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Typical Upper Bound–Lower Bound Mixed Mode Fracture Resistance Envelopes for Rock Material

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Abstract Mixed mode fracture experiments were conducted on Harsin marble using two disc-shape samples namely the Brazilian disc (BD) and the semi-circular bend (SCB) specimens. For each specimen, a complete fracture toughness envelope ranging from pure mode I to pure mode II was obtained. The experimental results indicate that the mixed mode fracture toughness depends on the geometry and loading conditions such that for any similar mode mixture, the BD test data were significantly greater than the SCB fracture toughness results. Therefore, the conventional fracture criteria which present a unique mixed mode fracture curve, fail to predict the test results. It is shown that a generalized criterion, which takes into account the effects of geometry and loading conditions, is able to provide individual fracture curves for theses specimens with very good estimates for the test results obtained from both BD and SCB specimens. The BD and SCB specimens can be suggested as appropriate specimens for obtaining typical upper bound and lower bound envelopes for mixed mode fracture toughness of rocks.

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1 Introduction

Fracture toughness is an important property in rock fracture mechanics, which defines the resistance of rock materials against the crack propagation. The determination of fracture toughness for rock materials requires suitable test specimens and reliable test methods. Disc-type specimens such as the center-cracked Brazilian disc (BD) specimen subjected to diametral compression and the edge cracked semi-circular bend (SCB) specimen under three-point bending are among favorite specimens for conducting the fracture toughness tests on rocks and geo-materials. Hence, rock fracture researchers have frequently used these two specimens in the past for investigating the crack growth behavior of various rock materials under mode I (Chong and Kuruppu 1984; Chong et al. 1987; Zhao et al. 1994; Lim et al. 1994a; Fowell 1994,1995; Funatsu et al. 2004; Aliha et al. 2006; Nasseri and Mohanty 2008) and mixed mode I/II loading (Awaji and Sato 1978; Lim et al. 1994b; Krishnan et al. 1998; Khan and Al-Shayea, 2000; Chang et al. 2002; Al-Shayea 2005; Liu et al. 2007; Ayatollahi and Aliha 2007a, 2008; Ke et al. 2008; Aliha et al. 2010).

There are two methods for introducing the initial crack in the BD and SCB specimens namely: (1) the straight thorough thickness crack and (2) the chevron notch method. The international society for rock mechanics (ISRM) has adopted the chevron notched BD specimen as a standard method for obtaining pure mode I fracture toughness of rocks. Therefore, many researchers have measured K_{Ic} of different rocks based on the ISRM suggested test method (Fowell 1994, 1995; Zhao et al. 1994; Khan and Al-Shayea 2000; Chang et al. 2002; Aliha et al. 2006; Nasseri and Mohanty 2008). Awaji and Sato (1978) were among the first researchers who used the BD specimen in their fracture studies. They used the chevron notched BD specimens made of marble to obtain mode I and mixed mode fracture toughness. Krishnan et al. (1998) employed the straight cracked BD specimens for evaluating the mixed mode fracture resistance of a soft sandstone with different bedding plane directions and under pure mode I, pure mode II and some combinations of modes I and II. The mixed mode fracture toughness of a soft limestone has been investigated by Khan and Al-Shayea (2000) and Al-Shayea (2005) using the straight cracked BD configuration. They studied the effects of specimen size, temperature and also the influence of confining pressure on mixed mode fracture toughness of Saudi Arabian limestone using several BD specimens. Chang et al. (2002) have also used the chevron notched BD specimens for mixed mode fracture toughness study on Korean marble and Korean granite. The SCB specimen was also first proposed by Chong and Kuruppu (1984) for measuring the mode I fracture toughness of rock materials. Then, Lim et al. used the SCB configuration with straight crack for investigating the crack growth behavior of a synthetic soft rock called Johnstone under mode I (Lim et al. 1994a) and mixed mode I/II (Lim et al. 1994b) as well. They studied the effects of different parameters such as crack length, specimen dimensions, water content and the span of loading supports on fracture resistance of Johnstone. Khan and Al-Shayea (2000) conducted also some mixed mode I/II fracture experiments using straight cracked SCB specimens made of limestone. Chang et al. (2002), attempted to study the mixed mode fracture behavior of rock materials using chevron notched SCB specimens. More recently, Aliha et al. (2010), used both BD and SCB specimens with straight cracks to investigate experimentally and theoretically fracture trajectory of a rock material obtained from UK (Guiting limestone).

Although, these two specimens have potential abilities for being used as standard test specimens in mixed mode rock fracture studies, a review of literature indicates that the available and reported experimental results using these two configurations are not sufficient for considering them as standard mixed mode I/II fracture test specimens. Indeed, the standardization of the procedures for evaluating the mixed mode fracture toughness of rocks needs extensive experimental data and test results using the suggested test configurations under different mixed mode conditions. However, there are still some unclear aspects when the BD and SCB specimens are used for evaluating the load bearing capacity of the cracked rock masses. For example, the influence of the specimen geometry and loading type on mixed mode crack growth behavior of rock materials have received very little attention by rock fracture researchers. Therefore, in this paper detailed experimental and theoretical fracture toughness studies are presented on a rock material using both BD and SCB specimens and for complete mode mixities ranging from pure mode I to pure mode II. It is shown that the mixed mode fracture toughness data obtained from these two specimens are significantly different. In other words, mixed mode fracture toughness depends on the geometry and loading condition of the tested specimen. The difference that exists in the test data are also justified using a fracture criterion proposed earlier by authors.

2 BD and SCB Specimens

Figure 1 shows the geometry and loading configurations for the BD and SCB specimens schematically. As can be seen from this figure, the BD specimen is a disc of radius R and thickness t that contains a center crack of length $2a$ and subjected to a diametral compressive force P. The SCB specimen is a half disc of radius R and thickness t, which contains an edge crack of length a. It is loaded under three-point bending with bottom support span of $2S$. In both BD and SCB samples, pure mode I takes place when the crack is vertical (or along the direction of applied load). But by changing the crack angle (α), the specimens experience mixed mode I/II loading. Pure mode II is also achieved in each specimen at a specific crack angle which depends on the ratios of a/R and S/R. Therefore, both specimens can provide complete mode mixtures from pure mode I to pure mode II.

