Review of Spalart-Allmaras Turbulence Model and its Modifications

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Abstract

Spalart-Allmaras (SA) is a turbulence model for modeling different type of turbulent flows, specifically aerodynamic flows. Different terms of the governing equation of this model (e.g. production, diffusion and destruction), just like the other turbulence models, were modified after 1992, when it was published by Spalart & Allmaras. In this literature review, we will first go through the details of the governing equation of this model and after understanding the role of each term and reasoning behind it, we will discuss the different proposed modifications for this model and concentrate on the one that we have used in my research and discuss how the modification effects the SA model by comparing the results from SA with other turbulence models.

1 Introduction

Computational Fluid Dynamics (CFD) is becoming more and more popular every day, since on one hand the advanced technology is providing desktop computers with huge memory capacity and powerful processors for unbelievably cheap prices, which are capable of solving large two and three dimensional problem numerically just in few days. On the other hand CFD commercial packages, like FLUENT, make performing numerical simulation easier than any other time.

However this easy access to the required tools for doing different type of CFD works, does not mean that anybody can produce correct and physically meaningful results for any problem. The important point for being able to have a successful and meaningful numerical simulation is to have sufficient core knowledge about the numerical model that is being used and being aware of choosing the right parameters and options for the numerical model. It is possible that one can run a simulation successfully, but the results does not make any physical sense or even be totally wrong!

In this paper we will review one of the common turbulence models, called Spalart-Allmaras model, which was basically developed to model aerodynamic flows. At first we will go through the details of each term of the its governing equation and then we will explain different proposed modification for that. In the end we are going to concentrate on one of those modifications, go through details of it and present some results to compare the modified model with other available turbulence models.

2 Spalart-Allmaras Governing Equation

2.1 The Basic Equations

Spalart-Allmaras is a one-equation turbulence model, that has been developed mainly for aerodynamic flows. This model is basically a transport equation for the eddy viscosity. The key idea that Spalart & Allmaras used to develop this equation is very similar to the model of Nee & Kovasznay.

In order to develop a closed system of governing equations for the mean motion of the flow, one should determine the distribution of the Reynolds stress. The various terms in the equation for the Reynolds stress can be identified as convection, diffusion, production and destruction. Both Nee-Kovasznay (NK) [1] and Spalart-Allmaras (SA) [2] made a simple "educated guess" concerning the various terms such as diffusion, production and destruction based on the physics of the turbulent flow. The dependent variable in both of these models is the eddy viscosity, which is related directly to Reynolds stress as follow:

$$\nu_t = \frac{\overline{u'v'}}{du/dy} \tag{1}$$

The eddy viscosity is considered to be governed by a rate equation and therefore it is dependent on the past history through convection and on nearby values through diffusion. Generally any transportable scalar quantity, like eddy viscosity, subject to the conversion laws is transported according to the following equation, which is the basic equation for both NK and SA model:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + (u \cdot \nabla)F = \text{Diffusion} + \text{Production} - \text{Destruction} \tag{2}$$

To construct the full model for a turbulent flow, each of the diffusion, production and destruction terms should be defined carefully. Defining these terms and make them non-dimensional will results to some additional constants and non-dimensional functions in each term. Those constants and functions will be defined based on the available experimental and numerical results. Nee & Kovasznay did a better job on describing how they defined each of these terms in compare to Spalart & Allmaras, therefore we will present both method for defining understanding the reasoning and philosophy behind the above terms and then we will concentrate on SA model and continue with their notation through the rest of this literature review.

2.2 Diffusion

NK consider the general definition for the diffusion of a scalar F by the following equation:

$$\text{Diffusion} = \nabla \cdot \varphi_F \tag{3}$$

where φ_F is the flux of quantity F due to diffusion and is given by $\varphi_F = D_F \nabla F$, where D_F is the coefficient of diffusion. They also consider the total viscosity $n = \epsilon + \nu$ as the transportable quantity, where ϵ is the eddy viscosity and ν is the molecular viscosity. They assume that since the turbulent motion diffuses by itself, it is reasonable to assume the coefficient of diffusion as $D_n = n$, therefore as a result in any case the turbulent Prandtl number and Schmidt number are approximately equal to one $D_n/n \approx 1$. Considering all of the above assumptions the diffusion term based on NK representation will be given as follow:

$$\text{Diffusion} = \nabla \cdot (n\nabla n) \tag{4}$$

SA representation for the diffusion term is a little different in compare to the one by NK. They consider the classical diffusion operator as $\nabla \cdot ([\nu_t/\sigma]\nabla \nu_t)$, where ν_t is the eddy viscosity and σ is a turbulent Prandtl number. Up to this point the two representations are exactly the same, considering that the turbulent Prandtl number is approximately equal to one and the molecular viscosity does not play a major role, since in such turbulent flows (e.g. aerodynamic flows) the energy and information cascades flow only from the larger scales to the small scales.

The difference comes from the point where SA argue that there is no reason why the integral of ν_t should be conserved. They point out the manipulations of two equation models often brings out diffusion terms that are not conservative, for example cross terms between ∇k and $\nabla \epsilon$. Therefore analogues to this explanation they allow a non-conservative diffusion term, involving first derivative of ν_t in their diffusion term and in the end their representation of diffusion term will be given as follow:

$$\text{Diffusion} = \frac{1}{\sigma}[\nabla \cdot (\nu_t \nabla \nu_t) + c_{b2}(\nabla \nu_t)^2] \tag{5}$$

Note that the first term on the right hand side of the above equation is the same as the diffusion representation by NK and the difference is due to the second term only.

2.3 Production

In NK model it has been stated that "The eddy viscosity can be regarded as the ability of a turbulent flow to transport momentum. The ability must be directly related to the general "level of activity" and therefore, to the turbulent energy". Based on this argument, they define the production term analogues to the production of turbulent energy, assuming that production term must increase monotonically with the magnitude of the mean vorticity $|\frac{\partial U}{\partial y}|$ and also with the increasing of the total viscosity. Therefore the most straight forward form for the production term can be represent as follow:

$$\text{production} = A(n - \nu)|\frac{\partial U}{\partial y}| \tag{6}$$

where A is a universal constant, n is the total viscosity and $|\frac{\partial U}{\partial y}|$ is the mean vorticity. Note that if we substitute the definition of n in the above equation, the production term will be linearly dependent on the eddy viscosity and mean vorticity, which is a very similar to the form of production term in SA model.

The main difference between NK and SA model for defining the production term is consideration the appropriate form of mean vorticity. Since NK model the turbulent shear flow, then the form of $|\frac{\partial U}{\partial y}|$ was the best choice for their model, but SA flow of interest is mainly the aerodynamic flows in which turbulence is found only where vorticity is. As a result they used the magnitude of the vorticity $\omega \equiv \sqrt{2\Omega_{ij}\Omega_{ij}}$ where $\Omega_{ij} \equiv \partial U_i/\partial x_j - \partial U_j/\partial x_i$, as the appropriate form of mean vorticity. Have this in mind that other forms of mean vorticity are also available and can be used in formulating the production term. SA has mentioned few of them in their original paper, but they said that they have not yet tasted the model with those proposed terms. The interesting point is that, one of the modifications that was considered for SA model was based on the choice of the mean vorticity in production term, which we will discuss in the later section of this literature review. In the end the production term defined by SA was as following, where c_{b1} is a constant:

$$\text{production} = c_{b1} S \nu_t \tag{7}$$

$$S \equiv \sqrt{2\Omega_{ij}\Omega_{ij}} \qquad \Omega_{ij} = \frac{1}{2}(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}) \tag{8}$$

2.4 Destruction

NK again used the same assumption that the eddy viscosity can be considered as the ability of a turbulent flow to transport momentum and the ability must be directly related to the general "level of activity", therefore to the turbulent energy to construct the destruction term. They argue that the rate of decay of the energy of high-intensity homogeneous turbulence is, to a very rough approximation inversely proportional to the square of the energy as follow:

$$\frac{d\overline{u^2}}{dt} = -\beta(\overline{u^2})^2 \tag{9}$$

Separating the terms of the above equation and integrating both sides, will lead to a decay law as follow:

$$\overline{u^2} \cong 1/\beta t \tag{10}$$

Since the rate equation (2), considering the quantity "F" to be the total viscosity "n", in the absence of the production and diffusion term will reduce to the following form:

$$\frac{\partial n}{\partial t} = -\text{Destruction} \tag{11}$$

based on NK assumption of similar behavior of total viscosity and turbulent energy, it can be considered that $Destruction \cong \beta n^2$. In the end based on the above assumptions and dimensional considerations the final form of destruction term will be as follow:

$$\text{Destruction} = \frac{B}{L^2} n(n - \nu) \tag{12}$$

where B is a universal constant and the characteristic length L has been introduced in order to make B a non-dimensional universal constant. Generally L is a function of y, but in the region of the outer edge of a turbulent flow L can be considered to be equal δ, where δ is the boundary layer thickness. However closer to the wall $L = y$ turn out to be a good approximation. The dependence of the destruction term on distance from the wall turns out to be quite necessary to account for the high rate of dissipation in the proximity of solid boundaries, where the maximum dimension of the dissipating eddies in the direction perpendicular to the flow must be the same as the distance from the wall.

Representation of SA for the destruction term is close to the way that NK present their destruction term. The only difference is that SA defined a non-dimensional function beside the constant in that term. SA argue that in a boundary layer the blocking effect of a wall is felt at a distance through the pressure term, which acts as the main destruction term for Reynolds shear stress, therefore based on this assumption and dimensional analysis, the first form of destruction term would be $-c_{w1}(\nu_t/d)^2$, where d is the distance to the wall and c_{w1} is a constant. Tests show that the model, on one hand when equipped with the destruction term, can produce an accurate log layer. On the other hand it produces too low a skin friction coefficient in a flat plate boundary layer. This shows that the destruction term as formulated, decays too slowly in the outer region of the boundary layer. To overcome this problem SA multiplied the first form of the destruction term by a non-dimensional function f_w which equals to 1 in log layer. In the end the final form of the destruction term will be as follow:

$$\text{Destruction} = -c_{w1} f_w (\frac{\nu_t}{d})^2 \tag{13}$$

From now on we will only discuss the SA model and stick to the corresponding notations that we introduced so far. In the next section we will talk about the calibration that SA did to find the best values for the defined constants and also the way that they defined their non-dimensionalized functions.

3 Calibration of Spalart-Allmaras Model

As it has been seen in the previous sections, each term in the main equation of SA model includes constants and non-dimensional functions. The procedure for evaluating and defining the appropriate form for these constants and non-dimensional functions respectively is called calibration. In the process of calibration, constants and functions will be defined with help of experimental and numerical results of the type of a flow that should be modeled. The basic assumptions that we talked about in previous sections are also helpful to recognize the range of constants.

SA calibrate the free shear flow version of the model by requiring correct levels of shear stress in two dimensional mixing layers and wakes. Appropriate values for the peak shear stress were considered to be $0.01(\Delta U)^2$ and $0.06(\Delta U)^2$ for the mixing layer and wake respectively, where ΔU is the peak velocity difference. This gave SA two conditions for three free constants c_{b1}, σ, and c_{b2}, and leaves a one dimensional family of solutions which is shown in Fig. 1 parametrized by the Prandtl number σ.

Based on the above results and observations in the numerical simulation results, SA suggested the

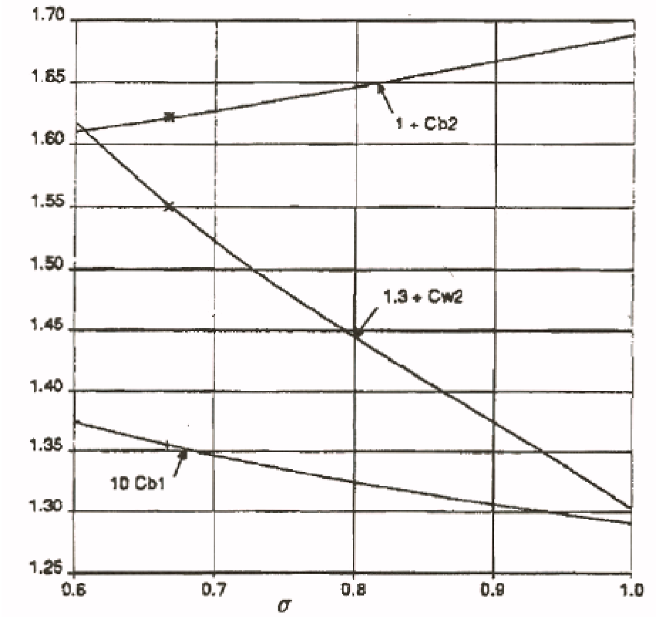

Figure 1: Calibrated model constants.

