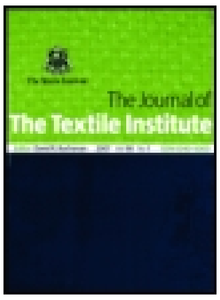


Journal of The Textile Institute
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tjti20

# 24—ANALYSIS OF THE DRY-RELAXED KNITTED-LOOP CONFIGURATION: PART I: TWO-DIMENSIONAL ANALYSIS

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Published online: 03 Nov 2008.

Vol. 58 No. 8 AUGUST, 1967

JOURNAL
*of the*
TEXTILE INSTITUTE



# 24—ANALYSIS OF THE DRY-RELAXED KNITTED-LOOP CONFIGURATION
## PART I: TWO-DIMENSIONAL ANALYSIS

By R. POSTLE and D. L. MUNDEN

Models of the relaxed plain-knit structure proposed by previous workers suffer from the disadvantage of being based on the assumption of some geometrical shape for the knitted loop and are not derived from equilibrium considerations of the forces and couples applied to one loop by its neighbours. In the work described in this paper, the dry-relaxed knitted-loop configuration is considered as a function of a system of localized forces and couples acting on the loop at the interlocking points in the fabric. The assumed system of forces and couples is derived from physical considerations of equilibrium and loop symmetry.

In this paper, the loop is assumed to be plane (or two-dimensional), and its shape is analysed as a function of the forces acting in the plane of the fabric. The resultant loop configuration is discussed in relation to experimental work previously done on relaxed plain-knit fabrics. The analysis is to be extended to three dimensions in Part II.



## 1. INTRODUCTION

### (a) Previous Experimental Work

Previous work on the physical and geometrical properties of the knitted structure can be traced back to Tompkins[1], who, in 1914, made the earliest recorded attempt to rationalize the properties of knitted fabrics. A major advance in this field occurred when Doyle[2] and Munden[3] reported that, for an extremely wide range of relaxed plain-knit fabrics, the fabric dimensions are completely determined by the knitted-loop length. The following relations were found to apply:

$$c = k_c/l, \qquad \text{......(1)}$$

$$w = k_w/l, \qquad \text{......(2)}$$

$$S = c \times w = k_s/l^2, \qquad \text{......(3)}$$

and

$$c/w = k_c/k_w = 1{\cdot}3, \qquad \text{......(4)}$$

where $c$ is the number of courses per unit length, $w$ the number of wales per unit length, $S$ the stitch density or number of loops per unit area, and $l$ the loop length, and $k_c$, $k_w$, and $k_s$ are constants such that

$$k_s = k_c \times k_w$$

and are termed the fabric dimensional parameters throughout the present series of papers.

The above relations apply only for plain-knit fabrics that are in their relaxed state. During the knitting process, strains are imparted to the fabric as it comes off the knitting machine, and subsequent recovery from these strains occurs until the fabric has attained its relaxed or stable-equilibrium state. Munden[3] defines two basic relaxed states: the dry-relaxed state and the wet-relaxed state. The yarn in a dry-relaxed fabric is in a condition of stress and when unravelled from the fabric tends to return to its natural stress-free or straight configuration. The stresses in the loop are largely released during wet relaxation. Setting then takes place during drying, and as a result the yarn retains its looped configuration when unravelled from a wet-relaxed fabric.

Equations (1)–(4) have been found by Munden[3] to apply for each of these relaxed states. For a wide range of plain-knit wool fabrics, he obtained the average numerical values of the fabric dimensional parameters given in Table I. For other plain-knit fabrics, similar values were obtained to those found for wool fabrics, except that for hydrophobic yarns the numerical values in the wet-relaxed state are the same as those in the dry-relaxed state.

Munden's interpretation of these observations is that, on relaxation, the knitted loop tends to take up a definite equilibrium configuration, which is independent of the tightness of construction of the fabric and of the physical properties of the yarn. This loop configuration is the same for all plain-knit fabrics in a given state of relaxation, and it governs the numerical values of the fabric dimensional parameters. The values of these parameters are constant, and, once they are determined, the fabric dimensions are functions only of the knitted-loop length.

**Table I**
**Numerical Values of the Fabric Dimensional Parameters for Plain-knit Wool Fabrics**

| | Dry-relaxed | Wet-relaxed |
|---|---|---|
| $k_c$ | 5·0 | 5·3 |
| $k_w$ | 3·8 | 4·1 |
| $k_s$ | 19·0 | 21·6 |
| $k_c/k_w$ | 1·3 | 1·3 |

Although the dimensions of relaxed knitted fabrics were found to be dependent only on the knitted-loop length and are independent of the yarn count, it is expected that other physical properties (such as the mechanical properties of the fabric, fabric stiffness, and pilling) will depend on the tightness of construction. Munden[4] has pointed out that the ratio of natural yarn diameter to the knitted-loop length can be used to specify the tightness of construction of a knitted fabric. From the proportionality between yarn diameter and count (or linear density), the parameter

$$K = \frac{\sqrt{T}}{l}, \qquad \ldots\ldots(5)$$

where $T$ is the yarn linear density in tex and $l$ is in cm, can be used as a measure of fabric tightness. This parameter, termed the 'cover factor' by several workers, has a value between 13 and 15 for plain-knit worsted fabrics of average tightness of construction.

### (b) Previous Analyses

Chamberlain[5], Shinn[6], Peirce[7], Leaf and Glaskin[8], Vekassy[9], and Leaf[10] have all made separate attempts to define geometrically the configuration of the relaxed knitted loop. None of these attempts to obtain a consistent geometry of the relaxed plain-knit structure is entirely satisfactory. These models are based on the assumption of some geometrical shape for the knitted loop, the parameters of which are then fitted to give the experimentally observed values of the fabric dimensional parameters. None of the above-mentioned models of the knitted loop is derived from considerations of equilibrium between the complex system of forces and couples applied to one loop by its neighbours.

For Leaf's geometry, the forces and couples were found[11] that must be applied at the interlocking points in the fabric to hold the loop in the assumed geometrical configuration. The main conclusion drawn from this work was that the yarn is held in the shape of Leaf's model primarily by the action of the couples applied to it at the points of loop-interlocking, and that the forces applied there play a relatively minor part. However, it will be shown in section 2 of the present paper (Fig. 4) that, because of symmetry, the largest component of the applied couple required to hold a yarn in the shape suggested by Leaf's model cannot exist in the knitted structure.

In 1958, Leaf[12] in an attempt to explain the experimental observations, showed that, when a homogeneous elastic rod is bent into a loop in one plane by bringing its two ends together and parallel, it takes up a definite configuration, which is independent of the length of the loop and the physical properties of the rod (provided that the rod is not plastically deformed by the bending). Nutting and Leaf[13] have extended this idea by investigating the conditions under which two straight elastic rods would take up similar configurations when they were deformed by forces and couples applied at their ends so that a three-dimensional loop was formed. By using the analysis of Love[14], it was found that, after deformation, the rods would take up a similar configuration in space only if they had the same ratio of flexural rigidity to torsional rigidity. However, the assumed conditions of application of the forces and couples are again not derived from equilibrium considerations within the knitted structure.

In the present work, the dry-relaxed knitted-loop configuration is determined as a function of the forces and couples acting on the loop. A similar approach to this has been employed by Peirce[15] and Olofsson[16] to determine the yarn-crimp configuration in a woven structure. The most satisfactory approach to the problem for the knitted structure would be to consider the general case of a yarn bent and twisted in three dimensions by forces applied over the regions of loop-interlocking, and to find the stable-equilibrium configuration of the yarn under these conditions. An approximate treatment is given in the present paper, in which the loop is considered initially to be a two-dimensional structure and its configuration is analysed by considering the force and couple components acting in the plane of the fabric. This analysis is then extended to three dimensions in Part II by considering the components acting perpendicular to the plane of the fabric. The yarn is assumed to behave as a homogeneous rod of perfectly elastic material and having a natural stress-free straight configuration. It follows that there can be no plastic deformation of the yarn when it is bent to form the loop; this assumption is reasonable for all knitted structures except those produced from yarns such as continuous-filament nylon, which are subjected to plastic deformation by the knitting action. It follows also that we exclude from consideration fabrics that have undergone setting treatments after knitting. The analysis thus deals only with knitted fabrics in their dry-relaxed state (where the natural stress-free configuration of the yarn is straight.)

## 2. FORCES ACTING ON THE LOOP IN THE PLANE OF THE FABRIC

A knitted fabric consists of a series of interlocking loops, which produce reaction forces along the regions of contact. Consider the plane loop whose central axis is represented by the full line in Fig. 1 and which interlocks with the loops represented by the broken lines. The structure is referred to the axes OX, OY, where OX is parallel to the line of the courses and OY parallel to the line of the wales, and the loop is assumed to lie in the XY plane. The abscissa OX cuts the loop at A, and the ordinate OY cuts the loop at C. Subsequently, the direction parallel to OX is termed horizontal and the direction parallel to OY vertical. By symmetry, it is necessary to consider only the quarter ABC of the central loop, with yarn cross-over points at D and E. Contact along the region of interlocking, DE, is made between loops of successive courses, and the point K is the centre point of the interlocking region, i.e., the mid-point of the line DE. The position of maximum loop width is at the point F.

