

Seismic behavior of RC coupled shear walls repaired with CFRP laminates having variable fibers spacing

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Abstract

The paper considers the seismic analysis of reinforced concrete (RC) coupled shear walls structures strengthened by bonded composite plates having variable fibers spacing. An efficient analysis method that can be used regardless of the sizes and location of the bonded plates is proposed in this study. In the numerical formulation, the adherents and the adhesives layers are all modeled as shear walls elements, using the mixed finite element method. Dynamic analysis was performed to investigate the influence of the fibers redistribution of the bonded plates on the lateral deflections of RC coupled shear walls. This conceptual study has demonstrated the feasibility of mitigating the seismic response of RC coupled shear walls building structures by using composites plates having variable fibers spacing. © 2006 Elsevier Ltd. All rights reserved.

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1. Introduction

In medium to high-rise buildings, reinforced concrete (RC) walls systems are commonly used to resist forces induced by earthquake. However, these structural systems are required to withstand earthquakes without collapsing and without incurring major damage. To accomplish this goals, the structure needs to have: (i) high lateral strength, (ii) high ductility, (iii) high energy dissipation capacity and (iv) sufficient shear stiffness to limit interstorey drifts.

Continuum approaches have been frequently proposed for the dynamic analysis of coupled shear walls, where the discrete system of connecting beams is replaced by homogeneous medium of equivalent properties [1,2]. Coupled shear walls have been also analyzed by standard numerical techniques such as the finite element method [3,4] and the finite strip method [5], which can cope with any type of material and geometric nonuniformity.

In order to achieve satisfactory earthquake response of RC coupled shear walls structures, three methods can be identified as being practical and efficient. These are structural isolation, energy absorption at plastic hinges and use of mechanical devices to provide structural control [6–10]. The use of those methods is very efficient but expensive and difficult to carry out.

From a technological point of view, the strengthening of RC coupled shear walls structures has been accomplished by adopting standard materials, mainly cement, concrete and steel. However, new reinforcement approaches are rising; they are based on the idea that the strengthening should be light and removable and, should not change the structural scheme of the construction. Composite materials appear to be good candidates to substitute standard materials. Since they are light, simple to install and are also removable. Moreover, composite materials are characterized by high strength, good durability and lower installation and maintenance cost.

Thus, one promising technique to improve the overall strength of RC coupled shear walls structures and to reduce their seismic vulnerability is to retrofit the RC coupled shear walls structures using fibers reinforced plastic (FRP).

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Extensive testing [11–15] has shown that externally bonded carbon fibers reinforced polymer (CFRP) laminates are particularly suited for improving the short-term behavior of deficient reinforced concrete beams and slabs.

Therefore, few researches on efficient analysis and experimental studies of strengthened shear wall has been undertaken [16–18].

In order to fill the gap between the technological interest and the development of appropriate design suggestions, this paper is aimed at investigating the seismic response of the RC coupled shear walls structures strengthened by means of CFRP composite plates. In conventional configuration, these plates are made of plies, the fibers within each ply being parallel and uniformly spaced. However, it is possible that significant increases in structural efficiency may be obtained by varying the fibers spacing of the bonded plates i.e., packing them closely together in regions where great stiffness is needed, but less densely in other regions. Leissa et al. [19] are the firsts to study the effect of variable fibres spacing on vibration and buckling problems.

If fibers spacing of the bonded plates varies, the analysis is considerably more complicated than for uniform spacing. Then the material must be treated as nonhomogeneous on the macroscopic scale, as well as on the microscopic.

In the numerical formulation of the present study, the adherents and the adhesives layers are all modelled as shear walls elements, by using a mixed finite element method [3] to deduce the stiffness matrix of the equivalent shear wall element having variable fibers spacing. The finite element method (FEM) is employed to carry out the dynamic analysis, in which a direct integration dynamic analysis was used to obtain the response of the structure under seismic loading. This analysis assemble the mass, stiffness and damping matrices and solve the equations of dynamic equilibrium at each point in time. The response of the structure is obtained for selected time steps of the input earthquake accelerogram. Numerical results are presented that relate to the performance of reinforced concrete coupled shear walls strengthened with composite sheets having parallel and variable fibers spacing.

2. Finite element for analysis of shear walls

Many different finite elements are now available. However none of them are suitable for shear/core walls analysis. For instance, some of the lower-order elements such as the bilinear element, are found to be afflicted by the shear locking problem which renders the elements over stiff under bending actions. Because displacement shape functions of this element are expressed in linear functions, deformation of element edges can be expressed by straight lines and the shear stress in an element are constant and cannot represent the actual stress distribution accurately if the finite element mesh is not fine. However, it is felt that the best method of dealing with parasitic shear is to avoid them by using elements that can exactly represent the strain state of pure bending.

To improve the computational efficiency of the finite element method, finite strip element [20], and higher order element [21,22] were developed to modelize the shear wall with the rotational degrees of freedoms (DOF) for represent the strain state of pure bending so to avoid parasitic shear problem.

