. (1997) | $V_s = 22N^{0.85}$ | – | – | – | |
| 13 | Kiku et al. (2001) | $V_s = 68.3N^{0.292}$ | – | – | – | |
| 14 | Hasancebi and Ulusay (2006) | $V_s = 90N^{0.309}$ | $V_s = 90.82N^{0.319}$ | – | $V_s = 97.89N^{0.269}$ | |
| 15 | Imai et al. (1975) | $V_s = 89.9N^{0.341}$ | | | | |
| 16 | Brandenberg et al. (2010) | | $\ln(V_s)_{ij} = \beta_0 + \beta_1 \ln(N_{60})_{ij} + \beta_2 \ln(\sigma'_v)_{ij}$ [a] | | | |

[a] $\beta$ coefficients were presented by Brandenberg et al. (2010) for any type of soils.

illustrates the measured values of shear wave velocity versus SPT-N values as well as the curves associated with the previous equations. According to Table 1, it should be noted that some of the previous models are independent of SPT-N values and, thus were not plotted in Fig. 1. The figure confirms that the measured data show a considerable scattering because their associated spots are widely distributed in the plot. Nevertheless, a single curve is resulted in for any model because they are only dependent to SPT-N value. In fact, majority of the equations reported in the literature (Table 1) has utilized only one parameter (often SPT-N value) to evaluate the values of shear wave velocity. The current study employs two input parameters for a more accurate prediction of the shear wave velocity.

Majority of the previous studies investigated the effect of the SPT-N on the shear wave velocity. A few researchers, however, studied the influence of other parameters like soil type on $V_s$ values. Iyisan (1996) used Turkey data and investigated how soil type affects the correlation between SPT-N value and shear wave velocity. The results showed that the equations developed for all soils, sands and clays obtain similar ($V_s$) values excepting gravels for which SPT results are questionable. Furthermore, Iyisan (1996) examined influence of the effective vertical stress on shear wave velocity and concluded that the correlation between ($V_s$) and $N_1$ would be more accurate provided the effective overburden pressure ($\sigma'_v$) is taken into account. Rollins et al. (1998) also noted that an improvement in shear wave velocity prediction can be achieved if the effective stress is considered in the regression equation. Importance of effective overburden pressure in the estimation of shear wave velocity was also highlighted in the recent study conducted by Brandenberg et al. (2010).

**Table 2**
Summary of case histories used in this study.

| | Site location | Reference | Number of boring | Number of cases |
|---|---|---|---|---|
| 1 | Argentina | Cetin (2000) | 5 | 5 |
| 2 | Guatemala | Cetin (2000) | 3 | 3 |
| 3 | Hyogoken-Nambu | Cetin (2000) | 24 | 56 |
| 4 | Imperial Valley | Cetin (2000) | 1 | 1 |
| 5 | Loma Prieta | Cetin (2000) | 1 | 1 |
| 6 | Nihonkai-Chubu | Cetin (2000) | 2 | 8 |
| 7 | Niigata | Cetin (2000) | 2 | 11 |
| 8 | Northridge | Cetin (2000) | 2 | 2 |
| 9 | San Fernando | Cetin (2000) | 2 | 2 |
| 10 | Turkey | PEER (2000) | 38 | 305 |
| | Sum= | | **80** | **394** |

**Table 3**
Ranges of parameters in new database.

| Statistical properties | Parameters | | | |
|---|---|---|---|---|
| | $N_{1,60}$ | $\sigma_v$ (kPa) | $\sigma'_v$(kPa) | $V_s$(m/s) |
| Min | 0 | 12.1 | 7.5 | 66 |
| Max | 75 | 408.9 | 233.7 | 363 |
| Mean | 15.35 | 135.06 | 78.36 | 169.49 |

## 4. Modeling using polynomial neural networks (PNN)

Conventional explicit modeling of complex systems requires recognition of a reasonable mathematical relationship between the inputs and outputs. This prejudgment on the model structure may introduce undesirable errors into the problem. Instead, soft computing methods are less affected by such drawbacks and, thus have received considerable attentions in the recent years (e.g., Baziar and Ghorbani, 2005; Baziar and Jafarian, 2007; Atashkari et al., 2007; Jafarian et al., 2010).

Many researchers have attempted to use such methods as effective tools for system identification. Polynomial Neural Network (PNN) is a hybrid self-organizing approach by which complicated models are generated based on the evaluation of their performances for a set of multi-input single-output data pairs (Onwubolu, 2009). PNN was initially developed by Ivakhnenko (1971) and applied to various problems such as data mining and knowledge discovery, forecasting and system modeling, optimization and pattern recognition. It provides possibility to find interrelations of data, to select optimal structure of model or network, and to increase the accuracy of existing algorithms. The main concept of this approach is to produce an analytical function in a feed forward neural network based on a quadratic node transfer function whose coefficients are obtained using regression technique.

In general, polynomial neural network involves some advantages over the other types of neural networks. First, it is capable to select the most suitable input variables in a set of applicants. By sorting different solutions, such networks aim to minimize

