Pergamon

Minerals Engineering, Vol. 7, No. 12, pp. 1459–1472, 1994
Elsevier Science Ltd
Printed in Great Britain
0892–6875(94)00084–0
0892–6875/94 $7.00+0.00

MATHEMATICAL MODELS OF THE COLUMN FLOTATION PROCESS A REVIEW

R.K. TUTEJA§, D.J. SPOTTISWOOD§ and V.N. MISRA†

§ WA School of Mines, Curtin University, PO Box 597, Kalgoorlie, WA 6430, Australia
† Kalgoorlie Metallurgical Laboratory, Chemistry Centre (WA), Department of Minerals & Energy, PO Box 881, Kalgoorlie, WA 6430, Australia
(Received 11 July 1994; accepted 21 August 1994)

ABSTRACT

Mathematical modelling is a valuable quantitative instrument, which is being effectively employed to the column flotation process in order to predict its metallurgical performance. This paper presents an up-to-date assessment of the various mathematical models for column flotation currently available under the categories of kinetic models and non-kinetic models and also identifies a number of associated shortcomings.

Keywords
Flotation column; modelling; kinetic models

INTRODUCTION

A mathematical model of the column flotation process should, for given feed characteristics and design and operating variables, predict metallurgical performance. The model may also be used for design, control and optimisation of the column flotation process. In general, the mathematical models may be divided into two broad categories namely, mechanistic models and empirical models. One of the main advantages of mechanistic models is that unlike empirical models they provide a sound basis for extrapolation [1].

Unfortunately, the mechanisms governing the column process are complex and in general not sufficiently well understood at the present time to formulate a mathematical model from theory. Therefore, all the mathematical models of the column flotation process which have been proposed to date are empirical models. The mathematical models available at present in the literature may further be divided into two categories namely, kinetic models and non-kinetic models. Kinetic models as the name suggests are models which are either wholly based on flotation kinetics, or flotation kinetics is one of the main components of the overall model form. In general, flotation kinetics is the study of the variation of froth overflow product with time, and the quantitative identification of all rate-controlling variables [2]. For flotation columns, following the study by Finch and Dobby [3], models based on axial dispersion theory have become the most common in this category.

Any model which does not fall under the category of a kinetic model is a non-kinetic model. Models based on the response surface methodology are in this category.

This paper discusses the various mathematical models for column flotation available under the categories of kinetic models and non-kinetic models. Also, the respective shortcomings of both model categories are identified.

Kinetic models

As has been indicated earlier, models under this category are based on axial dispersion theory. Therefore, axial dispersion theory is briefly discussed before proceeding to the details of various models in this category.

In models based on axial dispersion theory the recovery (or the change in concentration of pulp) is dependent on two main factors namely, flotation kinetics (assumed to be of first order) and degree of mixing (axial only). The models based on axial dispersion theory are in a sense one dimensional models, as they neglect the influence of radial mixing, non-uniform velocity profiles and short-circuiting on recovery [4]. If a tracer impulse is injected in a counter-current column at the top of a collection zone of length L, the mass transport equation that describes its concentration C at an axial distance x downstream from the point of injection at time t is given by:

$$D\frac{d^2C}{dx^2} - U_i\frac{dC}{dx} - \frac{dC}{dt} = 0 \qquad (1)$$

where D is the axial dispersion coefficient and U_i is the liquid (or solids) interstitial velocity.