The tendency of the yarn A′C′ to regain its natural straight configuration produces a reaction component of force acting on the centre loop and distributed over the region of contact DE. The resultant of this distributed force is assumed to have the same effect as that of a localized horizontal force *P*, whose line of action passes through the point K. The point B at which this localized force is applied to the loop is termed the *interlocking point*. The forces acting on the centre loop, shown by the full arrows in Fig. 1, are balanced by equal and opposite forces, which are derived from the reaction of the centre loop on its neighbours immediately above or below, and which are shown by broken arrows in Fig. 1.

In general, there is a vertical displacement between the point of maximum width of one loop, F, and the point of minimum width of the interlocking loop, F′ (as is shown in Fig. 1). If this displacement is zero, the point of application of the forces, i.e., the interlocking point, B, will be coincident with the point of maximum loop width, F; this limiting case is assumed in all the geometrical models proposed by other workers. However, microscopical examination of several relaxed plain-knit fabrics indicates that this assumption is not usually valid, and that in practice there is generally a vertical displacement between the points of maximum and minimum width of interlocking loops.

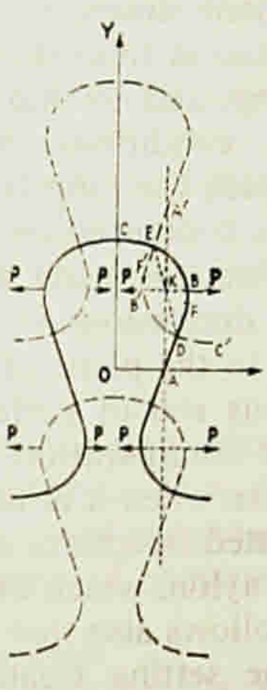



Fig. 1
Forces acting in the plain-knit structure

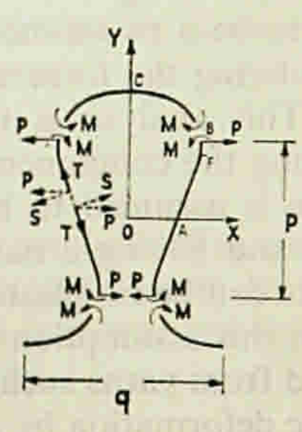



Fig. 2
Equilibrium of the loop segments

Consider the loop divided into segments as shown in Fig. 2. There is no force applied to the segment BC of the loop. Any vertical component of force acting on the segment BC is ruled out by equilibrium considerations. The application of a horizontal component of force to the segment BC would produce a resultant yarn tension or compression in the segment. Similar yarn tensions would exist in all other such segments throughout the fabric. These tensions would act in parallel for all the loops in the same wale, and their resultant would be an over-all fabric tension in the direction of its width, i.e., parallel to the line of the courses. This would require an external force applied to the fabric in the direction parallel to the line of the courses in order to hold it in its equilibrium state. No such external force is necessary to hold a fabric in its relaxed state, and the yarn tension in the loop segment BC must therefore be equal to zero. Thus, there can be no forces applied to the segment BC of the relaxed knitted loop.

By a similar argument, it can be shown that the only force that can act on the segment AB of the relaxed loop is the purely horizontal force $P$ (i.e., a force acting in the direction parallel to OX, the line of the courses). If at any point along this segment it is assumed that there is a tension $T$ acting parallel to the yarn axis and a shear $S$ acting perpendicular to the yarn axis, the resultant of which is always acting horizontally and is equal to the applied force $P$ (as shown in Fig. 2), any vertical component of force applied to the segment AB of the loop would result in an over-all fabric tension in the direction of its length. This would require an external force applied to the fabric in the length direction (i.e., parallel to the line of the wales) in order to hold it in its equilibrium configuration. Once again, this possibility is ruled out for a fabric in its relaxed state. It follows that the resultant force exerted by one loop on its neighbour at the interlocking point in a relaxed fabric must always act on the loop segment AB in a purely horizontal direction.

The validity of these assumptions is limited by geometrical considerations of jamming of the knitted structure in either the width or the length direction. Jamming of the structure occurs in the width direction when the two arms of the loop make contact at the point of minimum loop width, as shown in Fig. 3($a$), whereas jamming occurs in the length direction when loops of adjacent courses make contact at the back of the fabric, as shown in Fig. 3($b$). The assumptions described in the previous two paragraphs for the general case (where jamming of the structure does not occur) require that the tensions $T_1$ and $T_2$, shown

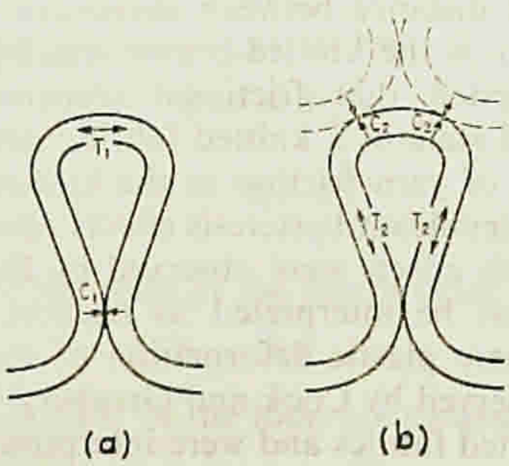



Fig. 3
Jamming of the plain-knit structure
($a$) in the width direction
($b$) in the length direction

in Fig. 3, must be equal to zero. However, the occurrence of jamming would give rise to yarn compressive forces, $C_1$ and $C_2$, as shown in Fig. 3 (*a*) and (*b*) for the conditions of jamming in the width and length directions, respectively. For a relaxed fabric, these compressive forces would have to be balanced by non-zero values of the yarn tensions, $T_1$ and $T_2$. Thus, the occurrence of jamming would considerably alter the forces acting on the loop, and the assumptions of the previous paragraphs would no longer apply.

The bending moment $M$ shown in Fig. 2 is derived from the action of the adjoining loop segment. There is no discontinuity in the bending moment at the interlocking points, and there can therefore be no applied couple acting in the plane of the fabric. That this must be so can be seen from Fig. 4. If there is a clockwise couple $L$ acting in the plane of the fabric about the point B on one loop, then there must be an anti-clockwise reaction couple $L$ acting about the point B′ on the interlocking loop. If symmetry is assumed and the figure is inverted, the loop segments AB and A′B′ are interchanged as shown by the letters given in brackets. For the inverted system, however, the couple $L$ acts in an anti-clockwise direction about the interlocking point B and in a clockwise direction about the interlocking point B′. Thus, the couples for the inverted system are acting in the opposite directions to those originally assumed. It follows that $L$ must be equal to zero.

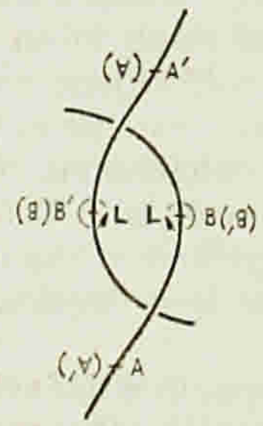



Fig. 4
Diagram showing that $L = 0$ by symmetry

If we now consider the segments AB of the loop, the condition of equilibrium is given by:

$$M = P \times \frac{p}{2}, \qquad \ldots\ldots(6)$$

where $p$ is the vertical distance between successive interlocking points (as shown in Fig. 2), i.e., $p$ is the knitted-course spacing.

Doyle[17] has suggested that frictional constraints are important in determining the relaxed state of a knitted fabric, particularly its dry-relaxed state. The importance of yarn friction in the knitted structure is indicated experimentally by the significant hysteresis effects observed in load–extension cycling. These hysteresis effects were observed by Doyle[2] for plain-knit and rib structures and must be interpreted as friction, especially for cycling between low loads, where plastic deformation of the yarn does not occur. Similar effects were observed by Cook and Grosberg[18] for the load–extension properties of warp-knitted fabrics and were interpreted as friction.

The rôle played by yarn friction at the interlocking points is shown in Fig. 5. When a fabric is taken from the knitting machine, it is highly distorted, as shown in Fig. 5(*a*), such that there is a considerable vertical displacement between the point of maximum width of one loop, F, and the point of

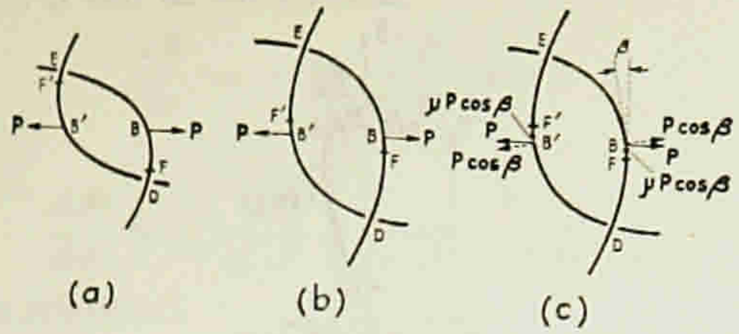


Fig. 5
Diagram showing the forces acting on the loop
(*a*) in a distorted condition
(*b*) in the relaxed condition
(*c*) in the relaxed condition, showing the elastic and frictional components

minimum width of the interlocking loop, F′. On relaxation, sliding of one loop over its neighbour would tend to occur, so that the vertical displacement between F and F′ is reduced as shown in Fig. 5(*b*). This sliding is opposed by frictional forces. The forces acting on the loop at the interlocking point B are therefore the reaction force and the frictional force tending to oppose sliding. The resultant force acting at B must always be a purely horizontal force acting on the segment AB in order that the over-all fabric tension is equal to zero. If it is assumed that the resultant force acting at each interlocking point is equal to $P$ as shown in Fig. 5(*c*), the reaction component of $P$ is equal to $P\cos\beta$ and acts perpendicular to the loop, whereas the frictional component acts tangentially to the loop and is equal to $\mu P\cos\beta$, where $\mu$ is the coefficient of static yarn–yarn friction and $\beta$ the angle that the tangent to the loop at the interlocking point makes with the vertical. The angle $\beta$ is termed the *interlocking angle* and is given by the following relation:

$$\tan\beta = \mu \qquad \ldots\ldots(7)$$

for knitted fabrics in their dry-relaxed state, where no external influence is provided to overcome the frictional constraints.

