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Linear control for mixed synchronization of a fractional-order chaotic system

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A B S T R A C T

This paper studies the mixed synchronization of a fractional-order chaotic system. By designing a simple linear controller, anti-phase synchronization of the first two pairs of output signals between the unidirectional systems are realized, and the third pair of signals is complete synchronous. The control scheme is theoretically proven based on final-value theorem of the Laplace transformation, numerical simulations are further provided to verify the feasibility of the theoretical analysis.

1. Introduction

Chaos is a sophisticated yet interesting phenomenon and has been intensively studied in theoretical and practical applications [1–4]. Fractional dynamics can be recollected to the contribution of Leibniz and L'Hospital 300 years ago [5,6], but its applications in concrete scientific discipline are just a recent topic of interests [7–10]. The fractional calculus is known to be a more appropriate mathematical tool for modeling the physical processes in reality, since it can provide a deeper insight and a long range memory behavior, in comparison with integer calculus. All natural phenomena exist virtually in fractional form with the special case of the integer one [11].

Synchronization is known to be one of the important branch of chaos theory, widely discussed recently in prospective applications including (but not limited to) power converters [12], biological systems [13], secure communications [14,15], chemical reactions [16] and information transmission [17], due to its major fundamental significance accompanied with continuous broadband frequency spectrum. There exist different types of chaotic synchronization discussed recently such as complete synchronization [18], anti-synchronization [19], phase synchronization [20], generalized synchronization [21], lag synchronization [22], projective synchronization [23], mixed synchronization [24]. Among these synchronous types of chaotic systems the mixed synchronization stands out due to its special features and potential applications, which combines the complete synchronization and anti-synchronization.

As we know that fractional-order chaotic systems can enrich the key space and thus be efficiently applied in encryption due to the more adjustable variables. Therefore, it's more significant to discuss the synchronization of fractional-order chaotic systems. Although there emerge many excellent contributions, it's still an unexpanded but deserved topic to synchronize a fractional-order chaotic system with challenge [25–28].

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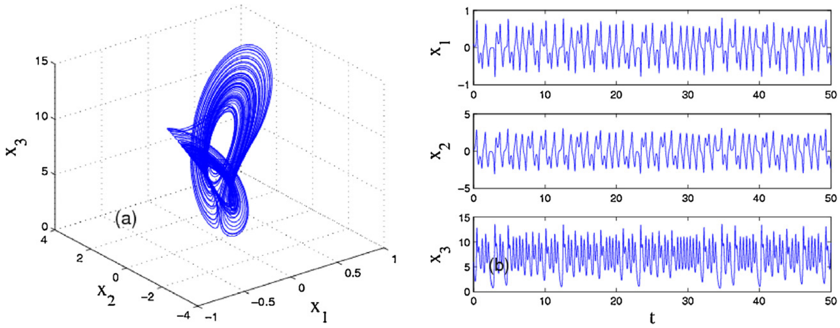

Fig. 1. (a) Chaotic attractor; and (b) time series of fractional system (1).

In this work, we discuss the issue of mixed synchronization for a fractional-order chaotic system. By designing a linear controller, anti-phase synchronization of the first two pairs of output signals between the unidirectional systems are realized, and the third pair of signals is complete synchronous. The control scheme contains only a single controller for expediently designing and implementing, and is theoretically proven based on final-value theorem of the Laplace transformation. Finally, numerical simulations are further provided to verify the feasibility of the theoretical analysis.

2. System description

The fractional chaotic system under investigation is described by Li et al. [25]

$$\begin{cases} D^{\beta_1}x_1 = -ax_1 + fx_2x_3 \\ D^{\beta_2}x_2 = cx_2 - dx_1x_3 \\ D^{\beta_3}x_3 = -bx_3 + ex_2{}^2 \end{cases} \tag{1}$$

System (1) possess three equilibrium points as

$$E_0(0,0,0),\ E_1(\frac{ef}{ab}x_o{}^3, x_o, \frac{e}{b}x_o{}^2),\ E_2(-\frac{ef}{ab}x_o{}^3, -x_o, \frac{e}{b}x_o{}^2)\text{with}x_o = (\frac{ab^2c}{de^2f})^{\frac{1}{4}}$$

When selecting the parameter set $a = 16$, $b = 5$, $c = 10$, $d = 6$, $e = 18$, $f = 0.5$, the characteristic values corresponding to different equilibrium points are

$$E_0(0,0,0):\quad \lambda_1 = -11.544,\ \lambda_2 = 5.544,\quad \lambda_3 = -5$$

$$E_{1,2}(\pm 0.325, \pm 1.4243, 7.303):\quad \lambda_1 = -2.257 + 27.068.\text{i},\ \lambda_2 = -2.257 - 27.068.\text{i},\quad \lambda_3 = -6.486$$

