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Experimental investigation of mixed convection heat transfer from longitudinal fins in a horizontal rectangular channel

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ABSTRACT

Mixed convection heat transfer from longitudinal fins inside a horizontal channel has been investigated for a wide range of modified Rayleigh numbers and different fin heights and spacings. An experimental parametric study was made to investigate effects of fin spacing, fin height and magnitude of heat flux on mixed convection heat transfer from rectangular fin arrays heated from below in a horizontal channel. The optimum fin spacing to obtain maximum heat transfer has also been investigated. During the experiments constant heat flux boundary condition was realized and air was used as the working fluid. The velocity of fluid entering channel was kept nearly constant ($0.15 \le w_{in} \le 0.16 \text{ m/s}$) using a flow rate control valve so that Reynolds number was always about Re = 1500. Experiments were conducted for modified Rayleigh numbers $3 \times 10^7 < Ra^2 < 8 \times 10^8$ and Richardson number 0.4 < Ri < 5. Dimensionless fin spacing was varied from S/H = 0.04 to S/H = 0.018 and fin height was varied from $H_f/H = 0.25$ to $H_f/H = 0.80$. For mixed convection heat transfer, the results obtained from experimental study show that the optimum fin spacing which yields the maximum heat transfer is S = 8-9 mm and optimum fin spacing depends on the value of Ra^* .

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HEAT M/

1. Introduction

Fins are often used to enhance the rate of heat transfer from heated surfaces to environment. They can be placed on plane surfaces, tubes, or other geometries. These surfaces have been used to augment heat transfer by adding additional surface area and encouraging mixing. When an array of fins is used to enhance heat transfer under mixed convection conditions, the optimum geometry of fins (corresponding to a maximum rate of heat transfer) should be used, provided this is compatible with available space and financial limitations. Advantages in printed circuit boards have yielded increasing power dissipation from surfaces in a channel. Rectangular fins are used extensively to increase the rates of convection heat transfer from systems, because such fins are simple and cheap, to manufacture. Providing adequate cooling of printed circuits boards has recently motivated experiments on the use of longitudinal fins to enhance heat transfer in rectangular channels. The heat transfer, to the fluid flowing through a channel by the heat dissipating surfaces can be obtained mainly by using the mechanisms of heat transfer by forced convection, natural convection and by radiative heat transfer. This paper deals with those issues related to the heat transfer obtained mainly by mixed convection.

An extensive review and discussion of work done on the convective heat transfer in electronic cooling were presented by Incropera [1], summarizing various convection cooling options. A great number of experimental and analytical work has been carried out on this problem since Elenbaas [2], who first introduced the problem of natural convection between parallel plates. Starner and McManus [3], who measured the average heat transfer coefficient not only in horizontal but also in 45° and vertical base positions, have performed the first work on horizontal rectangular fin arrays. They showed that incorrect application of fins to a surface actually might reduce the total heat transfer to a value below that of the base alone. Welling and Wooldridge [4] conducted a similar experimental study on rectangular vertical fins to determine maximum heat transfer and tried to find optimum values for the ratio of fin height to fin spacing. Harahap and McManus [5] observed the flow patterns in two series of horizontal rectangular fin arrays. Jones and Smith [6] investigated the effects of fin height and fin spacing on heat transfer coefficient. They concluded that fin spacing is the main geometrical parameter, and it should be chosen as characteristic length. Numerous experimental studies exist in the literature, that were carried out in recent years on natural convection heat transfer from rectangular fin arrays placed on either horizontal or vertical plates [7–9]. In these studies effects of fin height, fin spacing and the difference between the base surface temperature and ambient air temperature were investigated. As results of these