### 3. ANALYSIS OF THE TWO-DIMENSIONAL LOOP CONFIGURATION

With reference to Fig. 1, if the length of the loop segement AB is $l_1$ and the length of the segment BC is $l_2$, then by symmetry the knitted-loop length is given by:

$$l = 4\,(l_1 + l_2). \qquad \ldots\ldots(8)$$

Furthermore, the wale-spacing, $q$, is given by:

$$q = 4\,X_A \qquad \ldots\ldots(9)$$

or

$$q = 2(b-d), \qquad \ldots\ldots(10)$$

where $b$ is the width of the loop at the interlocking point B, i.e.:

$$b = 2\,X_B. \qquad \ldots\ldots(11)$$

The two segments AB and BC of the loop are analysed separately.

#### (a) Analysis of Segment AB

The analysis of this segment is the problem of a bent beam with ends parallel but not aligned, as shown in Fig. 6. A similar problem is considered in detail by Love[14] and by Southwell[19].

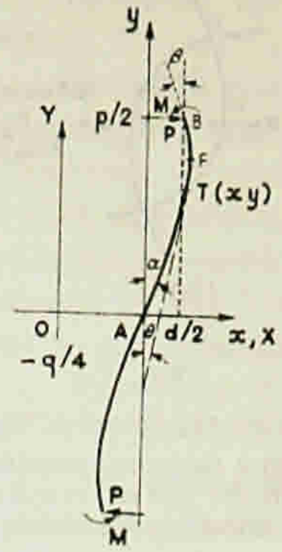



Fig. 6
Geometry of the loop segment AB



The origin of co-ordinates in Fig. 6 has been translated from O($X$, $Y$) to A($x$, $y$) such that:

$$X = x + q/4, \qquad \ldots\ldots(12)$$

and:

$$Y = y. \qquad \ldots\ldots(13)$$

The curvature of the central axis of the loop at the point T($x$,$y$) is given by ($-\mathrm{d}\theta/\mathrm{d}s$), where $\theta$ is the angle that the tangent to the loop at T makes with the positive direction of the $y$-axis (i.e., the line of the wales) and $s$ is the distance of T, measured along the loop, from the point A. The force acting at the point T($x$,$y$) must be $P$. There being an inflexion at the point A (by symmetry), there are zero moment and zero curvature at A. Hence, by considering the loop segment AT and taking moments about A, it follows that:

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} + \frac{P}{B}y = 0, \qquad \ldots\ldots(14)$$

where $B$ is the flexural rigidity of the yarn. This differential equation governs the equilibrium configuration of the loop segment AB and is the equation of an elastica. The parameters $\varepsilon$ and $\varphi$ are introduced such that:

$$\varepsilon = \sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \qquad \ldots\ldots(15)$$

and

$$\varepsilon \sin\varphi = \sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right), \qquad \ldots\ldots(16)$$

where $\alpha$ is the angle that the tangent to the centre point of the loop A makes with the line of the wales, and is subsequently termed the *loop angle*. Equations (15) and (16) show that $\varphi = \pi/2$ at A (where $\theta = \alpha$). Let $\varphi = \varphi_\beta$ at the interlocking point B (where $\theta = -\beta$), so that:

$$\varepsilon \sin\varphi_\beta = \sin\left(\frac{\pi}{4} - \frac{\beta}{2}\right)$$

or

$$\sin\varphi_\beta = \frac{1}{\varepsilon\sqrt{2}}\left\{\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right\}. \qquad \ldots\ldots(17)$$

From Southwell[19], it can be shown that:

$$\frac{d\theta}{ds} = -2\varepsilon\sqrt{\frac{P}{B}} \cdot \cos\varphi \qquad ......(18)$$

and

$$\frac{ds}{d\varphi} = -\sqrt{\frac{B}{P}} \cdot \frac{1}{\sqrt{(1-\varepsilon^2\sin^2\varphi)}} \cdot \qquad ......(19)$$

By using the relations:

$$dx = \sin\theta . ds$$

and

$$dy = \cos\theta . ds,$$

the Cartesian co-ordinates of any point T on the loop segment can be found and are given by:

$$x = \sqrt{\frac{B}{P}}\left\{ f(\varepsilon,\varphi) - 2e(\varepsilon,\varphi) \right\} \qquad ......(20)$$

and

$$y = 2\varepsilon\sqrt{\frac{B}{P}} \cdot \cos\varphi, \qquad ......(21)$$

where

$$f(\varepsilon,\varphi) = F(\varepsilon,\pi/2) - F(\varepsilon,\varphi), \qquad ......(22)$$

$$e(\varepsilon,\varphi) = E(\varepsilon,\pi/2) - E(\varepsilon,\varphi), \qquad ......(23)$$

$F(\varepsilon,\pi/2)$ and $F(\varepsilon,\varphi)$ are, respectively, complete and incomplete elliptic integals of the first kind, such that:

$$F(\varepsilon,\varphi) = \int_0^{\varphi} \frac{d\varphi}{\sqrt{(1-\varepsilon^2\sin^2\varphi)}},$$

and $E(\varepsilon,\pi/2)$ and $E(\varepsilon,\varphi)$ are, respectively, complete and incomplete elliptic integrals of the second kind, such that:

$$E(\varepsilon,\varphi) = \int_0^{\varphi} \sqrt{(1-\varepsilon^2\sin^2\varphi)}\, d\varphi.$$

These integrals do not give an analytical solution, but they have been solved numerically for various values of $\varepsilon$ and $\varphi$. Their numerical values are given in several sets of mathematical tables, such as those of Dale[20].

At the interlocking point B, $y = p/2$, and equation (21) leads to:

$$p = 4\varepsilon\sqrt{\frac{B}{P}} \cdot \cos\varphi_\beta .$$

Substituting for $\varepsilon$ and $\varphi_\beta$ from Equations (15) and (17) gives:

$$\frac{P}{B} = \frac{8(\sin\alpha + \sin\beta)}{p^2} . \qquad ......(24)$$

Substituting in Equations (20) and (21) to eliminate P/B gives:

$$\frac{x}{p} = \frac{1}{2\sqrt{2}\,(\sin\alpha + \sin\beta)}\left\{ f(\varepsilon,\varphi) - 2e(\varepsilon,\varphi) \right\} \qquad ......(25)$$

and

$$\frac{y}{p} = \frac{\varepsilon}{\sqrt{2(\sin\alpha + \sin\beta)}} \cos\varphi . \qquad \ldots\ldots(26)$$

Equations (25) and (26) are the equations of the loop segment AB in terms of the parameters $\varepsilon$ and $\varphi$. The distance $s$ measured along the loop from A to the point $T(\varepsilon,\varphi)$ can be found from Equation (19):

$$s = -\sqrt{\frac{B}{P}} \int_{\pi/2}^{\varphi} \frac{d\varphi}{\sqrt{(1 - \varepsilon^2 \sin^2\varphi)}} . \qquad \ldots\ldots(27)$$

Substituting from Equations (22) and (24) gives:

$$\frac{s}{p} = \frac{1}{2\sqrt{2(\sin\alpha + \sin\beta)}} f(\varepsilon,\varphi). \qquad \ldots\ldots(28)$$

The length $l_1$ of the loop segment AB is therefore given by:

$$\frac{l_1}{p} = \frac{1}{2\sqrt{2(\sin\alpha + \sin\beta)}} f(\varepsilon,\varphi_\beta). \qquad \ldots\ldots(29)$$

**(b) Analysis of Segment BC**

The bending moment, $M$, remains constant along the length of this segment. Hence the equilibrium configuration of BC is that of a segment of a circle with diameter $D$, where:

$$\frac{2}{D} = \frac{M}{B} . \qquad \ldots\ldots(30)$$

Substituting for $M$ from Equation (6) gives:

$$D = \frac{4}{p} \cdot \frac{B}{P} ,$$

and substitution for $P/B$ from Equation (24) gives:

$$\frac{D}{p} = \frac{1}{2\,(\sin\alpha + \sin\beta)} . \qquad \ldots\ldots(31)$$

The width $b$ of the loop at the interlocking point B is given by:

$$b = D \cos\beta$$

or

$$\frac{b}{p} = \frac{\cos\beta}{2\,(\sin\alpha + \sin\beta)} . \qquad \ldots\ldots(32)$$

The length $l_2$ of the segment BC is given by:

$$l_2 = \left(\frac{\pi}{2} - \beta\right)\frac{D}{2} .$$

Substituting for $D$ from Equation (31) gives:

$$\frac{l_2}{p} = \frac{\pi/2 - \beta}{4\,(\sin\alpha + \sin\beta)} . \qquad \ldots\ldots(33)$$

