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# Finite-time fractional-order adaptive intelligent backstepping sliding mode control of uncertain fractional-order chaotic systems

Nooshin Bigdeli*, Hossein Alinia Ziazi

*Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran*

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**Abstract**

This paper precedes chaos control of fractional-order chaotic systems in presence of uncertainty and external disturbances. Based on some basic properties on fractional calculus and the stability theorems, we present a hybrid adaptive intelligent backstepping-sliding mode controller (FAIBSMC) for the finite-time control of such systems. The FAIBSMC is proposed based on the concept of active control technique. The asymptotic stability of the controller is shown based on Lyapunov theorem and the finite time reaching to the sliding surfaces is also proved. Illustrative and comparative examples and simulation results are given to confirm the effectiveness of the proposed procedure, which consent well with the analytical results.


## 1. Introduction

One of the mathematical topics with more than three centuries history is fractional calculus theory which can be traced back to Leibniz, Riemann, Liouville, Grünwald, and Letnikov [1–3]. Although, it did not attract much attention for a long time, but, in the recent years, due to high modeling accuracy of real physical systems by fractional order equations, these systems have been increasingly used for modeling in many areas such as physics [4] and engineering [5]. In interdisciplinary fields, many systems have been found which can be described by fractional

*Corresponding author.
*E-mail addresses:* n.bigdeli@eng.ikiu.ac.ir, nooshin_bigdeli@yahoo.com (N. Bigdeli), Hossein.alinia.ziazi@gmail.com (H.A. Ziazi).

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differential equations. For instance, electrochemical processes [6], viscoelastic systems, dielectric polarization and electromagnetic waves [2,7], and some biological systems [8] can be encountered in this category. Applying latent possibilities of fractional calculus, in order to improve the performance of classical controllers, is one of the recent applications of this mathematical topic.

One of the phenomena which are frequently observed in the fractional order nonlinear systems is chaos phenomenon. Chaotic systems are a prominent class of nonlinear systems, which have various special properties, such as phenomenal sensitivity to system initial conditions, chaotic attractors, and fractal motions. Study of chaotic systems with fractional orders is one of the hottest research areas. Up to now, researchers have studied the fractional order nonlinear dynamics of many popular chaotic systems, such as fractional order Lorenz system [9–11], fractional order Rössler system [12,13], fractional order Chen system [12,14], fractional order Lü system [15,16] and fractional order Genesio-Tesi system [17] and some other chaotic systems. One of the most attractive topics in the nonlinear science which has been comprehensively studied in the recent decades is Chaos control and synchronization. Therefore, many control procedures such as sliding mode control (SMC) [18–22], linear control [23], adaptive control theory [24–26], backstepping control [27–30], active control [31], fuzzy sliding mode control [32], adaptive sliding mode control [33,34] and some other methods [35–38] have been successfully applied to chaos control and synchronization. Backstepping method is a recursive procedure which was proposed for designing controls for a special type of nonlinear dynamical systems by choosing Lyapunov functions skillfully and a systematic design approach, and it can guarantee global stability, good tracking and transient performance for most of strict-feedback systems. Therefore, it has become one of the important and popular approaches for nonlinear systems [39,40]. Sliding mode control technique has been known as a powerful strategy to design robust control for linear and nonlinear systems in recent years. The significant superiorities of SMC are its robustness to parameters uncertainty and its immutability to external disturbances. It is also a strong robust procedure for controlling the nonlinear dynamic systems [41–43], especially for uncertain systems. It has been studied extensively and received many applications due to its insensitivity to system parameter variations, external disturbances rejection, fast dynamic and good transient response which made it applicable for the control problem of chaotic system [18–22,32–34]. In this control scheme, for conducting an effective switching surface and guaranteeing the stability of an SMC system, a good estimation of the uncertainty bound, including the unknown dynamics, parameter variation, and external disturbance must be available at the outset of the design. Since, in practice, such bounds cannot be estimated easily, some conservative control strategies are also need to be applied for ensuring stability of the closed loop system.

Due to the universal approximation ability of fuzzy systems and neural networks it has been widely used in chaos control problems [44–46]. Recently, the learning abilities of neural networks to realize to design of fuzzy systems have been recognized as a powerful approach [47]. The neuro-fuzzy-network (NFN) possesses the merits of low-level learning and computation power of neural networks, and the high-level human-like thinking and reasoning of fuzzy theory. However, in complicated situations, where plant parameters are subject to perturbations or when the dynamics of the systems are too complex for a mathematical model to describe, adaptive schemes should be used online to gather data and adjust the control parameters, automatically.

Recently, many significant results have been obtained by using hybrid adaptive intelligent control techniques for uncertain chaotic systems [48–53]. For fractional-order systems, this idea

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has been extended to some extent, by the researchers, in the literature. For example, in [54], uncertain fractional-order Liu system is controlled via fuzzy fractional-order sliding mode method. In [55], adaptive fuzzy sliding mode control has been employed for synchronization of uncertain fractional order chaotic systems, in which, a fuzzy Lyapunov synthesis approach is proposed to tune free parameters of the adaptive fuzzy controller on line by output feedback control law and adaptive law. Synchronization of uncertain fractional order chaotic systems via adaptive interval type-2 fuzzy sliding mode control was considered in [56] to handle high level uncertainties facing the fuzzy logic controller (FLC) in dynamic fractional order chaotic systems such as uncertainties in inputs to the FLC, uncertainties in control outputs, linguistic uncertainties and uncertainties associated with the noisy training data. In [57], an adaptive fuzzy sliding mode control was designed to synchronize two different uncertain fractional-order time-delay chaotic systems, where, the adaptive time-delay fuzzy-logic system is constructed to approximate the unknown fractional-order time-delay-system functions. In [58], the design of adaptive fuzzy wavelet neural sliding mode controller for uncertain nonlinear systems was studied for a class of high-order nonlinear systems while the structure of the system is unknown and no prior knowledge about uncertainty is available. The proposed scheme was employed to construct equivalent control term and an Adaptive Proportional-Integral (A-PI) controller for implementing switching term to provide smooth control input. Next attempt ([59]) deals with the synchronization for Arneodo chaotic system and response system with unknown nonlinear function. Based on backstepping method, a fuzzy adaptive control scheme by combining fuzzy logic system with parameter was presented to achieve synchronization. In [60], a fractional-order adaptive intelligent controller was designed based active control method, in which, the unknown boundaries of the lumped uncertainties are estimated via an adaptive neuro-fuzzy approximator.

In this paper, in order to merge the capabilities of backstepping control and sliding mode control, an adaptive intelligent backstepping sliding mode controller is proposed for stabilization of fractional order chaotic systems in finite time. The intelligent neuro-fuzzy controller is used to estimate unknown functions. The use of auxiliary controller is in order to improve velocity and performance of the proposed control system and to dispel the uncertainties, external disturbances and error approximation. These three factors are unknown bounded, and so the controller will take advantage of robust adaptive design. In design of the control law, neuro-fuzzy network parameters and the uncertainties bound estimator, external disturbances bound estimator and approximation error, are adjusted as an adaptive scheme. The asymptotic stability of the controller is shown based on Lyapunov theorem and the finite reaching time to the sliding surfaces is also proved. The results are indicative of the Lyapunov stability of the closed loop system, robustness against uncertainties, external disturbances and approximation errors, while the control signal remains bounded.

The rest of this paper is organized as follows: In Section 2, the definitions and some basic properties of fractional calculus are introduced. Section 3, presents dynamic models of fractional order chaotic systems which are used in this paper and in Section 4 description of the neuro-fuzzy network structure is presented. Problem description is described in Section 5. Numerical simulation results which confirm the validity and feasibility of the designed controllers, are shown in Section 6. Finally, conclusions are given in Section 7.

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## 2. Preliminaries on fractional calculus

Three of the most prominent definitions which are used for fractional derivatives are: Riemann-Liouville, Grunwald-Letnikov, and Caputo definitions. Caputo definition is extensively utilized in engineering applications since it takes on the same form as for integer-order differential equations in the initial conditions. Therefore, the following sections are based on Caputo derivative definition.

**Definition 1.** The Caputo fractional derivative of order $\alpha$ of a continuous function $f : R^+ \rightarrow R$ is defined as:

$$\begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m} f(t), & \alpha=m \end{cases} \tag{1}$$

where $\Gamma$ is the well-known Gamma function, and

$$\Gamma(z) = \int_0^t e^{-t} t^{z-1} dt, \text{ and } \Gamma(z+1) = z\Gamma(z) \tag{2}$$

It should be noted that the fractional integral of order $\alpha > 0$ is denoted by $D_t^{-\alpha}$. In what follows, we list several basic facts of fractional derivatives and integrals.

**Fact 1.** For $\alpha = n$, where $n$ is an integer, the operation $D_t^n f(t)$ gives the same result as classical calculus of integer-order $n$. In particular, When $\alpha = 1$, the operation $D_t^1 f(t)$ coincides with the ordinary derivative $df(t)/dt$.

**Fact 2.** For $\alpha = 0$, the operation $D_t^\alpha f(t)$ is the identity operation:

$$D_t^0 f(t) = f(t) \tag{3}$$

**Fact 3.** Similar to integer-order calculus, fractional differentiation and fractional integration are both linear operations.

$$D_t^\alpha [af(t) + bg(t)] = aD_t^\alpha f(t) + bD_t^\alpha g(t) \tag{4}$$

where $a$ and $b$ are constants.