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Nomenclature

Ab	copper plate surface area, m ²	Re	Reynolds number, $Re = w_{in}D_h/v_{air}$
$A_{\rm c}$	channel cross sectional area, m ²	Ri T	Richardson number, $Ri = Gr/Re^2$
$A_{\rm R}$	radiation neat transfer surface area, m ⁻	I T	temperature, °C
	channel hydraulic diameter 44 /D m		inlot air temperature °C
$D_{\rm h}$	channel hydraulic diameter, 4A _c /P, m	I _{in}	iniet all'temperature, °C
r ~	gravitational accoloration m/s ²		average copper plate surface temperature, °C
g Cm	gravitational acceleration, m/s^2		iocal copper plate surface temperature, °C
Gr	dimensionless Grasnor number, $Gr = (g\beta(I_w - I_{in})D_h)/(v^2)$	I _{WZmax}	fin anaging m
Ь	(U_{air}^2)	3	IIII Spacifig, III
П _{аv} ப	channel beight m	W _{in}	voltage V
н Ц.	fin height m	V 147	width m
IIf I	electric current	7 [*]	dimensionless axial distance $7^* - 7/L$
I V	thermal conductivity W/mK	L	unitensionless axial distance, $Z = 2/L_{\rm f}$
к I	length m	Creak symbols	
L I.	fin length m	$r_{\rm r}$ thormal diffucivity m^2/c	
Lf	ini iciigtii, in	11	
Nu	dimensionless overage Nusselt number $N_{ij} = h D_{ij} / D_{ij}$	ß	thermal expansion coefficient 1/K
Nu _{av}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/l_{av}$	β	thermal expansion coefficient, 1/K
Nu _{av}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$	α β ε	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms
Nu _{av} Nu _{wz}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$	α β ε μ	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinomatic viscosity, m ² /c
Nu _{av} Nu _{wz} P	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux W/m^2	α β ε μ ν	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stafin Boltzmann constant W/m ² K ⁴
Nu _{av} Nu _{wz} P q _{con}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 convection heat transfer rate W	α β ε μ ν σ	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} Nu _{wz} P Q _{con} Q _{convectio}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 n convection heat transfer rate, W	β β μ ν σ	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} NU _{wz} P Q _{con} Q _{convectio} Q _{conductio}	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 n convection heat transfer rate, W m conduction heat transfer rate, W	β ε μ ν σ Subscrip	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} NU _{wz} P Qcon Qconvectio Qradiation	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 n convection heat transfer rate, W m conduction heat transfer rate, W radiation heat transfer rate, W	α β ε μ ν σ Subscrip air	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} NU _{wz} P Qcon Qconvectio Qconductio Qradiation Qrotal Ra*	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 <i>n</i> convection heat transfer rate, W <i>m</i> conduction heat transfer rate, W radiation heat transfer rate, W total power dissipation, W medified Rayleigh number $Ra^* = (\alpha\beta a D^4)/k_{av}$	β ε ν σ Subscrip air f	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} NU _{wz} P Gcon Qconvectio Qconductio Qradiation Qtotal Ra [*]	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 <i>n</i> convection heat transfer rate, W m conduction heat transfer rate, W radiation heat transfer rate, W total power dissipation, W modified Rayleigh number, $Ra^* = (g\beta q_{con}D_h^4)/(k_b q_{con}D_h^4)/k_{air}$	α β ε μ v σ Subscrip air f in	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴
NU _{av} NU _{wz} P Qcon Qconvectio Qconductio Qradiation Qtotal Ra [*]	dimensionless average Nusselt number, $Nu_{av} = h_{av}D_h/k_{air}$ local Nusselt number, $Nu_{wz} = \frac{Q_{convection}D_h}{A_b(T_{wz}-T_{in})k_{air}}$ perimeter, m convection heat flux, W/m^2 n convection heat transfer rate, W radiation heat transfer rate, W total power dissipation, W modified Rayleigh number, $Ra^* = (g\beta q_{con}D_h^4)/(k_{air}\alpha_{air}v_{air})$	β ε ν Subscrip air f in W	thermal expansion coefficient, 1/K heater surface emissivity dynamic viscosity, kg/ms kinematic viscosity, m ² /s Stefan–Boltzmann constant, W/m ² K ⁴

studies some correlation equations were proposed as a function of fin height and spacing to obtain the maximum heat transfer in the case of natural convection with fin arrays for a certain temperature difference between the fin base and ambient air. A survey of literature reveals some theoretical studies on horizontal fin arrays [10– 15]. Radiation heat transfer plays an important role in heat transfer from fin arrays. Some studies also exist in literature, which take into account the effects of radiation on convective heat transfer from fin arrays [16].