## 4. THE RELAXED-FABRIC PARAMETERS

The knitted-loop length, $l$, can be found by substituting from Equations (29) and (33) in Equation (8):

$$\frac{l}{p} = \frac{1}{(\sin\alpha + \sin\beta)} \left\{ (\pi/2 - \beta) + \sqrt{2\,(\sin\alpha + \sin\beta)}\, f(\varepsilon,\varphi_\beta) \right\} . \qquad \ldots(34)$$

In addition, the wale-spacing, $q$, given by Equation (10), can be found by using Equations (27) and (32):

$$\frac{q}{p} = \frac{1}{(\sin\alpha + \sin\beta)}\left[\cos\beta - \sqrt{2(\sin\alpha + \sin\beta)}\left\{f(\varepsilon,\varphi_\beta) - 2e(\varepsilon,\varphi_\beta)\right\}\right] \quad ..(35)$$

The fabric dimensional parameters, defined in Equations (1)–(4), are given by:

$$k_c = \frac{l}{p},$$

which, on substituting from Equation (34), becomes:

$$k_c = \frac{1}{(\sin\alpha + \sin\beta)}\left\{(\pi/2 - \beta) + \sqrt{2(\sin\alpha + \sin\beta)}\, f(\varepsilon,\varphi_\beta)\right\}; \quad ....(36)$$

and:

$$k_w = \frac{l}{q} = \frac{l}{p}\cdot\frac{p}{q},$$

which, on substituting from Equations (34) and (35), becomes:

$$k_w = \frac{(\pi/2 - \beta) + \sqrt{2(\sin\alpha + \sin\beta)}\, f(\varepsilon,\varphi_\beta)}{\cos\beta - \sqrt{2(\sin\alpha + \sin\beta)}\{f(\varepsilon,\varphi_\beta) - 2e(\varepsilon,\varphi_\beta)\}} \quad ......(37)$$

Expressions for the stitch-density parameter, $k_s$, and the ratio of courses to wales per unit length, $c/w$, can be found by using the relations:

$$k_s = k_c \times k_w$$

and

$$\frac{c}{w} = \frac{q}{p} = \frac{k_c}{k_w}.$$

It should be noted that the parameters $\varepsilon$ and $\varphi_\beta$ are fixed by Equations (15) and (17) for any given values of the loop angle $\alpha$ and the interlocking angle $\beta$. The fabric dimensional parameters are therefore completely determined by the values of these angles.

The force $P$ required to hold the loop in its equilibrium configuration is given by Equation (24), namely:

$$\frac{P}{B} = \frac{8(\sin\alpha + \sin\beta)}{p^2},$$

which, in terms of dimensionless parameters, becomes:

$$\frac{Pl^2}{B} = 8k_c^2(\sin\alpha + \sin\beta). \quad ......(38)$$

The bending moment, $M$, at the interlocking points is simply related to the curvature, $2/D$, of the segment BC and is given by Equation (6), namely:

$$M = \frac{Pp}{2}.$$

In terms of dimensionless parameters, this becomes:

$$\frac{Ml}{B} = \frac{2l}{D} = 4k_c(\sin\alpha + \sin\beta). \quad ......(39)$$

## 5. GEOMETRICAL LIMITATIONS IMPOSED BY LOOP-INTERLOCKING

There are two limitations that must be applied to the geometry of the loop, i.e., to the values of the geometrical parameters $\alpha$ and $\beta$, in order that the requirements of loop-interlocking are fulfilled. The two conditions are:

(*a*) limitation of jamming in the width direction; and

(*b*) limitation of jamming in the length direction.

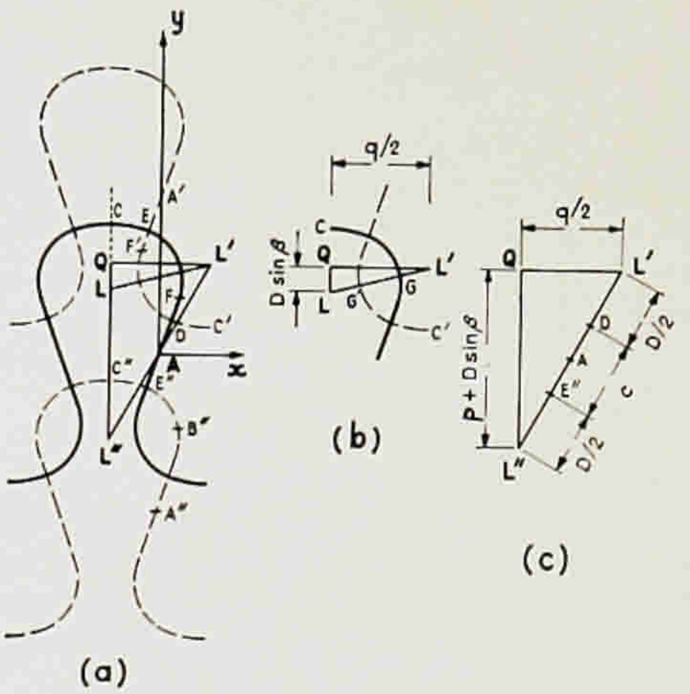



Fig. 7
Diagram showing in detail the geometry of loop-interlocking

### (a) Limitation of Jamming in the Width Direction

It can be seen from Fig. 7 that, in order that there should be sufficient space in the structure for interlocking of loops in the width direction:

$$\frac{q/2 - 2x_F}{d} \geqslant 1, \qquad \ldots\ldots(40)$$

where $x_F$ is the value of the abscissa $x$ (with origin at A) at the point of maximum loop width F ($\theta = 0$), and $d$ is the effective diameter of the yarn at the interlocking points in the fabric. Since interlocking loops must always make contact along the interlocking region in order to produce the reaction component of the applied force, it follows that the effective yarn diameter, $d$, is equal to the distance GG′ shown in Fig. 7(*b*). Considering the triangle LQL′ where L and L′ are the centre points of, respectively, the circular portions of the loops GC and G′C′ (both of diameter $D$), we have:

$$(D - d)^2 = D^2 \sin^2 \beta + \frac{q^2}{4},$$

from which it can be shown that:

$$\frac{l}{d} = k_c \Bigg/ \left\{ \frac{D}{p} - \sqrt{\frac{1}{4}\left(\frac{q}{p}\right)^2 + \left(\frac{D}{p}\right)^2 \sin^2\beta} \right\}, \qquad \ldots\ldots(41)$$

where $D/p$ and $q/p$ are given by Equations (31) and (35), respectively.

The ratio given on the left-hand side of Relation (40) is a direct measure of the openness of the knitted structure in the width direction. The inequality of Relation (40) requires that the two arms of the loop do not make true contact with each other at the position of minimum loop width. The limiting case, given by the equality of Relation (40), occurs when the two arms of the loop do make contact; in this case, the knitted structure becomes jammed in the direction of its width.

Relation (40) can be rewritten in the form:

$$\left\{ \frac{1}{2k_w} - \frac{2(x_r/p)}{k_c} \right\} \frac{l}{d} \geqslant 1, \qquad \ldots\ldots(42)$$

where $k_c$, $k_w$, and $l/d$ are given by Equations (36), (37), and (41), respectively, and $(x_r/p)$ is given by Equation (25) when $\theta = 0$ (i.e., $\varepsilon \sin \varphi = 1/\sqrt{2}$).

The relation between $\alpha$ and $\beta$ for the condition of jamming in the width direction, i.e., when the equality of Relation (42) holds, is shown graphically by the curve labelled I in Fig. 8. This curve shows that width-jamming occurs over a very small range of values of $\alpha$ (from about 26 to 28·5°). Any loop configuration having a combination of values of the angles $\alpha$ and $\beta$ that falls below this curve gives a structure for which jamming in the width direction does not occur, i.e., the inequality of Relation (42) holds. Combinations of $\alpha$ and $\beta$ falling above the curve are geometrically impossible for the knitted structure; in these cases, the restriction of Relation (42) is not satisfied.

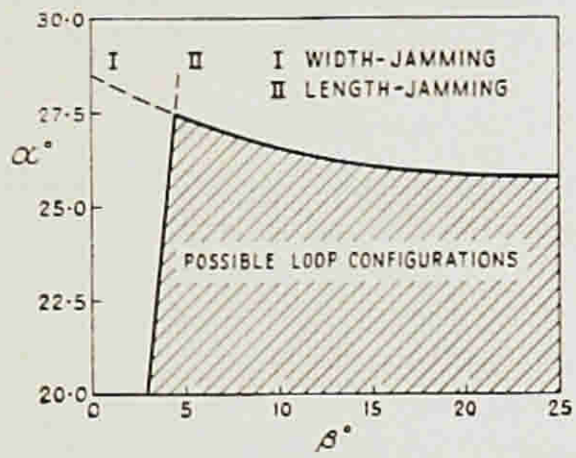



Fig. 8
Relation between $\alpha$ and $\beta$ for the jamming conditions

**(b) Limitation of Jamming in the Length Direction**

Interlocking of loops of successive courses can occur only if the distance between the central yarn axes of adjacent courses at the back of the fabric is greater than or equal to the effective diameter of the yarn. With reference once again to Fig. 7, this condition requires that the length DE″ is greater than or equal to the effective diameter of the yarn at the cross-over points, D and E″. For a first approximation, it is assumed that the effective yarn diameter remains constant along the length of the loop and is equal to $d$, where $l/d$ is given by Equation (41). If the length DE″ is denoted by $c$, then:

$$c/d \geqslant 1, \qquad \ldots\ldots(43)$$

where $c/d$ is a direct measure of the openness of the knitted structure in the length direction. The inequality of Relation (43) indicates that there is room for adjacent courses at the back of the fabric to fit together without making contact with each other at the yarn cross-over points. The equality implies that contact is made, i.e., the fabric becomes jammed in the direction of its length.