**Fact 4.** The additive law of exponents (semi-group property) holds.

$$D_t^\alpha D_t^\beta f(t) = D_t^\alpha D_t^\beta f(t) = D_t^{\alpha+\beta} f(t) \tag{5}$$

**Fact 5.** For $\alpha > 0$, the following equation holds:

$$D_t^\alpha D_t^{-\alpha} f(t) = D_t^0 f(t) = f(t) \tag{6}$$

Which means that the fractional differentiation operator is a left inverse to the fractional integration operator of the same order $a$.

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**Lemma 2.1.** (Fractional order extension of Lyapunov direct method [61]). Let $x=0$ be an equilibrium point for either Caputo or RL fractional non-autonomous system:

$$D_t^q x(t) = f(x,t) \tag{7}$$

where $q \in (0,1)$ and $f(x,t)$ satisfies the Lipschitz condition with Lipschitz constant $l>0$. Assume that there exits a Lyapunov function $V(t,x(t))$ satisfying

$$\begin{aligned} &\alpha_1\|x\|^a \le V(t,x(t)) \le \alpha_2\|x\| \\ &\dot{V}(t,x) \le -\alpha_3\|x\| \end{aligned} \tag{8}$$

where $\alpha_1, \alpha_2, \alpha_3$ and $a$ are positive constants and $\|.\|$denotes an arbitrary norm. Then the equilibrium point of system (7) is asymptotically stable.

**Lemma 2.2.** (Barbalat's lemma [62]). If $\eta{:}R\to R$ is a uniformly continuous function for $t\ge 0$ and if the limit of the integral $\int_0^t \eta(\omega)d\omega$ exists and is finite, then $\lim_{t\to\infty}\eta(t)=0$.

## 3. The example systems description

### 3.1. The fractional order energy resources demand–supply system

The fractional order energy resources demand–supply system is described as follows [63]:

$$\begin{aligned} &D^{q_1}x_1 = a_1x_1\left(1-x_1/M\right) - a_2(x_2+x_3) \\ &D^{q_2}x_2 = b_1x_2 - b_2x_3 + b_3x_1(N-(x_1-x_3)) \\ &D^{q_3}x_3 = c_1x_3(c_2x_1-c_3) \end{aligned} \tag{9}$$

where $(a_1,a_2,b_1,b_2,b_3,c_1,c_2,c_3,M,N) = (0.1, 0.3, 0.01, 0.02, 0.2, 0.5, 0.8, 0.1, 2, 1)$ and $q=[0.98, 0.85, 0.92]$, the system shows chaotic behavior as shown in Fig. 1.

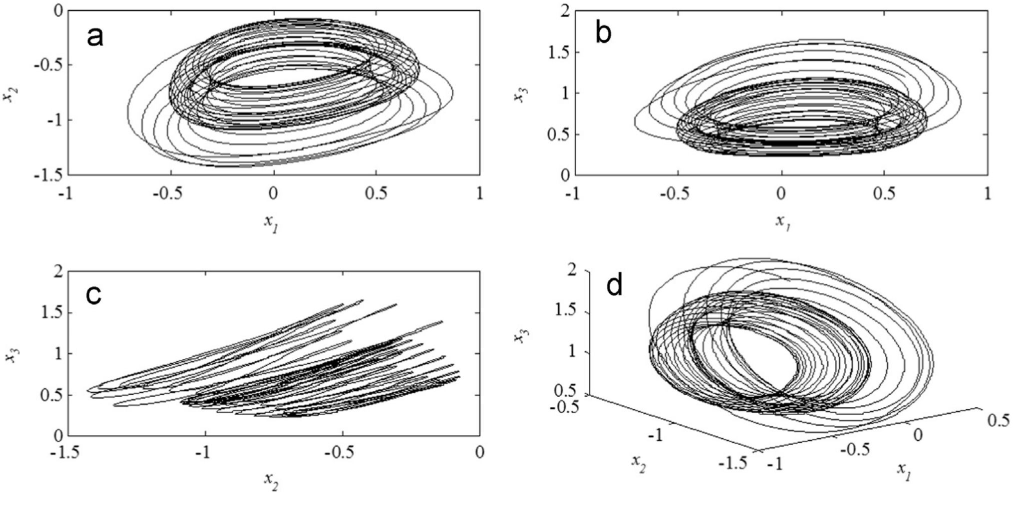


Fig. 1. The chaotic attractor of the fractional order energy resources demand–supply system.

### 3.2. The fractional order Chen system

The fractional order Chen system is given as follows [14]:

$$\begin{aligned} D^q x_1 &= \beta_1 (x_2 - x_1) \\ D^q x_2 &= (\beta_2 - \beta_1) x_1 - x_1 x_3 + \beta_2 x_2 \\ D^q x_3 &= x_1 x_2 - \beta_3 x_3 \end{aligned} \tag{10}$$

where $\beta_1 = 35, \beta_2 = 28, \beta_3 = 3$, By choosing $q = 0.995$, system of Eq. (10) has chaotic attractor as seen in Fig. 2.

## 4. Neuro-fuzzy network estimator

Recently, neuro-fuzzy networks have been demonstrated in a lot of researches such as control application and information processing [46,47,64]. Neuro-fuzzy networks have the advantages of fuzzy systems and neural networks, simultaneously: one is the inference characteristic of the fuzzy system; and the other one is the learning ability of the neural network which can be applied for the adjustment of the fuzzy rules. The architecture of a hybrid neuro-fuzzy system is shown in Fig. 3. Fig.4 shows the structure of a hybrid five layer NFN, which is comprised of the input, the membership, the rule, the normalization, and the output layers. In following, the operation functionalities of the nodes in each layer of the NFN model are described, in short. In the following description, $y^{(l)}$ denotes the output of a node in the $l$th layer.

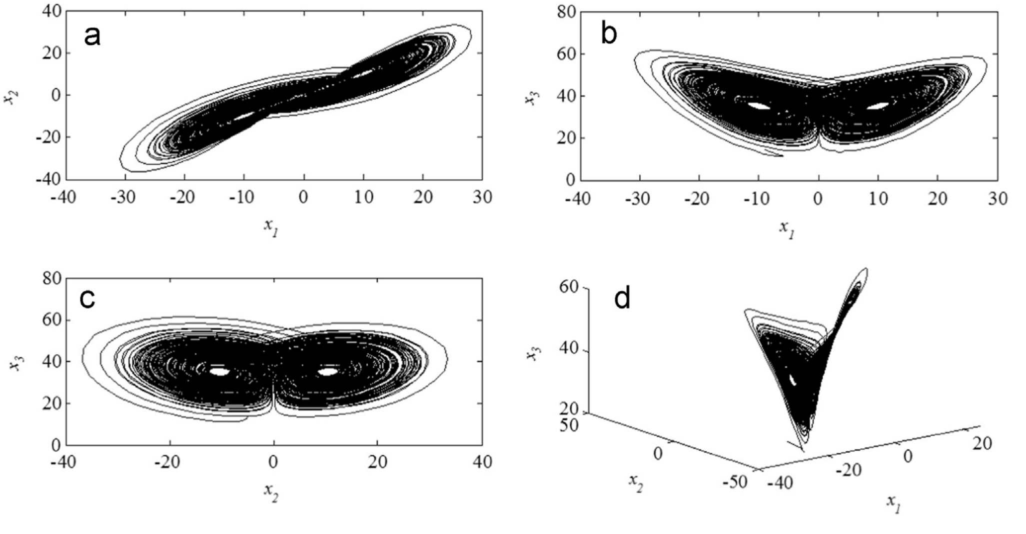


Fig. 2. The chaotic attractor of the fractional order Chen system.

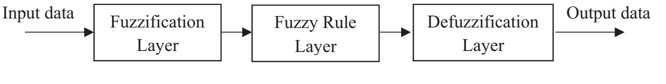


Fig. 3. The schematic diagram of a hybrid neuro-fuzzy network.

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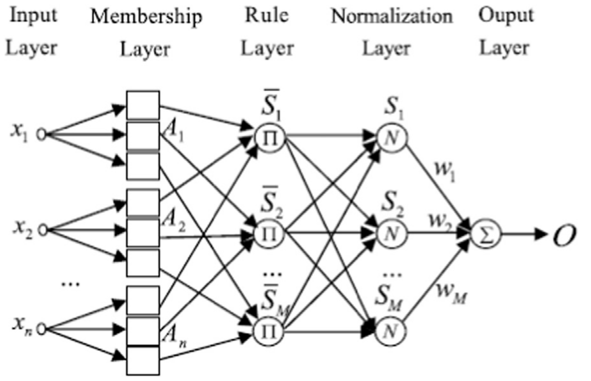


Fig. 4. The structure of a hybrid five-layer NFN.