Under mixed convection conditions, flow is driven by an externally imposed pressure gradient, as well as by buoyancy forces. For horizontal channels if the heating occurs at the bottom surface, buoyancy may induce a secondary flow which is in combination with the main flow. In the experiments performed for laminar flow between asymmetrically heated parallel plates (Osborne and Incopera [17]), convection coefficients at the bottom plate were found to exceed these corresponding to pure forced convection by up to a factor of six. Extended surfaces were used to enhance mixed convection heat transfer by Zhang and Patankar [18].

An analytical study was made to investigate the effects of buoyancy on laminar mixed convection in a shrouded fin array by Acharya and Patankar [19]. They concluded that the buoyancy forces significantly affect the heat transfer characteristics of laminar mixed convection in a shrouded fin array, buoyancy forces induce secondary flows and the increase in effects of natural convection, causes the buoyancy driven secondary flow to develop strongly and therefore causes an increase in the rate of heat transfer. Additional numerical investigations on mixed convection with fins were performed by Mahaney et al. [20] for a horizontal channel and by Al-Sarki et al. [21] for a shrouded fin array. Results suggest that Nusselt number is significantly enhanced by the effects of buoyancy in the mixed convection regime. Laminar mixed convection in tubes with longitudinal internal fins was investigated experimentally by Rustum and Soliman [22]. Results of experiments showed that natural convection has a strong influence on heat transfer in finned tubes and the enhancement in Nusselt number due to natural convection starts at high values of Rayleigh number. The effects of fin geometries and also the fin tip to shroud clearance on the heat transfer from longitudinal rectangular fin array in a horizontal channel were investigated experimentally by El-Sayed et al. [23].

Maughan and Incropera [24] presented some experimental results on the mixed convection heat transfer with longitudinal fins in a horizontal parallel plate channel. Nusselt numbers for the finned surfaces were in some instances in this study lower than those for mixed convection without fins. This effect was most pronounced for smaller fin heights. Mixed convection heat transfer with longitudinal fins in a horizontal channel was numerically investigated by Maughan and Incropera [25] and Mahaney et al.[26].

From the above literature review one can see that only little information can be found in literature for mixed convection heat transfer from fin arrays in a horizontal channel. A few of mixed convection studies, however, were carried out in channels at constant base plate temperatures. This paper reports the results of an experimental investigation of mixed convection heat transfer from rectangular fin arrays which are mounted on the bottom wall of a horizontal channel. To achieve maximum heat transfer, the selection of fin spacing and fin height must be optimized for a constant heat flux condition. In this paper, therefore, the effects of fin height, fin spacing, and heat flux on mixed convection are investigated experimentally.

2. Experimental set-up and data reduction

Detailed information on the experimental apparatus, devices used and procedures followed are given by Dogan [27]. A summary of information is presented below.





Fig. 1. Schematic diagram of the experimental set-up, (measures are in mm).

2.1. Experimental set-up

A schematic drawing of the experimental set-up is shown in Fig. 1. The experimental set-up consists mainly of a filter, a nozzle with flow straighteners, the test section, a damping chamber, a diffuser, a flow control valve and an AC fan. A data acquisition system with a PC was used for the measurement and storage of temperatures. A variac was used to adjust the electric current supplied to the heater plate. The test section is a rectangular duct heated at the bottom with a cross-section of 300 mm in width and 100 mm in height. The top, sides and bottom, entry and exit regions of the duct are constructed of 5 mm thick plexi-glass. The unheated entrance region of the duct is 600 mm, the heated test section involving a copper plate and rectangular longitudinal fins is 600 mm and unheated exit region is 300 mm. At the channel entrance a nozzle with flow straighteners has been used to suppress turbulence and achieve steady, laminar flow conditions with a uniform velocity distribution. The flow straightener was made of 5 mm diameter and 50 mm long plastic hoses. In addition, a filter was used to filter incoming air. The nozzle is made of 0.5 mm thick aluminum sheet and has been designed to eliminate flow separation, minimize turbulence, and provide a uniform velocity profile at the channel entrance. The test section is a bottom heated rectangular duct. It consists of $5 \times 300 \times 600$ mm copper plate, aluminum longitudinal fins of 1 mm in thickness and 600 mm in width heater plate, and thermocouples. Aluminum fins were attached to the copper plate by machining longitudinal grooves 1 mm wide and 2.5 mm deep. Initially, the grooves were filled with a thermally-conductive paste. Then the fin edges were coated with conducting paste, and the fins were press fitted into the length of the slots. The copper plate surfaces were polished to minimize radiation losses. The surface emissivities were measured to be 0.05 and 0.04 for the polished copper and aluminum surfaces, respectively. All surfaces were checked for flatness and were carefully cleaned and polished. A sheet heater plate was placed under the copper plate, with a size equal to the size of copper plate having dimensions of 300×600 mm. Electric current was provided to the heater plate via a variac, providing a heat flux boundary condition specified for a decided experimental case.