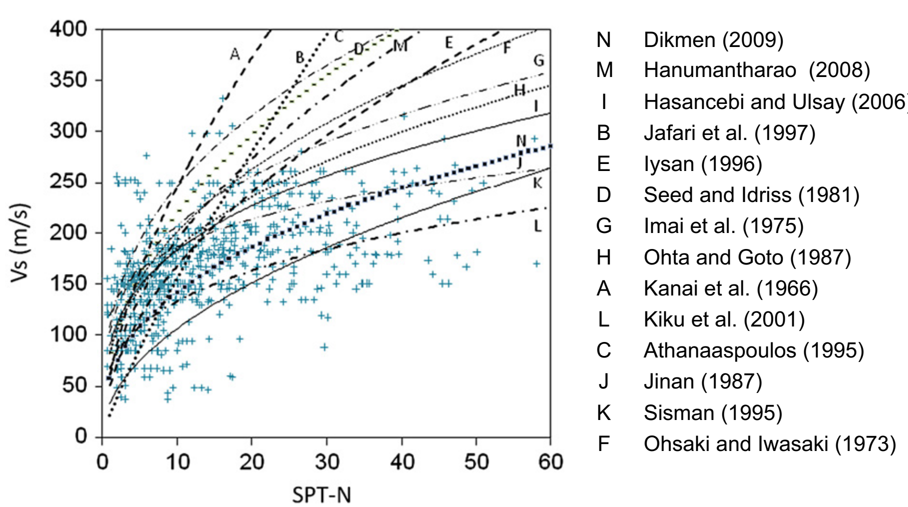


**Fig. 1.** Performance of the previous equations of shear wave velocity against the compiled database.

influence of the developer on the results of modeling. Computer automatically finds the optimal structure of the model and the laws acting on the system. This could be an important feature of PNN knowing that design of the conventional neural networks includes difficulties such as finding the most appropriate topology and initial coefficients (weights). Therefore, their performance may be noticeably impacted by the model developer (Onwubolu, 2009).

PNN represents an ANN in which different pairs of neurons in each layer are connected through a quadratic polynomial, and thus they produce new neurons for the next layer. The formal definition of the identification problem is to find a function $\hat{f}$ that approximates an actual function, $f$, so that predicts output $\hat{y}$ for a given input vector $X=(x_1, x_2, x_3, \ldots, x_n)$ as close as possible to actual output $y$ (Sanchez et al., 1997). More details about the mathematical basis of polynomial neural networks and optimizing its structure and parameters is beyond the scope of this paper and can be found in the relevant references (e.g., Nariman-Zadeh et al., 2003).

### 4.1. Data division

In the soft computing applications, the database is commonly divided into two parts including training and testing subsets. The testing data set tries to obtain a more generalized model while it is not incorporated in the training procedure. According to an accepted rule, training and testing data sets must be similar in terms of their statistical properties such as mean and standard deviation (Tokar and Johnson, 1999). Data division can be carried out using genetic algorithm, fuzzy clustering, and random selecting techniques.

In the present study, training and testing sets with similar statistical properties were achieved using the random selecting technique. A sensitivity analysis was then performed to examine how the developed model is affected by the different combinations of training and testing sets.

### 4.2. Optimized model architecture

Determination of the optimized network architecture is an important and difficult step in neural network modeling. It involves selection of the number of nodes and hidden layers. Since the PNN method is a self-organizing approach, optimized model architecture is achieved properly easier than the ANN method, thereby saving much time in this stage.

### 4.3. Weight optimization (training)

The procedure of optimizing the connection weight is known as training or learning process. Back-propagation algorithm is commonly used in the conventional neural networks for finding the optimum weights. In contrast, PNN uses stochastic technique that is better than traditional gradient based techniques.

### 4.4. Model validation

After obtaining the optimal model architecture, performance of the trained model is validated to ensure its ability for the prediction of unseen cases, which have not participated in the training procedure. The validation process is carried out using some controlling measures like the coefficient of determination ($R^2$), root mean squared error (RMSE), and mean absolute error (MAE). The coefficient of determination ($R^2$) determines the relative correlation between two sets of variables and is defined as:

$$R^2 = \frac{\sum_j (o_j)^2 - \sum_j (t_j - o_j)^2}{\sum_j (o_j)^2} \tag{1}$$

where $t$ and $o$ stand for target and output values, respectively.

The RMSE is a popular error measure and has the advantage that large errors receive greater than smaller ones. In contrast, the MAE eliminates the emphasis given the large errors. However, both RMSE and MAE are desired when the evaluated data are smooth or continuous. These two coefficients are defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_j (t_j - o_j)^2}{p}} \tag{2}$$

$$\mathrm{MAE} = \frac{\sum_p |t_j - o_j|}{p} \tag{3}$$

where $p$ is the patterns number.

## 5. Results and discussion

### 5.1. The proposed equation

Out of 394 case histories 307 cases (78% of all data) were used for training stage and the rest (22%) were considered as testing set. The optimum structure was obtained using PNN. Corrected SPT blow counts ($N_{1,60}$), and effective vertical overburden pressure ($\sigma'_v$) were recognized by PNN to be more effective than the other parameter (i.e., ($\sigma_v$)), and thus they were selected as input variables. This is in accord with the findings of the recent researches that developed equations for $V_s$ (Brandenberg et al., 2010). In addition, the experimental studies conducted by resonant column tests on the samples of soils have frequently reported strong dependency of small strain shear modulus on soil density and effective confining pressure (Towhata, 2008).

The optimum network which includes two hidden layers and three nodes obtained the following empirical equation for prediction of shear wave velocity:

$$\mathbf{V_s} = 3.02 + 1.8839\mathbf{Y}_2 - 0.9307\mathbf{Y}_3 + 0.33683\mathbf{Y}_2^2 + 0.35324\mathbf{Y}_3^2 - 0.68995\mathbf{Y}_2\mathbf{Y}_3 \tag{4}$$

where:

$$Y_3 = -157.27 - 1.184\sigma'_v + 3.3944Y_1 - 0.00198\sigma'^2_v - 0.00891Y_1^2 + 0.0086\sigma'_v Y_1$$

$$Y_2 = 1.62 + .935Y_1 + 0.551N_{1,60} - 0.00036Y_1^2 + 0.00372N_{1,60}^2 - 0.00396Y_1 N_{1,60}$$

$$Y_1 = 106.27 + 2.34N_{1,60} + 0.48\sigma'_v - 0.021N_{1,60}^2 + 0.00052\sigma'^2_v - 0.00204N_{1,60}\sigma'_v$$

In Eq. (4), $N_{1,60}$ is the SPT blow counts corrected for effective overburden pressure and hammer energy, ($\sigma'_v$) is the effective overburden stress in kPa, and ($V_s$) is the predicted shear wave velocity in m/s.