Equation (1) has been solved analytically by Wehner and Wilhelm [5], assuming first order reactions, and is valid for all entrance and exit conditions. The solution is:

$$Recovery\ (\%) = 100\left(1 - \frac{4a\exp\left(\frac{1}{2N_p}\right)}{(1+a^2)\exp\left(\frac{a}{2N_p}\right) - (1-a^2)\exp\left(\frac{-a}{2N_p}\right)}\right) \qquad (2)$$

where

$$a = \sqrt{1+4ktN_p}$$

and

$$Np = \frac{D}{U_i L}$$

Also, if the concentration of tracer is measured at the tailings discharge with time (time=0 when tracer impulse injected at top of the collection zone), then a residence time distribution of liquid (or solids) can be obtained. The residence time distribution can be modelled using two parameters: mean residence time (t) and dimensionless vessel dispersion number (N_p) to describe mixing conditions [3]. In the literature, sometimes the inverse of the vessel dispersion number (N_p) is used and referred to as the Peclet number (P_e). The main reason for measuring the mixing parameters is to quantify the influence of mixing on recovery.

For plug flow, N_p tends to zero and Equation (2) can be simplified to:

$$\text{Recovery} = 1 - \exp(-kt) \qquad (3)$$

and for perfectly mixed flow, N_p tends to infinity and Equation (2) simplifies to:

$$Recovery = \frac{kt}{1+kt} \qquad (4)$$

In general, the flow conditions in a laboratory column (5 cm diameter) approach plug flow while in a plant column (> 1 m diameter) the flow conditions approach between plug and perfectly mixed conditions. More information on axial dispersion theory is available from Finch and Dobby [3], Levenspiel [6] and Alford [7].

a) Finch and Dobby model

Finch and Dobby [9] first proposed a column flotation process model on the basis of axial dispersion theory. They proposed a two phase model for calculation of overall recovery. The overall recovery is not only dependent on the behaviour in the collection zone but also is governed by behaviour in the froth zone. Therefore, ideally there should be two separate models for the collection and froth zones, from which the overall recovery may be computed as follows:

$$\textit{Overall recovery} = 100\left(\frac{R_c R_f}{1-R_c+R_c R_f}\right) \tag{5}$$

where R_c is recovery in collection zone and R_f is recovery in froth zone.

However, since at the froth zone mechanisms are not well understood, and no comprehensive model of the froth zone is available, Finch and Dobby [3] assumed for simulation exercises that froth zone recovery was equal to 100%. In this case Equation (5) may be simplified to:

$$\text{Overall recovery} = 100\ R_c \tag{6}$$

that is, the column performance is assumed to be governed by collection zone behaviour alone.

More recently, it has been suggested that the froth zone recoveries may vary between 40 and 80% in pilot scale columns and may be even lower for plant scale columns. However, no froth zone model has been suggested [3, 8, 9].

In order to compute the recovery in the collection zone, Finch and Dobby [3] have proposed the following equations:

$$\textit{Recovery}\ (\%) = 100\left(1-\frac{4a\exp\left(\frac{1}{2N_p}\right)}{(1+a^2)\exp\left(\frac{a}{2N_p}\right)-(1-a^2)\exp\left(\frac{-a}{2N_p}\right)}\right) \tag{7}$$

where

$$a = \sqrt{1+4ktN_p}$$

$$N_p = \frac{0.063 D_c\left(\frac{J_g}{1.6}\right)^{0.3}}{\left(\frac{J_{sl}}{1-e_g}+U_{sp}\right)H_c} \tag{8}$$

$$U_{sp} = \frac{g d_p^2(\partial_p - \partial_{sl})(1-\phi_s)^{2.7}}{18\mu_f(1+0.15Re_p^{0.687})} \tag{9}$$

$$Re_p = \frac{d_p U_{sp} \partial_l (1-\phi_s)}{\mu_f} \tag{10}$$

$$t_p = t_l\left(\frac{\dfrac{J_{sl}}{1-e_g}}{\dfrac{J_{sl}}{1-e_g}+U_{sp}}\right) \tag{11}$$

$$t_l = \frac{H_c(1-e_g)}{J_{sl}} \tag{12}$$

It should be noted that many of these equations are based on work done in other applications of bubble columns.