The cross-section view of the rectangular duct is schematically shown in Fig. 2. As can be seen from this figure, the test section of the duct was insulated with 15 mm Glasswool (k = 0.048 W/m K) and 50 mm Styrofoam (k = 0.028 W/m K) and was mounted on a rigid supporting frame. The test section is isolated from the fan acoustically and mechanically with a damping chamber and flexible hosing. A control valve was used to control the flow rate.

A digital multimeter with accuracy of ±1% for voltage and ±1% for resistance was utilized. Sixty four copper-constant thermocouples were used in total. All thermocouples were separately calibrated. The test section was instrumented with 44 thermocouples located on the copper plate. The thermocouples on the heated section were inserted through holes drilled in the insulation and were pushed into drilled cavities placed inside the copper plate and soldered for rigidity. Temperatures were also measured at the inlet, outlet, ambient at several locations on the top, bottom and side walls of the insulation applied. The temperature signals were transferred to two 32-channel data acquisition unit, and finally, sent to a PC for further processing. Temperatures measured for 15 s time intervals were collected, stored and analyzed in this PC. The air velocity is measured with a hot-wire anemometer with an accuracy of ±0.015 m/s. It was observed that experimental conditions reach a steady-state condition about approximately 7-8 h. After conditions had been steady for sometime and differences in temperatures between two intervals become negligible $(\Delta T < 0.1 \text{ K})$, all temperatures were collected, averaged and stored.

2.2. Processing of the experimental data

The data obtained during the experiments are values of temperature, velocity, voltage drop across the heater, and electric current.



Fig. 2. Cross-section view of the channel.

Using these values, the average Nusselt number and local Nusselt number for the heated bottom plate is defined as

$$Nu_{\rm av} = \frac{h_{\rm av}D_{\rm h}}{k_{\rm air}} \tag{1}$$

$$Nu_{wz} = \frac{Q_{\text{convection}}D_{\text{h}}}{A_{\text{b}}(T_{wz} - T_{\text{in}})k_{\text{air}}}$$
(2)

where h_{av} , D_h , T_{wz} and k_{air} are the average heat transfer coefficient, characteristic length, local copper plate surface temperature and thermal conductivity of air, respectively. The average heat transfer coefficient h_{av} and D_h are defined below.

$$h_{\rm av} = \frac{Q_{\rm convection}}{A_{\rm b}(T_{\rm w} - T_{\rm in})} \tag{3}$$

$$D_{\rm h} = \frac{4A_{\rm c}}{P} \tag{4}$$

In Eq. (2) and Eq. (3), $Q_{\text{convection}}$ is convection heat transfer rate to the fluid. $A_b = L^*W$ is unfinned surface area of heated plate. T_w and T_{in} are average copper plate surface temperature and inlet temperature of air, respectively. A_c is channel cross sectional area and P is the perimeter of the channel. Convection heat transfer from both the bottom copper plate and fins ($Q_{\text{convection}}$) was determined from an energy balance

$$Q_{\text{total}} = Q_{\text{convection}} + Q_{\text{convection}} + Q_{\text{radiation}}$$
(5)

where Q_{total} is total dissipated energy from the surface heater source, $Q_{\text{convection}}$ is the total conduction heat loss through the insulation of test section, and $Q_{\text{radiation}}$ is the total radiation heat loss from bottom plate and fin surfaces. The total dissipated energy was determined from Ohm's law, $Q_{\text{total}} = VI$ at the heater source. The voltage drop *V* and current *I* were measured during the experiment. The heat loss through the insulation of test section was calculated from

$$Q_{\text{conduction}} = k_{\text{insulation}} A_{\text{insulation}} \frac{\Delta T_{\text{insulation}}}{L_{\text{insulation}}}$$
(6)