Therefore, with the parameter set, the necessary condition for the existence of chaos for commensurate fractional system (1) is

$$\beta > \frac{2}{\pi} atan(\frac{|Im(\lambda*)|}{Re(\lambda*)}) = \frac{2}{\pi} atan(\frac{27.0687}{\text{-}2.257}) = 0.947$$

Choosing the parameter set and $\beta_1 = \beta_2 = \beta_3 = 0.995$, system (1) displays chaotic. The corresponding phase portrait and time series are depicted in Fig. 1.

3. Mixed synchronization of the fractional system

3.1. Synchronization analysis

In order to observe the mixed synchronization behavior of the presented chaotic systems, we assume that the drive system is governed as (1), and the response system is given by

$$\begin{cases} D^{\beta_1}y_1 = -ay_1 + fy_2y_3 \\ D^{\beta_2}y_2 = cy_2 - dy_1y_3 + u \\ D^{\beta_3}y_3 = -by_3 + ey_2{}^2 \end{cases} \tag{2}$$

in which u is the controller added to the second term of the system equation.

The aim is to choose a simple control scheme u such that the states of the drive and response systems are synchronized, depicted by

$$\begin{cases} \lim_{t\to\infty} e_1 = \lim_{t\to\infty}(x_1+y_1)=0 \\ \lim_{t\to\infty} e_2 = \lim_{t\to\infty}(x_2+y_2)=0 \\ \lim_{t\to\infty} e_3 = \lim_{t\to\infty}(x_3-y_3)=0 \end{cases} \tag{3}$$

Theorem 1. *For the fractional-order systems (1) and (2), if the controller is designed as $u = k(x_2 + y_2)$, $k + c \neq 0$, then one can obtain the mixed synchronization described by (3).*

Before begin our proof, we recall the following preliminaries.

(1) Denote $E_i(s) = L\{e_i(t)\}$, where L is the Laplace transformation of time-domain signal.

(2) Final-value theorem of fractional signal

$$L\left\{D^{\beta}e(t)\right\} = s^{\beta}E(s) - s^{\beta-1}e(0) \text{ and } \lim_{t\to\infty} e(t) = \lim_{s\to 0^+} sE(s)$$

Proof. The fractional error dynamical system can be expressed as

$$\begin{cases} D^{\beta_1}e_1 = -ae_1 + fx_2x_3 + fy_2y_3 \\ D^{\beta_2}e_2 = (c+k)e_2 - dx_1x_3 - dy_1y_3 \\ D^{\beta_3}e_3 = -be_3 + ex_2{}^2 - ey_2{}^2 \end{cases} \tag{4}$$

Taking the Laplace transformation on both sides of error system (4) yields

$$\begin{cases} (s^{\beta_1}+a)E_1(s) = s^{\beta_1-1}e_1(0) + L[fx_2x_3 + fy_2y_3] \\ (s^{\beta_2}-c-k)E_2(s) = s^{\beta_2-1}e_2(0) - L[dx_1x_3 + dy_1y_3] \\ (s^{\beta_3}+b)E_3(s) = s^{\beta_3-1}e_3(0) + L[ex_2{}^2 - ey_2{}^2] \end{cases}$$

So, when $k + c \neq 0$, one gets

$$\begin{cases} E_1(s) = \dfrac{s^{\beta_1-1}e_1(0)}{s^{\beta_1}+a} + \dfrac{L[fx_2x_3 + fy_2y_3]}{s^{\beta_1}+a} \\ E_2(s) = \dfrac{s^{\beta_2-1}e_2(0)}{s^{\beta_2}-c-k} - \dfrac{L[dx_1x_3 + dy_1y_3]}{s^{\beta_2}-c-k} \\ E_3(s) = \dfrac{s^{\beta_3-1}e_3(0)}{s^{\beta_3}+b} + \dfrac{L[ex_2{}^2 - ey_2{}^2]}{s^{\beta_3}+b} \end{cases}$$