following values for constants:

$$\sigma = 2/3 \qquad c_{b1} = 0.1355 \qquad c_{b2} = 0.6220 \qquad (1 + c_{b2})/\sigma \approx 2.4 \tag{14}$$

Now the only remained constant and non-dimensional function to be defined is the one in the definition of destruction term. SA argued that in a classical log layer, where with friction velocity u_τ and turbulent kinetic energy k, the strain rate tensor and eddy viscosity are defined as $S = u_\tau/(kd)$ and $v_t = u_\tau kd$ respectively, consideration of equilibrium between the production, diffusion and the destruction term is possible, provided that the constant c_{w1} in the destruction term is defined as function of other constants as follow:

$$c_{w1} = c_{b1}/k^2 + (1 + c_{b2})/\sigma \tag{15}$$

SA defined the non-dimensional function f_w inspired by algebraic models, in which the mixing length plays a major role near the walls. This length scale is defined by $l \equiv \sqrt{\nu_t/S}$ and square of l/kd was used to make the following non-dimensionalization:

$$r \equiv \frac{\nu_t}{Sk^2d^2} \tag{16}$$

Note that both r and f_w equal to one in the log layer and decrease in the outer region. In the end a satisfactory form for the non-dimensional function f_w for SA has been defined as follow:

$$f_w(r) = g\left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6}\right]^{1/6}, \qquad g = r + c_{w2}(r^6 - r) \tag{17}$$

The above function is shown in Fig. 2. The results are most sensitive to the slope of f_w at $r = 1$, which is controlled by c_{w3}. The step from g to f_w works as a limiter that prevents large value of f_w, which could be problematic for the numerical simulations and give an undeserved importance to the fact that S may vanish. The region $r > 1$ is exercised only in adverse pressure gradients, and then rarely beyond $r = 1.1$. Having $f_w(0) = 0$ is not essential, since in free shear flows the destruction term vanishes on account of the d^2 in its denominator. According to calibration reasonable values for c_{w3} and c_{w2} is 2 and 0.3 respectively.

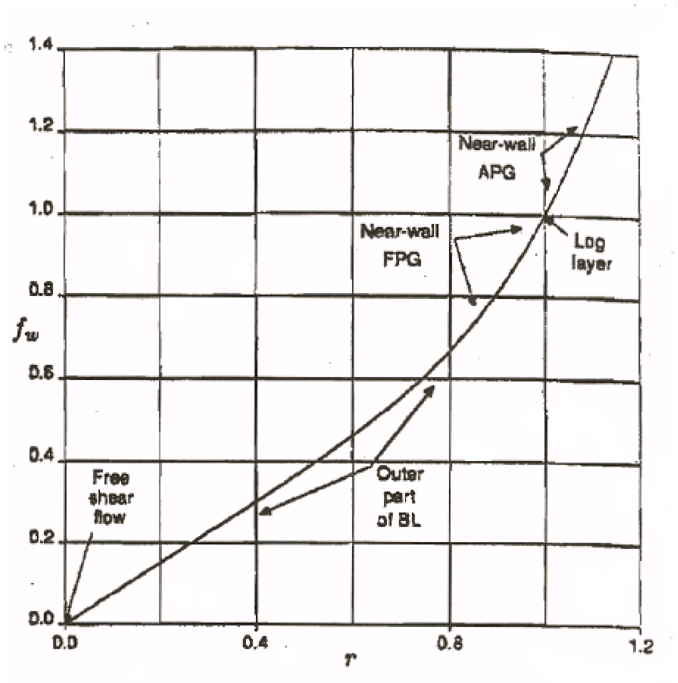

Figure 2: f_w function involved in the destruction term.