To estimate the rate constant (k), Finch and Dobby [3] recommend that the overall rate constant be determined (ie obtained in presence of froth) by varying the residence time while keeping the other variables at predetermined levels. In large columns the residence time may be varied by manipulating tailings flow rate and in laboratory columns residence time may be varied by collection and recycling of tailings. In both cases the overall rate constant is estimated from the slope of the (100-R) vs t curve (Figure 1), where R is cumulative percentage recovery and t is residence time.

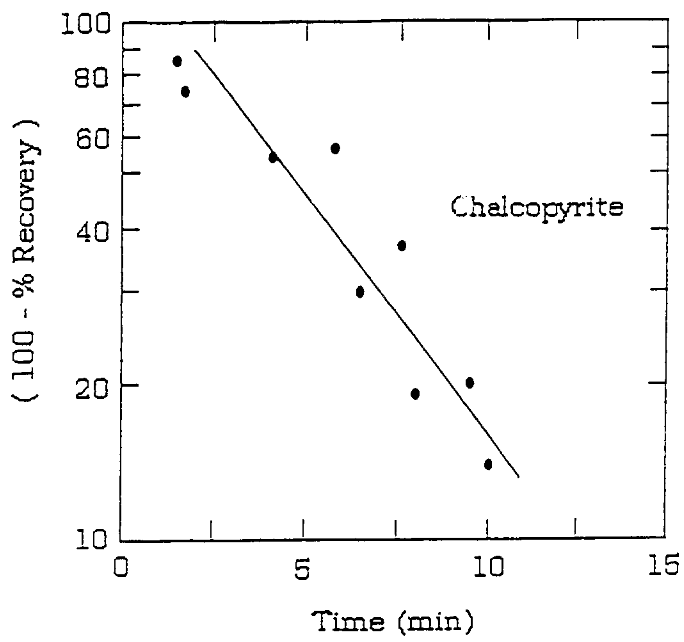

Fig. 1 Determination of rate parameter [3]

Further, based on maximum carrying capacity measurements from a variety of columns (different operating conditions and column diameter), Finch and Dobby [3] have suggested the following correlation:

$$C_{max} = 0.068\ \partial_c\ d_{80} \tag{13}$$

where d_{80} is 80% passing size of concentrate and ∂_c is mean particle density in the concentrate. Interestingly, Finch and Dobby [3] have noted that the maximum carrying capacity is not significantly dependent on air rate.

In addition, Finch and Dobby [3] have successfully used the drift-flux model to calculate the mean bubble size.

b) VPI model

Yoon et al., [10, 11] have been conducting research work on mathematical modelling of flotation columns for over a decade. Their model is also based on axial dispersion theory. However, they have used the Peclet number as a parameter to quantify mixing conditions in the column. The mathematical relationship between Peclet number and vessel dispersion number is that the former is the inverse of the latter. The following model equations to predict recovery are suggested:

$$Recovery\ (\%) = 100\left(1 - \frac{4a\exp\left(\frac{P_e}{2}\right)}{(1+a^2)\exp\left(\frac{aP_e}{2}\right) - (1-a^2)\exp\left(\frac{-aP_e}{2}\right)}\right) \tag{14}$$

where

$$a = \sqrt{1 + \frac{4kt}{P_e}}$$

$$P_e = 0.6\left(\frac{H_c}{D_c}\right)^{0.63}\left(\frac{J_t}{J_g(1-e_g)}\right)^{0.5} \tag{15}$$

$$k = \frac{3P}{2d_b}J_g \tag{16}$$

$$t_l = \frac{H_c(1-e_g)}{J_t} \tag{17}$$

As can be noted from the model equations, Yoon et al. [11] have related the flotation rate constant to column operating variables. P, the probability of particle capture, has been shown to be inversely proportional to the square of bubble diameter for small particle and bubble sizes. As a consequence, Yoon et al. [11] suggest that even a small change in bubble diameter can lead to a significant increase in the flotation rate parameter. Hence, bubble size plays an important role in governing flotation process performance. In order to estimate bubble size, Yoon et al. [12] have developed a mathematical model relating the air hold-up in the flotation column to bubble size distribution present under a variety of flow conditions. They have taken into account the loss of fine bubbles through tailings flow. For further details refer to Mankosa et al. [13].