**Layer 1** (Input layer): No computation is performed in this layer. Each node in this layer is an input node, which corresponds to one input variable, and only transmits input values to the next layer directly:

$$y_i^{(1)} = x_i, i = 1, 2, \ldots, n \tag{11}$$

**Layer 2** (Membership function layer): Nodes in this layer correspond to a single linguistic label of the input variables in Layer 1. Therefore, the calculated membership value specifies the degree to which an input value belongs to a fuzzy set in layer 2. The implemented Gaussian membership function in layer 2 is:

$$y_{ij}^{(2)} = \exp\left( -\frac{\left[y_i^{(1)} - m_{ij}\right]^2}{2\sigma_{ij}{}^2} \right), i = 1, 2, \ldots, n; j = 1, 2, \ldots, M \tag{12}$$

where $m_{ij}$and $\sigma_{ij}$ are the mean and variance of the Gaussian membership function, respectively, related to the $j^{th}$ term of the $i^{th}$ input variable $x_i$.

**Layer 3** (Rule layer): Nodes in this layer represent the preconditioned part of a fuzzy logic rule. They receive one-dimensional membership degrees of the associated rule from the nodes of a set in layer 2. Here, the product operator is adopted to perform the IF condition matching of the fuzzy rules. As a result, the output function of each inference node is:

$$S_j^{(3)} = y_j^{(3)} = \prod_i y_{ij}^{(2)}, i = 1, 2, \ldots, n; j = 1, 2, \ldots, M \tag{13}$$

**Layer 4** (Normalization layer): Nodes in this layer are called normalization nodes. The inputs to a node in layer 4 are the outputs of layer 3. For such a node, we have:

$$S_j^{(4)} = y_j^{(4)} = y_j^{(3)} \Big/ \left( \sum_{k=1}^{M} y_k^{(3)} \right), j = 1, 2, \ldots, M \tag{14}$$

**Layer 5** (Output layer): Each node in this layer corresponds to a single output variable. The node integrates all of the actions recommended by linear combination of outputs of layers

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4 and parameter vector $W=[w_1,w_2,\ldots,w_M]^T$ and acts as a defuzzifier with:

$$O=\sum_{k=1}^{M} w_k y_k^{(4)} = W^T S \tag{15}$$

**Remark 1.** It is assumed that the structure of the NFN system and the membership function parameters are properly specified in advance by the designer. This means that the designer decision is needed to determine the structure of the NFN system, that is, pertinent inputs, number of membership functions for each input, membership function parameters, and number of rules and the consequent parameters, i.e., $W$, must be calculated by learning adaptive algorithms.

## 5. Problem formulation and the proposed hybrid adaptive intelligent controller design

Consider a class of fractional uncertain chaotic systems which can be described by the following differential equation:

$$\begin{aligned} D^{q_1}x_1 &= f_1(x_1,x_2,x_3)+\Delta f_1(x)+d_1(t)+U_1(t) \\ D^{q_2}x_2 &= f_2(x_1,x_2,x_3)+\Delta f_2(x)+d_2(t)+U_2(t) \\ D^{q_3}x_3 &= f_3(x_1,x_2,x_3)+\Delta f_3(x)+d_3(t)+U_3(t) \end{aligned} \tag{16}$$

where $x_i(i=1,2,3)$ are the system states, $f_i(i=1,2,3)$ are unknown continuous functions, and $U_i(t)(i=1,2,3)$ denote the control inputs. Besides, $\Delta f_i(x)(i=1,2,3)$ are the plant uncertainties and $d_i(t)(i=1,2,3)$ denote the unknown, but bounded external disturbance applied to the system. It is desired to design a robust controller against the uncertainties in the system model, uncertainties and the disturbances. For this purpose, the fractional adaptive intelligent back stepping-sliding mode controller (FAIBSMC) is proposed. The controller design procedure can be described as:

**Step 1.** Assume that $z_1=x_1$, then its derivative can be obtained as:

$$D^{q_1}z_1 = f_1\left(z_1,\varphi_1(z_1),\varphi_2(z_1,z_2)\right)+l_1(t,z)+U_1(t) \tag{17}$$

where $l_1(t,z)=\Delta f_1(x)+d_1(t)$ refers to the lumped uncertainty. Besides, $x_2=\varphi_1(z_1)$ and $x_3=\varphi_2(z_1,z_2)$ are considered as virtual controllers which should be determined. The sliding surface can be chosen as:

$$s_1=k_1D^{-1}z_1+D^{q_1-1}z_1 \tag{18}$$

where $k_1>0$ is the sliding surface parameter which will be chosen, later. According to sliding mode method, the condition which guarantees the trajectory of the system arrives at the sliding surface is $s_1\dot{s}_1<0$, and the existence of the sliding mode requires the following conditions to be satisfied:

$$\mathrm{s}_1=0 \tag{19}$$

$$\dot{\mathrm{s}}_1=0 \tag{20}$$

That is,

$$\dot{s}_1=k_1z_1+D^{q_1}z_1=0$$

$$=f_1\left(z_1,\varphi_1(z_1),\varphi_2(z_1,z_2)\right)+l_1(t,z)+U_1(t)+k_1z_1=0 \tag{21}$$

Then, the ideal equivalent control law $U_1{}^*(t)$ can be derived as follows:

$$U_1{}^*(t)=-f_1\left(z_1,\varphi_1(z_1),\varphi_2(z_1,z_2)\right)-l_1(t,z_1)-k_1z_1 \tag{22}$$

It has been shown that there exists an ideal sliding mode controller for achieving control objectives for the mentioned subsystem in (22). Now, it is turn to show how to develop a neuro-fuzzy network system to adaptively approximate the unknown continuous function. If it is considered that

$$D^{q_1}\varphi_1(z_1)=-k_{21}z_1 \tag{23a}$$

$$D^{q_2}\varphi_2(z_1,z_2)=-k_{31}z_1-k_{32}z_2 \tag{23b}$$

where $k_{21}$, $k_{31}$, and $k_{32}$ are constants and $z_2=x_2-\varphi_1(z_1)$ (which is ideally equal to zero). Therefore, it is concluded that $f_1\left(z_1,\varphi_1(z_1),\varphi_2(z_1,z_2)\right)=f_1(z_1)$. By the universal approximation theory, there exists an ideal NFN estimator $W_1{}^{T^*}\phi(z_1)$ such that

$$f_1(z_1)=W_1{}^{T^*}\phi(z_1)+\delta_1 \tag{24}$$

where $\delta_1$ is an approximation error and $W_1{}^*$ is the optimal value of the parameter $W_1$. Since the optimal NFN cannot be obtained, an NFN estimator is used to estimate the optimal NFN; this NFN estimator is defined as:

$$\hat{f}_1(z_1)=\hat{W}_1^T\phi(z_1) \tag{25}$$

where $\hat{W}_1$ is the estimated matrix of $W_1{}^*$. Parameter $W_1{}^*$ is determined through the following optimization problem:

$$W_1{}^*\triangleq argmin\left\{sup\left|W_1{}^T\phi(z_1)-f_1(z_1)\right|\right\} \tag{26}$$

**Assumption 1.** We consider that the unknown uncertainties $\Delta f_i(x)$ and the external disturbances $d_i(t)$ are to be bounded as:

$$\left|\Delta f_i(x)\right|\le\alpha_{1i}|z_i|+\alpha_{2i} \tag{27}$$

$$\left|d_1(t)\right|\le\alpha_{3i} \tag{28}$$

where $\alpha_{1i}$, $\alpha_{2i}$ and $\alpha_{3i}$ are unknown positive parameters.

**Assumption 2.** The approximation error $\delta_i$ and the lumped uncertainty $l_i(t,z)$ satisfy the following conditions:

$$\left|l_i(t,z)\right|\le\lambda_{1i}|z_i|+\lambda_{2i} \tag{29}$$

$$\left|\delta_i\right|\le\lambda_{3i} \tag{30}$$

where $\lambda_{1i}$, $\lambda_{1i}$ and $\lambda_{3i}$ are unknown positive parameters.

Then, the following robust controller based on neuro-fuzzy network can be constructed as:

$$U_1=U_{11}+U_{NFN_1}+U_{Rob_1} \tag{31}$$

In Eq. (26), the signals $U_{11}$, $U_{NFN_1}$ and $U_{Rob_1}$ can be designed as:

$$U_{11} = -k_1 z_1 - \gamma_1 s_1 \tag{32}$$

$$U_{NFN_1} = -\hat{W}_1^T \phi(z_1) \tag{33}$$

$$U_{Rob_1} = -\left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] Sgn(s_1) \tag{34}$$

In the robust controller designed in (30), $U_{11}$ is a state feedback controller, which is used to control the nominal error system, $U_{NFN_1}$is a neuro-fuzzy network controller which is used to control the unknown assistant function and $U_{Rob_1}$ is an adaptive controller which is used to compensate the approximation errors, lumped uncertainty and effects of the functions approximations in the design process on the controlled system.

**Theorem 1.** Consider the fractional-order $z_1$-subsystem of Eq. (17) with unknown bounded uncertainties and external disturbances. The controller structure is designed as described in Eqs. (31)–(34). If the on-line adapting laws for parameters are as follows:

$$\dot{\hat{W}}_1 = \theta_1 s_1 \phi(z_1) \tag{35}$$

$$\dot{\hat{\lambda}}_{11} = l_{11}|s_1||z_1| \tag{36}$$

$$\dot{\hat{\lambda}}_{21} = l_{21}|s_1| \tag{37}$$

$$\dot{\hat{\lambda}}_{31} = l_{31}|s_1| \tag{38}$$

where $\theta_1$, $l_{11}$, $l_{21}$, $l_{31}$ are the learning rates with positive constants, then the tracking error converge asymptotically to the origin and all signals in the closed loop system are bounded.