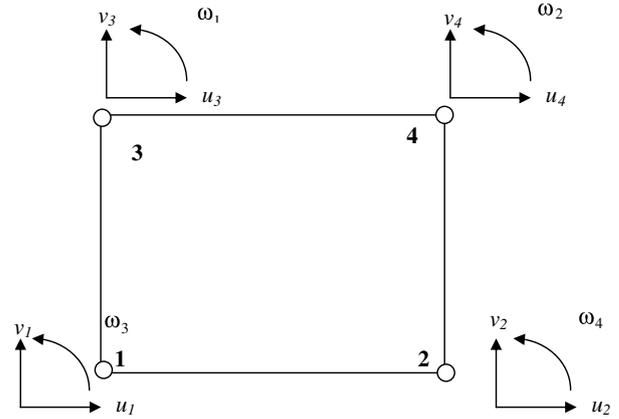


Fig. 1. Cheng's beam type element (12 DOF).

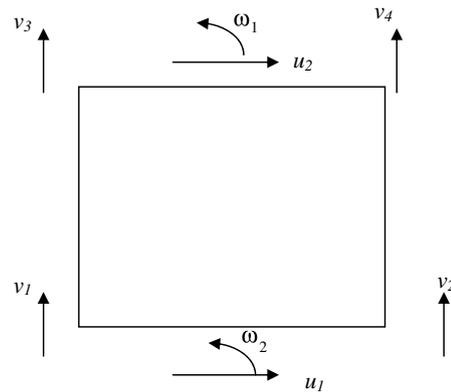


Fig. 2. Kwan's strain based element (8 DOF).

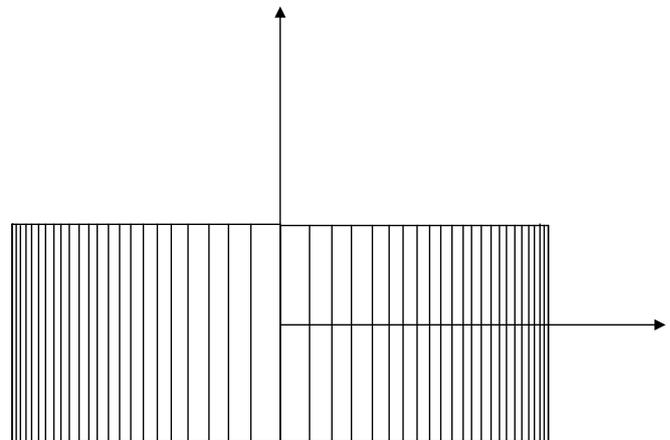


Fig. 3. Nonhomogeneous orthotropic composite plate.

Therefore, the 12 DOF plane stress element as [23] and Lee element [22] with drilling DOF (Fig. 1) was used in many research works as [24].

As suggested by Kwan [3,4], by neglecting the lateral strain in the wall, which are generally of little significance. The DOF can be reduced from 12 to 8 as shown in Fig. 2.