Performance of the proposed model for training and testing sets was examined using two criteria: (1) the statistical parameters mentioned through Eqs. (1–3) the residual plots shown in Fig. 2(a) and (b). For the training set coefficient of determination, root mean squared error, and mean absolute error were obtained as $R^2 = 0.96$, $\mathrm{RMSE} = 35\ m/s$, and $\mathrm{MAE} = 26\ m/s$, respectively. Furthermore, for the testing set the mentioned values were obtained as $R^2 = 0.95$, $\mathrm{RMSE} = 37.2\ m/s$, and $\mathrm{MAE} = 28.2\ m/s$, which are very close to the values belong to the training set. Residuals are defined as the difference between the observed and predicted values as illustrated in Fig. 2(a) and (b) versus the input parameters i.e., $N_{1,60}$ and ($\sigma'_v$), respectively. The plots include both

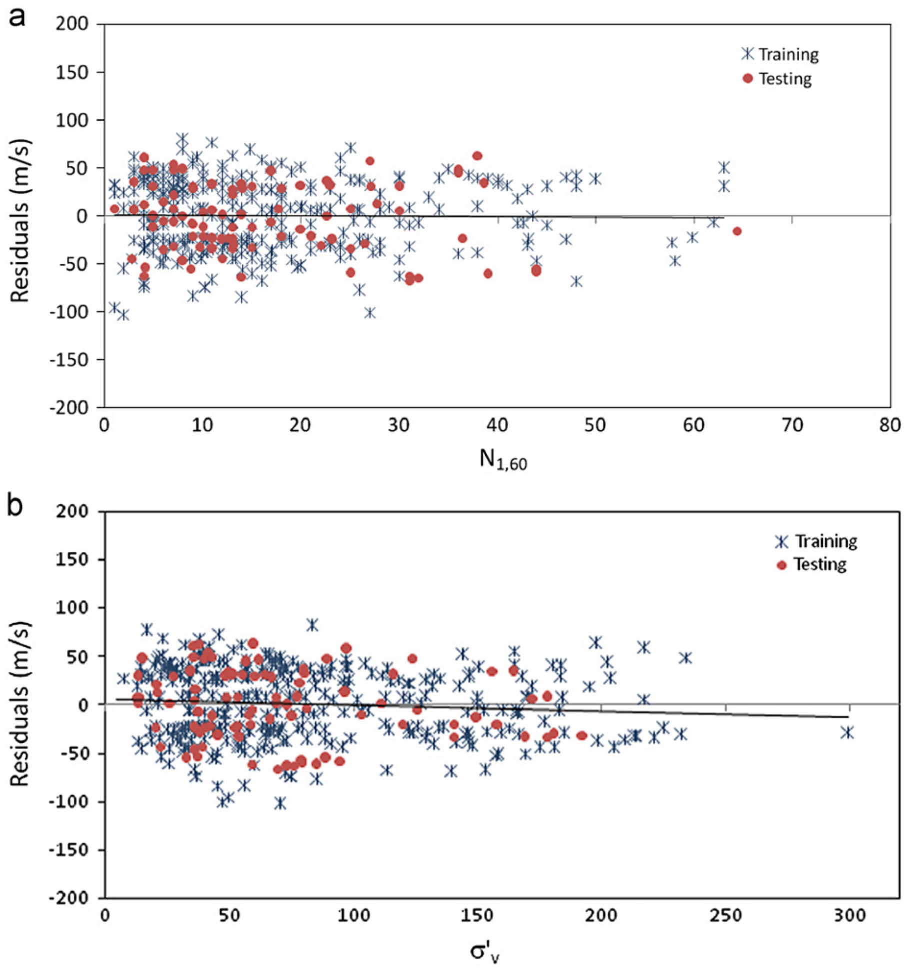


**Fig. 2.** Residuals of training and testing sets versus (a) corrected SPT blow counts, and (b) effective overburden pressure.

training and testing datasets. In addition, a linear trend line was plotted throughout each figure in order to emphasize the negligible bias of the predicted values versus the input variables.

Both shear wave velocity ($V_s$) and standard penetration resistant ($N_{60}$) are commonly corrected for effective overburden stress. To consider this effect, these parameters are commonly modified with the following equations:

$$(N_1)_{60} = N_{60}\left\{\frac{P_a}{\sigma'_v}\right\}^n \quad (5)$$

$$V_{s1} = V_s\left\{\frac{P_a}{\sigma'_v}\right\}^m \quad (6)$$

where $(\sigma'_v)$ is effective overburden pressure and $n$ and $m$ exponents vary based on soil type, cementation, and plasticity index. Typical values of $n$ change from 0.5 for sand and 1 for clay. Also, the value of $m$ can be considered 0.25 for clean sands and a maximum value of 0.5 for cohesive soils (Robertson et al., 1992; Yamada et al., 2008; Brandenberg et al., 2010).

Because there are different normalized equations for SPT and $V_s$ (as mentioned above), it is required to include effective stress as a variable for any equation which correlates corrected values of SPT resistance and shear wave velocity. Although this important parameter has not been considered in majority of the previous studies, effective stress was included as an important variable in the proposed PNN-based model.