**Proof.** Let the Lyapunov functional candidate be given by:

$$V_1(t) = \frac{1}{2}{s_1}^2 + \frac{1}{2\theta_1}\tilde{W}_1^T\tilde{W}_1 + \frac{1}{2l_{11}}\tilde{\lambda}_{11}^T 2 + \frac{1}{2l_{21}}\tilde{\lambda}_{21}^T 2 + \frac{1}{2l_{31}}\tilde{\lambda}_{31}^T 2 \tag{39}$$

where $\tilde{W}_1 = W_1 - \hat{W}_1$, $\tilde{\lambda}_{i1} = \lambda_{i1} - \hat{\lambda}_{i1} (i = 1, 2, 3)$. The time derivative of $V_1$ along the trajectories of the $z_1$-subsystem of Eq. (17) is obtained as:

$$\dot{V}_1(t) = s_1\dot{s_1} - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \tag{40}$$

Substituting (21) into (40), it results in:

$$\dot{V}_1(t) = s_1[k_1 z_1 + D^{q_1} z_3] - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \tag{41}$$

By augmentation of Eqs. (17) and (41), it yields:

$$\dot{V}_1 = s_1\left[k_1 z_1 + f_1\left(z_1, \varphi_1(z_1), \varphi_2(z_1, z_2)\right) + l_1(t, z) + U_1(t)\right] - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \tag{42}$$

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where $\varphi_1(z_1)$ and $\varphi_2(z_1, z_2)$ are considered as in Eqs. (23a) and (23b) and from Eqs. (24) and (25), with applying $U_{11}$ and ideal intelligent estimator, the following result can be obtained as:

$$\dot{V}_1(t) = s_1\left[{W_1}^{T^*}\phi(z_1) + \delta_1 + l_1(t,z) - \gamma_1 s_1 + U_{NFN_1} + U_{Rob_1}\right] - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \quad (43)$$

With applying $U_{Rob_1}$ and $U_{NFN_1}$, Eq. (43) can be rewritten as:

$$\dot{V}_1(t) = s_1\left[{W_1}^{T^*}\phi(z_1) + \delta_1 + l_1(t,z) - \gamma_1 s_1 - \hat{W}_1^T\phi(z_1) - \left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right]Sgn(s_1)\right] - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \quad (44)$$

Accordingly, one obtains:

$$\dot{V}_1(t) = s_1\left[\tilde{W}_1^T\phi(z_1) + \delta_1 + l_1(t,z) - \gamma_1 s_1 - \left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right]Sgn(s_1)\right] - \frac{1}{\theta_1}\tilde{W}_1^T\dot{\hat{W}}_1 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \quad (45)$$

Using the adaptive laws in (35)–(38) and substituting $s_1.Sgn(s_1)$ with $|s_1|$, it is clear that Eq. (45) can be rewritten as:

$$\dot{V}_1 = \tilde{W}_1^T\left[s_1\phi(z_1) - \frac{1}{\theta_1}\dot{\hat{W}}_1\right] + s_1[\delta_1 + l_1(t,z)] - |s_1|\left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] - \gamma {s_1}^2 - \frac{1}{l_{11}}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - \frac{1}{l_{21}}\tilde{\lambda}_{21}\dot{\hat{\lambda}}_{21} - \frac{1}{l_{31}}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} \quad (46)$$

Eq. (46) can be rewritten as:

$$\dot{V}_1(t) = s_1[\delta_1 + l_1(t,z)] - |s_1|\left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] - \gamma {s_1}^2 - |s_1|\left[\tilde{\lambda}_{11}|z_1| + \tilde{\lambda}_{21} + \tilde{\lambda}_{31}\right] \quad (47)$$

It is obvious that:

$$\dot{V}_1 \le |s_1|\left[|\delta_1| + |l_1(t,z)|\right] - |s_1|\left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] - \gamma {s_1}^2 - |s_1|\left[\tilde{\lambda}_{11}|z_1| + \tilde{\lambda}_{21} + \tilde{\lambda}_{31}\right] \quad (48)$$

where $\tilde{\lambda}_{i1} = \lambda_{i1} - \hat{\lambda}_{i1} (i = 1, 2, 3)$, we have:

$$\dot{V}_1 \le |s_1|\left[|\delta_1| + |l_1(t,z)|\right] - |s_1|\left[\hat{\lambda}_{11}|z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] - \gamma {s_1}^2 - |s_1|\left[\tilde{\lambda}_{11}|z_1| + \tilde{\lambda}_{21} + \tilde{\lambda}_{31}\right] \quad (49)$$

Eq. (49) can be rewritten as:

$$\dot{V}_1(t) \le |s_1|\left[|l_1(t,z)| - \lambda_{11}|z_1| - \lambda_{21}\right] + |s_1|\left[|\delta_1| - \lambda_{31}\right] - \gamma {s_1}^2 \quad (50)$$

According to Assumption 2, Eq. (50) can be rewritten as:

$$\dot{V}_1(t) \le |s_1|\left[|l_1(t,z)| - \lambda_{11}|z_1| - \lambda_{21}\right] + |s_1|\left[|\delta_1| - \lambda_{31}\right] - \gamma {s_1}^2 \le 0 \quad (51)$$

Since $\dot{V}_1$ is negative semi definite, the stability in the Lyapunov sense is proved. It can imply that all signals in the closed loop system are bounded. That is:

$$\dot{V}_1(t) \le |s_1|\left[\left|l_1(t,z)\right| - \lambda_{11}|z_1| - \lambda_{21}\right] + |s_1|\left[\left|\delta_1\right| - \lambda_{31}\right] - \gamma {s_1}^2 \equiv -Q(t) \le 0 \quad (52)$$

Integrating $Q(t)$ with respect to time, it gives:

$$\int_0^t Q(t)dt \le V_1(0) - V_1(t) \quad (53)$$

Because $V_1(0)$ is bounded, and $V_1(t)$ is nonincreasing and bounded (based on Eq. (51), the following result can be obtained:

$$\int_0^t Q(t)dt \le \infty \quad (54)$$

In addition, since $\dot{Q}(t)$ is bounded, according to Lemma 2.2 (Barbalat's Lemma), it can be shown that:

$$\lim_{t\to\infty} Q(t) = 0 \quad (55)$$

Eq. (55) implies that, $s_1(t) \to 0$ as $t \to \infty$. Therefore, the state trajectories of the controlled $z_1$-subsystem of Eq. (17) can be forced onto the predefined sliding surface. This proved that the fractional $z_1$-subsystem of Eq. (17) with uncertainty and external disturbance can be stabilized via the proposed control law of Eq. (31). Therefore, the proof is completed.

**Remark 2.** It is worth mentioning that the learning rates are selected by means of trial and error considering the design requirements. The selection of these parameters affects the convergence rate of the adaptive parameters, and consequently the convergence of the system's response. □

**Theorem 2.** For the given fractional subsystem of Eq. (17), when the sliding surface is defined as Eq. (18), and $U_1$ defined as in Eqs. (31)–(34), then the system trajectories converge to the sliding surface in finite time.

**Proof.** Choosing a Lyapunov candidate function in the following form:

$$V_{11}(t) = \frac{1}{2}{s_1}^2 \quad (56)$$

Taking the time derivative of $V_{11}$ and following the same procedure of Eqs. (40)–(51), we have:

$$\dot{V}_{11}(t) \le |s_1|[|l_1(t,z)| - \lambda_{11}|z_1| - \lambda_{21}] \le -|s_1| = -\frac{\sqrt{2}}{2}V_{11}^{\frac{1}{2}}(t) \quad (57)$$

From simple calculation, we get:

$$dt \le -\sqrt{2}V_{11}^{-\frac{1}{2}}\mathrm{d}V_{11} \quad (58)$$

Taking integral of both sides of (58) from $t_0$ to the reaching time $t_r$ and letting $V_{11}(t_r) = 0$ (from Theorem 1), we have:

$$t_r \le t_0 + 2\sqrt{2}V_{11}(t_0) \quad (59)$$

Denoting the reaching time as $T_1(t_0)=t_r-t_0$, the proof results in:

$$T_1(t_0)\le 2\sqrt{2}V_{11}(t_0) \tag{60}$$

Therefore, the state trajectories of the mentioned fractional subsystem converge to the surface $s_1=0$ in finite time and the proof is completed. □

**Step 2.** Since the virtual control function $\varphi_1(z_1)$ is estimative, the error between $x_2$ and $\varphi_1(z_1)$ is defined as follows:

$$z_2=x_2-\varphi_1(z_1) \tag{61}$$

Thus, based on Eqs. (16), (17) and (23), the $(z_1-z_2)$-subsystem dynamics can be represented as:

$$\begin{aligned}D^{q_1}z_1 &=f_1\big(z_1,\varphi_1(z_1),\varphi_2(z_1,z_2)\big)\\ &\quad +l_1(t,z)-k_1z_1-\gamma_1 s_1-\hat{W}_1^T\phi(z_1)-[\hat{\lambda}_{11}|z_1|+\hat{\lambda}_{21}+\hat{\lambda}_{31}]Sgn(s_1)\\ D^{q_2}z_2 &=f_2\big(z_1,z_2+\varphi_1(z_1),\varphi_2(z_1,z_2)\big)+k_{21}z_1+l_2(t,z)+U_2(t)\end{aligned} \tag{62}$$