The mode I and mode II stress intensity factors (K_I and K_{II}) in the BD and SCB specimens are functions of the specimen geometry, the crack length, the applied load, the

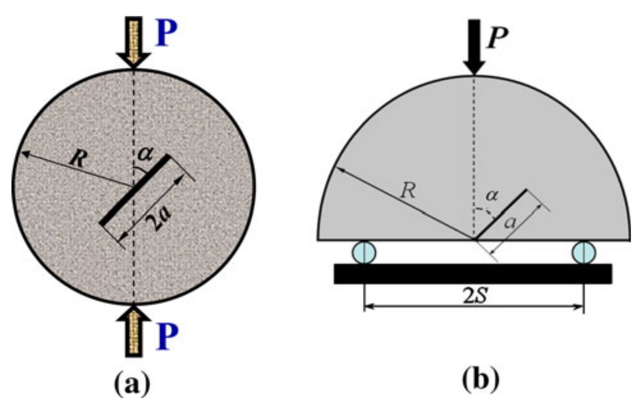


Fig. 1 Geometry and loading conditions for **a** Brazilian disc (BD) and **b** semi-circular bend (SCB) specimens

location of loading supports (2S) and the crack orientation angle. Thus, K_{I} and K_{II} are often written as:

$$K_i = \begin{cases} Y_i \frac{P}{Rt}\sqrt{\frac{a}{\pi}} & \text{for BD specimen} \\ Y_i \frac{P\sqrt{\pi a}}{2Rt} & \text{for SCB specimen} \end{cases} \quad i = \mathrm{I}, \mathrm{II} \tag{1}$$

where, Y_{I} and Y_{II} are the geometry factors corresponding to mode I and mode II, respectively. The dimensionless parameters Y_{I} and Y_{II} in these two specimens have been calculated by Ayatollahi and Aliha (2007b) for different geometry parameters. In the next section, the experimental procedure used for conducting the fracture tests using the BD and SCB specimens are described.

3 Rock Fracture Toughness Experiments

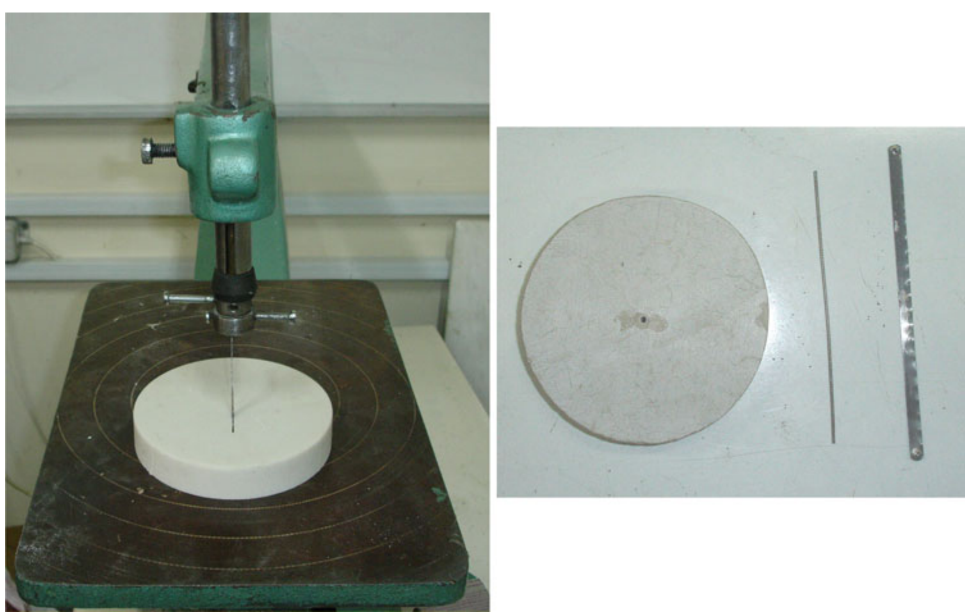
3.1 Specimen Preparation

For preparing the BD and SCB specimens, first a few cylinders with a diameter of 110 mm were cut from a marble block. The selected marble was from Harsin area (a region in the west of Iran) and was homogonous and white in color. Then the rock cylinders were sliced by a rotary diamond saw. The thicknesses of sliced discs were approximately 25 mm. Furthermore, for manufacturing the SCB samples the disc specimens were split in two halves by using a very narrow rotary saw blade. A thin fret saw blade of 0.5 mm thickness was then used for creating the edge cracks in the SCB specimens along the required inclination angles. For introducing the center crack in the BD specimen, first a small hole of 2 mm diameter was drilled in the center of disc and then the same fret saw blade (used for cracking the SCB specimens) was utilized for creating two same line cracks from the edge of center hole (Fig. 2).

3.2 Fracture Tests

For both specimens, the ratio of a/R was chosen as 0.3 and for the SCB specimen, the loading span ratio (S/R), was chosen as 0.43. For the mentioned geometry and loading conditions, pure mode II angles are 27° and 50° for the BD and SCB specimens, respectively (Ayatollahi and Aliha 2007b). Thus, the following crack inclination angles were considered in the experiments: (i) for the BD specimen: α = [0° (pure mode I), 4°, 8°, 12°, 16°, 20°, 24° and 27° (pure mode II)] and (ii) for the SCB specimen: α = [0° (pure mode I), 10°, 20°, 30°, 40°, 43°, 47° and 50° (pure mode II)]. For each crack angle, four BD and four SCB samples were tested. The BD test samples were located between two plates and the SCB specimens were placed inside a three-point bend fixture. Both specimens were loaded by a servo hydraulic compressive test machine with a crosshead speed of 0.5 mm/min. The monotonic loading was increased until the final fracture, and the load–displacement data were recorded during each test. The load–displacement curves for all the samples were linear and the test samples fractured suddenly from the crack tip implying that the tested rock behaved predominantly as a linear elastic material and failed in a brittle manner. Figure 3 shows a typical load–displacement curve for one of the tested samples and Fig. 4 displays the loading setup and the fracture paths for sample BD and SCB specimens.