Considering the Final-value theorem of the Laplace transformation, we have

$$\begin{cases} \lim\limits_{t\to\infty} e_1(t) = \lim\limits_{s\to 0^+} sE_1(s) = \lim\limits_{s\to 0^+} \dfrac{s^{\beta_1}e_1(0)}{s^{\beta_1}+a} + \lim\limits_{s\to 0^+} \dfrac{sL[fx_2x_3 + fy_2y_3]}{s^{\beta_1}+a} = 0 \\ \lim\limits_{t\to\infty} e_2(t) = \lim\limits_{s\to 0^+} sE_2(s) = \lim\limits_{s\to 0^+} \dfrac{s^{\beta_2}e_2(0)}{s^{\beta_2}-c-k} - \lim\limits_{s\to 0^+} \dfrac{sL[dx_1x_3 + dy_1y_3]}{s^{\beta_2}-c-k} = 0 \\ \lim\limits_{t\to\infty} e_3(t) = \lim\limits_{s\to 0^+} sE_3(s) = \lim\limits_{s\to 0^+} \dfrac{s^{\beta_3}e_3(0)}{s^{\beta_3}+b} + \lim\limits_{s\to 0^+} \dfrac{sL[ex_2{}^2 - ey_2{}^2]}{s^{\beta_3}+b} = 0 \end{cases}$$

This completes the proof of Theorem 1.

3.2. Numerical simulation

In this section, we execute the mixed synchronization of fractional-order chaotic systems (1) and (2) to demonstrate the effectiveness of the proposed scheme.

In the numerical simulation, the system parameters are set as $a = 16, b = 5, c = 10, d = 6, e = 18, f = 0.5$, the orders of fractional system are $\beta_1 = \beta_2 = \beta_3 = 0.995$, the initial conditions for the drive and response systems are set to $X(0) = (1, 2, 0.1)$, $Y(0) = (-1, -2, 6)$. When substituting the controller with $k = 6$ into the response system, the mixed synchronization is achieved. The trajectory time curves and synchronization errors are displayed in Fig. 2. As we expected, the trajectories of the drive system synchronize those of response system rapidly.

When taking another controller with $k = 8$ into the response system, the mixed synchronization is also achieved. The trajectory time curves and synchronization errors are depicted in Fig. 3. It's known that the transient period is shorter when a bigger gain k is adopted.

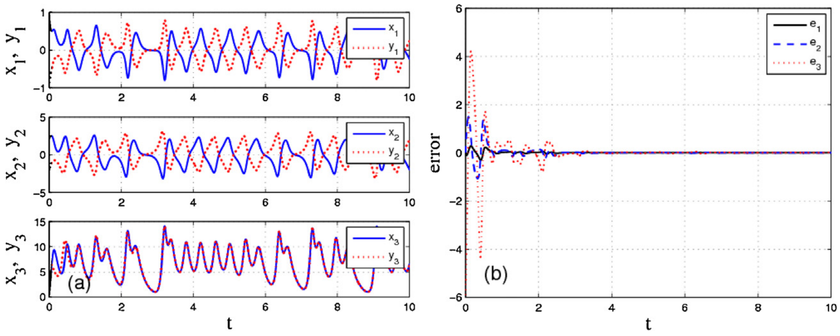

Fig. 2. Control result with $k = 6$ (a) state trajectory; (b) synchronization error.

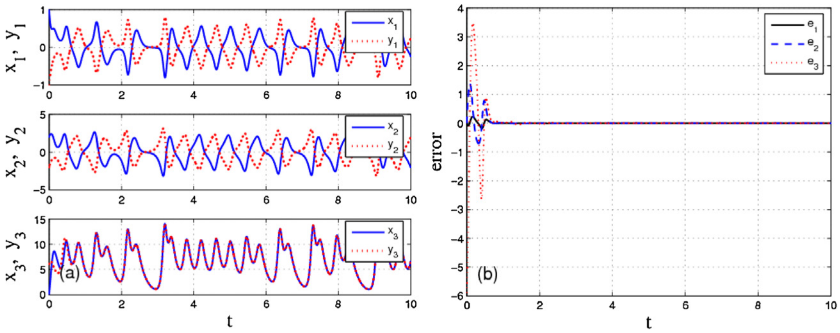

Fig. 3. Control result with $k = 8$ (a) state trajectory; (b) synchronization error.

4. Conclusion

In this work, we discuss the issue of mixed synchronization for a fractional-order chaotic system. By designing a linear controller, anti-phase synchronization of the first two pairs of output signals between the unidirectional systems are realized, and the third pair of signals is complete synchronous. The control scheme contains only a single controller for expediently designing and implementing, and is theoretically proven based on final-value theorem of the Laplace transformation. Finally, numerical simulations are further provided to show that the theoretical result is practicable, and a bigger gain k can obtain a shorter transient period.

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