Further, Yoon et al. [10] have recently suggested that the theoretical carrying capacity (C_a) is the product of superficial bubble surface area rate (S_b) and the mass of attached particles per unit of bubble surface area (M_b) moving through the froth phase, that is:

$$C_a = M_b S_b = \frac{2d_p \partial_p \beta}{3} \frac{6J_g}{d_b} = \frac{4d_p \partial_p \beta J_g}{d_b} \tag{18}$$

In practice, there is a limit to how high J_g can be increased. At very high J_g, pronounced bubble coalescence, increased bubble diameter and slugging can occur. Therefore, in general there is an upper limit to carrying capacity often referred to as maximum carrying capacity (C_{max}). Yoon et al. [10] have recently proposed the following equation relating C_{max} to column diameter.

$$C_{max} = \frac{4M_f Y}{\pi N D_c^2} \tag{19}$$

where

$$M_f = \frac{Q_f}{\frac{1}{\partial_f} + \frac{1-S}{S}}$$

Substituting the value of M_f in Equation 19 and simplifying in terms of D_c gives,

$$D_c = \left(\frac{4Y}{\pi C_{max}} \left(\frac{1}{\partial_f} + \frac{1-S}{S} \right)^{-1} \frac{Q_f}{N} \right)^{\frac{1}{2}} \tag{20}$$

Equation (20) is useful for scale-up as it gives D_c as a function of Q_f and N for desired values of C_{max} and Y.

To calculate column length H_c, Yoon et al. [10] have proposed the following equations:

$$t_l = \frac{H_c(1-e_g)}{J_t} \tag{21}$$

where $J_t = J_f + \alpha J_w$

$$t_p = t_l \left(\frac{\frac{J_t}{1-e_g}}{\frac{J_t}{1-e_g} + U_{sp}} \right) \tag{22}$$

Substituting Equation (21) into Equation (22) and simplifying gives,

$$H_c = t_p \left(\frac{J_f + \alpha J_w}{1-e_g} + U_{sp} \right) \tag{23}$$

However, before using Equation (23) for scale-up, the influence of mixing on retention time should be quantified using the axial dispersion model (Equation 14). That is, as the column diameter increases the

Peclet number decreases and mixing increases. Therefore, the recovery would decrease if k and t_p are held constant. To maintain the same recovery, t_p must be increased (assuming k is constant). Thus, a new column length would need to be calculated using Equation (23). Because of the interdependence among various column variables, an iterative process would need to be used for calculating column length H_c.

c) JKMRC model

Alford [7], conducted a study with the objective of validating the column flotation model based on axial dispersion theory proposed by Finch and Dobby [3]. Experimental data were generated from a number of plant columns operating under a variety of conditions. Based on the data generated Alford modelled the flotation rate parameter 'k' in terms of operating variables such as air rate, viscosity etc. and proposed the following correlation:

$$k_i = \frac{C_i\left(J_a - \frac{bA_c J_a^{0.75}}{Q_c}\right)^{0.75}}{\mu} \tag{24}$$

Alford noted that the froth depth and wash water flow rate did not have any significant influence on the flotation rate parameter (under normal operating range).

With regard to the validation of the Finch and Dobby model, Alford [7] reported that best results were obtained using the single zone model (ie overall recovery$=R_c$). Further, for laboratory and pilot columns (up to 1.5 m diameter), it was found that the axial dispersion plug flow model gave satisfactory results while for larger diameter plant columns the axial dispersion perfectly mixed model was applicable.