Fig. 2 Center crack preparation in the BD specimen using very thin fret saw blade



3.3 Experimental Results

The critical stress intensity factors at the onset of fracture were determined for each tested sample using Eq. 1 and the geometry factors Y_I and Y_{II} given in Ayatollahi and Aliha (2007b). The variations of Y_I and Y_{II} versus crack angles (α) extracted from Ayatollahi and Aliha (2007b) are presented in Fig. 5 for the tested BD and SCB specimens.

The mode I fracture toughness (K_{Ic}) was obtained about $1\,\mathrm{MPa}\sqrt{\mathrm{m}}$ from the BD specimens as recommended by ISRM (Fowell 1994). Figure 6 shows the test results obtained for both BD and SCB specimens in a non-dimensional K_{II}/K_{Ic}–K_I/K_{Ic} diagram. According to Fig. 6, the test data are consistent only for predominantly mode I loading conditions (i.e. the horizontal axis in Fig. 6); but not for other combinations of mode I and mode II. It is also seen that the BD mixed mode test results are significantly higher than the SCB test data. This indicates that the mixed mode fracture toughness depends on the geometry and loading conditions of the tested rock specimen.

Also shown in Fig. 6 are the predictions of the conventional fracture criteria i.e. the maximum tangential stress (MTS) criterion (Erdogan and Sih 1963), the minimum strain energy density (SED) criterion (Sih 1974), the maximum energy release rate (G) criterion (Hussain et al. 1974) and the cohesive zone model (CZM) criterion (Gómez et al. 2009). It is seen from Fig. 6 that the experimental results obtained for the SCB specimens are

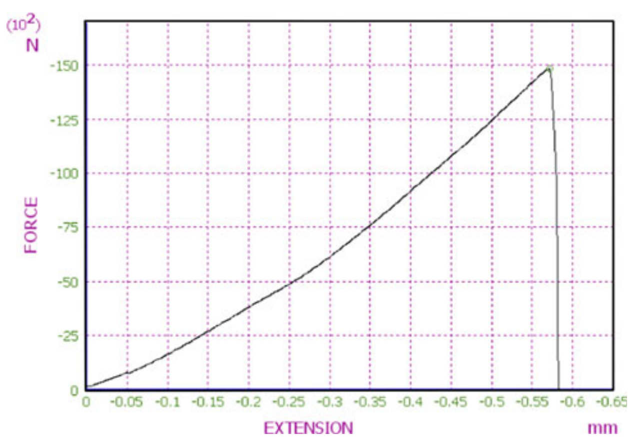


Fig. 3 Typical load–displacement curve for one of the test samples made of Harsin marble

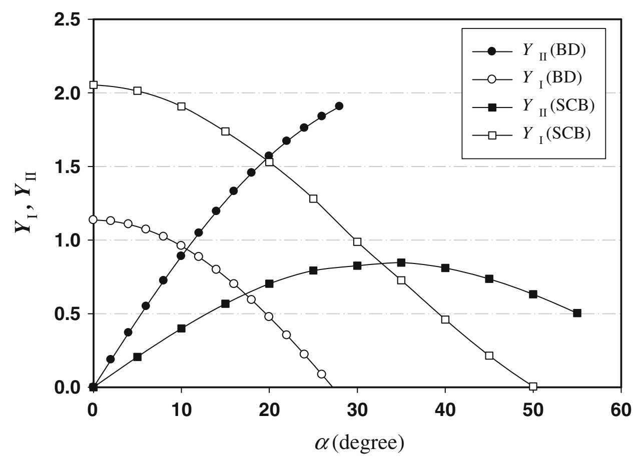


Fig. 5 Mode I and mode II geometry factors (Y_I and Y_{II}) for the tested BD and SCB specimens

Fig. 4 Loading setup and fracture path for one of the tested BD and SCB specimens

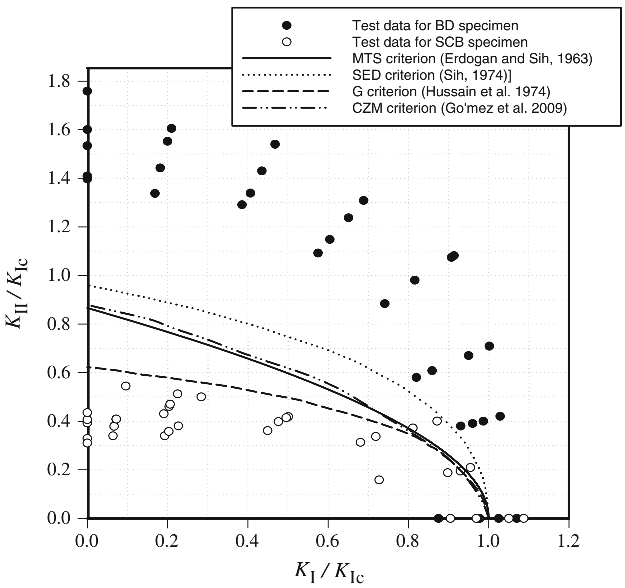


Fig. 6 Mixed mode fracture toughness results obtained for the tested marble using the BD and SCB specimens in comparison with predictions of the conventional fracture criteria

generally lower than the estimates of the conventional fracture criteria (like MTS criterion) especially for predominantly mode II loading conditions and conversely the BD test data are significantly higher than the predictions of these fracture criteria. In other words, the conventional fracture criteria significantly overestimate and underestimate the mixed mode fracture toughness of marble obtained from the SCB and BD specimens, respectively.