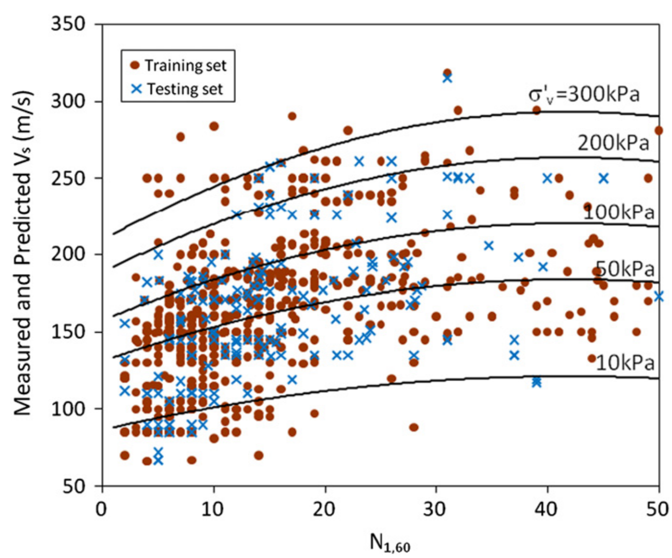


**Fig. 3.** Parametric study of the developed equation (curves) and measured values (data points).

### 5.2. Parametric study

Variations of the shear wave velocity against $N_{1,60}$ and $(\sigma'_v)$parameters cannot be directly explained from the equation due to multivariate form of the proposed equation. Thus, shear wave velocity was considered as a function of the $N_{1,60}$ in the different levels of effective overburden stress ($\sigma'_v$). Accordingly, effective overburden pressure ($\sigma'_v$) was assumed equal to 10, 50, 100, 200, and 300 kPa and the resulted values of shear wave velocity were plotted against corrected $N$ value as depicted in Fig. 3. As seen, values of shear wave velocity increase versus $N_{1,60}$ for a given constant effective overburden stress, while the incremental trend is more pronounced for higher levels of stress. It is also obvious from Fig. 3 that deeper soil strata possess larger values of shear wave velocity. The measured values of ($V_s$) were also superimposed in the figure as individual data points in order to demonstrate that the entire range of the measured points can be covered by the proposed equation and various combinations of $N_{1,60}$ and ($\sigma'_v$).

## 6. Comparison with the previous models

Variations of the model prediction along the ground depth are investigated through the recorded data belong to some randomly selected boring logs. The results of this evaluation have been shown in Fig. 4(a)–(e). For comparison, predictions of some previously published equations which are only dependent on depth are also shown in the figures. This comparison confirms that the values predicted by PNN can reasonably trace the field measurements, while the previous depth-based equations generally over-predicted or under-predicted the measured values of shear wave velocity along the depth.

In Table 4, accuracy of the developed model (i.e., Eq. (4)) is compared with the most recent equation which was developed by Brandenberg et al. (2010). As seen in this table, Brandenberg et al. (2010) fitted their equation (see Table 1) to the shear wave velocity data of Caltrans bridge sites for three types of soils (i.e., sands, silts, and clays). Since, in reality, natural soils commonly exist in mixed texture, the authors have proposed a single equation for all types of soils. Furthermore, broad scattering of the available field data might not allow for such a precise categorization of the soil types. Table 4 confirms that the proposed equation has more accuracy than Brandenberg et al. (2010)'s relationships for the database compiled in the current study.

## 7. Application of the proposed model in liquefaction analysis

Shear wave velocity of soils has many applications in the geotechnical problems. This parameter has been recommended as an index to evaluate the liquefaction resistance of sands

**Table 4**
Comparison between the accuracy of Brandenberg et al. (2010)'s equation and the current study.

| Soil type | Brandenberg et al. (2010) | | | This study | | |
|---|---|---|---|---|---|---|
| | $R^2$ | RMSE (m/s) | MAE (m/s) | $R^2$ | RMSE (m/s) | MAE (m/s) |
| Sand | 0.93 | 46.82 | 37.95 | – | – | – |
| Silt | 0.94 | 42.17 | 34.05 | – | – | – |
| Clay | 0.92 | 50.36 | 39.94 | – | – | – |
| All soils[a] | 0.93 | 45.60 | 36.62 | 0.95 | 37.75 | 32.14 |

[a] For Brandenberg et al. (2010), "All soils" is an average given for the predicted values of the soil types.

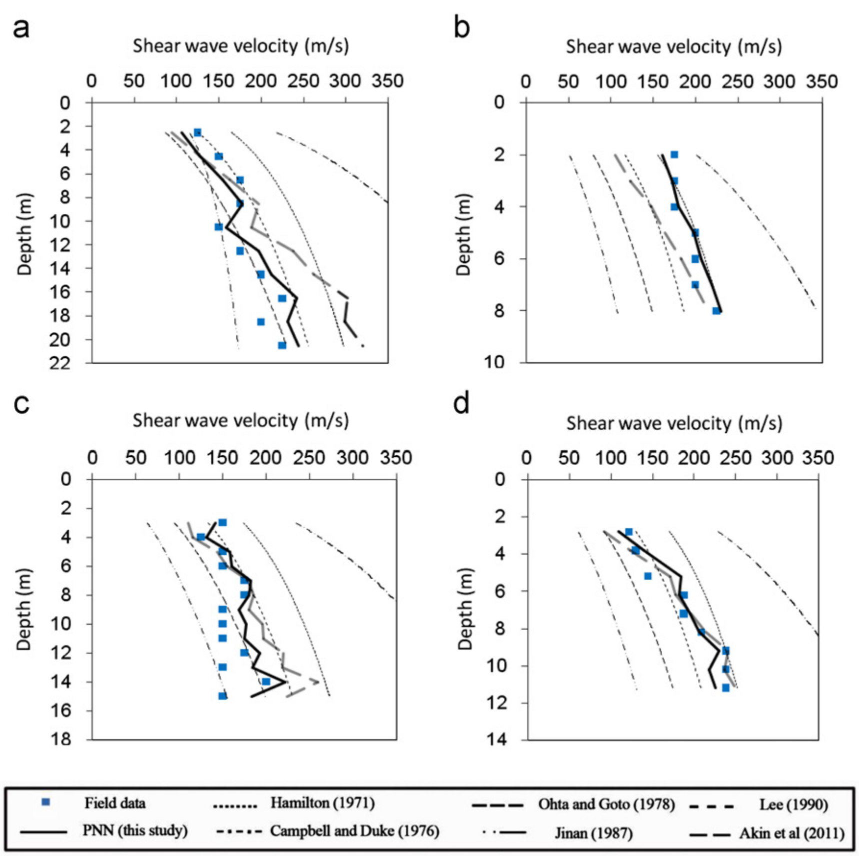


**Fig. 4.** Evaluating of the developed model in depth for some sites including (a) Niigata, (b) Nihonaki-Chubu, (c) Hyogoken-Nanbu, (d) Taiwan, and (e) Turkey.