In addition, Alford [7] conducted parallel pilot and plant column surveys to study the influence of column diameter on metallurgical performance. After data analysis, Alford concluded that the axial dispersion model could not be used to predict the plant scale column metallurgical performance. It was found that the best indication of plant scale metallurgical performance was the pilot-scale grade-recovery curve. However, full-scale columns may be sized using the axial dispersion model. According to Alford [7], the variation in column diameter and uncertainty in model parameters may be accounted for by using the lowest parameters in the 90% statistical confidence interval, and relatively low air rates (ie "worst case" scenario). Alford notes that the above observations are only valid for a scavenging configuration and need to be validated for other types of circuit configuration(s). It should be noted that Alford [7] did not maintain similar operating conditions in both pilot column and plant column during the testwork.

Shortcomings of the kinetic models

- Models based on axial dispersion theory are not applicable for the froth zone and as yet no comprehensive model for the froth zone is available.

- Models based on axial dispersion theory do not take into account the influence of radial mixing and non-uniform velocity profiles on column metallurgical performance [3, 14].

- Models based on the axial dispersion model are inappropriate to use for vessel dispersion numbers greater than 0.2 because of the underlying assumption of the model [15]. This approximation assumes the end effects are negligible, which is clearly not the case at low aspect ratios. In the recent past, investigations by O'Connor et al., [14] have shown that the pulp vessel dispersion number in flotation columns is often greater than 0.3.

- Most of the models based on axial dispersion theory have used three-phase bubble correlations for bubble columns to model the solid phase dispersion coefficient in flotation columns. These correlations are unlikely to be valid for the column flotation process, since the correlation for bubble columns was derived for quite different conditions [14, 16].

- When applying the model based on axial dispersion theory where backmixing is large and the system is not closed, Levenspiel [6] advises caution. This is not only because the assumption of the axial dispersion model might not be satisfied under these conditions, but also in open vessels different ways of introducing and measuring tracers may lead to different residence time distribution curves. Levenspiel further notes that in general, the signal injected may not be the signal measured. One reason for this is that tracer may move upstream and reappear at a later time at the injection point [6].

- None of the models available at present under this category predicts concentrate grade (or has concentrate grade as a response parameter).

- All the models available under this category neglect feed characterisation. (In comparison, it is normal to have sink-float analysis for feed characterisation in gravity-based processes like jigs, heavy medium baths etc. or size analysis for feed characterisation in screens, hydrocyclones etc.). One implication of this is that it is not possible to infer the theoretical maximum recovery possible at a given product grade specification. Hence, the metallurgical efficiency cannot be quantified (and compared) under various conditions.

Non-kinetic models

In general, a non-kinetic model is a model which is not based on flotation kinetics. In other words, any model which does not fall under the category of kinetic model is a non-kinetic model. All the non-kinetic models which have been proposed to date are based on response surface methodology (RSM). Therefore, before discussing the various models available in this category, it might be useful to briefly discuss the response surface method.

Response surface methodology consists of a group of techniques used in the empirical study of relations between one or more measured responses, such as recovery and concentrate grade on the one hand, and the number of input variables such as air rate, feed rate etc., on the other. The RSM technique can be applied whenever cause-and-effect relationships between variables are being investigated. Broadly the RSM techniques consists of the following steps [1, 17]:

1. Experiment design.

2. Determining a mathematical model that best fits the data collected from the experiment design chosen in step 1, by conducting appropriate tests of hypotheses concerning model parameters.

3. Conducting simulation studies, that is, how a particular response is affected by a given set of input variables over the desired range.

4. If required, determining the optimal settings of various input variables that produce the maximum (or minimum) value of a response.