4 4 Geometry Dependent Fracture Criterion

The conventional fracture criteria are often derived based only on the singular stress terms which depend on the stress intensity factors and generally provide a fixed curve for predicting the onset of brittle fracture in different cracked specimens. For example, the mixed mode fracture curve of the MTS criterion is obtained from the following equation (Erdogan and Sih 1963):

$$\frac{K_I}{K_{Ic}} \cos^3 \frac{\theta_0}{2} - \frac{3}{2} \frac{K_{II}}{K_{Ic}} \sin \theta_0 \cos \frac{\theta_0}{2} = 1 \quad (2)$$

where θ_0 is the direction of fracture initiation. Since the conventional fracture criteria like the MTS criterion failed to provide acceptable predictions for the mixed mode fracture toughness of the tested BD and SCB specimens, a generalized MTS criterion (called the GMTS criterion) (Smith et al. 2001) was employed in this research.

The GMTS criterion is able to consider the effect of specimen geometry and its loading conditions for predicting the onset of mixed mod fracture, since in addition to the conventional stress intensity factors it takes into account the influence of T-stress, which is dependent on the specimen geometry and the type of loading. Thus as shown later, this criterion can provide individual fracture curves for the BD and SCB test samples. In comparison with the conventional MTS criterion, the GMTS criterion uses a more accurate description of the tangential stress $\sigma_{\theta\theta}$ in front of the crack tip as (Williams 1957):

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] + T \sin^2 \theta + O\left(r^{1/2}\right) \quad (3)$$

where r and θ are the conventional crack tip co-ordinates. T is a non-singular and constant stress term which is independent of the distance from the crack tip, usually called the T-stress. The magnitude of T-stress may vary in a wide range for different test specimens. The higher order terms $O(r^{1/2})$ represent the remaining terms of the series expansion which are negligible near the crack tip.

The GMTS criterion proposes that the crack growth initiates radially from the crack tip along the direction of MTS θ_0. Also the crack extension takes place when the tangential stress $\sigma_{\theta\theta}$ along θ_0 and at a critical distance r_c from the crack tip attains a critical value $\sigma_{\theta\theta c}$. Both r_c and $\sigma_{\theta\theta}$ are assumed to be material constants. According to the GMTS criterion, the direction of fracture initiation angle θ_0 and the onset of mixed mode I/II brittle fracture can be found from (Smith et al. 2001):

$$\left. \frac{\partial \sigma_{\theta\theta}}{\partial \theta} \right|_{\theta=\theta_0} = 0 \quad \Rightarrow \quad [K_I \sin \theta_0 + K_{II}(3\cos \theta_0 - 1)] - \frac{16T}{3} \sqrt{2\pi r_c} \cos \theta_0 \sin \frac{\theta_0}{2} = 0 \quad (4)$$

$$K_{Ic} = \cos \frac{\theta_0}{2} \left[K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} K_{II} \sin \theta_0 \right] + \sqrt{2\pi r_c} T \sin^2 \theta_0. \quad (5)$$

More details about how Eqs. 4 and 5 are derived can be found in Smith et al. (2001). If the effect of T in the above equations is ignored, the GMTS criterion will be identical to the conventional MTS criterion. Based on the GMTS criterion, the negative T-stress in a cracked geometry increases the mixed mode I/II fracture resistance in brittle materials and conversely a positive T-stress decreases it (Smith et al. 2001).

5 Application of GMTS Criterion for BD and SCB Specimens

5.1 T-Stress Data

In order to use the GMTS criterion, the value of T-stress should be known for each test specimen. The T-stresses in the BD and SCB specimens are functions of crack length ratio (a/R), loading span to radius ratio (S/R) and the crack inclination angle (α); and can be written as:

$$T = \begin{cases} \frac{T^* P}{\pi R t (R-a)} & \text{for BD specimen} \\ \frac{T^* P}{2Rt} & \text{for SCB specimen} \end{cases} \quad (6)$$

where T^* is the non-dimensional form of T-stress. Figure 7 shows the variations of T^* with the crack inclination angle in the tested BD specimen (with $a/R = 0.3$) and SCB specimen (with $a/R = 0.3$ and $S/R = 0.43$), as extracted from (Ayatollahi and Aliha 2007b). While its magnitude is always negative in the BD specimen, the T-stress is considerably positive in the SCB specimen and especially for predominantly mode II conditions. Furthermore, the T-stress in both specimens increases noticeably by moving from pure mode I to pure mode II.

5.2 Extraction of Mixed Mode Fracture Curves

In order to estimate the onset of brittle fracture for the BD and SCB specimens using the GMTS criterion, the relevant parameters K_I, K_{II} and T are extracted for different mode mixities from Figs. 5 and 7. Then, by substituting these parameters into Eqs. 4 and 5, the direction of fracture initiation θ_0 and the onset of mixed mode fracture can be determined for any combinations of mode I and mode II in the BD and SCB specimens. Since for any given mode mixity the values of K_I, K_{II} and T corresponding to the BD and SCB specimens differ noticeably, it is expected that two different fracture curves are obtained for these specimens.

However, in order to use Eqs. 4 and 5 the value of r_c (i.e. the critical distance from the crack tip) should also be known. For rock materials, the critical distance can be related to the size of fracture process zone in front of the crack tip. The fracture process zone is developed due to the nucleation and coalescence of microcracks in front of the crack tip. According to a maximum principle stress model suggested by Schmidt (1980) the size of fracture process zone in rock materials can be estimated from:

$$r_c = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 \quad (7)$$

where σ_t is the rock tensile strength. By using the average values of $K_{Ic} = 1\,\text{MPa}\sqrt{\text{m}}$ and $\sigma_t = 7.2$ MPa obtained from our experiments conducted on the Harsin marble (based on the ISRM (1994, 1978) suggested methods), the value of r_c is found as 3 mm for the tested rock. Now, if $r_c = 3$ mm is used in conjunction with the fracture parameters K_I, K_{II} and T, the estimates of GMTS criterion are obtained for the BD and SCB specimens for various mode mixities. Then a curve can be plotted for each set of calculated results. Figure 8 shows the GMTS curves obtained for mixed mode fracture toughness of the two marble specimens. It is seen that the GMTS criterion provides very good estimates for the experimental results obtained from both BD and SCB test specimens.