In Fig. 7, the point L″ is the centre point of the circular loop segment B″C″. Because of symmetry, the hypotenuse of the triangle QL′L″ must pass through the point A of the loop. It should be noted that the distance $c$ is actually equal to the length measured along the hypotenuse of the triangle between the central axes of adjacent courses at the back of the fabric. This distance is very nearly equal to the length DE″. This assumption is used merely for the purpose of clarity and does not affect the derivation given below. On considering the triangle QL′L″, it follows from Fig. 7(*c*) that

$$(c + D)^2 = q^2/4 + (p + D \sin \beta)^2,$$

from which it can be shown that:

$$\frac{c}{d} = \left\{ \sqrt{\frac{1}{4}\left(\frac{q}{p}\right)^2 + \left(1 + \frac{D}{p}\sin\beta\right)^2} - \frac{D}{p} \right\} \frac{l/d}{k_c}. \qquad \ldots\ldots(44)$$

The relation between $\alpha$ and $\beta$ for the jamming condition in the length direction is shown graphically by the curve labelled II in Fig. 8, from which it can be seen that length-jamming occurs for only a very small range of values of the interlocking angle $\beta$ (in the region of 3–4·5°). On combining the restrictions on the angles $\alpha$ and $\beta$ imposed by the two geometrical limitations, as given by curves I and II of Fig. 8, only those values lying in the shaded area of the figure are geometrically possible for the plain-knit structure. It is immediately evident that jamming in the length direction of the fabric would prevent the attainment of the two-dimensional loop configuration in which the interlocking angle, $\beta$, is equal to zero. From Equation (44) it can be shown that in this case

$$c/d \approx 1/2$$

for all values of the loop angle, $\alpha$. Hence, in order to attain the loop configuration in which $\beta$ is equal to zero, the effective diameter of the yarn at the cross-over points would have to be approximately half that at the points of application of the forces (or the interlocking points, as they are termed in the present work). This is physically unrealistic, since it would involve extremely high compressive forces between loops of adjacent courses at the back of the fabric.

The point of intersection of curves I and II of Fig. 8 corresponds to the condition of jamming in both length and width directions of the two-dimensional structure. This occurs when:

$$\alpha = 27{\cdot}5^\circ$$

and

$$\beta = 4{\cdot}5^\circ$$

and gives the completely jammed plain-knit structure.

## 6. DISCUSSION

### (a) Dependence of the Fabric Dimensional Parameters on the Loop Shape

Equations (36) and (37) show that the relaxed-fabric dimensional parameters are functions only of the geometrical parameters of the loop, $\alpha$ and $\beta$. The actual shape of the loop is determined by the loop angle, $\alpha$, and the point at which interlocking occurs is determined by the interlocking angle, $\beta$. By the geometrical considerations of jamming given in the previous section, the possible combinations of $\alpha$ and $\beta$ have been restricted.

The variation of the relaxed-fabric dimensional parameters with loop shape (i.e., with the loop angle $\alpha$) within the limits allowed by the geometry is shown in Figures 9–12 for constant values of $\beta$ between 0 and 25°. In each figure, the limiting conditions of jamming in the width and length directions are given by the bold curves labelled I and II, respectively. The significance of the curve marked $l/d = 20$ in these figures will be discussed in section 6(*b*). Only those regions of the graphs shown by the unbroken curves are geometrically feasible for the relaxed plain-knit structure. The broken portion of the curves for $\beta \geqslant 5^\circ$ corresponds to structures that lie beyond the

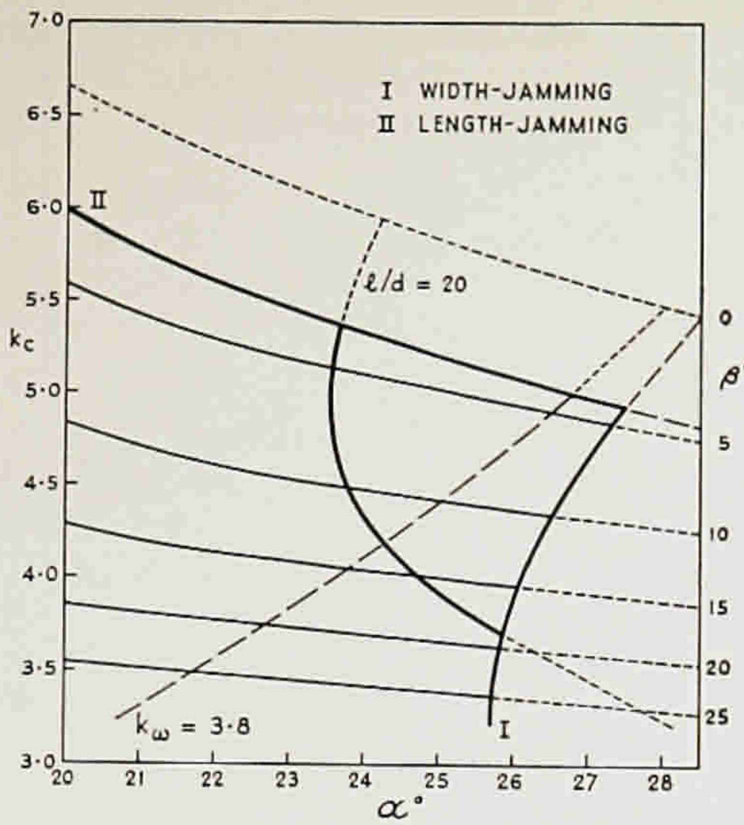



Fig. 9
Relation between $\alpha$ and $k_c$

condition of jamming in the width direction. The curves for $\beta = 0$ are shown broken because, for all values of $\alpha$, the resulting structure lies beyond the condition of jamming in the length direction.

Figures 9–12 show that the actual values of the dry-relaxed-fabric parameters are very critically dependent on $\alpha$ and $\beta$. Figures 9 and 10 indicate that the value of the parameter $k_c$ decreases, whereas that of $k_w$ increases for increasing values of either $\alpha$ or $\beta$. This is accompanied by a rapid decrease in the ratio of courses to wales per unit length, $k_c/k_w$, for increasing values of either $\alpha$ or $\beta$, as is shown in Fig. 11. The stitch-density parameter, $k_s$, as is shown in Fig. 12, varies with $\alpha$ in the same sense as $k_w$, but with $\beta$ in the same sense as $k_c$. However, $k_s$ is much less critically dependent on $\alpha$ and $\beta$ than are the linear fabric dimensional parameters, $k_c$ and $k_w$.