One of the main advantages of the RSM strategy when compared to the classical one-variable-at-a-time strategy is that it takes into account the interaction among variables and their influence on the response variables [18]. The one-variable-at-a-time strategy fails because it assumes each variable is independent of the others. This, however, is usually not true and many researchers have reported interactions among various flotation variables [18,19]. Further, the RSM strategy has an advantage over traditional regression analysis in that RSM strategy employs additional techniques, before and after the regression analysis is performed on the data [17]. Preceding regression analysis experiments have to be carefully designed (by using factorial or semi-factorial experiment design). After the regression analysis is performed, certain model testing procedures are applied and if required, additional optimization techniques are employed. Hence, the RSM strategy provides a better understanding of the system under consideration.

a) Musara and Mular model

Musara and Mular [20] conducted experiments with fine coal (96% <600 μm) on a 6.35 cm diameter laboratory column (5.5 m in height). They studied the influence of seven parameters namely, feed rate, feed percent solids, air flow rate, wash water flow rate, frother dosage, collecter dosage and froth depth, on the efficiency index. The efficiency index is defined as:

$$E = 100\left(\frac{A_f - A_t}{A_c - A_t}\right)\left(\frac{A_t}{A_c}\right) \tag{25}$$

where A_f is feed ash content, A_c is concentrate ash content and A_t is tails ash content.

They designed experiments using the semi-factorial method so that first order, second order, two term and three term interactions could be estimated. The raw data obtained from the experiments were mass-balanced using the simplex direct search routine which minimises the relative error between measured and calculated values, that is

$$Objective\ Function = \sum\left(\frac{M_o - M_c}{M_o}\right) \tag{26}$$

where M_o and M_c are observed and calculated values of the variables, respectively.

A second order model of the following form was fitted using the mass-balanced data to examine the influence of various column parameters on the efficiency index:

$$E = A_o + \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N} A_{ii} x_i^2 + \sum_{i<j}^{N} \sum A_{ij} x_i X_j + e \tag{27}$$

where A_o, A_i and A_{ii} are model constants, X_i is the ith input variable and e is the error term.

The full model equation for their data is available [20].

The analysis of variance (ANOVA) tests were carried out to identify 'terms' in the model having significant influence on the efficiency index. They found that air rate, froth depth, wash water flow rate and collector dosage did not have significant influence on efficiency index. Table 1 shows the terms in the model that were found to be statistically significant ie having significant influence on the efficiency index.

TABLE 1 Statistically significant flotation column parameters

a) First order parameters Feed rate, feed percent solids and frother dosage.
b) Two parameter interaction terms Feed percent solids and air rate, feed percent solids and frother dosage, feed rate and froth depth, froth depth and collecter dosage, frother dosage and collecter dosage and wash-water rate and collecter dosage.
c) Three parameter interaction terms Feed rate, feed percent solids and air rate and feed percent solids, air rate and froth depth.

One of the main drawbacks of evaluating flotation performance in terms of efficiency index is that it is possible to have similar values of the efficiency index under different conditions.

b) Crozier model

Crozier [18] recommends the use of Box-Wilson semi-factorial experiment design. The reason for semi-factorial or fractional-factorial design is that if the number of variables or levels to be tested exceeds three, the number of experiments required by a full factorial design becomes excessive. The Box-Wilson semi-factorial design provides an easy way to calculate a five level system which gives similar information as a full-factorial design, but with a minimum number of experiments [18].

In addition, Crozier [18] provides some general rules on design of experiments which can provide more efficient information:

- Symmetry in experimental layout - simplifies calculation greatly in coded form and helps in visualisation of responses,
- the number of levels fixes the shape of response curve that can be detected,
  2 levels, detects trends only
  3 levels, detects existence of response but no shape
  >3 levels, progressively refines the shape of response curve.

Once the experiments have been conducted and the relevant data generated, a second order model of the following form may be fitted:

$$R = A_o + \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N} A_{ii} X_i^2 + \sum \sum_{i<j}^{N} A_{ij} X_i X_j + e \qquad (28)$$

where A_o, A_i and A_{ii} are model constants, X_i is the ith input variable and e is the error term.