(Kayen et al., 1992;Tokimatsu and Uchida, 1990; Andrus and Stokoe, 2000; Youd et al., 2001). Liquefaction phenomenon has been widely observed in loose to medium non-cohesive soils during the past earthquakes (e.g., Alaska 1964 and Tohoku 2011 earthquakes) and produced tremendous failures in the ground and civil engineering structures. Evaluation of liquefaction potential is commonly performed via the empirical criteria obtained from the liquefaction/non-liquefaction case histories. Factor of safety against liquefaction initiation is obtained by dividing the cyclic resistance ratio (CRR) to cyclic stress ratio (CSR). CSR and CRR, respectively stand for the driving and resisting shear strengths normalized by the effective overburden pressure. Several empirical relationships can be found in which cyclic resistance ratio (CRR) is correlated to the shear wave velocity of the soil. This part of the study aims to demonstrate how the developed PNN-based ($V_s$) model is in accord with the ($V_s$)-based studies of liquefaction potential assessment. Hence, three practical examples are given in the subsequent parts.

### 7.1. Example 1

Participants of NCEER 2001 Workshop (Youd et al., 2001) recommended the following equation for estimating cyclic resistance ratio (CRR) of clean sands in terms of $N_{1,60}$:

$$\mathrm{CRR}_{7.5} = \frac{1}{34 - N_{1,60}} + \frac{N_{1,60}}{135} + \frac{50}{[10N_{1,60} + 45]^2} - \frac{1}{200} \tag{7}$$

where $\mathrm{CRR}_{7.5}$ stands for cyclic resistance ratio at the earthquake moment magnitude of 7.5.

The proposed $V_s$ model (i.e., Eq. (4)) is employed and $N_{1,60}$ in Eq. (7) is substituted with shear wave velocity for a reference effective overburden pressure, $(\sigma'_v)$ , of 100 kPa. Hence, the magnitude-scaled cyclic resistance ratio, $\mathrm{CRR}_{7.5}$, is obtained as a function of equivalent shear wave velocity. The heavy solid curve (curve $A$) in Fig. 5 illustrates variations of the equivalent $V_s$-based $\mathrm{CRR}_{7.5}$, which was obtained from the mentioned procedure, versus the shear wave velocity normalized by effective overburden pressure ($V_{sl}$). In addition, the available $V_{sl}$-CRR correlations are also superimposed in this figure in order to be compared by the equivalent $V_{s1}$-CRR correlation, which was obtained by the proposed $V_s$ model. As seen in the figure, use of the proposed $V_s - N_{1,60}$ correlation yields a liquefaction state boundary curve which is in accord with the previously published recommendations.

### 7.2. Example 2

The most common relationship used to evaluate the liquefaction resistance of sands from shear wave velocity is the equation proposed by Andrus and Stokoe (2000), which was developed based on a database including 26 earthquakes and more than 70 measurement sites. Their relationship, which was also recommended by the participants of NCEER workshop (Youd et al., 2001), has the following form:

$$\mathrm{CRR} = \left\{ a\left(\frac{V_{s1}}{100}\right)^2 + b\left(\frac{1}{V^*_{s1} - V_{s1}} - \frac{1}{V^*_{s1}}\right) \right\}\mathrm{MSF} \tag{8}$$

where $V^*_{s1}$ is the limiting upper value of $V_{sl}$ for cyclic liquefaction occurrence which depends on fines contents, $F_c$, (soil particles smaller than 0.075 mm).

Also, $a$ and $b$ are curve fitting parameters and were considered 0.022 and 2.8, respectively (Andrus and Stokoe, 2000). MSF is the magnitude scaling factor to account for the effect of earthquake magnitude and can be calculated as below:

$$\mathrm{MSF} = \left(\frac{M_w}{7.5}\right)^n \tag{9}$$

where $M_w$ is moment magnitude and $n$ is equal to $-2.56$ (Andrus and Stokoe, 2000).

Fig. 6(a) shows three liquefaction evaluation charts presented by Andrus and Stokoe (1997) for different levels of fines content. The data points shown in this figure were obtained from the in-situ measurement of shear wave velocity using the techniques such as SASW, crosshole, downhole, and SCPT. Cetin (2000) re-analyzed a worldwide liquefaction catalog including 200 liquefaction case histories for which corrected SPT values, $N_{1,60}$, were measured. The $V_s$ model proposed in the current study is applied to the Cetin (2000)'s SPT database in order to convert the measured $N_{1,60}$ values of the whole 200 case histories to equivalent $V_s$ values with the reference $\sigma'_v$ of 100 kPa. Fig. 6(b) illustrates the converted $V_s$ values of these case histories versus cyclic shear stress ratio (CSR). The triple boundary curves of Andrus and Stokoe (1997) were also superimposed in this figure. As seen

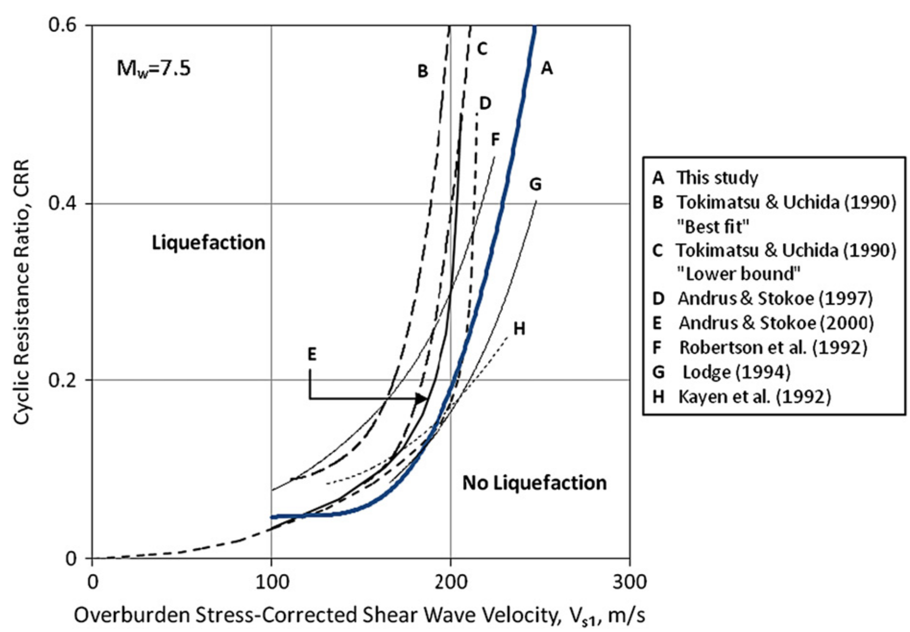


**Fig. 5.** Comparison of available $V_{sl}$-CRR correlations and correlation derived by new $V_s$ equation developed in this study.