In order to stabilize the $(z_1-z_2)$-subsystem of Eq. (62), the sliding surface can be chosen as:

$$s_2=k_2D^{-1}z_2+D^{q_2-1}z_2 \tag{63}$$

where $k_2$ is a positive constant which to be designed later. Similar to first step, the condition which guarantees the trajectory of the system arrives at the sliding surface is $s_2\dot{s}_2<0$, that requires the following conditions to be satisfied:

$$s_2=0 \tag{64}$$

$$\dot{s}_2=0 \tag{65}$$

$$\begin{aligned}\dot{s}_2=k_2z_2+D^{q_2}z_2 &=f_2\big(z_1,z_2+\varphi_1(z_1),\varphi_2(z_1,z_2)\big)+k_{21}z_1+l_2(t,z)+U_2(t)\\ &\quad +k_2z_2=0\end{aligned} \tag{66}$$

From Eq. (23), the ideal equivalent control $U_2{}^*(t)$ is calculated by:

$$U_2{}^*(t)=-f_2(z_1,z_2)-l_2(t,z_2)-k_2z_2-k_{21}z_1 \tag{67}$$

Similarly will be shown that how to develop a neuro-fuzzy network system to adaptively approximate the unknown continuous function, there exists an ideal NFN estimator $W_2{}^{T^*}\phi(z_1,z_2)$ such that:

$$f_2(z_1,z_2)=W_2{}^{T^*}\phi(z_1,z_2)+\delta_2 \tag{68}$$

where $\delta_2$ is an approximation error, and $W_2{}^*$ is the optimal value of the parameter $W_2$. Since the optimal NFN cannot be obtained, an NFN estimator is used to estimate the optimal NFN. This NFN estimator is defined as:

$$\hat{f}_2(z_1,z_2)=\hat{W}_2^T\phi(z_1,z_2) \tag{69}$$

And $\hat{W}_2$ is the estimated matrix of $W_2{}^*$. Parameter $W_2{}^*$is determined through following optimization problem:

$$W_2{}^*\triangleq arg\ min\ \left\{sup\left|W_2{}^T\phi(z_1,z_2)-f_2(z_1,z_2)\right|\right\} \tag{70}$$

Next, the robust controller based on neuro-fuzzy network is computed by:

$$U_2(t) = U_{21} + U_{NFN_2} + U_{Rob_2} \tag{71}$$

In Eq. (66), the signals $U_{21}$, $U_{NFN_2}$ and $U_{Rob_2}$ can be designed as:

$$U_{21} = -k_{21}z_1 - k_2z_2 - \gamma_2 s_2 \tag{72}$$

$$U_{NFN_2} = -\hat{W}_2^T\phi(z_1, z_2) \tag{73}$$

$$U_{Rob_2} = -\left[\hat{\lambda}_{12}|z_2| + \hat{\lambda}_{22} + \hat{\lambda}_{32}\right] Sgn(s_2) \tag{74}$$

In the robust controller designed in Eq. (71), $U_{21}$ is a state feedback controller, which is used to control the nominal error system, $U_{NFN_2}$ is a neuro-fuzzy network controller which is employed to control the unknown assistant function and $U_{Rob_2}$ is an adaptive controller which is employed to compensate the approximation errors, lumped uncertainty and effects of the function approximations in design process on the controlled system.

**Theorem 3.** Consider the fractional-order $(z_1 - z_2)$-subsystem of Eq. (62) with unknown bounded uncertainties and external disturbances. Then, the controller structure is designed as described in Eqs. (72)–(74). If the on-line adapting laws for parameters are as follows:

$$\dot{\hat{W}}_2 = \theta_2 s_2 \phi(z_1, z_2) \tag{75}$$

$$\dot{\hat{\lambda}}_{12} = l_{12}|s_2||z_2| \tag{76}$$

$$\dot{\hat{\lambda}}_{22} = l_{22}|s_2| \tag{77}$$

$$\dot{\hat{\lambda}}_{32} = l_{32}|s_2| \tag{78}$$

where $\theta_2$, $l_{12}$, $l_{22}$, $l_{32}$ are the learning rates with positive constants, then the tracking error converge asymptotically to origin and all signals in the closed loop system are bounded. Besides, the system trajectories converge to the sliding surface in finite time.

**Proof.** Let the Lyapunov functional candidate be given by:

$$V_2 = V_1 + \frac{1}{2}{s_2}^2 + \frac{1}{2\theta_2}\tilde{W}_2^T\tilde{W}_2 + \frac{1}{2l_{12}}\tilde{\lambda}_{12}^T 2 + \frac{1}{2l_{22}}\tilde{\lambda}_{22}^T 2 + \frac{1}{2l_{32}}\tilde{\lambda}_{32}^T 2 \tag{79}$$

where $\tilde{W}_2 = W_2 - \hat{W}_2$, $\tilde{\lambda}_{i2} = \lambda_{i2} - \hat{\lambda}_{i2} (i = 1, 2, 3)$. Similar to the proof of Theore.1, it can be shown that by applying controller $U_2(t)$ to the $(z_1 - z_2)$-subsystem, $\dot{V}_2 \le 0$ which will be shown that following $(z_1 - z_2)$-subsystem is stable and state trajectories of the controlled $(z_1 - z_2)$-subsystem (62) can be forced onto the predefined sliding surface. This proved that the fractional $(z_1 - z_2)$-subsystem (62) with uncertainty and external disturbance can be stabilized via the proposed control law (71).

Next, by choosing an auxiliary Lyapunov candidate function in the following form:

$$V_{22}(t) = \frac{1}{2}{s_2}^2 \tag{80}$$

and following the same procedure as the proof of Theorem 2, it can be shown that the trajectories converge to the sliding surface $s_2$ in finite time, as well.

**Step 3.** Suppose that $z_3 = x_3 - \varphi_2(z_1, z_2)$, then the dynamic of $(z_1 - z_2 - z_3)$-system is represented as:

$$\begin{aligned} D^{q_1} z_1 &= f_1(z_1, \varphi_1(z_1), \varphi_2(z_1, z_2) \\ &\quad + l_1(t,z) - k_1 z_1 - \gamma_1 s_1 - \hat{W}_1^T \phi(z_1) - \left[\hat{\lambda}_{11} |z_1| + \hat{\lambda}_{21} + \hat{\lambda}_{31}\right] Sgn(s_1) \\ D^{q_2} z_2 &= f_2\left(z_1, z_2 + \varphi_1(z_1), \varphi_2(z_1, z_2)\right) + l_2(t,z) - k_2 z_2 - \gamma_2 s_2 - \hat{W}_2^T \phi(z_1, z_2) \\ &\quad - \left[\hat{\lambda}_{12} |z_2| + \hat{\lambda}_{22} + \hat{\lambda}_{32}\right] Sgn(s_2) \end{aligned}$$

$$D^{q_3} z_3 = f_3\left(z_1, z_2 + \varphi_1(z_1), z_3 + \varphi_2(z_1, z_2)\right) + l_3(t,z) + U_3(t) + k_{31} z_1 + k_{32} z_2 \tag{81}$$

The sliding surface can be chosen as:

$$s_3 = k_3 D^{-1} z_3 + D^{q_3 - 1} z_3 \tag{82}$$

where $k_3$ is a positive constant which is to be designed later. Similar to previous steps, the condition which guarantees the trajectory of the system attains to the sliding surface is $s_3 \dot{s}_3 < 0$, that requires the following conditions to be satisfied:

$$s_3 = 0 \tag{83}$$

$$\dot{s}_3 = 0 \tag{84}$$

$$\dot{s}_3 = k_3 z_3 + D^{q_3} z_3 = f_3(z_1, z_2, z_3) + l_3(t,z) + U_3(t) + k_{31} z_1 + k_{32} z_2 + k_3 z_3 = 0 \tag{85}$$

Then, the ideal equivalent control $U_3{}^*(t)$ is computed as:

$$U_3{}^*(t) = -f_3(z_1, z_2, z_3) - l_3(t,z) - k_{31} z_1 - k_{32} z_2 - k_3 z_3 \tag{86}$$

Following the same procedure as the past steps, it will be shown that a neuro-fuzzy network system which approximate the unknown continuous function, an ideal NFN estimator $W_3{}^{T^*} \phi(z_1, z_2, z_3)$ exists such that:

$$f_3(z_1, z_2, z_3) = W_3{}^{T^*} \phi(z_1, z_2, z_3) + \delta_3 \tag{87}$$

In Eq. (87), $\delta_3$ is an approximation error and $W_3{}^*$ is the optimal value of the parameter vector of $W_3$. A NFN estimator is used to estimate the optimal NFN defined as:

$$\hat{f}_3(z_1, z_2, z_3) = \hat{W}_3^T \phi(z_1, z_2, z_3) \tag{88}$$

where $\hat{W}_3$ is the estimated matrix of $W_3{}^*$. Parameter $W_3{}^*$ is determined through the following optimization problem:

$$W_3{}^* \triangleq argmin \left\{ sup \left| W_3{}^T \phi(z_1, z_2, z_3) - f_3(z_1, z_2, z_3) \right| \right\} \tag{89}$$

The, the robust controller based on neuro-fuzzy network is calculated as:

$$U_3(t) = U_{31} + U_{NFN_3} + U_{Rob_3} \tag{90}$$

In (90), the signals $U_{31}$, $U_{NFN_3}$ and $U_{Rob_3}$ can be designed as:

$$U_{31} = -k_{31} z_1 - k_{32} z_2 - k_3 z_3 - \gamma_3 s_3 \tag{91}$$

$$U_{NFN_3} = -\hat{W}_3^T \phi(z_1, z_2, z_3) \tag{92}$$

$$U_{Rob_3} = -\left[\hat{\lambda}_{13} |z_3| + \hat{\lambda}_{23} + \hat{\lambda}_{33}\right] Sgn(s_3) \tag{93}$$

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where $U_{31}$ is a state feedback controller, $U_{NFN_3}$ is a neuro-fuzzy network controller which is used to control the unknown assistant function. Besides, $U_{Rob_3}$ is an adaptive controller which is used to compensate for the approximation errors, lumped uncertainty and effects of the functions approximation in design process on the controlled system.