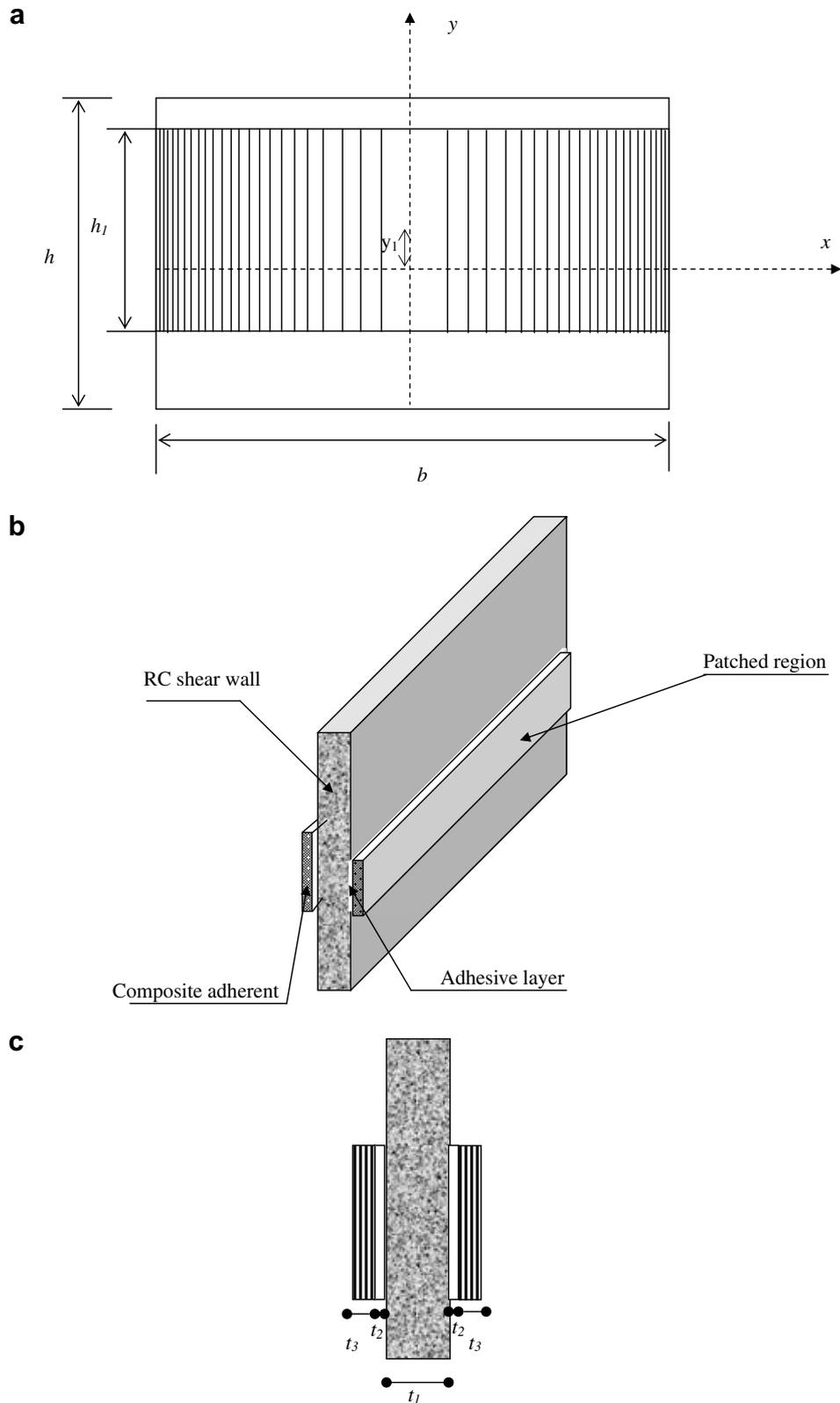


Fig. 4. A strengthened RC shear wall element: (a) front view; (b) perspective view and (c) lateral view.

Use of this simplified Cheung’s element, which is computationally more efficient, is recommended rather the original Cheung’s element.

Using the mixed finite element method, Kwan [3] developed a wall element with the eight DOF. This element included two existing elements, namely the simplified Cheung’s element [20] and Kwan’s strain based element [4].

3. Theory and solution procedure

3.1. The elastic modulus of composite plates having variable spacing

The elastic modulus E_y and G_{xy} for the composite material may be expressed in terms of the properties of the fibre and the matrix material by applying the law of mixtures [25]:

$$E_y = E_f \left(V_f + \frac{1 - V_f}{R_1} \right) \tag{1}$$

$$G_{xy} = \frac{E_f}{2(1 + \nu_f)[R_3(1 - V_f) + V_f]} \tag{2}$$

where

$$R_1 = \frac{E_f}{E_m}, \quad R_3 = \frac{G_f}{G_m} \tag{3}$$

In the above equations the subscripts f and m are used to denote properties of the fiber and matrix respectively, and V_f is the volume fraction of the fibers in the composite material. For material having variable fiber spacing, V_f is a function of x , and therefore E_y and G_{xy} are each functions of x .

Suppose, for example, the fiber volume fraction varies parabolically as $V_f = \xi^2$. Where $\xi = 2x/b$ is the nondimensional coordinate having its origin at the shear wall element centre. With this severe variation in the fibers spacing, the material at the wall edges $\xi = \pm 1$ is all fibers, whereas at the centre it is all matrix, as shown in Fig. 3.

3.2. Stiffness matrix of a strengthened shear wall element and connecting beam

As shown in Fig. 4, consider a strengthened shear wall system, which is described as a combination of composite thin plates having variable fibres spacing and a reinforced concrete wall, attached to the two sides using adhesives plates. The adhesives are assumed to be isotropic shear

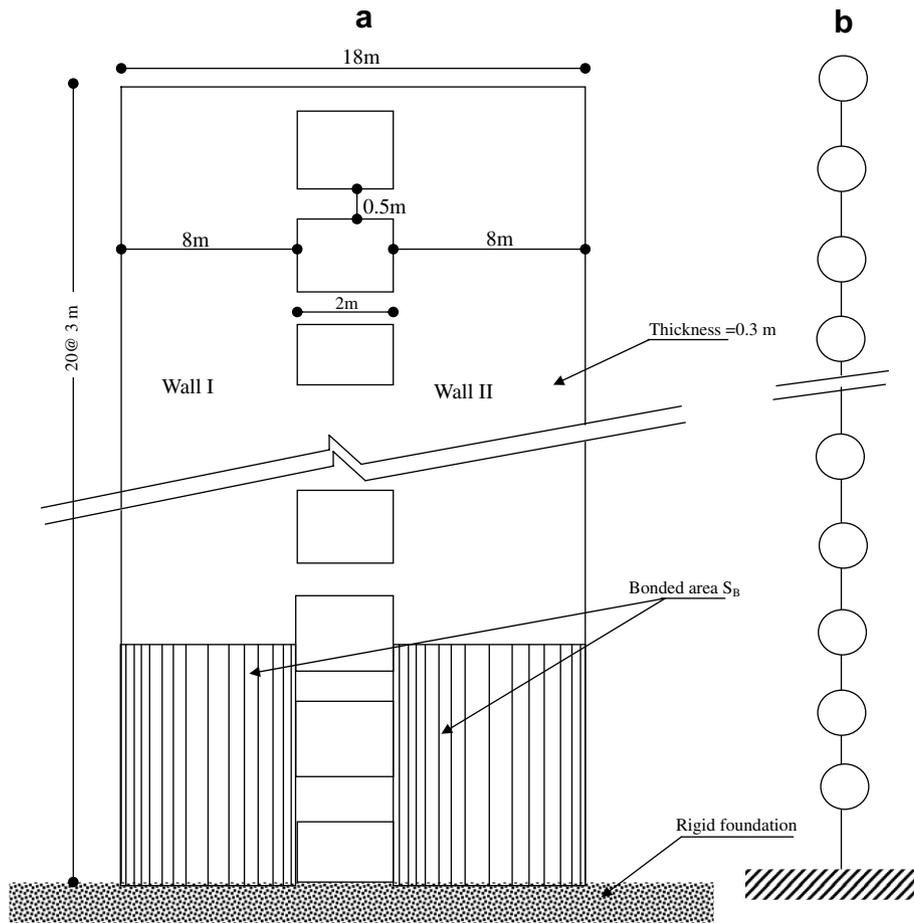


Fig. 5. (a) Strengthened coupled shear wall considered in the parametric study. (b) Lumped mass model for coupled shear walls.

walls of thickness t_2 , the external adherents are considered to be composite shear walls of thickness t_3 bonded to the area $s_b^e = bh_1$. In static analysis problem, the shear wall element which has the total area $s_f^e = bh$ is subjected to lateral load. Let us denote by u the deflection, v the vertical displacement and ω the rotation of the vertical fibers.