6 Discussion

Based on the fracture results obtained for the two different test specimens (i.e. BD and SCB) made of the same material (Harsin marble), the mixed mode fracture

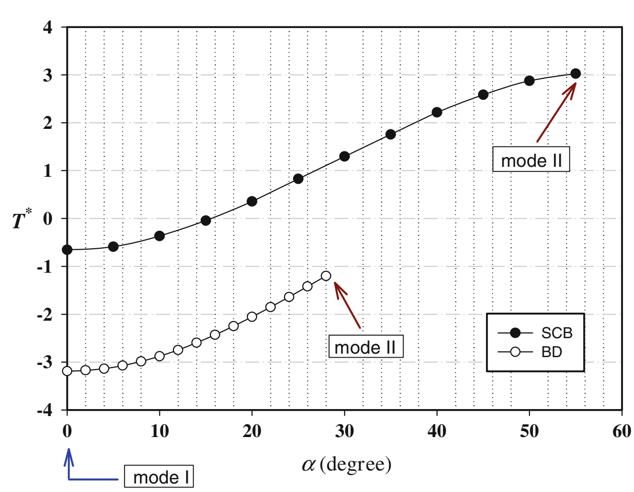


Fig. 7 Variations of non-dimensional T-stress (T^*) with crack inclination angle (α) in the tested BD and SCB specimens

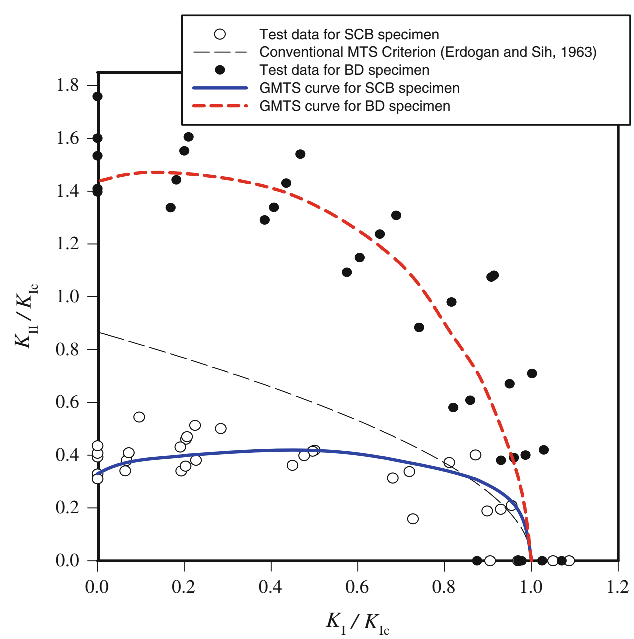


Fig. 8 The curves of GMTS criterion for mixed mode fracture resistance of the tested SCB and BD specimens

resistance was strongly dependent on the type of specimen and its geometry and loading conditions. These differences in the fracture resistance data reveal that for the real rock fracture applications such as mining, tunneling, rock cutting and stability analysis of rock slopes, the effects of geometry and loading conditions of the rock masses must be taken into account for estimating the load bearing capacity of the cracked rock structures.

According to Fig. 8, mixed mode fracture toughness envelopes obtained for the SCB specimen lies considerably lower than the curve of the BD specimen. In other words, for any similar mode mixture (or any given ratio of K_I/K_{II}), the fracture resistance of the BD specimen is significantly higher than the values obtained for the SCB specimen. The variations of mixed mode fracture toughness ratio K_{BD}/K_{SCB} (i.e. the fracture toughness of BD specimen over the SCB specimen) for the tested marble is shown in Fig. 9 for different mode mixtures (M^e), where

$$M^e = \frac{2}{\pi}\arctan(K_I/K_{II}) \quad (8)$$

$$K_{BD} = \left(\sqrt{K_I^2 + K_{II}^2}\right)_{BD} \quad (9)$$

$$K_{SCB} = \left(\sqrt{K_I^2 + K_{II}^2}\right)_{SCB} \quad (10)$$

As it is seen from Fig. 9, the mixed mode fracture toughness ratio (K_{BD}/K_{SCB}) is significantly greater than 1. For example, under pure mode II conditions (i.e. where $M^e = 0$), the fracture resistance of the same marble material tested with the BD specimen is almost 4.2 times the value obtained from the SCB configuration. On the other hand, the mixed mode fracture resistance envelopes

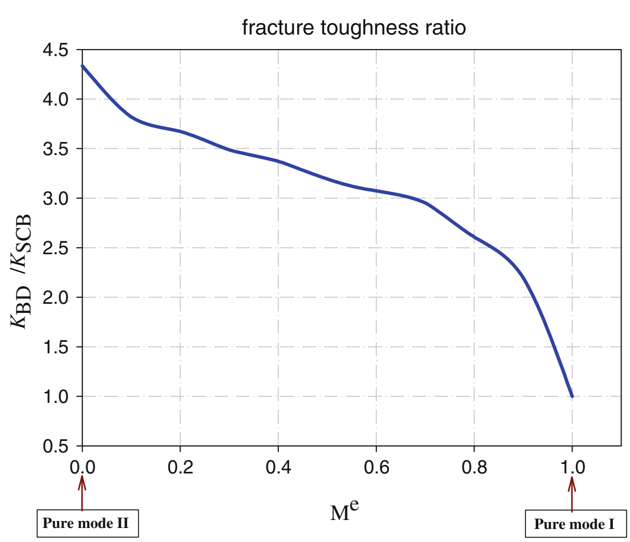


Fig. 9 Variations of mixed mode fracture toughness ratio (K_{BD}/K_{SCB}) obtained from the BD and SCB specimens made of Harsin marble and for different mode mixities