**Theorem 4.** Consider the fractional-order $(z_1-z_2-z_3)$-subsystem of Eq. (81) with unknown bounded uncertainties and external disturbances, then, the controller structure is designed as described in Eqs. (91)–(93). If the on-line adapting laws for parameters are as follows:

$$\dot{\hat{W}}_3=\theta_3 s_3\phi(z_1,z_2,z_3) \tag{94}$$

$$\dot{\hat{\lambda}}_{13}=l_{13}|s_3||z_3| \tag{95}$$

$$\dot{\hat{\lambda}}_{23}=l_{23}|s_3| \tag{96}$$

$$\dot{\hat{\lambda}}_{33}=l_{33}|s_3| \tag{97}$$

where $\theta_3$, $l_{13}$, $l_{23}$, $l_{33}$ are the learning rates with positive constants, then the tracking error converge asymptotically to origin and all signals in the closed loop system are bounded. Additionally, the system trajectories converge to the sliding surface in finite time.

**Proof.** Let the Lyapunov functional candidate be given by:

$$V_3=V_2+\frac{1}{2}{s_3}^2+\frac{1}{2\theta_3}\tilde{W}_3^T\tilde{W}_3+\frac{1}{2l_{13}}\tilde{\lambda}_{13}^T2+\frac{1}{2l_{23}}\tilde{\lambda}_{23}^T2+\frac{1}{2l_{33}}\tilde{\lambda}_{33}^T2 \tag{98}$$

where $\tilde{W}_3=W_3-\hat{W}_3$, $\tilde{\lambda}_{i3}=\lambda_{i3}-\hat{\lambda}_{i3}(i=1,2,3)$. Similar to the past steps, it will be proved that applying the controller $U_3(t)$ to the $(z_1-z_2-z_3)$-subsystem, results in $\dot{V}_3\le 0$. That is, it represents however that $(z_1-z_2-z_3)$-subsystem is stable and all the state trajectories of the controlled $(z_1-z_2-z_3)$-subsystem of Eq. (81) can be forced onto the predefined sliding surface. In this manner, it is proved that the fractional $(z_1-z_2-z_3)$-subsystem of Eq. (81) with uncertainty and external disturbance can be stabilized via the proposed control law of Eq. (90). It means that in the $(z_1-z_2-z_3)$-system coordinates the equilibrium $(0,0,0)$ is stable.

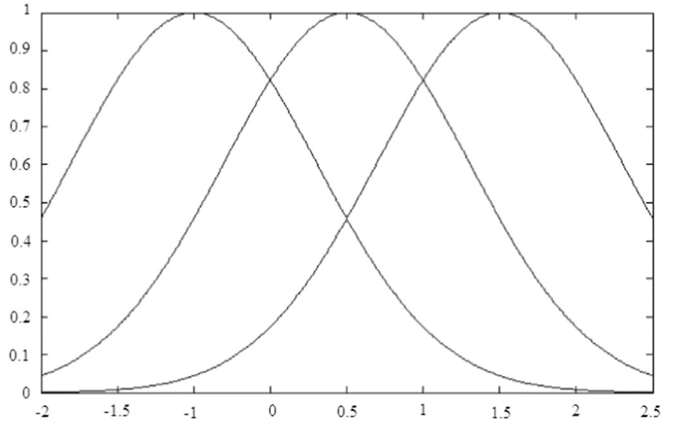


Fig. 5. The normalized Fuzzy membership function for the inputs of NFN system in provided examples.

Consequently, by considering the definitions of $z_1$, $z_2$ and $z_3$, the state variables $x_1, x_2, x_3$ converge to zero, as well.

Next, the auxiliary Lyapunov candidate function is selected as:

$$V_{33}(t) = \frac{1}{2}{s_3}^2 \tag{99}$$

Following the same procedure as the proof of Theorem 2, it can be shown that the sliding surface $s_3$is reached in finite time.

## 6. Simulation results

In this section, two numerical examples are presented to illustrate the performance of the proposed fractional hybrid adaptive intelligent controller. In all two examples, for each input variable to the NFN system, three Gaussian membership functions have been defined which are uniformly distributed in the interval $[-2.5,2.5]$ as shown in Fig. 5 where the system inputs has been scaled accordingly, in these examples.

**Example 1. The fractional-order energy resources demand-supply system:** In order to validate the efficiency and effectiveness of the proposed control scheme numerical simulations are made for chaotic fractional order energy resources demand–supply system with parameters $(a_1, a_2, b_1, b_2, b_3, c_1, c_2, c_3, M, N) = (0.1, 0.3, 0.01, 0.02, 0.2, 0.5, 0.8, 0.1, 2, 1)$. This system is non-commensurate and has fractional orders of $q = [0.98, 0.85, 0.92]$. The initial conditions of the system are assumed to be $x_0 = [-15,\ -18,\ 15,\ 10]^T$. At first, the following model uncertainty vector is added to the system:

$$\Delta g(x) = [0.1\, Sin\,(\pi x_2 x_3)\, Cos\,(\pi x_1), 0.1\, Sin\,(\pi x_1 x_3)\, Cos\,(\pi x_2), 0.1\, Sin\,(\pi x_2 x_1)\, Cos\,(\pi x_3)]$$

In all cases, the external disturbances vector is defined as:

$$d(t) = [0.1\, Cos\,(t), 0.15\, Cos\,(t), 0.2\, Cos\,(t)]$$

In this case, the control parameters have been set as:

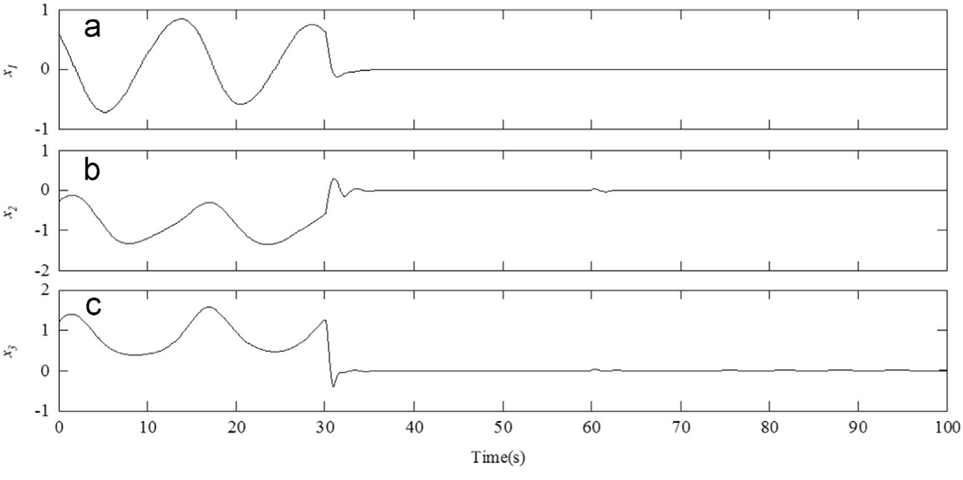


Fig. 6. The states trajectories of the closed-loop fractional order energy resources demand–supply system; (a) $x_1$, (b) $x_2$, (c) $x_3$.

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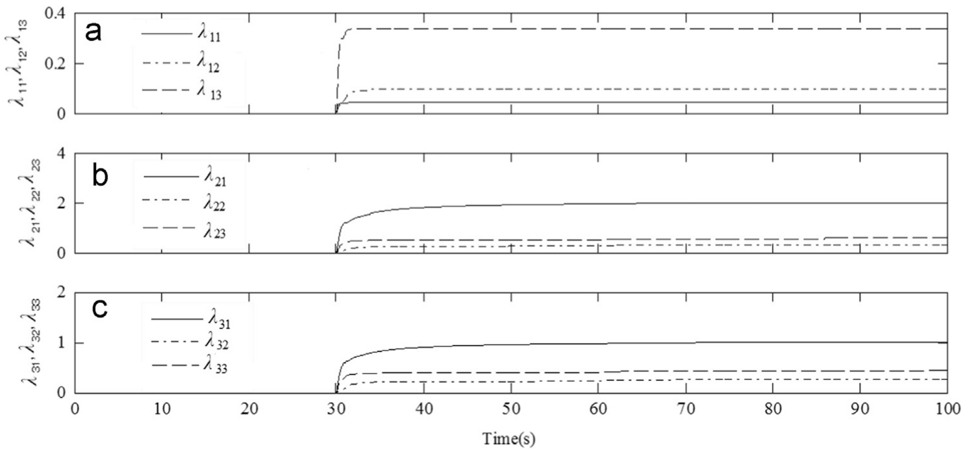


Fig. 7. The time responses of the update vector parameters; (a) $\lambda_1$, (b) $\lambda_2$, (c) $\lambda_3$.