In this paper, the mixed finite element method established by Kwan [3] was deployed to deduce the stiffness matrix of a proposed strengthened shear wall element. Hence, the displacement components at any point within the wall element may be expressed in the terms of the nodal DOF of the element as follows:

$$v = v_1 \left(\frac{1-x}{2} - \frac{y}{h} \right) \left(\frac{1-y}{2} - \frac{x}{b} \right) + v_2 \left(\frac{1+x}{2} + \frac{y}{h} \right) \left(\frac{1-y}{2} - \frac{x}{b} \right) + v_3 \left(\frac{1-x}{2} - \frac{y}{h} \right) \left(\frac{1+y}{2} + \frac{x}{b} \right) + v_4 \left(\frac{1+x}{2} + \frac{y}{h} \right) \left(\frac{1+y}{2} + \frac{x}{b} \right) + \left(\frac{6}{h}(u_1 - u_2) - 3(\omega_1 + \omega_2) \right) \left(\frac{1}{4} - \left(\frac{y}{h} \right)^2 \right) \left(\frac{x}{2} + \frac{b}{4} \right) \quad (4)$$

$$u = u_1 \left(\frac{1}{2} - \frac{3}{2} \left(\frac{y}{h} \right) + 2 \left(\frac{y}{h} \right)^3 \right) + \omega_1 h \left(-\frac{1}{8} + \frac{1}{4} \left(\frac{y}{h} \right) + \frac{1}{2} \left(\frac{y}{h} \right)^2 - \left(\frac{y}{h} \right)^3 \right) + u_2 \left(\frac{1}{2} + \frac{3}{2} \left(\frac{y}{h} \right) - 2 \left(\frac{y}{h} \right)^3 \right) + \omega_2 h \left(\frac{1}{8} + \frac{1}{4} \left(\frac{y}{h} \right) - \frac{1}{2} \left(\frac{y}{h} \right)^2 - \left(\frac{y}{h} \right)^3 \right) \quad (5)$$

The strain energy for each wall element can be written as $U^e = U_B^e + U_S^e$ (6)

where U_B^e and U_S^e are the strain energy due to the bending and shear effects, respectively, which are written as a function of the strains on the shear wall element. The strain energy considering only the bending effect U_B^e is done as

$$U_B^e = \frac{1}{2} \sum_{i=1}^3 \int_{vol} E_y^{(i)} (\epsilon_y)^2 dvol_{(i)} \quad (7)$$

According to the relationships (1), the strain energy U_B^e is expressed as

$$U_B^e = \frac{1}{2} \left[E_y^{(1)} t_1 \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\epsilon_y)^2 dx dy + 2t_2 E_y^{(2)} \int_{y_1 - \frac{h_1}{2}}^{y_1 + \frac{h_1}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} (\epsilon_y)^2 dx dy + 2E_f t_3 \int_{y_1 - \frac{h_1}{2}}^{y_1 + \frac{h_1}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left(V_f + \frac{1 - V_f}{R_1} \right) (\epsilon_y)^2 dx dy \right] \quad (8)$$

in which

$$\epsilon_y = \frac{dv}{dy} \quad (9)$$

the expression of the strain energy which relate to the shear effect may be written as

$$U_S^e = \frac{1}{2} \sum_{i=1}^3 \int_{vol} G_{xy}^{(i)} (\gamma_{xy})^2 dvol_{(i)} \quad (10)$$

The insertion of the relationship (2) in the above equation leads to the following expression:

$$U_S^e = \frac{1}{2} \left[G_{xy}^{(1)} t_1 \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\gamma_{xy})^2 dx dy + 2t_2 G_{xy}^{(2)} \int_{y_1 - \frac{h_1}{2}}^{y_1 + \frac{h_1}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} (\gamma_{xy})^2 dx dy + 2E_f t_3 \int_{y_1 - \frac{h_1}{2}}^{y_1 + \frac{h_1}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{(\gamma_{xy})^2}{2(1 + \nu_f)[R_3(1 - \nu_f) + \nu_f]} dx dy \right] \quad (11)$$