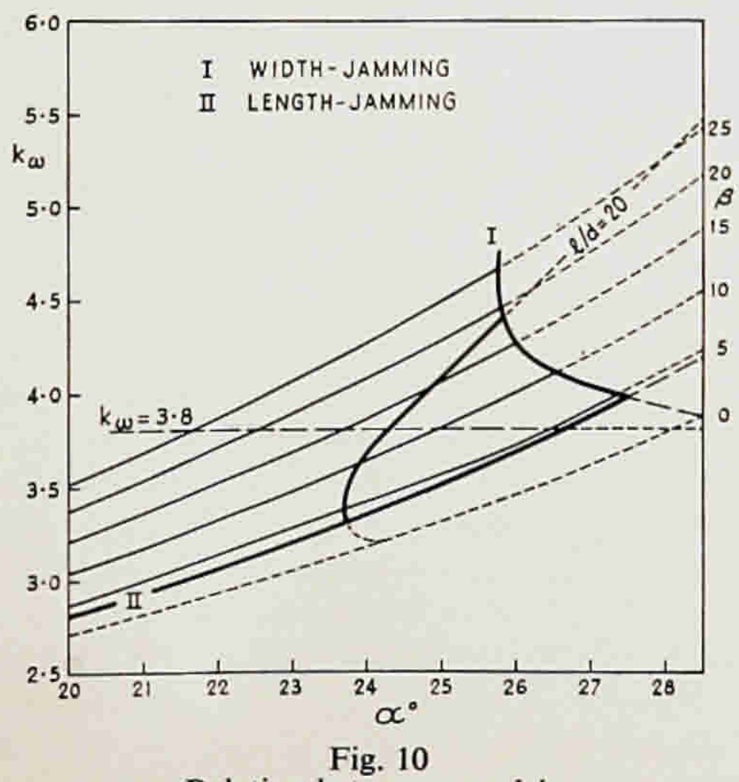



Fig. 10
Relation between $\alpha$ and $k_w$

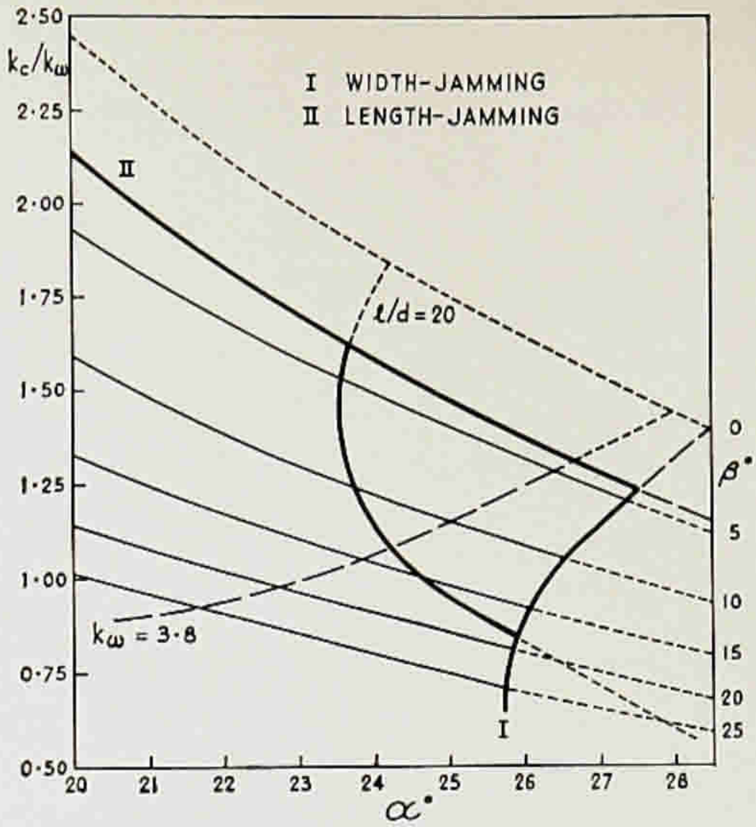



Fig. 11
Relation between $\alpha$ and $k_c/k_w$

The curve labelled $k_w = 3{\cdot}8$ in Figures 9–12 gives an estimate of the average values of $\alpha$ and $\beta$ for dry-relaxed plain-knit fabrics. Any equilibrium loop configuration with values of $\alpha$ and $\beta$ that fall on this curve would give a dry-relaxed fabric having a value of $k_w$ equal to 3·8, this being the average value experimentally observed by Munden[3] for wool fabrics (Table I). The parameter $k_w$ is chosen as a guide for the estimation of $\alpha$ and $\beta$ because its value is unaffected by the extension of the analysis to three dimensions, whereas $k_c$, and hence $k_c/k_w$ and $k_s$, are increased by an amount governed by the curvature of the loop out of the plane of the fabric; this will be fully discussed in Part II of this series.

In each figure, the curve for $k_w = 3{\cdot}8$ converges towards the corresponding curve for the limiting condition of jamming in the width direction as $\beta$ decreases towards zero. Constant values for both $k_c$ and $k_w$ are obtained only

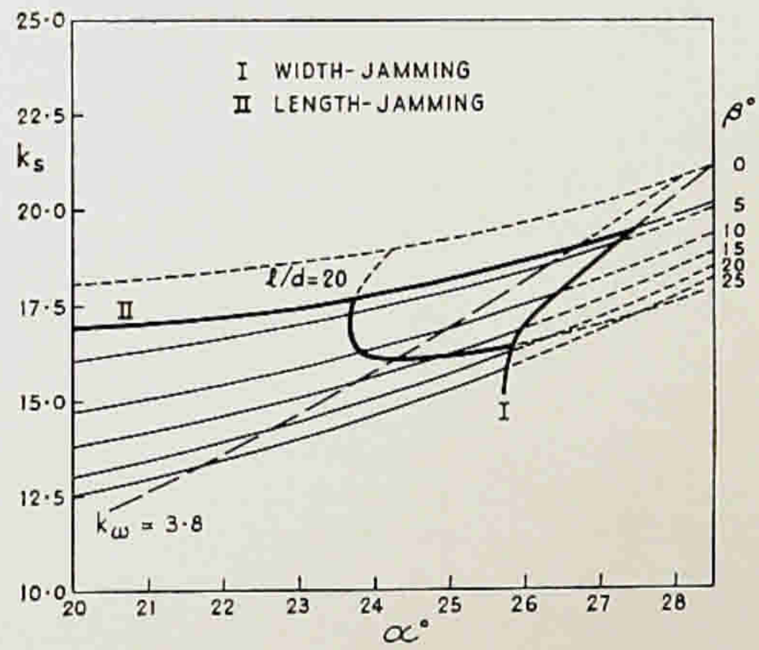



Fig. 12
Relation between $\alpha$ and $k_s$

if $\alpha$ and $\beta$ are fixed, i.e., constant values of $k_c$ and $k_w$ imply that the knitted-loop configuration is similar for all dry-relaxed fabrics, independently of their tightness of construction (in accordance with the ideas of Munden[3]).

### (b) The Effect of Fabric Tightness

The tightness of a plain-knit fabric is generally specified by the 'cover factor', $K$, where $K$ is given by Equation (5), i.e.:

$$K = \frac{\sqrt{T}}{l},$$

and where $T$ is the yarn linear density in tex.

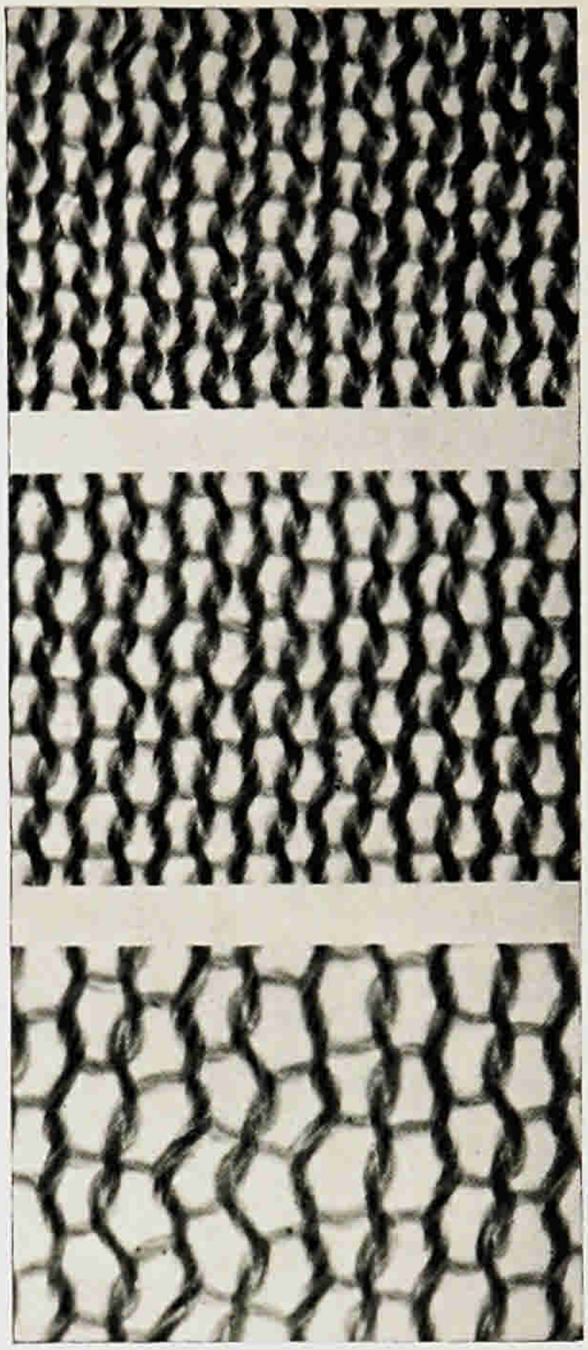
The cover factor $K$ is proportional to the ratio $\bar{d}/l$, where $\bar{d}$ is the average natural yarn diameter and $l$ the knitted-loop length. This ratio is related in an inverse manner to the parameter $l/d$ given by Equation (41) of the theory, where $d$ is the *effective* diameter of the yarn at the interlocking points in the fabric. The average natural yarn diameter is a fixed quantity for any given yarn, whereas the effective diameter is not a constant but varies with the knitted-loop length for fabrics knitted from the same yarn. A decrease in loop length produces a tighter fabric, and hence more yarn compression at the interlocking points; this causes a reduction in the effective diameter of the yarn. This effect has been noted qualitatively[21,22] for plain-knit wool fabrics.

#### (*i*) *Experimental*

The dependence of the parameter $l/d$ on the loop length was investigated experimentally by measuring the variation of $d$ with $l$ for six plain-knit fabrics, the cover factor of which lay between 10·0 and 16·2 tex$^{½}$ cm$^{-1}$. They were all knitted from 260-den (29-tex) silk yarn, silk being chosen because of its high elasticity and regularity and clarity of diameter. Five of these fabrics were knitted on a 3½-in.-diameter, 18-gauge machine, and the slackest was knitted on a 4½-in.-diameter, 12-gauge machine.

The fabrics were allowed to dry-relax in a standard atmosphere (20°C, 65% r.h.) for seven days. The loop lengths of the samples were measured on a H.A.T.R.A. Course-length Tester, a tension of 10 g being used; this tension was found from the load–elongation curve of the yarn unravelled from the fabrics. Fifty measurements of the effective yarn diameter were made on each sample by means of a screen-projection microscope; 25 measurements were made on each side of the loop. Each measurement was the average of the two effective yarn diameters at the mid-point of the interlocking region; this point corresponds to the interlocking point B of the theory.