obtained for both BD and SCB specimens did not comply with the theoretical fracture curves proposed by the conventional fracture criteria (Erdogan and Sih 1963; Sih 1974; Hussain et al. 1974; Gómez et al. 2009). While the empirical fracture curve of the BD specimen was significantly higher than the curves related to fracture criteria like MTS (Erdogan and Sih 1963), SED (Sih 1974), G (Hussain et al. 1974) and CZM (Gómez et al. 2009), the empirical fracture envelope obtained for the SCB specimen was generally lower than the curve of the mentioned fracture criteria. However, it was demonstrated that a generalized criterion (GMTS criterion; Smith et al. 2001) was able to provide very good estimates for fracture behavior and load bearing capacity of both BD and SCB specimens. Based on the GMTS criterion, the magnitude and the sign of T-stress can influence significantly the fracture toughness of cracked specimens subjected to mixed mode loading. According to Fig. 7, the T-stress in the tested BD specimen is considerably negative and conversely its magnitude is significantly positive for the tested SCB specimen especially for dominantly mode II loading conditions. Therefore, the noticeable differences that exist between the BD and SCB test data for any combinations of mode I and mode II could be related to the different effects of the T-stress in these specimens. Consequently, decrease in mixed mode fracture resistance of the SCB specimen is mainly due to very large positive T-stresses that exist in the tested SCB specimens. Conversely, the significant negative T-stress in the tested BD specimen is the main reason for increasing the mixed mode fracture resistance of the BD specimen.

6.1 Fracture Initiation Angle

It should also be noted that the crack growth paths and the fracture trajectories of the tested BD and SCB specimens were not the same for mixed mode I/II loading conditions. Figure 10 shows the fracture paths observed for some of the tested BD and SCB specimens. It can be seen in this Figure that except for pure mode I conditions ($\alpha = 0°$) where the crack growth was self-similar, for any other crack angles (α) the fracture paths differed. For example, a comparison between the mode II fracture paths in both BD and SCB specimens indicates that the deviation of crack path from the initial crack line is more pronounced for the SCB geometry. Accordingly for any similar mode mixture, the mixed mode fracture initiation angles in the SCB samples were significantly greater than the observed initiation angles for the BD specimens. This implies that the fracture path and especially the initial crack growth direction is dependent also on the geometry and loading and is not necessarily the same for any given mode mixtures obtained from different test configurations. Therefore,

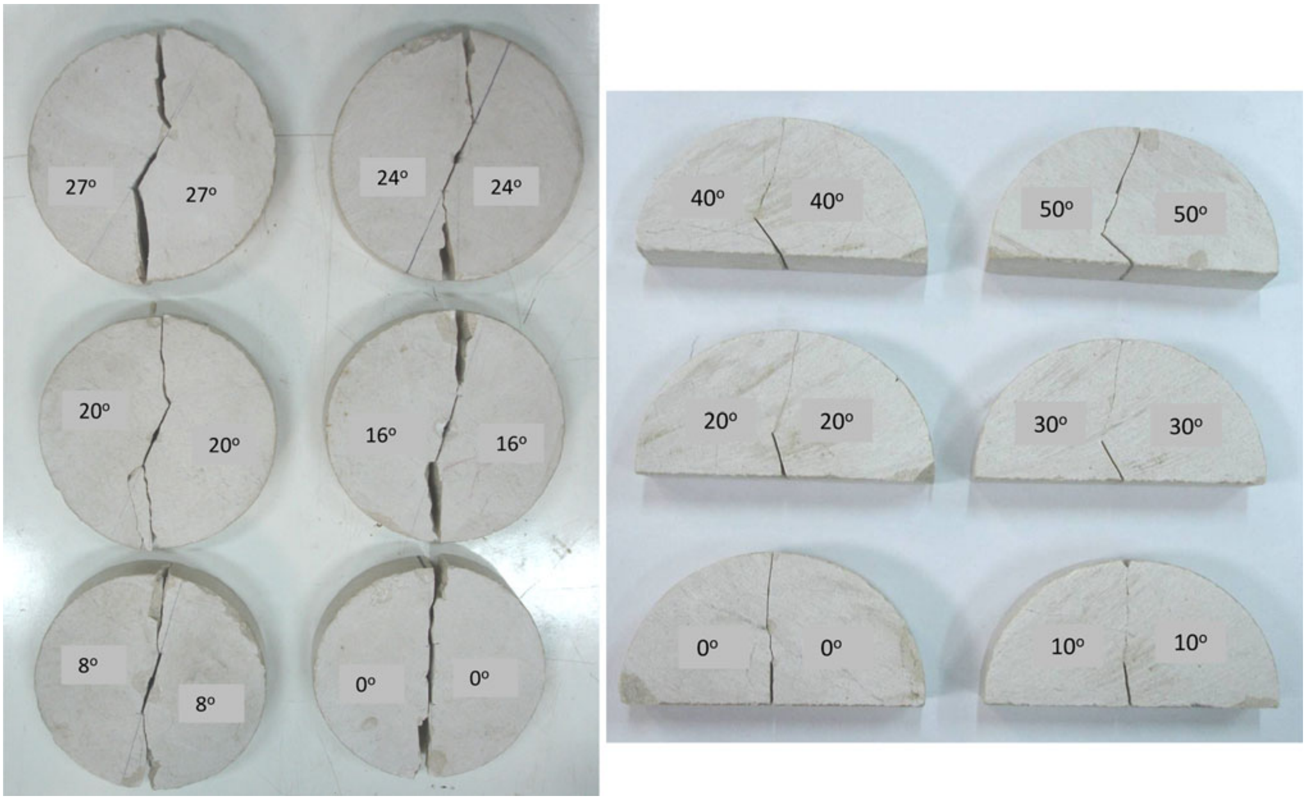


Fig. 10 Mixed mode fracture Paths observed for some of the broken SCB and BD samples made of Harsin marble

the path of crack growth in practical applications like cutting of rock masses or stability of rock slopes, tunnels and mines can be significantly affected by the type of applied loads and geometry of the cracked rock masses. Hence for simulating and predicting the fracture trajectories of cracked rock structures subjected to any combination of tensile-shear loads, the effect of geometry and loading conditions should be considered.