in which

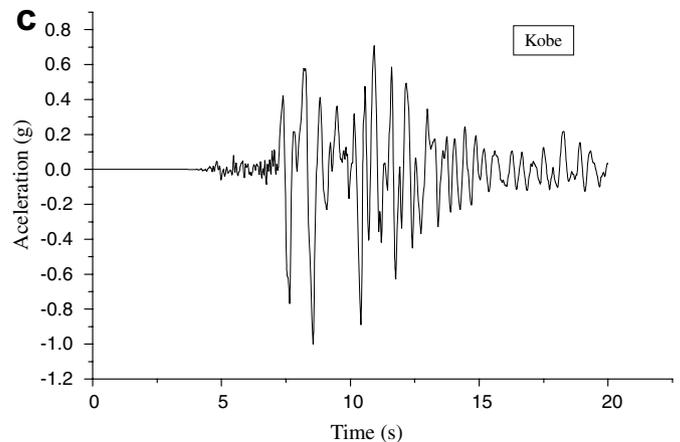
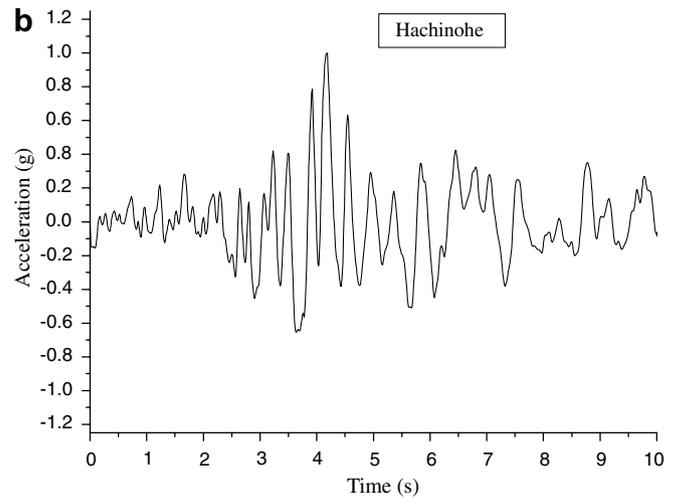
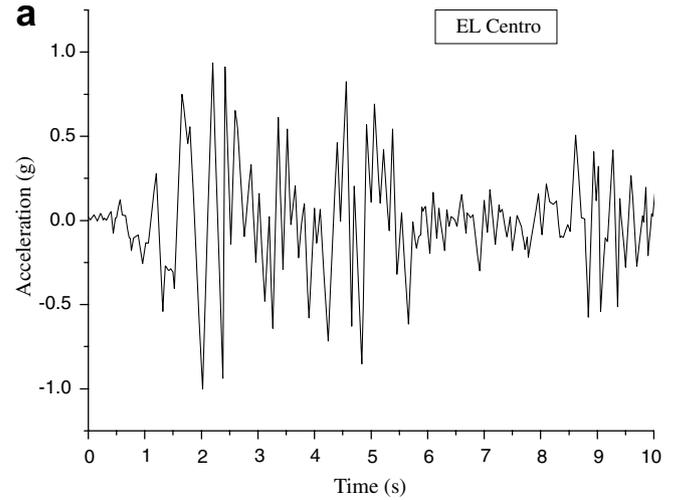


Fig. 6. Earthquake records: (a) El Centro; (b) Hachinohe and (c) Kobe.

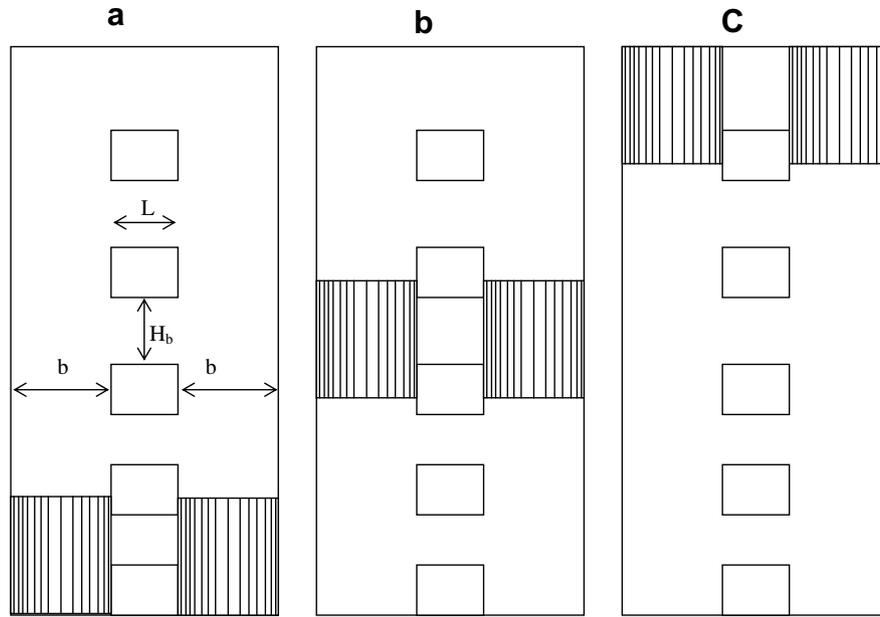


Fig. 7. Strengthened coupled shear walls: (a) plates bonded at the base; (b) plates bonded at the centre and (c) plates bonded at the top.

$$\gamma_{xy} = \frac{dv}{dx} + \frac{du}{dy} \tag{12}$$

where $E_y^{(i)}$ and $G_{xy}^{(i)}$ ($i = 1 \dots 3$) are the Young's modulus in the y direction and the shear modulus, of the RC shear wall, adhesive and bonded material respectively. As shown in Fig. 4, the patched region is applied along the wide b and from y_1 about the shear wall element centre.