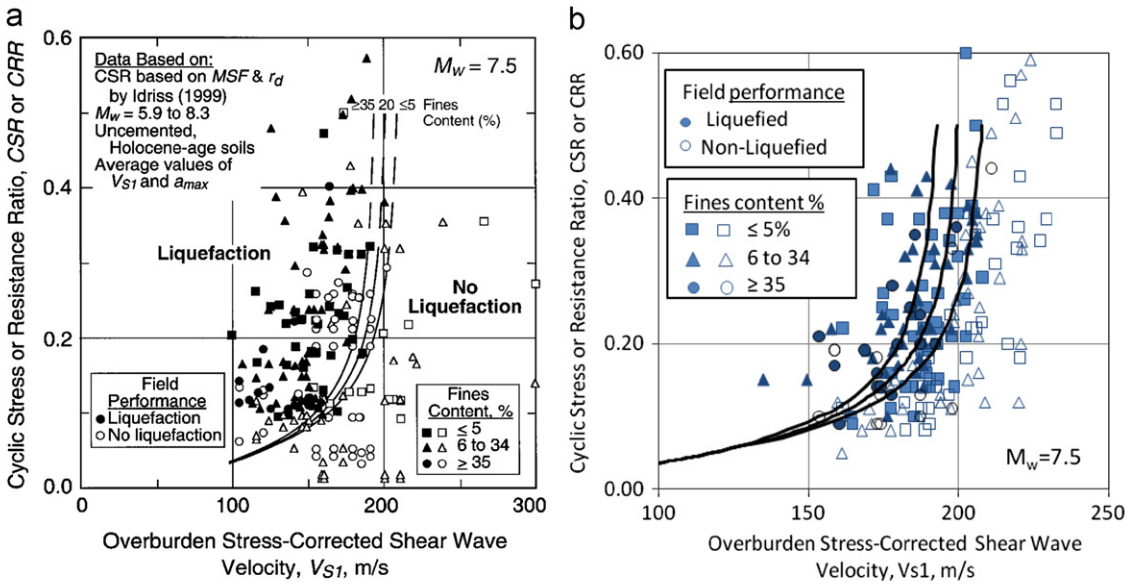


**Fig. 6.** CRR-$V_{s1}$ curves for different fines content and evaluation of proposed $V_s$ equation for conversion of SPT resistance to shear wave velocity, (a) Andrus and Stokoe (2000)'s $V_s$ database, (b) Cetin (2000)'s SPT database.

in Fig. 6(b), the case histories have relatively categorized into liquefied/non-liquefied conditions based the $N_{1,60}$-to-$V_s$ conversion made by the new equation. Furthermore, the triple boundary curves have relatively located between the liquefied and non-liquefied points. Comparison between Fig. 6(a) and (b) confirms that the equivalent $V_s$-based case histories together with the boundary curves are comparable with the plot of original $V_s$-based case histories, as shown in Fig. 6(a).

### *7.3. Example 3*

The procedure described in Example 2 was applied to a special "small magnitude" catalog of liquefaction case histories, which was compiled by Prof. T.L. Youd at Brigham Young University and reported by Cetin (2000). These data are potentially valuable due to the infrequency of small magnitude ($M_w < 6.2$) liquefaction case histories. The database includes 44 cases; majority of them are non-liquefied events and involve the geotechnical parameters that are required to apply the procedure described in Example 2.

The original SPT-based data points were converted to equivalent $V_s$ values and were plotted, as shown in Fig. 7. According to this figure, almost all of the points have located in the right hand side of the liquefaction curves; confirming that liquefaction was not occurred. This is in accordance with the field observations for these low magnitude liquefaction case histories.

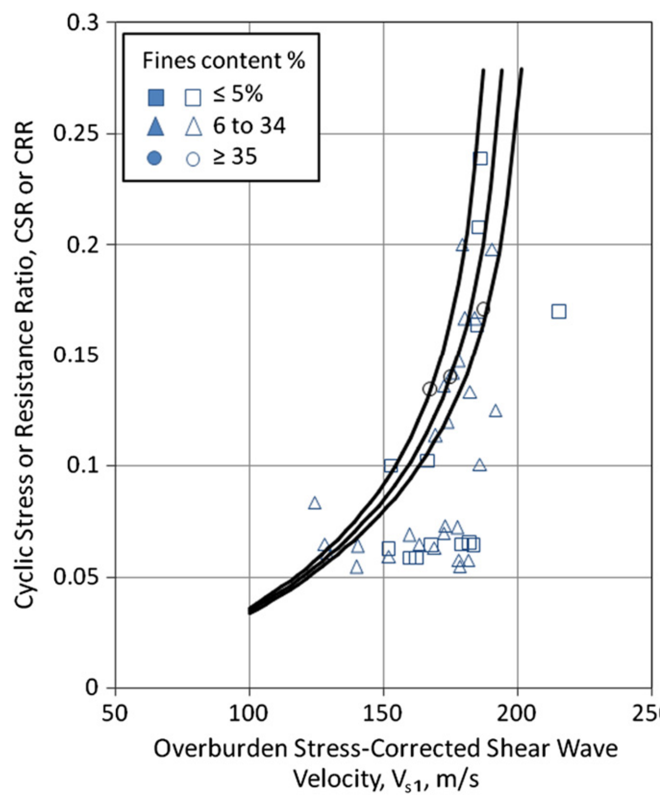


**Fig. 7.** Evaluating performance of proposed $V_s$ equation in liquefaction potential for a "small magnitude" database compiled by Prof. T.L. Youd at Brigham Young University and reported by Cetin (2000).