where $k_{\text{insulation}}$ is thermal conductivity of the insulation and $\Delta T_{\text{insulation}}$ is the difference between internal and external surface temperatures of the insulation. Radiation losses were determined from the assumption of isothermal, gray, diffuse and opaque surfaces. These losses were determined by using the method of Ellison [16] and the following equation:

$$Q_{\text{radiation}} = FA_R \sigma (T_w^4 - T_b^4) \tag{7}$$

where *F* is the gray body shape factor, A_R is the surface area for radiation heat transfer and σ is Stephan–Boltzman constant. The results of experiments showed that the radiation losses never exceeded 6% of the total power dissipated. Total heat losses, however, were calculated to be always less than 13% of the total power dissipated. The dimensionless number affecting the heat transfer are, The Reynolds number:

$$Re = \frac{W_{inD_h}}{v_{air}} \tag{8}$$

The modified Rayleigh number:

$$Ra^* = \frac{g\beta q_{\rm con} D_{\rm h}^4}{k_{\rm air} \alpha_{\rm air} v_{\rm air}} \tag{9}$$

The Grashof number Gr:

$$Gr = (g\beta(T_{\rm w} - T_{\rm in})D_{\rm h}^3)/(\upsilon_{\rm air}^2)$$
⁽¹⁰⁾

The Richardson number:

$$Ri = \frac{Gr}{Re^2} \tag{11}$$

where $q_{\text{con}} = \frac{Q_{\text{convection}}}{A_b}$ is average convection heat flux transferred to the fluid. The fluid properties used in these definitions were determined at the arithmetic average of average copper plate and fluid inlet temperatures $(\frac{T_w+T_{\text{in}}}{T_w+T_{\text{in}}})$.

In order to determine the reliability of the experimental results an uncertainty analysis was conducted on all measured quantities as well as the quantities calculated from the measurement results. Uncertainties were estimated according to the standard procedures reported in the literature [28,29]. Overall, the uncertainty in the Nusselt number is around $\pm 6\%$ and for the modified Grashof number it is around $\pm 4\%$, which is primarily due to uncertainties in the convective heat flux values. Uncertainty in the Reynolds number is around $\pm 2.5\%$.

3. Results and discussion

The mixed convection heat transfer with longitudinal fins in a horizontal channel under bottom wall constant heat flux conditions has been investigated experimentally. By adjusting the flow control valve, the fluid velocity at the inlet of test section was obtained as $0.15 < w_{in} < 0.16$ m/s so that Reynolds number was always around Re = 1500 during the experiments. Experiments were conducted under various heat flux conditions. The experimental results are presented using the modified Rayleigh number Ra^* based on heat flux and the channel geometry see Eq. (8). As a consequence of the above mentioned experimental conditions the Richardson number was obtained between 0.4 and 5, which corresponds mixed convection regime. Measurements were taken with non-dimensionalized fin spacings ranging between $0.04 \le S/H \le 0.18$ and for non-dimensionalized fin heights of 0.25, 0.50 and 0.80. The fin-array configurations studied are listed in Table 1.

Table 1Fin-arrays configurations.

Fin-array set	Fin height (H_f/H)	Fin spacing (S/H)	Fins number (N)
1	0.25	0.04	59
2	0.25	0.08	32
3	0.25	0.12	22
4	0.25	0.18	15
5	0.50	0.04	59
6	0.50	0.08	32
7	0.50	0.12	22
8	0.50	0.18	15
9	0.80	0.04	59
10	0.80	0.08	32
11	0.80	0.12	22
12	0.80	0.18	15



Fig. 3. Variation of average convection heat transfer coefficient with fin spacing at different fin heights (average modified Rayleigh number, $Ra^* = 4 \times 10^7$).



Fig. 4. Variation of average convection heat transfer coefficient with fin spacing at different fin heights (average modified Rayleigh number, $Ra^* = 2 \times 10^8$).

Figs. 3–7 present variations of average convection heat transfer coefficient with fin spacing for different modified Rayleigh numbers and fin heights, $H_f/H = 0.25$, 0.50 and 0.80. As can be seen from these figures, the increase or decrease in fin spacing after a fixed value does not cause an enhancement in heat transfer coefficient. On the contrary, it causes a reduction in heat transfer coefficient. The average heat transfer coefficient first increases with fin spacing up to a maximum value and then it decreases with the increase in fin spacing. The value of fin spacing at which the heat transfer takes its maximum value, is defined as optimum fin spacing, S_{opt} .