4 Final Form of Spalart-Allmaras Model

For the buffer layer and viscous sublayer, SA introduced two new variables $\tilde{v}$, which is equal to v_t except in the viscous range and $X \equiv \tilde{v}/\nu$. They followed Baldwin and Barth[3] in choosing a transported quantity $\tilde{v}$, which behaves linearly near the wall. This is beneficial numerical solutions, since $\tilde{v}$ is easier to resolve than U itself, in contrast with other variables like dissipation rate ϵ or turbulent kinetic energy k. Therefore SA model doesn't require a finer grid in compare to $k - \omega$ or $k - \epsilon$ models. In order to arrive at this SA, considered the classical log layer and devise near-wall "damping functions" that are compatible with know results. These functions are different than f_w which was a near wall inviscid destruction term.

As it was mentioned before the reason for introducing new variable $\tilde{v}$, was that the eddy viscosity v_t equals kyu_τ in the log layer but not in the buffer layer, therefore $\tilde{v}$ was defined so that it equals to kyu_τ all the way to the wall. This result to the following equation:

$$v_t = \tilde{v} * f_{v1}, \qquad f_{v1} = \frac{X^3}{X^3 + c_{v1}^3} \tag{18}$$

SA state that f_{v1} function has been borrowed from Mellor and Herring and the appropriate value for c_{v1} was chosen to be equal to 7.1.

Considering $\tilde{v}$ as a new variable, affects the definition of production term by changing the way that scalar norm of the strain rate tensor S was defined. The new definition for S will be denoted by $\tilde{S}$ and is defined as follow:

$$\tilde{S} \equiv S + \frac{\tilde{v}}{k^2 d^2} f_{v2}, \qquad f_{v2} = 1 - \frac{X}{1 + X f_{v2}} \tag{19}$$

where f_{v2} has been constructed, just like f_{v1}, so that $\tilde{S}$ would maintain its linear behavior in log layer all the way to the wall. $\tilde{S}$ is singular at the wall, but as long as $\tilde{v}$ would be be zero there (at $d = 0$), therefore the production term is well behaved. The other quantities involved in the "inviscid" model will be defined in terms of $\tilde{v}$ instead of v_t, for example $r \equiv \tilde{v}/(\tilde{S}k^2d^2)$. In the end SA insert the molecular viscosity in a convenient place in order pay little attention to a factor of σ and the final form of the transport equation and basically governing equation of Spalart-Allmaras will be as follow:

$$\frac{D\tilde{v}}{Dt} = c_{b1}\tilde{S}\tilde{v} + \frac{1}{\sigma}\left[\nabla \cdot ((\nu + \tilde{v})\nabla\tilde{v}) + c_{b2}(\nabla\tilde{v})^2\right] - c_{w1} f_w \left[\frac{\tilde{v}}{d}\right]^2 \tag{20}$$

5 Modifications for Spalart-Allmaras Model

In the very last section of the original paper by Spalart and Allmaras[2], they talked about the potential modifications and alternative options that can be considered for each of the terms in the one equation that governs their model. For example, they stated that there is a modification for the non-dimensional function f_w in the region $r > 1$ or there is the choice of S, between the vorticity, the strain rate, or another scalar norm of deformation. These comments state that, the values of constants or definition of the non-dimensional functions are not universal and can be modified in future. Sometimes it might turn out that the evaluated constant or defined non-dimensional function is under- or overproducing a specific term in the turbulent flow. This can be regarded as the nature of turbulent modeling and what makes this field of study interesting and attractive for scientists to learn more about different aspects of this field of study and try to model this chaotic flows as close to the real world and possible. In the next two sections we will first briefly explain proposed modifications and second concentrate and discuss the works that has been done recently to look at the modification of variable S in the production term, which turn out to be a problem specially when SA model is being used for modeling highly vortical flows (e.g. wake of a wind or tidal turbine).