The following form of the strain energy for each wall element is expressed by

$$U^e = \frac{1}{2} d_e^t K_w d_e \tag{13}$$

where the nodal displacement vector $d_e^t = \{u_1 \omega_1 v_1 v_2 u_2 \omega_2 v_3 v_4\}$

To carry out numerical analysis, we employ the standard finite element formulation to determine the stiffness matrix K_w of a strengthened shear wall element. No explicit procedure to determine the stiffness matrix needs to be

Table 1
Mechanical properties of materials

Material	E (MPa)	ν
Epoxy	3445	0.35
Graphite	275 000	0.28

Table 2
Fibers distribution

$V_f(\xi)$	$(V_f)_{av}(\%)$	$(V_f)_{max}(\%)$
ξ^2	0.33	100
$1/2 + 1/2\xi^2$	0.67	100
$1/2 + 1/4\xi^2$	0.58	75

given here. However, mores detailed information can be found in reference [26].

The coupling beam may be modelling by a standard two-nodes beam element with shear deformation taken into account.

4. Dynamic analysis

For dynamic analysis of the structure, it is required to determine both the mass and stiffness matrices. In order to determine the mass matrix, the structure is considered as a discrete lumped mass system as shown in Fig. 5b. It is assumed that the floors slabs are considered to be rigid in their planes and they move horizontally and vertically as a rigid bodies, i.e rotatory inertia is neglected in comparison to the lateral and vertical inertias effects. The lumped mass matrix of this equivalent multi-degree-of-freedom system commonly used for dynamic analysis of structure in engineering is taken as a diagonal matrix.

The damping matrix of the model is assumed to be proportional to the stiffness and mass matrices by the Rayleigh's proportionality factors [27]. In the present analysis, the 1st and 2nd vibration modes are used, and the damping ratio is taken as 5%. The Newmark- β step-by-step time-integration method [28] is employed to obtain the solution of the dynamic equation.

Results (obtained via computer programs prepared in FORTRAN) are presented in this section for coupled shear walls strengthened by bonded composite thin plates having variable fibers spacing. Many examples that relates on the applicability of the present concept have already been given in [16,17]. No attempt is made here to add any more examples.

4.1. Earthquake records

In general, earthquakes have different properties such as peak acceleration, duration of strong motion and different ranges of dominant frequencies and therefore have differ-

ent influences on the structure. Three earthquake excitations are used in this study. For more consistent comparison, all earthquake records were scaled to the peak acceleration of 1g. El Centro; Hachinohe and Kobe earthquake records (see Fig. 6) have been selected to investigate the dynamic response of the structure.

4.2. Numerical example

A 20-storey RC coupled shear wall structure strengthened by composite plates having variable fibers spacing as shown in Fig. 5a was considered for a numerical study. The purpose of this study is to illustrate how the fibers

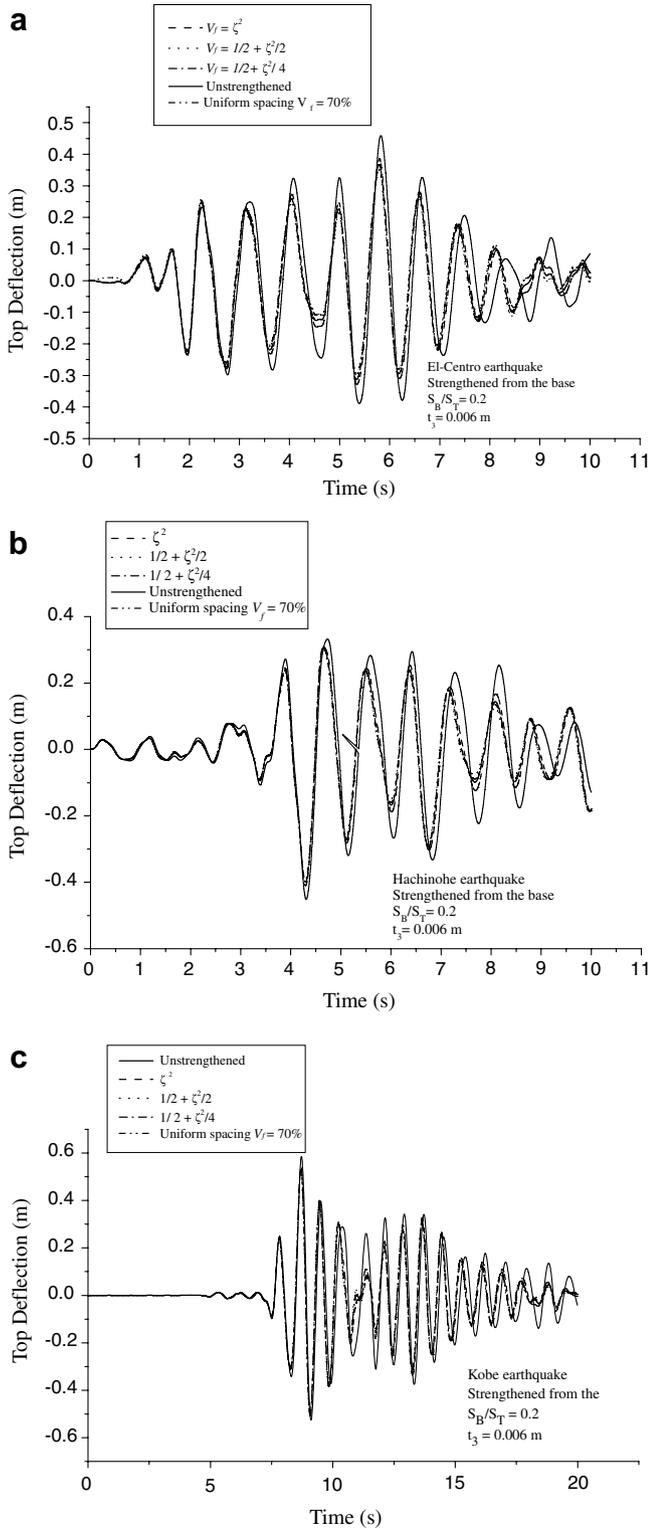


Fig. 8. Effect of the fibers distribution on the top deflection: (a) El Centro; (b) Hachinohe and (c) Kobe.