The measured fracture initiation angles in terms of M^e have been presented in Fig. 11 for the tested BD and SCB specimens. Again, it is seen that a great discrepancy exists between the fracture initiation angles obtained from the BD and SCB experiments. For example, for the case of $K_I/K_{II} \approx 1$ (or $M^e \approx 0.5$) it is seen from Fig. 11 that the difference between the fracture initiation angles of the BD and SCB specimens is surprisingly more than 50°. Also shown in Fig. 11 are the predictions of the MTS and GMTS criteria for the fracture initiation angles. While the MTS criterion, fails in predicting both sets of BD and SCB results, it is seen that the fracture initiation angles can be predicted theoretically very well by the GMTS criterion and by considering the same values of K_I, K_{II}, T and r_c (=3 mm) previously used for evaluating the onset of fracture in each specimen. Based on the GMTS criterion, a negative T-stress decreases the fracture initiation angle and conversely a positive T-stress increases it. This phenomenon is in very good agreement with the fracture initiation angles observed for the tested BD and SCB specimens as seen from Fig. 11.

6.2 Upper Bound and Lower Bound Mixed Mode Fracture Curves

A measure of rock mass resistance against crack growth is usually a required data for two general categories of rock mechanics applications. In some situations like drilling, rock cutting, rock mass fragmentation and excavation of tunnels using tunnel boring machines (TBM), the minimum force required for initiating a fracture in rock should be known as an input parameter for designing and selecting rock cutters and tunnel excavation tools. Conversely, in the second category of rock fracture mechanics applications such as stability and integrity assessment of underground rock structures (like mines and subway tunnels) and stability analyses of jointed rock slopes, the maximum load bearing capacity of the field rock masses is needed to predict the onset of failure and collapse in the rock masses.

Since the real jointed rock masses are usually subjected to complex mixed mode deformations, an estimate of the resultant mixed mode fracture resistance should be known. However, according to the findings of this research, mixed mode fracture resistance of rock materials obtained from

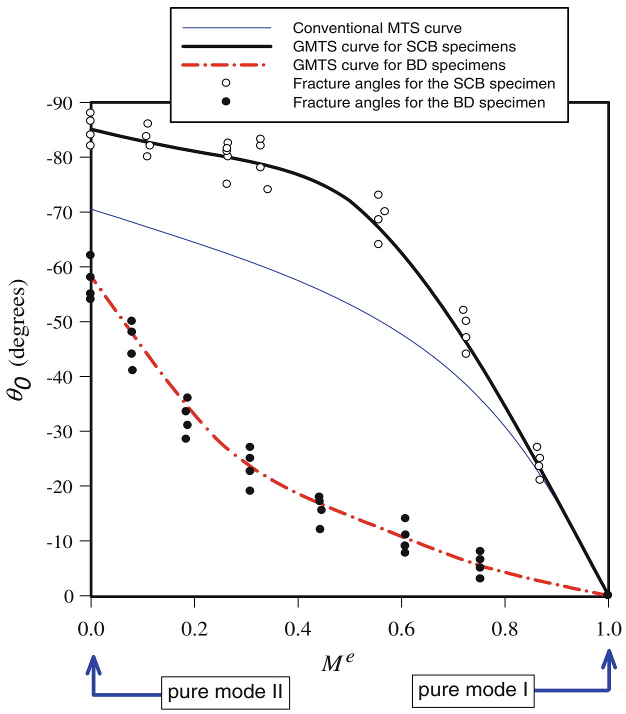


Fig. 11 Predictions of the MTS and GMTS fracture criteria for fracture initiation angles of the BD and SCB specimens made of Harsin marble

laboratory test specimens is not a unique value and depends on the geometry and loading conditions. Consequently, a fracture resistance bound can be obtained for any given mode mixity of rock materials. It was shown that the value of fracture resistance is controlled by the magnitude and the sign of T-stress. A review of literature shows that the absolute values of T-stresses that exist in the BD and SCB specimens are significantly greater than the T-stress of other laboratory test samples used for brittle fracture toughness studies (Kharazi 2008; Aliha et al. 2008; Ayatollahi and Aliha 2009). Thus, the corresponding curves of the T-stress obtained for the BD and SCB specimens can be considered as two typical upper bound and lower bound curves, respectively, for any desired combinations of modes I and II. Since the values of T-stresses in real cracked rocks are not usually known, the mixed mode fracture resistance envelopes obtained from the BD and SCB experiments can be used as upper bound and lower bound values according to the GMTS criterion.

Consequently, for estimating a reasonable value for the load bearing capacity and fracture resistance of cracked rocks subjected to any desired combinations of tensile–shear loads, it is recommended to determine two typical upper bound and lower bound benchmark fracture resistance values by field coring of cylindrical rock samples and then testing some BD and SCB specimens under the required mode mixture. These values can then be used as input design parameters in rock-related projects. For applications like rock cutting and rock fragmentations, the upper bound value obtained from the BD test should be used for evaluating the minimum cutting force and conversely for applications like stability of tunnels and mines where the maximum resistance against the crack growth is required, the lower bound fracture toughness value determined from the SCB test should be used as a safe design value.

7 Conclusions

- The mixed mode fracture resistance results obtained from two test specimens (BD and SCB) made of the same material (Harsin Marble) were strongly dependent on the type of specimen and its geometry and loading conditions.
- The obtained mixed mode fracture toughness envelope for the BD specimen is considerably higher than the corresponding curve of the SCB specimen. Moreover, all the available mixed mode fracture criteria fail to provide acceptable predictions for both BD and SCB test data.
- The differences that were observed between the fracture toughness of BD and SCB specimens for any similar mode mixture was found to be related to the influence of T-stress in the tested BD and SCB specimens.
- The generalized maximum tangential stress (GMTS) criterion (which takes into account the geometry and loading conditions of the test specimen) can provide an individual mixed mode curve for any given test specimen. Consequently, the generalized criterion was able to predict very well the BD and SCB test results.
- The BD mixed mode fracture toughness envelope represents a typical upper bound value for the fracture resistance of a given material because of its very high negative T-stress.
- Conversely, due to very positive T-stresses that exist in the SCB specimen, it is expected that the mixed mode fracture toughness envelope of the SCB specimen would provide a benchmark lower bound curve for estimating the minimum fracture resistance of real cracked rock structures.