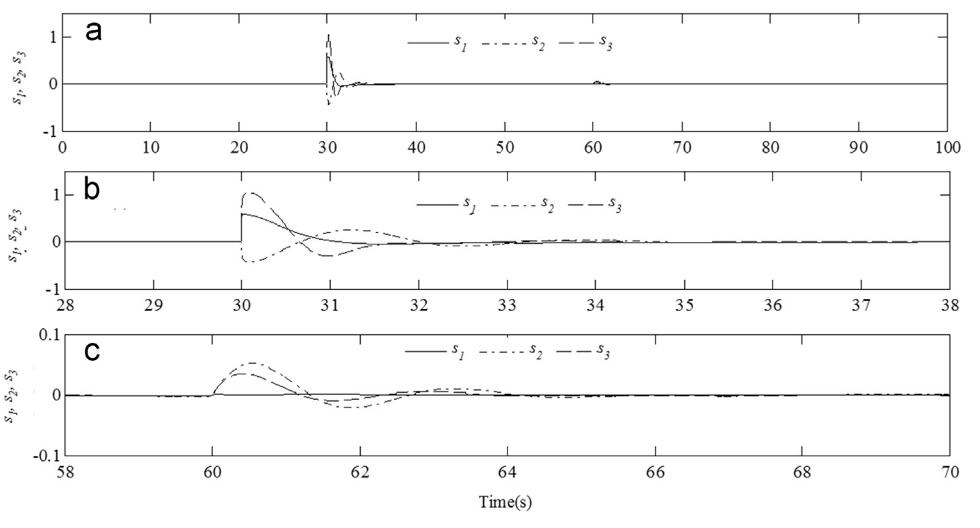


Fig. 8. (a) The time evolution of sliding mode surfaces, $s_i, i = 1, 2, 3$, (b) and (c) some parts of Fig. 8(a) zoomed out.

$l_1 = [0.4, 0.9, 0.7]$, $l_2 = [4, 5, 8]$, $l_3 = [2, 4, 6]$.
$\gamma = [25–27]$, $k = [5, 7, 10]$.

Numerical simulations are carried out using the MATLAB software. The ode45 solver is used for solving differential equations. Control scheme on energy resources demand–supply fractional order chaotic system are shown in Figs. 6–9. Fig. 6 illustrates the trajectories of the system states, where the FAIBSMC inputs are activated at $t = 30$s. Next, at $t = 60$s the applied uncertainties and disturbances have been changed to:

$$\Delta f(x) = [0.2\, Cos\,(\pi x_2 x_3)\, Cos\,(\pi x_1), 0.2\, Cos\,(\pi x_1 x_3)\, Cos\,(\pi x_2), 0.2\, Cos\,(\pi x_2 x_1)\, Cos\,(\pi x_3)]$$

And, the external disturbances vector is applied as:

$$d(t) = [0.1\, Sin\,(t), 0.1\, Sin\,(t), 0.1\, Sin\,(t)]$$

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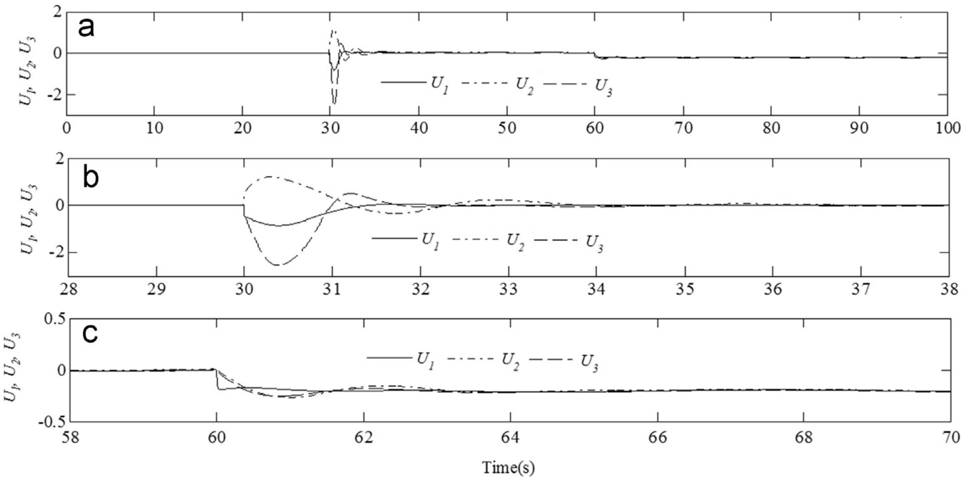


Fig. 9. Control input of the closed-loop fractional order energy resources demand–supply system, $U_i, i=1,2,3$, (b) and (c) some parts of (a) zoomed out.

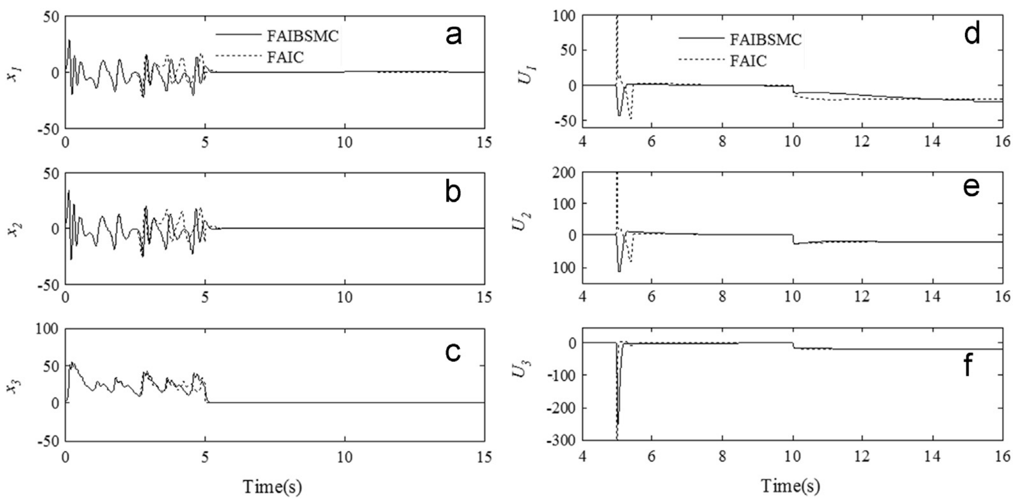


Fig. 10. The states trajectories and control inputs of the closed-loop fractional order Chen system with the proposed FAIBSMC in comparison with FAIC [60]; (a) $x_1$, (b) $x_2$, (c) $x_3$, (d) $U_1$, (e) $U_2$, (f) $U_3$.

As we can see, the state trajectories converge to the zero after applying the control inputs and the variations in uncertainties are rejected by the controller as well. The time responses of the update vector parameters $\lambda_1$, $\lambda_2$ and $\lambda_3$, are depicted in Fig. 7, respectively. Obviously, all of the update parameters approach to determined constants which verify the feasibility of the proposed method. In Figs. 8 and 9 the time evolution of sliding mode surfaces and the input control signals are brought, respectively. To make the figures more traceable, Figs. 8(a) and 9(a) have been zoomed out for various time scales in Figs. 8(b, c) and 9(b, c), respectively. From these figures, it

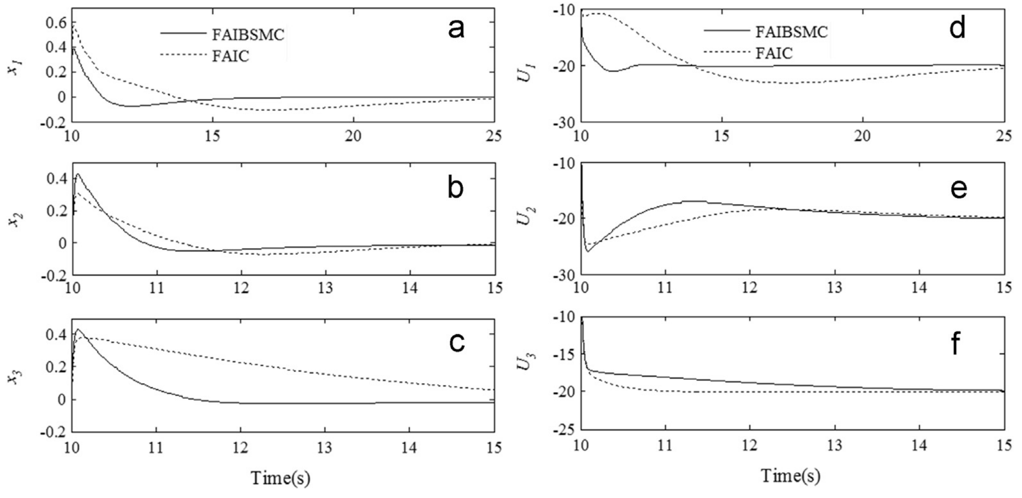


Fig. 11. The states trajectories and control inputs of the closed-loop fractional order Chen system with the proposed FAIBSMC in comparison with FAIC [60] zoomed out for $t \geq 10$ s; (a) $x_1$, (b) $x_2$, (c) $x_3$, (d) $U_1$, (e) $U_2$, (f) $U_3$.