Table II gives the knitted-loop length for each sample, together with its cover factor $K$ in units of tex$^{½}$ cm$^{-1}$. The values of $d_1$ and $d_2$ shown in the table are the mean effective yarn diameters on each side of the loop. When the mean values are taken, the left-hand side of one loop interlocks with the left-hand side of the other (this giving a measure of $d_1$), and there is a similar situation for the right-hand side (this giving a measure of $d_2$). In general, both $d_1$ and $d_2$ decrease as the cover factor increases, i.e., as the structure becomes tighter. The coefficient of variation of both $d_1$ and $d_2$ is between 5 and 7% for all the samples. The marked difference for each sample between the values of $d_1$ and $d_2$ can be explained by the effect of the twist induced in different directions in the arms of the loop when it is bent in three dimensions (as will be shown in Part II). This results in an asymmetrical structure, having a different value of $l/d$ ($l/d_1$ and $l/d_2$ as given in Table II) for each side of the loop. This asymmetrical structure can be seen in the photographs of the silk fabrics given in Fig. 13 (and is a common feature of plain-knit fabrics).



(*a*)

(*b*)

(*c*)

Fig. 13
Dry-relaxed silk fabrics
(*a*) sample 1 ($K = 16{\cdot}1$)
(*b*) sample 3 ($K = 14{\cdot}0$)
(*c*) sample 6 ($K = 10{\cdot}0$)

Averaging the values of $l/d_1$ and $l/d_2$ gives a measure of $l/d$ for the fabric as a whole. A systematic relation between $l/d$ and tightness is obtained such that $l/d$ decreases as the structure becomes tighter. The range of values of $l/d$ is relatively small, and in commercially acceptable fabrics would be smaller still, since here samples 1 and 6 both lie well outside the commercial range. In practice, therefore, $l/d$ will usually be less than 20.

**Table II**
**Silk-fabric Parameters**

| Sample | $l$ (cm) | $K$ ($\text{tex}^{\frac{1}{2}}\text{cm}^{-1}$) | $d_1$ (mm) | $d_2$ (mm) | $l/d_1$ | $l/d_2$ | $l/d$ |
|---|---|---|---|---|---|---|---|
| 1 | 0·335 | 16·1 | 0·209 | 0·185 | 16·0 | 18·1 | 17·1 |
| 2 | 0·361 | 14·9 | 0·220 | 0·184 | 16·4 | 19·6 | 18·0 |
| 3 | 0·383 | 14·0 | 0·231 | 0·192 | 16·6 | 20·0 | 18·3 |
| 4 | 0·429 | 12·5 | 0·252 | 0·208 | 17·0 | 20·6 | 18·8 |
| 5 | 0·439 | 12·2 | 0·251 | 0·197 | 17·5 | 22·3 | 19·9 |
| 6 | 0·538 | 10·0 | 0·267 | 0·222 | 20·2 | 24·2 | 22·2 |

*(ii) Theoretical*

The theoretical relation between the loop shape (as defined by the loop angle, $\alpha$) and the parameter $l/d$ is given by Equation (41) and is shown graphically in Fig. 14 for constant values of the interlocking angle, $\beta$, between 0 and 25°. The limiting condition of jamming in the width direction is once again shown by the curve labelled I, but for the sake of clarity the length-jamming condition is not shown in this figure. Only regions of the graph shown by the unbroken curves are allowed by the geometry. The curve for $k_w = 3{\cdot}8$ intersects with the curve for the width-jamming condition; this is because there is a minimum value of $l/d$ for a given value of $\alpha$ when $\beta$ is between 5 and 10°.

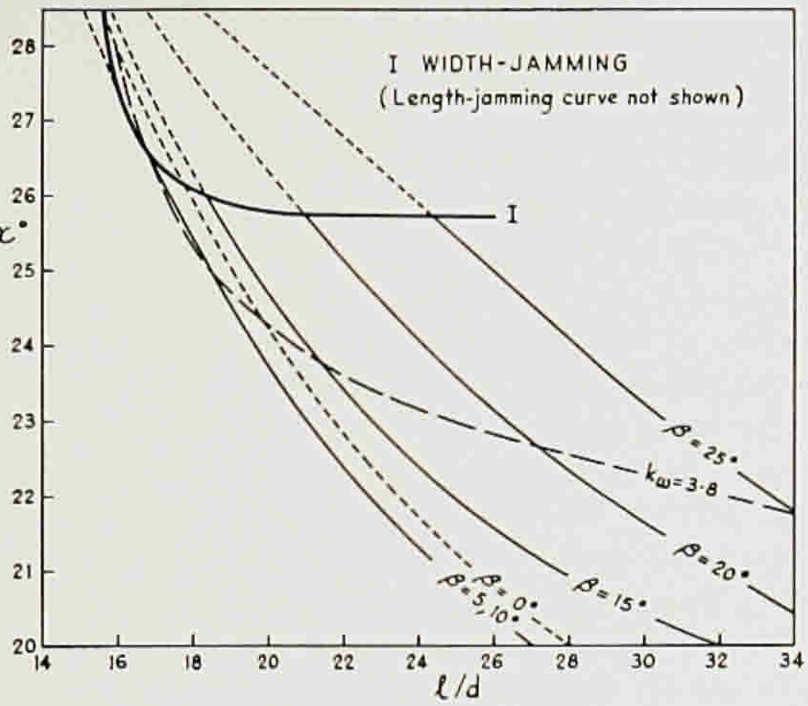



Fig. 14
Relation between $l/d$ and $\alpha$

The data of Fig. 14 show that the minimum value of $l/d$ allowed by the geometry is equal to 16·0 (corresponding to the completely jammed structure); this value of $l/d$ gives a dry-relaxed fabric of the tightest possible construction. The values of the fabric parameters for this structure of maximum tightness (as given by Figures 9–12) are shown in Table III.

**Table III**
**Theoretical Values of the Fabric Parameters**

| Tight Fabrics | Slack Fabrics |
|---|---|
| $l/d = 16$ | $l/d = 20$ |
| $\alpha = 27{\cdot}5°$ | $23{\cdot}7° < \alpha < 25{\cdot}8°$ |
| $\beta = 4{\cdot}5°$ | $3{\cdot}7° < \beta < 18{\cdot}0°$ |
| $k_c = 4{\cdot}96$ | $5{\cdot}4 > k_c > 3{\cdot}8$ |
| $k_w = 3{\cdot}95$ | $3{\cdot}3 < k_w < 4{\cdot}4$ |
| $k_c/k_w = 1{\cdot}25$ | $1{\cdot}6 > k_c/k_w > 0{\cdot}9$ |
| $k_s = 19{\cdot}6$ | $17{\cdot}7 > k_s > 16{\cdot}5$ |

The slackest dry-relaxed structure likely to be met in practice corresponds to a value of $l/d$ of about 20 (as obtained from the experimental data of Table II). The variation of the fabric parameters for very slack dry-relaxed structures is therefore represented by the bold curve for $l/d = 20$ shown in Figures 9–12. Table III gives the maximum range of values of the fabric parameters that are allowed by the geometry when $l/d = 20$. The first value in this range corresponds to the limiting condition of jamming in the length direction when $l/d = 20$, and the second value corresponds to the limiting condition of jamming in the width direction.

Thus, the region of Figures 9–12 that is of practical importance is the area enclosed by the curve for $l/d = 20$ and the two jamming curves. The range of values of the fabric parameters allowed by the geometry for a given value of $l/d$ is largest for very slack fabrics ($l/d = 20$), and decreases as the structure becomes tighter until, for the tightest possible construction ($l/d = 16$), the values of the fabric parameters are fixed by the concurrence of the two jamming conditions.

It can be seen from Table III that the whole practical range of $l/d$ ($16 \leqslant l/d \leqslant 20$) represents a maximum variation in the loop angle, $\alpha$, of only 3·8° (from 23·7 to 27·5°), whereas the interlocking angle, $\beta$, exhibits a much larger range of values, from 3·7 to 18·0°. Munden[3] has suggested that all plain-knit fabrics in a given state of relaxation have a similar loop configuration, which is independent of the loop length and of the physical properties of the yarn. The theory presented here indicates that this suggestion seems a reasonable approximation for dry-relaxed fabrics. However, the values of the fabric dimensional parameters for dry-relaxed fabrics are not necessarily constant. The curves for $k_w = 3{\cdot}8$ in Figures 9–12 have shown that, even if $k_w$ is assumed constant, the values of the other fabric parameters are very critically dependent on the loop shape and exhibit relatively large variations for the small changes in $\alpha$ accompanying changes in fabric tightness. Furthermore, Table III has shown that, for the practical range of dry-relaxed fabrics, there are relatively large variations of the fabric parameters inherent in the dry-relaxed knitted structure. The values of the fabric dimensional parameters given in Table III encompass corresponding variations for dry-relaxed fabrics observed experimentally by several workers[22–25].