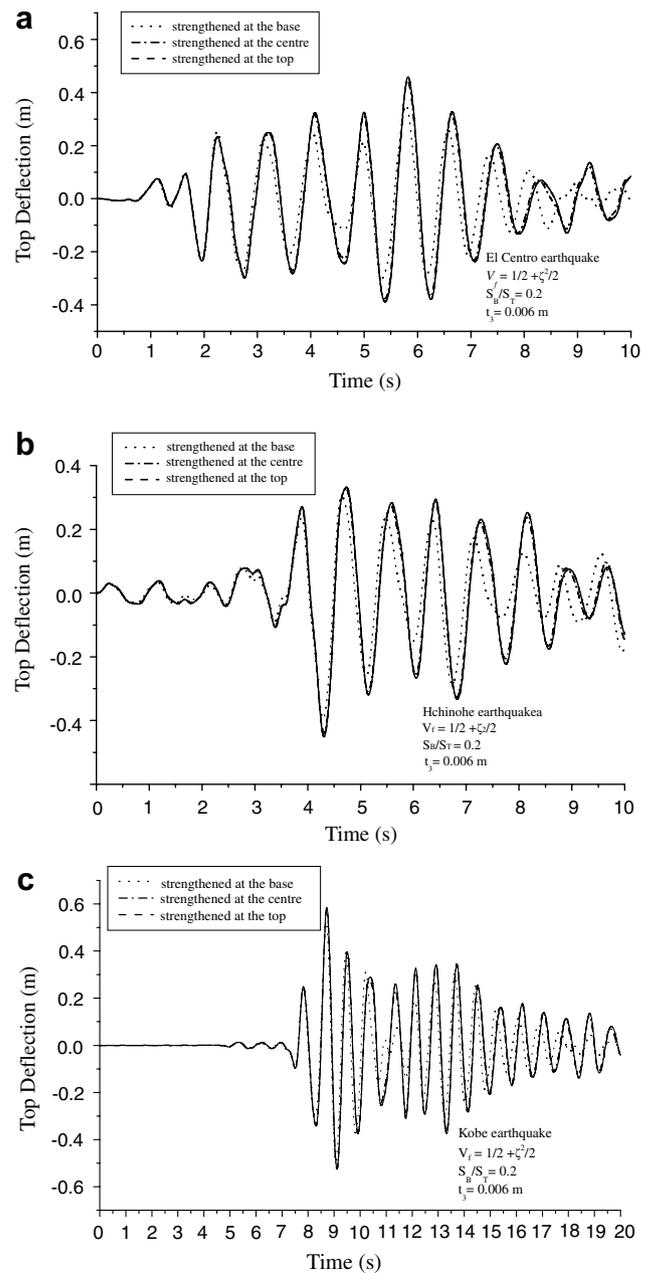


Fig. 9. Effect of the bonded plates location on the lateral displacement: (a) El Centro; (b) Hachinohe and (c) Kobe.

distribution; the location and the thickness of the composite sheets (see Fig. 7) affect the top deflection under the three selected earthquake records.

We note that S_B is the area of the bonded composite plates and S_T is the total area of the two walls (Wall I and Wall II as shown in Fig. 5a).