## 8. Summary and conclusions

A new correlation was presented for shear wave velocity, $V_s$, of soil deposits as a function of corrected SPT blow counts, $N_{1,60}$, and effective overburden stress, $(\sigma'_v)$. The correlation was developed using polynomial neural network (PNN) and a newly compiled database of shear wave velocity measurement including 10 different sites, 80 boreholes, and totally 394 data pairs. Prior to the model development, numerous existing equations, which were previously proposed for specified site and soil conditions, were examined via the compiled database. Significant scatter and errors were observed for the previous models because majority of them were just dependent to penetration resistance and ignore effective overburden pressure. In addition, most of the previous models were developed based on the measurements of shear wave velocity in a specific site, and thus they might not be useful for various sites.

The developed PNN model represents a good performance for both training ($R^2 = 0.96$, RMSE $= 35\ m/s$, and MAE $= 26\ m/s$) and testing ($R^2 = 0.95$, RMSE $= 37.2\ m/s$, and MAE $= 28.2\ m/s$) data sets. Parametric study was carried out and the effect of $(\sigma'_v)$ on the shear wave velocity of soils was shown. Moreover, the equation was validated in depth by the comparisons made between the model predictions and measured shear wave velocity of some boring logs.

More precision of the PNN-based $V_s$ model, compared with the previous equations, comes from the facts that SPT-N value cannot be sufficient to determine shear wave velocity and it should be accompanied with effective overburden pressure to enhance accuracy of the prediction. In fact, effective overburden pressure was found to be an important parameter for the estimation of shear wave velocity because the developed model, which considers this parameter, yields superior performance compared with the previously published equations. This finding is in accord with the small strain laboratory tests that have shown significant dependency of small strain shear modulus ($G_{max}$) to effective stress; considering the fact that ($G_{max}$)and shear wave velocity are rigorously dependent together.

Three applicable examples are demonstrated in the final part of the paper in order to show applications of the shear wave velocity model in the liquefaction potential assessment of soils. Reasonable performance of the developed model is confirmed in the examples since it agrees with the available $V_s$-based charts of liquefaction potential assessment.

## References


Akin, M.K., Karamer, S., Topal, T., 2011. Empirical correlations of shear wave velocity ($V_s$) and penetration resistance (SPT-N) for different soils in an earthquake-prone area (Erbba–Turkey). Engineering Geology 119, 1–17.

Andrus, R.D., Stokoe, K.H., 1997. Liquefaction resistance based on shear wave velocity. In: Proceedings of the NCEER Workshop on Evaluation of Liquefaction Resistance in Soils. Technical Report NCEER-97-0022, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Buffalo, NY, pp. 89–128.

Andrus, R.D., Stokoe II, K.H., 2000. Liquefaction resistance of soils from shear-wave velocity. Journal of Geotechnical and Geoenvironmental Engineering 126 (11), 1015–1025.

Atashkari, K., Nariman-Zadeh, N., Golcu, M., Khalkhali, A., Jamali, A., 2007. Modelling and multi-objective optimization of a variable valve-timing spark-ignition engine using polynomial neural networks and evolutionary algorithms. Energy Conversion and Management 48 (3), 1029–1041.

Athanasopoulos, G.A., 1995. Empirical correlations Vs-NSPT for soils of Greece: a comparative study of reliability. In: Proceedings of the 7th International Conference on Soil Dynamics and Earthquake Engineering, Chania, Crete, pp. 19–26.

Baziar, M.H., Ghorbani, A., 2005. Evaluation of lateral spreading using artificial neural networks. Soil Dynamics and Earthquake Engineering 25, 1–9.

Baziar, M.H., Jafarian, A., 2007. Assessment of liquefaction triggering using strain energy concept and ANN model: capacity energy. Soil Dynamics and Earthquake Engineering 27, 1056–1072.

Brandenberg, S.J., Bellana, N., Shantz, T., 2010. Shear wave velocity as a statistical function of standard penetration test resistance and vertical effective stress at Caltrans bridge sites. PEER Report 2010/03, Pacific Earthquake Engineering Research Center.

Campbell, K., Duke, C., 1976. Correlations among seismic velocity, depth and geology in the Los Angeles area. UCLS, UCLA-ENG-766 2, 54 pp, University of California at Los Angeles School of Engineering and Applied Science.

Cetin, K.O., 2000. Reliability-based assessment of seismic soil liquefaction initiation hazard. Ph.D. Dissertation, University of California, Berkeley, CA, May 2000, 602 pp.

Choi, Y., Stewart, J.P., 2005. Nonlinear site amplification as function of 30 m shear wave velocity. Earthquake Spectra 21 (1), 1–30.

Dikmen, U., 2009. Statistical correlations of shear wave velocity and penetration resistance for soils. Journal of Geophysics and Engineering 6, 61–72.

Dobry, R., Stokoe, K.H., II, Ladd, R.S., Youd, T.L., 1981. Liquefaction susceptibility from S-wave velocity. In: Proceedings of In Situ Tests to Evaluate Liquefaction Susceptibility, ASCE National Convention, St. Louis, USA, preprint 81-544.

Dobry, R., Borcherdt, R.D., Crouse, C.B., Idriss., I.M., Joyner, W.B, Martin, G.R., Power, M.S., Rinne, Seed, R.B., 2000. New site coefficients and site classification system used in recent building seismic code provisions. Earthquake spectra 16(1), 41-67.

Hamilton, E., 1976. Shear wave velocity versus depth in marine sediments: a review. Geophysics 41 (5), 985–996.

Hanumantharao, C., Ramana, G.V., 2008. Dynamic soil properties for microzonation of Dehli, India. Journal of Earth System Science 117, 719–730.

Hasancebi, N., Ulusay, R., 2006. Empirical correlations between shear wave velocity and penetration resistance for ground shaking assessments. Bulletin of Engineering Geology and the Environment 66 (2), 203–213.

Imai, T., Fumoto, H., Yokota, K., 1975. The relation of mechanical properties of soil to P-and S-wave velocities in Japan. In: Proceedings of the 4th Earthquake Engineering Symposium, Japan, pp. 89–96 (in Japanese).