In one of our recent presentations at American Physical Society conference[4], we received a comment on the turbulent model that we were using for modeling the turbulent wake of the NREL phase VI wind turbine. The comment basically was that Spalart-Allmaras, the chosen turbulence model in our work, is not capable of modeling highly vortical flows accurately. Although at that time we had validated our numerical simulations against the famous report by NREL, which was about experimental results of testing the NREL

Phase VI turbine in a NASA Ames wind tunnel, we found that comment very interesting and start doing research about it to find out whether this comment is valid or not? During our research we found a paper by Potsdam & Pulliam[5], which was presented at the AHL Specialist's Conference on Aeromechanics that was addressing the problem that non-modified SA model has with highly vortical flows. They stated in their paper that the typical dependence of the eddy viscosity production term on vorticity is problematic for highly vortical flows. They claim that the eddy viscosity can be drastically over-produced in the vortex core with non-modified SA model or any other model in which the production term depends on vorticity solely. In order to ameliorate this and other difficulties resulted from this over-predication, Potsdam & Pulliam proposed five different modifications for this type of turbulent flows and ran different type of simulations for different cases of highly vortical flows to study the effect of each modification. Going through the details of each of these modifications and modelings is beyond the scope of this literature review, therefore we will just briefly explain each of them in the following section and then concentrate on the one, that we used for our simulations and work, in the section after that and in the end show how that particular modification modified SA model in compare to other turbulence models.

5.1 Proposed Modifications by Potsdam & Pulliam

Following is a brief explanation of existing turbulence models and modifications from Potsdam & Pulliam paper to include the effect of rotation in highly vortical flows and account for this non-physical dependence:

1- Turbulence models which explicitly include rotation and curvature terms do not suffer from the over-production of eddy viscosity. This includes the full Reynolds Stress turbulence models (RSM). They are however, still quite expensive, and yet to fully show improved accuracy.

2- Rotation and curvature have been developed for application to a range of turbulence models. Spalart proposes an empirical alteration to account for rotation and curvature by introducing additional Galilean invariant, higher derivative terms[6].

3- A simpler correction has been proposed by Dacles-Mariani and was originally implemented for the production term, P, of the Baldwin-Barth model as follow:

$$P = c_1 v R_t S \tag{21}$$

where c_1 is a constant, v is the laminar viscosity, R_t is the turbulent Reynolds number and S is a scalar measure of the deformation tensor. This modification is very similar to the one that we used and validated in our research against experimental data form NREL and other turbulence models. We will go to details of this modification in the next section.

4- A simple alternative is to turn off the production terms in the off-body grid wake region. This is typically only possible in multi-block or overset mesh systems. Based on this assumption the turbulence that is generated elsewhere can still convect, dissipate, and diffuse, but it can not be produced. This modification has some physical basis, as experimental studies on wing tip vortices have found reduced turbulence levels and analytical studies indicate laminar diffusion mechanisms due to the solid body rotation in the vortex core.

5- An even simpler ad hoc modification includes completely turning off the turbulence model in the wake such that the eddy viscosity remains at its freestream value. The freestream value of eddy viscosity sets the boundary condition to any near-body grid blocks where the turbulence model is activated.

From my personal point of view the first proposed modification is the most expensive one and it is expected to work very well, but since computational time and expenses are two major factor in CFD works, it will not be the first choice to go with. The fourth and fifth modifications will not be a good choice for modeling turbulent flows at all, since these modification partly or completely ignore the nature of the flow, which is being turbulent. Therefore a good choice would be either the second or third one, since they will consider modifications to the model that we are using, instead solving the flow directly, which is very expensive or simplify it the flow significantly by ignoring the nature of the flow. In the following section we go through the details of the third modification proposed here.

5.2 Modification of Production Term in Spalart-Allmaras Turbulence Model

As it has been mentioned before the idea for the modification that we used in our work is the same as the third modification discussed by Potsdam & Pulliam. The key point is to define the scalar measure of the deformation tensor, S, in the production term in a way that it will not only depends on vorticity. As a recall the defined production term for SA model was as follow:

$$\text{Production} = c_{b1}\tilde{S}\tilde{v}, \qquad \text{where} \qquad \tilde{S} \equiv S + \frac{\tilde{v}}{k^2 d^2} f_{v2}, \quad f_{v2} = 1 - \frac{X}{1 + x f_{v2}} \tag{22}$$

In this modification scalar measure of the deformation tensor, S, will be defined as function of both vorticity and strain rate as follow:

$$S \equiv |\Omega_{ij}| + C_{\text{prod}} \min(0, |S_{ij}| - |\Omega_{ij}|) \tag{23}$$

$$C_{\text{prod}} = 2.0, \qquad |\Omega_{ij}| \equiv \sqrt{2\Omega_{ij}\Omega_{ij}}, \qquad |S_{ij}| \equiv \sqrt{2S_{ij}S_{ij}} \tag{24}$$

$$\Omega_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}) \quad S_{ij} = \frac{1}{2}(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}) \tag{25}$$

Form equation (24) it can be seen that in regions where the vorticity is the dominant term and it exceeds strain, such as tip vortices, the production term will be reduced. In the regions where strain and vorticity are of similar magnitude, such as boundary and sheer layers, there is no effect as the correction term goes to zero.