The interaction among variables may be limited to first order as most physical situations can be approximated by a quadratic function over a reasonable range of the input variables [18]. Also, in many situations the experimental accuracy may not permit reliable identification of higher order interactions among variables.

After the coefficients of the response function have been determined, an analysis of variance should be carried out to evaluate the statistical significance of the mathematical model and the significance of the various input variables. Once this has been done and if the model is found to be significantly representative of the system, an analysis can be made of the shape of the response function. This may be done either by contour plotting or a more sophisticated non-graphical analysis such as conical analysis.

c) CIMM model

A study was conducted at CIMM using Cu/Mo ore (96% < 74 μm) on a 5 cm diameter (2 m in height) laboratory column [18]. The influence of feed rate, air rate, froth depth and bias was investigated on column metallurgical performance. One of the objectives of the study was to mathematically model the experimental data and determine optimum column operating conditions.

Once the desired range of input variables was chosen, the experiments were designed using the semi-factorial approach. However, it should be noted that the decision regarding operating at extreme levels was taken on site on the basis of assay results, how stable the column was at previous operating conditions, as well as visual inspection of froth quality. As the pulp characteristics changed during the testwork, statistical analysis was carried out to evaluate the influence of following input variables on the metallurgical performance (concentrate grade and recovery).

Input variables:

X1 = Feed percent solids (by weight)
X2 = Feed grade (Cu%/ Mo%)
X3 = Apparent average residence time of the pulp in column (min.)
and X4 = Apparent average residence time of air in column (min.).

Response variables:

Y1 = Concentrate grade (Cu%/ Mo%)
and Y2 = Recovery (%).

Unlike Musara and Mular [20], CIMM have not reported the fitted model equations. They have, however, commented that for most linear polynomial equations R^2 was greater than 95% [18]. Further, on the basis of model equations, CIMM have suggested optimum operating conditions for 5 cm columns which will yield best metallurgical performance (ie. maximum recovery at specified concentrate grade).

Shortcomings of the Non-kinetic models

- All models based on RSM are valid only in a limited range of the input variables, and do not provide a sound basis for extrapolation [1].
- All the models available under this category lack in feed characterisation. One of the implications of this is that it is not possible to infer the theoretically maximum recovery possible at a given product grade specification. Hence, the metallurgical efficiency cannot be quantified (and compared) under various conditions.
- Box et al. [1] on the use of statistical techniques have commented that statistical techniques are most effective when combined with subject matter knowledge. They caution against the over-zealous use of some statistical tools or methodology and advise that the statistical techniques should be used as an important adjunct to, not a replacement for, the skill of the experimenter.

SUMMARY

The mathematical models of the column flotation process available in the literature can broadly be divided into two categories namely, kinetic models and non-kinetic models. Kinetic models as the name suggests are models based either wholly on flotation kinetics or flotation kinetics is one of the main components of the overall model form. On the other hand, non-kinetic models are models which are not based on flotation kinetics.

Under the category of kinetic models, Finch & Dobby were the first to propose and model the column flotation process based on axial dispersion theory. Since then studies have been conducted to validate the axial dispersion model and model improvements suggested. At VPI, Yoon and his co-workers have related the flotation rate parameter (k) to some column operating variables. In addition, Yoon et. al. have recently suggested an alternative scale-up method to the one proposed by Finch & Dobby. Further, Yoon and co-workers have suggested a correlation for carrying capacity from theoretical consideration. Also, Yoon et al. have related the maximum carrying capacity to column diameter. At JKMRC, Alford has also suggested correlation between flotation rate parameter and column operating variables. Further, Alford found that the axial dispersion model cannot be used to predict plant scale metallurgical performance based on pilot scale results. However, Alford has suggested an empirical method for sizing of full-scale columns based on pilot results using the axial dispersion model. From the literature the VPI model developed by Yoon and his co-workers appears to be the best under this category.