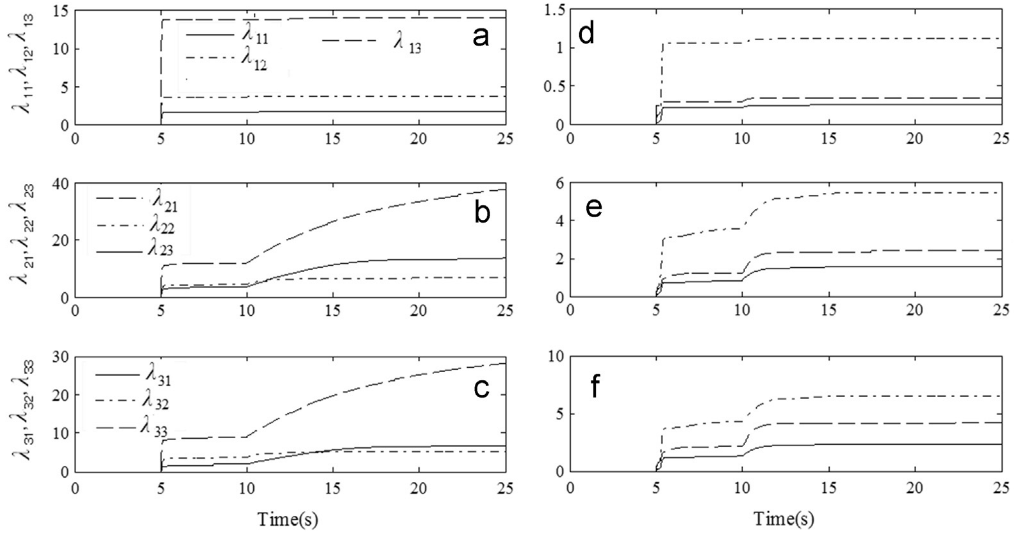


Fig. 12. The time responses of the update vector parameters of the closed-loop fractional order Chen system; (a), (b), (c) with the proposed FAIBSMC in comparison with (d), (e) and (f) FAIC [60]; (a , d) $\lambda_1$, (b, e) $\lambda_2$, (c ,f) $\lambda_3$.

is shown that the proposed controller shows good convergence properties while the control efforts remain bounded and applicable.

**Example 2. Fractional-order Chen system:** Now, consider the Chen fractional order chaotic system with parameters $\beta_1 = 35, \beta_2 = 28, \beta_3 = 3$. This system is commensurate and its fractional

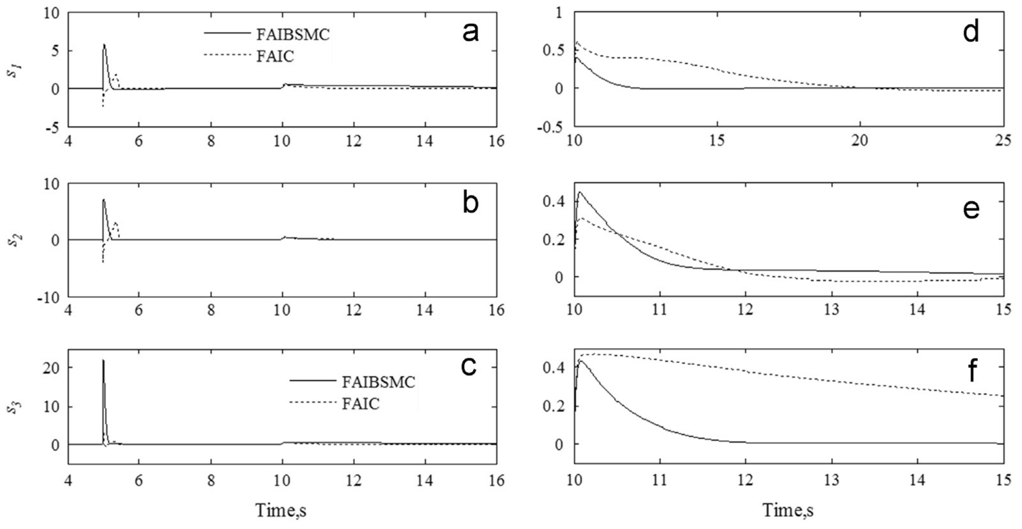


Fig. 13. (a), (b), (c) The time evolution of sliding mode surfaces $s_i, i=1,2,3$, of the closed-loop fractional order Chen system with the proposed FAIBSMC in comparison with FAIC [60]; (d) $s_1$, (e) $s_2$, (f) $s_3$zoomed out for $t \geq 10$ sec.

order is $q=0.9$. The initial conditions of the system are assumed to be $x_0=[-15,-18,15,10]$. At the beginning of simulation, the following model uncertainty vector is added to the system:

$$\Delta f(x)=[2\,Sin\,(\pi x_2 x_3)\,Cos\,(\pi x_1), 2in(\pi x_1 x_3)\,Cos\,(\pi x_2), 2\,Sin\,(\pi x_2 x_1)\,Cos\,(\pi x_3)]$$

In this case, the external disturbances vector is defined as:

$$d(t)=[0.1\,Cos\,(t), 0.1\,Sin\,(t), 0.1\,Cos\,(t)]$$

The control parameters are set as:
$l_1=[0.5,0.9,0.7]$, $l_2=[2,4,5]$, $l_3=[0.01,0.04,0.07]$.
$\gamma=[25\text{–}27]$, $k=[5,7,10]$.

The results of the designed controller on the fractional-order chaotic Chen system are shown in Figs. 10–13; where the performance of the proposed FAIBSMC has been compared with Fractional Adaptive Intelligent Controller (FAIC), introduced in [60]. Fig. 10 shows the state trajectories as well as control inputs of the closed-loop fractional order Chen system with the proposed FAIBSMC in comparison with FAIC, where the control inputs are activated at $t=5$s. As we can see, the state trajectories are stabilized to zero by applying the control inputs very well. Next, at $t=10$s the applied uncertainties and disturbances have been changed to:

$$\Delta f(x)=[5\,Cos\,(\pi x_2 x_3)\,Cos\,(\pi x_1), 5\,Cos\,(\pi x_1 x_3)\,Cos\,(\pi x_2), 5\,Cos\,(\pi x_2 x_1)\,Cos\,(\pi x_3)]$$

And,

$$d(t)=[0.2\,sin\,(t), 0.2\,Sin\,(t), 0.2\,sin\,(t)]$$

It should be noted that to make the chaotic signals distinguishable, the parameters of the Chen system have been taken slightly different in the two cases (the parameters are different within the order of $10^{-2}$). In Fig. 11, the figures of Fig. 10 are zoomed out to show the effect of such variations in system uncertainties and disturbances. Form these figures it is obvious that both of the controllers have been able to overcome the uncertainties and disturbances very well. But, the FAIBSMC response is considerably faster as its settling time is less than that of the FAIC.

Besides, the overshoot/under-shoot of the FAIBSMC control inputs is noticeably less than that of FAIC, while the control efforts are reasonably bounded and convergent, in both cases. The time responses of the update vector parameters of $\lambda_1$, $\lambda_2$ and $\lambda_3$ for both of the controllers are shown in Fig. 12. Obviously, all of the update parameters approach to constant values which verify the feasibility of the proposed method, at the time period of $5 \le t < 10$ and try to cope with the system variations, in continue ($10 \le t$). In Fig. 13 the time evolution of sliding mode surfaces of both controllers are shown. Besides, the plots in Fig. 13(a), (b), (c) have been zoomed out in Fig. 13(d), (e), and (f) for better clearance. The results are representative of the good convergence of the sliding surfaces to zero in both cases, but faster convergence is observed for FAIBSMC, once again.

## 7. Conclusions

In this paper, in order for stabilization of the fractional uncertain chaotic systems, a novel hybrid fractional-order robust adaptive intelligent control scheme which is comprised of sliding mode control, backstepping control, adaptive control, and neuro-fuzzy network is proposed. An SMC law has been synthesized to guarantee the reachability of the specified sliding surface. The neuro-fuzzy network is employed to estimate the unknown continuous function. To cope with lumped uncertainties generated by NFN approximation errors and extra disturbances a robust structure with adaptive gains is used which on-line adaptive laws of the control system are derived based on the Lyapunov stability theorem so that the global asymptotic stability of the dynamical system can be achieved. Furthermore, the finite reaching time to the sliding surfaces has been proved. As some examples, the proposed technique is applied to control the energy resources demand–supply fractional order chaotic system and Chen fractional order chaotic system, these examples demonstrate the validity, effectiveness and good performance of the proposed FAIBSMC method. Based on the formulations, presented approach can be applied for stabilization for a large class of fractional uncertain chaotic systems with unknown system dynamics.

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