5.3 Results and Conclusions

Figures three and four on the next page show results of numerical modeling of NREL phase VI turbine using modified version of Spalart-Allmaras and $k - \omega$ turbulence model. All the boundary conditions, domain grid size and etc. are the same for both cases and the only difference is the turbulence model that has been used. These colorful plots are showing the axial velocity of the flow, normalized by the inlet velocity equal to $6.8(\frac{m}{sec})$ on a X-cut plane along the wind tunnel. Turbine blade is at position $\frac{Y}{R} = 0$ and velocity profiles are shown on Y-cuts planes along the wind tunnel. Fig. 3 shows the case of SA turbulence model, whereas Fig. 4 shows the case of $k - \omega$ model with velocity profiles from SA superimposed in blue on it. As it can be seen, both cases models near wake (up to 5 radius downstream of the blade) very similar to each other. Velocity profiles at each section are in a very good agreement with each other except near the hub of the turbine and the edge of the turbine wake. Note that we are not modeling the hub at all. We are considering an empty volume starting from inlet and go through the wind tunnel to the outlet and the boundary condition there is free sheer wall. The main conclusion is that the modified SA model is doing a very good job, especially at the near wake where the vorticity is dominating strain rate, which means that the modification is taking a good care of overproduction of the eddy viscosity. One more difference between the two results is the obvious change of both background velocity contour as well as velocity profiles at the far wake (after 5 radius downstream). This can be linked to the decay term of the SA model. As it can be seen the flow modeled by $k - \omega$ is decaying much faster in compare to the one modeled with SA. By looking at Fig. 4, one can say that velocity profiles from $k - \omega$ model are recovering to the shape of the velocity profile at the inlet in a much faster rate in compare to the one from SA model. Unfortunately based on limited data that we have, we can not say which model is doing a better job in modeling the far wake. For sure one model is doing a better job in compare to the other one, but the only conclusion that we can come up with, is that the dissipation rate in $k - \omega$ model is much faster in compare to SA model.