References

Aliha MRM, Ashtari R, Ayatollahi MR (2006) Mode I and mode II fracture toughness testing for a coarse grain marble. J Appl Mech Mater 5–6:181–188

Aliha MRM, Ayatollahi MR, Pakzad R (2008) Brittle fracture analysis using a ring shape specimen containing two angled cracks. Int J Fract 153(1):63–68

Aliha MRM, Ayatollahi MR, Smith DJ, Pavier MJ (2010) Geometry and size effects on fracture trajectory in a limestone rock under mixed mode loading. Eng Fract Mech 77:2200–2212

Al-Shayea NA (2005) Crack propagation trajectories for rocks under mixed mode I–II fracture. Eng Geol 81(1):84–97

Awaji H, Sato S (1978) Combined mode fracture toughness measurement by the disc test. J Eng Mater Technol 100:175–182

Ayatollahi MR, Aliha MRM (2007a) Fracture toughness study for a brittle rock subjected to mixed mode I/II loading. Int J Rock Mech Min Sci 44(4):617–624

Ayatollahi MR, Aliha MRM (2007b) Wide range data for crack tip parameters in two disc-type specimens under mixed mode loading. Comput Mater Sci 38(4):660–670

Ayatollahi MR, Aliha MRM (2008) On the use of brazilian disc specimen for calculating mixed mode I–II fracture toughness of rock materials. Eng Fract Mech 75:4631–4641

Ayatollahi MR, Aliha MRM (2009) Analysis of a new specimen for mixed mode fracture tests on brittle materials. Eng Fract Mech 76(11):1563–1573

Chang SH, Lee CI, Jeon S (2002) Measurement of rock fracture toughness under modes I and II and mixed-mode conditions by using disc-type specimen. Eng Geol 66:79–97

Chong KP, Kuruppu MD (1984) New specimen for fracture toughness determination for rock and other materials. Int J Fract 26:59–62

Chong KP, Kuruppu MD, Kuszmual JS (1987) Fracture toughness determination of layered materials. Eng Fract Mech 28(1):43–54

Erdogan F, Sih GC (1963) On the crack extension in plates under plane loading and transverse shear. J Basic Eng Trans ASME 85:519–525

Fowell RJ (1994) The use of the cracked Brazilian disc geometry for rock fracture investigations. Int J Rock Mech Min Sci Geomech Abstr 31(6):571–579

Fowell RJ (1995) ISRM-Suggested methods for determining mode I fracture toughness using cracked chevron notched Brazilian disk (CCNBD) specimens. Int J Rock Mech Min Sci Geomech Abstr 32:57–64

Funatsu T, Seto M, Shimada H, Matsui K, Kuruppu M (2004) Combined effects of increasing temperature and confining pressure on the fracture toughness of clay bearing rocks. Int J Rock Mech Min Sci 41(6):927–938

Gómez FJ, Elices M, Berto F, Lazzarin P (2009) Fracture of U-notched specimens under mixed mode: experimental results and numerical predictions. Eng Fract Mech 76(2):236–249

Hussain MA, Pu SL, Underwood J (1974) Strain energy release rate for a crack under combined mode I and Mode II. Fracture analysis, ASTM STP 560. American Society for Testing and Materials, Philadelphia

ISRM (1978) Suggested methods for determining tensile strength of rock materials. Int J Rock Mech Min Sci Geomech Abstr 15:99–103

Ke CC, Chen CS, Tu CH (2008) Determination of fracture toughness of anisotropic rocks by boundary element method. Rock Mech Rock Eng 41(4):509–538

Khan K, Al-Shayea NA (2000) Effect of specimen geometry and testing method on mixed I–II fracture toughness of a limestone rock from Saudi Arabia. Rock Mech Rock Eng 33(3):179–206

Kharazi B (2008) Mechanical Engineering Department. MSc Thesis, Iran University of Science and Technology, Tehran

Krishnan GR, Zhao XL, Zaman M, Roegiers JC (1998) Fracture toughness of a soft sandstone. Int J Rock Mech Min Sci 35(6):695–710

Lim IL, Johnston IW, Choi SK, Boland JN (1994a) Fracture testing of a soft rock with semi-circular specimens under three-point bending, Part 1:mode I. Int J Rock Mech Min Sci Geomech Abstr 31(3):185–197

Lim IL, Johnston IW, Choi SK, Boland JN (1994b) Fracture testing of a soft rock with semi-circular specimens under three-point bending, Part 2: mixed mode. Int J Rock Mech Min Sci Geomech Abstr 31(3):199–212

Liu HY, Kous Q, Lindqvist PA, Tang CA (2007) Numerical modelling of the heterogeneous rock fracture process using various test techniques. Rock Mech Rock Eng 40(2):107–144

Nasseri MHB, Mohanty B (2008) Fracture toughness anisotropy in granitic rocks. Int J Rock Mech Min Sci 45(2):167–193

Schmidt RA (1980) A microcrack model and its significance to hydraulic fracturing and fracture toughness testing. In: Proceedings of the 21st U.S. rock mechanics symposium, Missouri, Rolla, 27–30 May 1980, pp 581–590

Sih GC (1974) Strain-energy-density factor applied to mixed mode crack problems. Int J Fract 10:305–321

Smith DJ, Ayatollahi MR, Pavier MJ (2001) The role of *T*-stress in brittle fracture for linear elastic materials under mixed mode loading. Fatig Fract Eng Mater Struct 24:137–150

Williams ML (1957) On the stress distribution at the base of a stationary crack. J Appl Mech 24:109–114

Zhao XL, Fowell RJ, Roegiers JC, Xu C (1994) Rock fracture-toughness determination by the Brazilian test. Eng Geol 38(1–2):181–184