Fig. 5. Variation of average convection heat transfer coefficient with fin spacing at different fin heights (average modified Rayleigh number, $Ra^* = 4 \times 10^8$).



Fig. 6. Variation of average convection heat transfer coefficient with fin spacing at different fin heights (average modified Rayleigh number, $Ra^* = 6 \times 10^8$).



Fig. 7. Variation of average convection heat transfer coefficient with fin spacing at different fin heights (average modified Rayleigh number, $Ra^* = 7 \times 10^8$).

These figures show that the optimum fin spacing is between 8 and 9 mm. As can be seen from these figures, when the heat transfer surface area is made larger (increasing number of fins or decreasing fin spacing), the heat transfer coefficient decreases. The decrease in fin spacing causes an intersection between boundary layers developed on fin surfaces. The intersection of boundary

layers, causing the velocity of fluid flowing through fin arrays to decrease, prevents the cold fluid to enter fin grooves and thus the hot fluid stays much longer between fin arrays.

The decrease in fluid velocity results in a deficiency in removing heat energy from heated surfaces, and therefore causes a large part of the duct to be occupied by heated fluid and prevents heat transfer to be achieved efficiently. After the optimum value of spacing, the average heat transfer coefficient decreases with the increase in fin spacing as shown from these figures. Because increasing the fin spacing (decreasing the number of fins), causes a decrease in total heat transfer surface area, which results in a decrease in the rate of heat transfer. As seen from these figures in the design of fin arrays on a horizontal surface, to achieve maximum amount of heat transfer, the fin spacing must have an optimum value. As can be seen from Figs. 3–5, the optimum fin spacing is near $S_{opt} = 8 \text{ mm}$ (for the modified Ray-leigh numbers, $Ra^* = 4 \times 10^7$, $Ra^* = 2 \times 10^8$ and $Ra^* = 4 \times 10^8$ respectively and for non-dimensionalized fin heights of $H_f/H = 0.25, 0.50$ and 0.80). In Figs. 6 and 7, it is seen that for $H_f/H = 0.25$, 0.50 and 0.80, the optimum fin spacing is near $S_{opt} = 9 \text{ mm}$ and the optimum fin spacing increases from $S_{opt} = 8 \text{ mm}$ to $S_{opt} = 9 \text{ mm}$ with the modified Rayleigh number at each value of fin height. If we consider the value of Richardson number it is about 0.4 for $Ra^* = 4 \times 10^7$, but 5.0 for $Ra^* = 7 \times 10^8$. From this we can say that increasing the modified Rayleigh number causes greater effects of natural convection on heat transfer. If fin spacing is sufficiently large the increase in effects of natural convection, causes the buoyancy driven secondary flow to develop strongly and therefore causes an increase in the rate of heat transfer. When the modified Rayleigh number is $Ra^* = 7 \times 10^8$, the buoyancy forces do not become strong enough for small values of spacing (such as S = 8 mm). Thus, the optimum fin spacing rises from $S_{opt} = 8 \text{ mm}(\text{seen in Figs. 3-5}) \text{ to } S_{opt} = 9 \text{ mm}(\text{seen in Figs. 6 and 7}) \text{ as}$ a result of the increase in modified Rayleigh number.

The variation of average heat transfer coefficient with fin heights are presented in Figs. 8–12 for different modified Rayleigh numbers and for fin spacings of S/H = 0.04, 0.08, 0.12 and 0.18. The average heat transfer coefficient increases with the increase in fin height for each fin spacing. Increasing the fin height for a fix value of channel height causes an increase in the heat transfer surface area. Consequently surface temperatures of fins and copper plate decrease, which results in an increase in the heat transfer coefficient. In addition, increasing the fin height causes the buoyancy driven secondary flows to be developed greatly for the cases at which natural convection effects are important. This results in a further increase in heat transfer coefficient.

For a fixed value of channel height and constant mass flow rate, if the fin height is increased, fins approaching or touching the top



Fig. 8. Variation of average convection heat transfer coefficient with fin height at different fin spacings (average modified Rayleigh number, $Ra^* = 4 \times 10^7$).



Fig. 9. Variation of average convection heat transfer coefficient with fin height at different fin spacings (average modified Rayleigh number, $Ra^* = 2 \times 10^8$).