Ivakhnenko, A.G., 1971. Polynomial theory of complex systems. IEEE Transactions on Systems, Man, and Cybernetics, SMC 1 (1), 364–378.

Iyisan, R., 1996. Correlations between shear wave velocity and in-situ penetration test results. Teknik Dergi 7 (2), 1187–1199. ([Chamber of Civil Engineers of Turkey, in Turkish])s.

Jafari, M.K., Asghari, A., Rahmani, I., 1997. Empirical correlation between shear wave velocity ($V_s$) and SPT-N value for south of Tehran soils. In: Proceedings of the 4th International Conference on Civil Engineering, Tehran, Iran (in Persian).

Jafarian, Y., Kermani, E., Baziar, M.H., 2010. Empirical predictive model for $V_{max}/a_{max}$ ratio of strong grpund motions using genetic programming. Computers and Geosciences 36 (12), 1523–1531.

Jinan, Z., 1987. Correlation between seismic wave velocity and the number of blow of SPT and depth. Chinese Journal of Geotechnical Engineering, 92–100.

Kanai, K., Tanaka, T., Morishita, T., Osada, K., 1966. Observation of microtermors. Bulletin of the Earthquake Research Institute 44, 1297–1333.

Kayen, R.E., Mitchell, J.K., Seed, R.B., Lodge, A., Nishio, S., Coutinho, R., 1992. Evaluation of SPT-, CPT-, and shear wave-based methods for liquefaction potential assessment using Loma Prieta data. In: Proceedings of the 4th Japan–U.S Workshop on Earthquake Resistant Design of Lifeline Facilities and Countermeasures for Soil Liquefaction, Honolulu, Hawaii, pp. 177–204.

Kiku, H., Yoshida, N., Yasuda, S., Irisawa, T., Nakazawa, H., Shimizu, Y., Ansal, A. Erkan, A., 2001. In-situ penetration tests and soil profiling in Adapazari, Turkey. In: Proceedings of the ICSMGE/TC4 Satellite Conference on Lessons Learned from Recent Strong Earthquakes, pp. 259–65.

Lee, S.H., 1990. Regression models of shear wave velocities. Journal of the Chinese Institute of Chemical Engineers 13, 519–532.

Lodge, A.L., 1994. Shear wave velocity measurements for subsurface characterization. Ph.D. Dissertation, University of California at Berkeley, CA, 305 pp.

Nariman- Zadeh, N., Darvizeh, A., Ahmad-Zadeh, G.R., 2003. Hybrid genetic design of GHDH-type neural networks using singular value decomposition for modeling and prediction of the explosive cutting process. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 217, 779–790.

Ohsaki, Y., Iwasaki, R., 1973. On dynamic shear moduli and Poisson's ratio of soil deposits. Soils and Foundations 13, 61–73.

Ohta, Y., Goto, N., 1978. Empirical shear wave velocity equations in terms of characteristic soil indexes. Earthquake Engineering & Structural Dynamics 6, 167–187.

Onwubolu, G.C. (Ed.), 2009. Hybrid Self-Organizing Modeling Systems. Springer-Verlag, Berlin, Heidelberg, Chennai, India.

Pacific Earthquake Engineering Research (PEER), 2000. Documenting incidents of ground failure resulting from the Aug. 17, 1999, Kocaeli, Turkey Earthquake. ⟨http://peer.Berkeley.edu⟩.

Robertson, P.K., Woeller, D.J., Finn, W.D., 1992. Seismic cone penetration test for evaluating liquefaction potential under cyclic loading. Canadian Geotechnical Journal 29 (4), 686–695.

Rollins, K.M., Evans, M.D., Diehl, N.B., Daily., W.D., 1998. Shear modulus and damping relationships for gravels. Journal of Geotechnical and Geoenvironmental Engineering 124, 396–405.

Sanchez, E., Shibata, T., Zadeh, L.A., 1997. Genetic Algorithms and Fuzzy Logic Systems. World Scientific, River Edge, NJ 250 pp.

Seed, H.B., Idriss, I.M., 1981. Evaluation of liquefaction potential sand deposits based on observation of performance in previous earthquakes. ASCE National Convention, Missouri, 81–544.

Seed, H.B., Idriss, I.M., Arangol, I., 1983. Evaluation of liquefaction potential using field performance data. Journal of Geotechnical Engineering, ASCE, 109 (3), 396–405.

Sisman, H., 1995. The relation between seismic wave velocities and SPT, Pressuremeter Tests. M.Sc. Thesis, Ankara University, Geophysical Engineering Department, Ankara, 75 pp. (in Turkish).

Tokar, S.A., Johnson, P.A., 1999. Rainfall-runoff modelling using artificial neural networks. Journal of Hydrologic Engineering 4 (3), 232–239.

Tokimatsu, K., Uchida, A., 1990. Correlation between liquefaction resistance and shear wave velocity. Soils and Foundations 30 (2), 33–42.

Towhata, I., 2008. Geotechnical Earthquake Engineering. Springer-Verlag, Berlin, Heidelberg 698pp.

Yamada, S., Hyodo, M., Orense, R.P., Dinesh, S.V., Hyodo, T., 2008. Strain-dependent dynamic properties of remolded sand–clay mixtures. Journal of Geotechnical and Geoenvironmental Engineering 134 (7), 972–981.

Youd, T.L., Idriss, I.M., Andurus, Ronald D., Arango, I., Castro, G., Christian, J.T., Dobry, R., Finn, W.D.L., Harder, L.F., Haymes, M.E., Ishihara, K., Koester, J.P., Liao, S.S.C., Marcusson, W.F., Martin, G.R., Mitchell, J.K., Moriwaki, Y., Power, M.C., Robertson, P.K., Seed, R.B., Stokoe, K.H., 2001. Liquefaction resistance of soils: summary report from the 1996 NCEER and 1998 NCEER/NSF Workshops on Evaluation of Liquefaction Resistance of Soils. Journal of Geotechnical and Geoenvironmental Engineering 127 (10), 817–833.