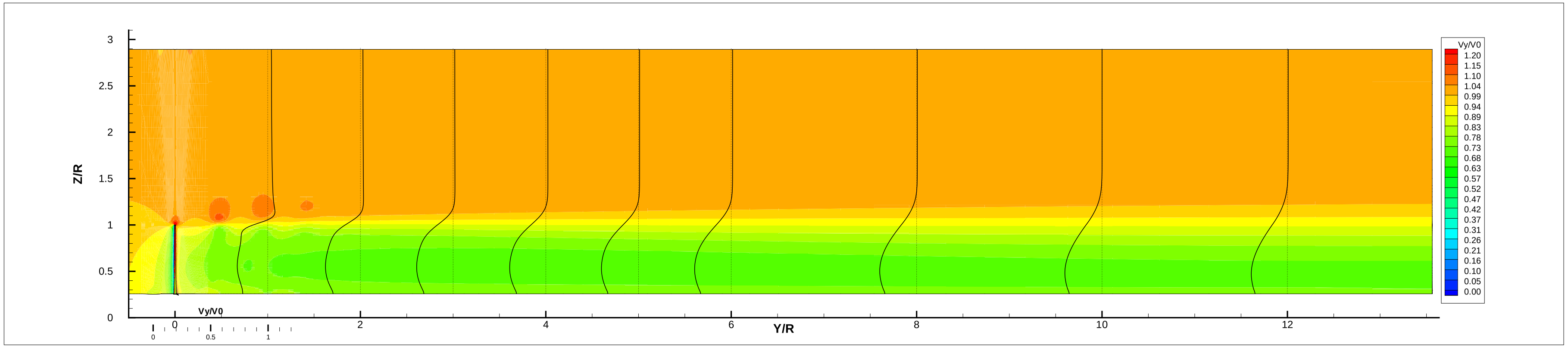

Figure 3: Contours of dimensionless axial velocity, plane $x = 0$, SA model

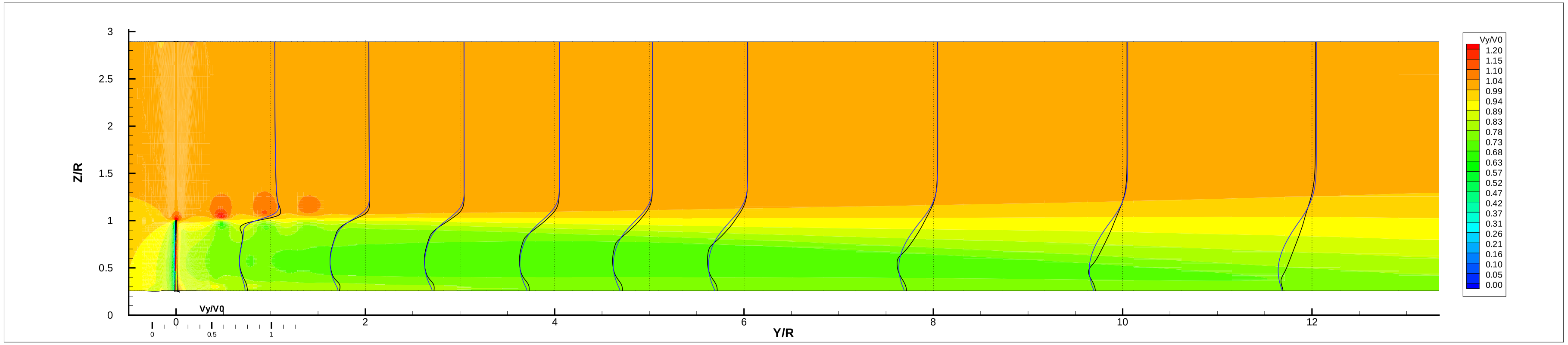

Figure 4: Contours of dimensionless axial velocity, plane $x = 0$, $k - \omega$ model

6 Criticisms

The main focus of this literature review was on the original paper by Spalart & Allmaras[2] and paper by Potsdam & Pulliam [5]. The main problem with SA paper was in giving a clear explanation for the different terms of the governing equation. Therefore, based on professor Riley's advise, we start working with the paper by Nee & Kovasznay[1] who did an excellent job in explaining those terms from fundamental equations with reasoning behind constructing each term of the equation.

There were two problems with Potsdam & Pulliam paper. First one was changing the notations and also form of the SA governing equation, although they referred readers to the original paper by SA[2]. The other problem was a wrong citation for the third proposed modification. They cited the paper by Dacles-Mariani and claimed that those were the people who originally implemented that modification of the production term in SA transport equation, but after referring to that paper, I found out that they are just using the results from a private communication between Spalart and Boeing company! However, except these minor points, both of the papers did a very good job in explaining the philosophy of the work and guiding reader toward the right way for understanding the key idea of the paper.

7 Future Work

During this literature review there were two points that I could not find an exact answer for them because of lack of time. Therefore I think that they can be a good choice for future work to make this literature review more complete.

The first point was the definition of the second term in the the diffusion term, $\frac{1}{\sigma}\left[c_{b2}(\nabla\tilde{v})^2\right]$, in the main equation. The hint for understanding this term is the following statement in SA paper "However there is no reason why the integral of v_t should be conserved. Manipulations of two-equation models often bring out diffusion terms that are not conservative, for instance cross terms between ∇k and $\nabla epsilon$". Referring to Baldwin & Barth paper[3] will give the readers an idea of the manipulations of two-equation models and spending some time on doing the basic manipulations will help the reader to come up with this term and find out about the details and meaning of it.

The other point was the obvious difference in both colorful velocity contours and velocity profiles in results from NREL phase VI turbine simulations at the far wake of the turbine. Finding out about the reason behind this differences and that which model is modeling the flow closer to the real case would be a great idea, which needs the detail knowledge about they way that $k-\omega$ model works.

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