Fig. 10. Variation of average convection heat transfer coefficient with fin height at different fin spacings (average modified Rayleigh number, $Ra^* = 4 \times 10^8$).

surface cause significant enhancement in the heat transfer coefficient for high values of fin height by forcing all the flow into the channels formed by the fins. If the fin height is decreased, fluid flows through the inner fin region causing the fluid to escape away from the inner fin region to outside region through the fin tips and



Fig. 11. Variation of average convection heat transfer coefficient with fin height at different fin spacings (average modified Rayleigh number, $Ra^* = 6 \times 10^8$).



Fig. 12. Variation of average convection heat transfer coefficient with fin height at different fin spacings (average modified Rayleigh number, $Ra^* = 7 \times 10^8$).

therefore heat transfer decreases. This effect can be seen in Figs. 8– 12. When the fin height is increased from $H_f/H = 0.25$ to $H_f/H = 0.50$, for the same mass flow rate at every fin spacings, the rate of increase in average heat transfer coefficient is obtained as about 50%. But as the fin height is increased from $H_f/H = 0.50$ to $H_f/H = 0.80$, the rate of increase in average heat transfer coefficient is about 100%.

As can be seen from these figures, for the fin height $H_f/H = 0.25$ and $H_f/H = 0.50$ the value of average heat transfer coefficient is greater for the fin spacing S/H = 0.18 than for the fin spacing S/H = 0.04. But, the average heat transfer coefficient obtained for the fin spacing of S/H = 0.04 is greater than the average heat transfer coefficient obtained for S/H = 0.18 for the fin height $H_f/H = 0.80$. This is due to the fact that the buoyancy driven forces become strong enough with the increase in the fin height, and therefore buoyancy driven secondary flows have more important effects on the rate of heat transfer.

Figs. 13–15 present the variation of average heat transfer coefficient with fin spacing for different modified Rayleigh numbers. As can be seen from these figures, for each of the modified Rayleigh numbers, the average heat transfer coefficient increases with fin spacing up to the optimum value of fin spacing and after that it decreases. In Fig. 13, the optimum value of fin spacing for the maximum average heat transfer coefficient increases (from about 8 mm to 9 mm). The reason of this, as explained above, is the small value of fin height and spacing, which are insufficient to develop buoy-



Fig. 13. Variation of average convection heat transfer coefficient with fin spacing (fin height, $H_f/H = 0.25$).



Fig. 14. Variation of average convection heat transfer coefficient with fin spacing (fin height, $H_{\rm f}/H$ = 0.50).



Fig. 15. Variation of average convection heat transfer coefficient with fin spacing (fin height, $H_f/H = 0.80$).

ancy driven secondary flow at high values of modified Rayleigh numbers. Fig. 13 shows, the fin spacing does not have a very significant effect on heat transfer from fin array for lower fin height. But, as can be seen in Figs. 14 and 15, for higher fin heights, the fin spacing has significant effect on heat transfer. This is because, as the fin height decreases, fluid escapes away from the inter fin region and flows over the fin tips.

In Figs. 16–18 variation of the dimensionless temperature and local Nusselt number distributions with the stream-wise direction (z) are presented for different modified Rayleigh numbers, fin heights, $H_f/H = 0.25$, 0.50, 0.80 and the fin spacing S/H = 0.08. Maximum temperatures on the base plate were obtained at the points $Z^* = 0.6$, 0.8 and 0.85 for fin heights $H_f/H = 0.25$, 0.50 and 0.80 respectively. After these points, the base plate temperature decreases in the main flow direction. But, for the fin height $H_f/H = 0.25$, the base plate temperature after the point $Z^* = 0.6$ does not change much in the main flow direction for small values of fin height. This effect is due to fact that natural convection which becomes important by the onset of secondary flow after this point. As can be seen from Figs. 16-18, Nusselt number variations show a forced convection characterisic for each modified Rayleigh numbers. The Nusselt number in main flow direction does not change much for fin height of $H_f/H = 0.25$ (see Fig. 16) as for other fin heights of $H_f/H = 0.50$ and 0.80 (see Fig. 17 and Fig. 18).