The analysis of the knitted loop put forward in the present work assumes that the yarn is perfectly elastic; as a result, the theoretical loop configuration is not subject to any residual fabric tensions introduced during the knitting process. In practice, however, the dry-relaxed loop configuration may be modified by the effect of tensions imparted to the fabric on the knitting machine, which distort the loop by an amount depending on the fabric tightness and which are not completely recovered during dry relaxation. These residual fabric tensions usually tend to augment the inherent effects of loop shape and fabric tightness on the dry-relaxed values of the fabric dimensional parameters and result in greater variations for fabrics knitted from yarns of relatively low elasticity (such as cotton) compared with fabrics knitted from yarns of relatively high elasticity (such as wool)[22,23].

### (c) Friction Considerations

The interlocking angle, $\beta$, is determined by the coefficient of static yarn friction, $\mu$, according to Equation (7), i.e.:

$$\tan \beta = \mu \, .$$

The relation between $\mu$ (or $\beta$) and the resultant force parameter, $Pl^2/B$, given by Equation (38), is shown graphically in Fig. 15 for all equilibrium loop configurations having values of $\alpha$ in the range 20–28·5° and $\beta$ lying between 0 and 30° (corresponding to values of $\mu$ less than about 0·5). It can be seen that this relation is unaffected by $\alpha$, and that the force parameter $Pl^2/B$ is dependent only on the coefficient of yarn friction (and hence on the interlocking angle, $\beta$). Increasing $\mu$ causes a decrease in $Pl^2/B$. This relation holds up to the jamming condition, beyond which other forces come into play (the broken portion of the curve).

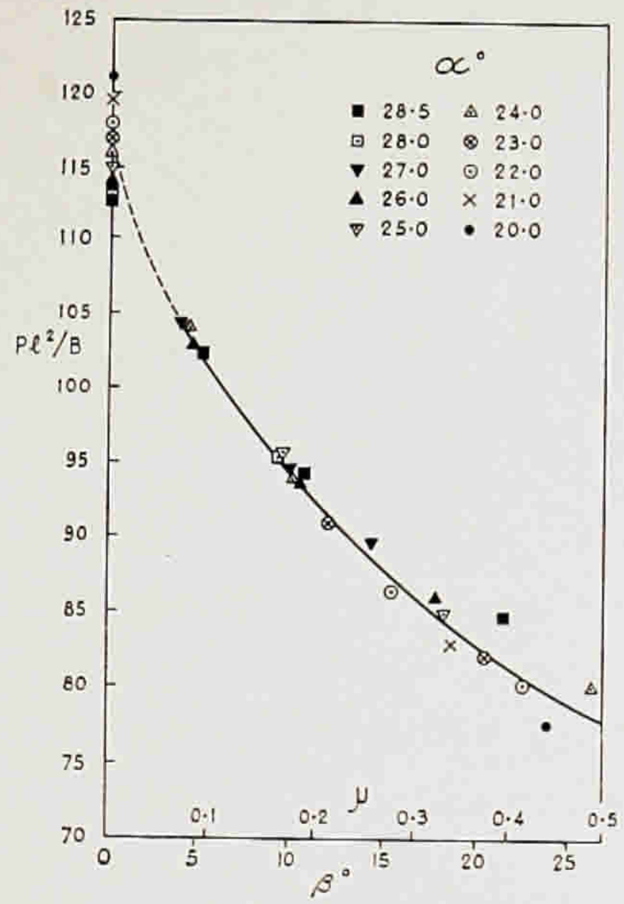



Fig. 15
Relation between μ (or β) and $Pl^2/B$



**(d) Knitted-fabric Cover**

Knitted fabrics are characterized by their extremely good covering properties. The fabric cover, $C$, is generally defined as the fraction of the fabric area that is actually occupied by the yarn, i.e.:

$$C = \frac{\text{area of yarn}}{\text{area of fabric}}.$$

Assuming, for a first approximation, that the effective yarn diameter remains constant over the whole loop length, we have:

$$C = \frac{ld - 4d^2}{1/S},$$

where $S$ is the stitch density, and the second term in the numerator is the effect that the four cross-over points per stitch have on the yarn area. From Equation (3), namely,

$$S = k_s/l^2,$$

it follows that:

$$C = \frac{k_s}{l/d}\left(1 - \frac{4}{l/d}\right). \qquad \text{......(45)}$$

The fabric cover, $C$, is plotted against α in Fig. 16 for constant values of β between 0 and 25°. Once again, the jamming curves are shown, and only the unbroken portions of the curves are allowed by the geometry. Increasing α or decreasing β gives an increase in the fabric cover. As in Figures 9–12 for the fabric dimensional parameters, the region of the graph that is of practical importance is the area bounded by the two jamming curves and the curve for $l/d = 20$; the curve for $k_w = 3{\cdot}8$ passes through the centre of this region.

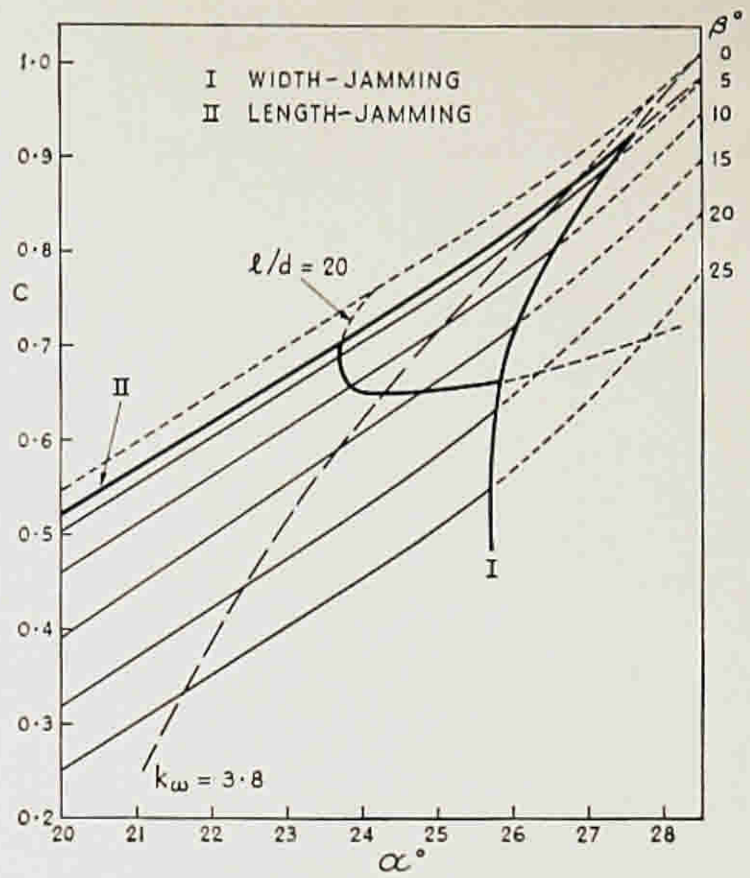



Fig. 16
Relation between α and knitted-fabric cover, *C*

By definition, the maximum value of the fabric cover possible is unity, and in this case the yarn would occupy the whole fabric area. The practical range of values of *C* is from 0·65–0·70 for very slack fabrics ($l/d = 20$) to a maximum of about 0·9 for the completely jammed structure ($l/d = 16$).

The fabric 'cover factor', *K*, defined by Equation (5) is something of a misnomer, since this parameter is not so much a direct measure of fabric cover as one of fabric tightness. In practice, the 'cover factor', *K*, is an easier parameter to evaluate than the parameter *C* given by Equation (45). The use of *K* as a measure of cover depends on the existence of a direct relation between fabric cover and tightness. This is generally true of any one state of relaxation, but *K* could not be used to compare the cover of fabrics in different states of relaxation.

## 7. CONCLUSIONS

The dry-relaxed knitted loop can be considered as a force-determined structure and can be analysed as a function of the forces acting in the plane of the fabric at the points of loop-interlocking. The limiting cases for which the analysis is applicable are governed by the conditions for which the structure becomes jammed in either the width or the length direction.

The geometry of the plain-knit structure is completely specified by the values of the loop angle, α, and the interlocking angle, β (which are shown in Fig. 6). The value of α determines the actual shape of the loop, and the value of β determines the point on the loop at which interlocking occurs.

The fabric dimensional parameters can be expressed as functions merely of α and β. This is true also of the fabric cover, *C*, and the parameter $l/d$ (the ratio of the knitted-loop length to the effective diameter of the yarn at the interlocking points in the fabric). This latter parameter is related to the fabric slackness, and for the practical range of plain-knit fabrics $16 \leqslant l/d \leqslant 20$. For any value of $l/d$ in this range, the interlocking angle, β, is a function of μ (the coefficient of static yarn friction), and the loop angle, α, can acquire any value that is compatible with the limitations of jamming. This allows a maximum variation in $\alpha$, for all dry-relaxed plain-knit fabrics, of less than 4°. However, even this small change in loop shape is accompanied by marked variations in the values of the fabric dimensional parameters and the fabric cover. These variations are greatest for slack fabrics and are inherent in the dry-relaxed knitted structure.

## ACKNOWLEDGEMENTS

The authors are indebted to Professor P. Grosberg and Dr. K. Hepworth, of the University of Leeds, for helpful discussions and suggestions that have been incorporated in this work, and to Dr. K. Baird, of the Division of Textile Physics, C.S.I.R.O., Australia, for his help in preparing this paper.

The work described in Part I and Part II of this series was done during the tenure of a research scholarship awarded to one of us (R.P.) by the Commonwealth Scientific and Industrial Research Organization, Australia.

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*Received* 27.2.1967
*Accepted for publication* 6.6.1967

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