The non-kinetic models that have been proposed to date are all based on response surface methodology (RSM). Lekki and Laskowski were the first to use this methodology in modelling the flotation process. Since then a few researchers have reported using response surface methodology for modelling the column flotation process. Musara & Mular have studied the influence of seven column parameters and their interactions (limited to three parameter) on efficiency index. Crozier in his book Flotation: Theory, Reagents and Testing has discussed in some detail and given guide-lines on how the response surface methodology may be applied to model the flotation process. Further, as an example Crozier has presented CIMM studies on column flotation. The CIMM model studied the influence of four column parameters on concentrate grade and recovery using the response surface methodology with one of the objectives being to determine the optimum operating conditions. As the response surface methodology consists of a well established set of statistical techniques with applications not only in the field of Engineering but also in many other areas e.g. Social sciences, Biological sciences etc., there is no good or bad model under this category. Nonetheless, the approach taken by Crozier appears to be most appropriate for modelling of the flotation process.

The main shortcomings of the kinetic models based on axial dispersion theory are that the models are not applicable for the froth zone or the influence of radial mixing, and non-uniform velocity profiles have been neglected. In addition, none of the kinetic models have concentrate grade as a response parameter. The main limitations of non-kinetic models based on response surface methodology are that the models are applicable to a limited range of input variables and provide no sound basis for extrapolation. All the models available to date, both kinetic and non-kinetic, lack feed characterisation and hence the metallurgical efficiency cannot be quantified.

ACKNOWLEDGMENTS

The authors would like to thank the Minerals and Energy Research Institute of Western Australia (MERIWA) for providing financial support. The help rendered by Harbour Lights plantsite, Chemistry Centre (WA) and WA School of Mines personnel is gratefully acknowledged. The authors would in particular like to mention Mr Joe Ariti and Mr Bruce Lumley of Harbour Lights, Dr John Hosking and Dr Tony Bagshaw of Chemistry Centre (WA) and Dr Denis Yan and Dr Jeff Vaughan of the WA School of Mines. Thanks are also due to Prof. Irvine Reilly for providing valuable advice on column flotation. Finally, we owe thanks to Ms Natalie McBeath for her patience and skilful typing of this paper. This paper is published with the permission of the Director, Chemistry Centre (WA).

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NOMENCLATURE

A_c	column cross-sectional area
C_a	concentrate carrying capacity
C_i	mineral specific term
$C_{max.}$	maximum concentrate carrying capacity
D	axial dispersion cofficient
D_c	column diameter
d_b	bubble diameter
d_{80}	80% passing particle size
e_g	air hold-up
g	gravitational acceleration, cm/s^2
H_c	collection zone height
H_f	froth height
J_a	superficial air flow rate $= Qa/A_c$
J_f	superficial air flow rate $=Qf/A_c$
J_{sl}	superficial slurry velocity $= Qsl/A_c$
J_t	superficial tailings rate $= Qt/A_c$
J_w	superficial wash water rate $= Qw/A_c$
k	rate constant
M_b	mass of particles attached per unit of bubble surface area
M_f	mass feed rate of solids
N	number of columns
N_p	vessel dispersion number

P	probability of particle capture
Pe	Peclet number
Q_a	volume air flow rate
Q_f	volume feed flow rate
Q_{sl}	slurry flow rate
Q_t	tailings flow rate
Q_w	wash water flow rate
R_c	pulp zone recovery
R_{ep}	reynolds number of particle
R_f	froth zone recovery
S	fractional solids content
S_b	superficial bubble surface area rate
t_l	liquid residence time
t_p	particle residence time
U_i	liquid (or solid) interstitial velocity
U_{sp}	slip velocity between particle and water
Y	product yield
α	bias factor
β	particle packing factor
∂_c	mean particle density in concentrate
∂_l	liquid density
∂_p	particle density
∂_{sl}	pulp density
ϕ_s	volume fraction solids in slurry
t	mean residence time
μ	pulp viscosity
μ_f	feed viscosity