As the fin height is lowered fluid begins traveling through a region between fin tips and the top surface of the channel since a



Fig. 16. (a) Variation of the temperature distribution and (b) variation of Nusselt Number with the main flow direction (z) (S/H = 0.08 and H_f/H = 0.25).



Fig. 17. (a) Variation of the temperature distribution and (b) variation of Nusselt Number with the main flow direction (z) (S/H = 0.08 and H_f/H = 0.50).

resistance against the fluid flow is formed between fins. For the same modified Rayleigh number, fin spacing and height of the channel, forced convection effects on heat transfer increase with the increase in fin height. This effect can be seen clearly from Figs. 16–18. As a result of the increase in fin height, the flow goes through the inner fin region, and thus the rate of heat transfer increases.

It has been shown in the present work that the average Nusselt number depends on the Reynolds number, modified Grashof number, fin spacing, fin height, tip clearance and fin length. Based on the Levenberg–Marquardt algorithm, and considering the effects of above parameters, an empirical equation has been derived to correlate the average Nusselt number. The resulting plot is shown in Fig. 19. The correlation could be expressed by the equation

$$Nu_{\rm av} = 0.06312 (Re)^{0.5639} (Gr^*)^{0.0843} \left(\frac{s}{H_{\rm f}}\right)^{-0.0312} \left(\frac{c}{H_{\rm f}}\right)^{-0.621} \left(\frac{s}{H_{\rm f}}\right)^{0.50}$$
(12)

within the ranges $250 \le Re \le 2300$, $5 \times 10^7 \le Gr^* \le 1 \times 10^9$, $0.05 \le S/H_f \le 0.72$, $0.25 \le c/H_f \le 3$ and $7.5 \le L_f/H_f \le 24$ with a rela-

tive error of $\pm 18\%$. The reader should note that this correlation is only valid for thin fins which do not cause much effect on longitudinal air flow. And this correlation does not show effects of fin cross sectional area, thermal conductivity and heat transfer coefficient of the fin.

4. Conclusions

Mixed convection heat transfer from longitudinal fins in a horizontal channel with a uniform heat flux boundary condition at the bottom surface has been studied experimentally. Experimental results for bottom heated fin arrays have been presented for different fin spacings, fin heights, and modified Rayleigh numbers, and the effects on heat transfer have been investigated.

It has been determined that the mixed convection heat transfer depends on the fin height and spacing. The effects of fin spacing on heat transfer have been investigated by conveying experiments at four different fin spacings (S/H = 0.04, 0.08, 0.12 and 0.18). The average convection heat transfer coefficient increases first with fin spacing and then it takes its maximum value after which it



Fig. 18. (a) Variation of the temperature distribution and (b) variation of Nusselt Number with the main flow direction (z) (S/H = 0.08 and H_f/H = 0.80).

starts to decrease with the increase in fin spacing. When the fin spacing is smaller than the required value, the resistance against the flow is formed due to the intersection of boundary layers developed on fin surfaces and as a result, the rate heat transfer from fin arrays decreases. For large values of fin spacing (causing a small number of fins for fixed fin base area), however, the decrease in the total heat transfer area causes the rate of heat transfer to decrease. Results of experiments have shown that to obtain maximum amount of heat transfer from fin arrays, the fin spacing should be at an optimum value. The optimum fin spacing has been obtained in this study as $S_{opt} = 8-9$ mm.

The optimum value of fin spacing depends mainly on modified Rayleigh number. The buoyancy driven secondary flows which augment the heat transfer, cannot develop, if the fin height H_f/H is not sufficiently large. The rise in the fin height for the same fin spacing, provides the required pressure decrease in the region bounded by fin plates, which causes the secondary flows to develop. The rise in the fin height, however, increases heat transfer from fin arrays by causing an increase in total heat transfer surface area. Besides, for a fixed channel height and constant mass flow rate, as a result of increase in fin height, fins approaching or touching the top surface of the channel cause significant enhancement in heat transfer by forcing all the flow into the channels formed by the fins. Results of experiments have shown that the effect of the fin spacing on heat transfer increases with increasing fin height.



Fig. 19. Comparison of the correlation obtained from the present study with experimental results.

A correlation for average Nusselt number has also been presented to relate the heat transfer from fin arrays in channel with dimensionless, Reynolds number, modified Grashof number and other experimental parameters such as fin spacing, fin height, tip clearance and fin length.

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