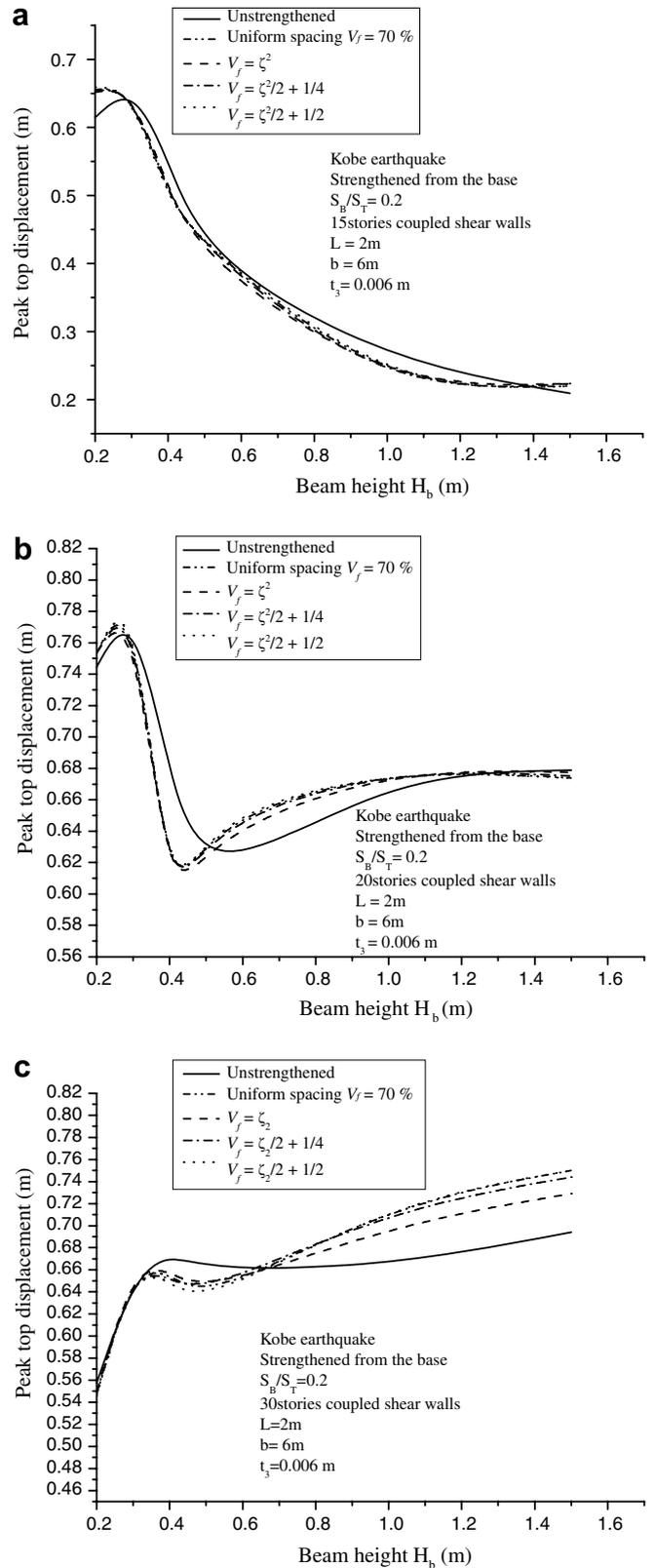
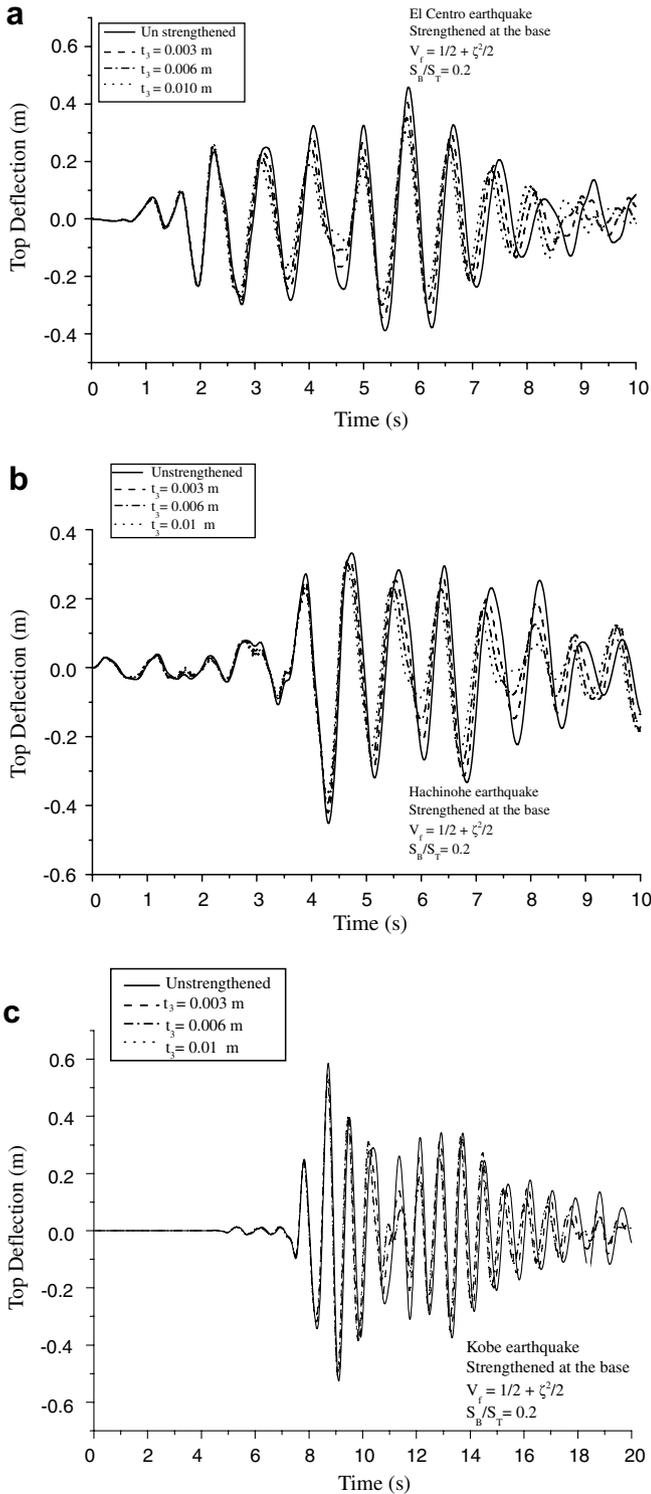


Fig. 10. Effect of the thickness of the bonded plates on the lateral displacement: (a) El Centro; (b) Hachinohe and (c) Kobe.

Fig. 11. Typical plot of peak top response versus H_b : (a) 15 stories; (b) 20 stories and (c) 30 stories.

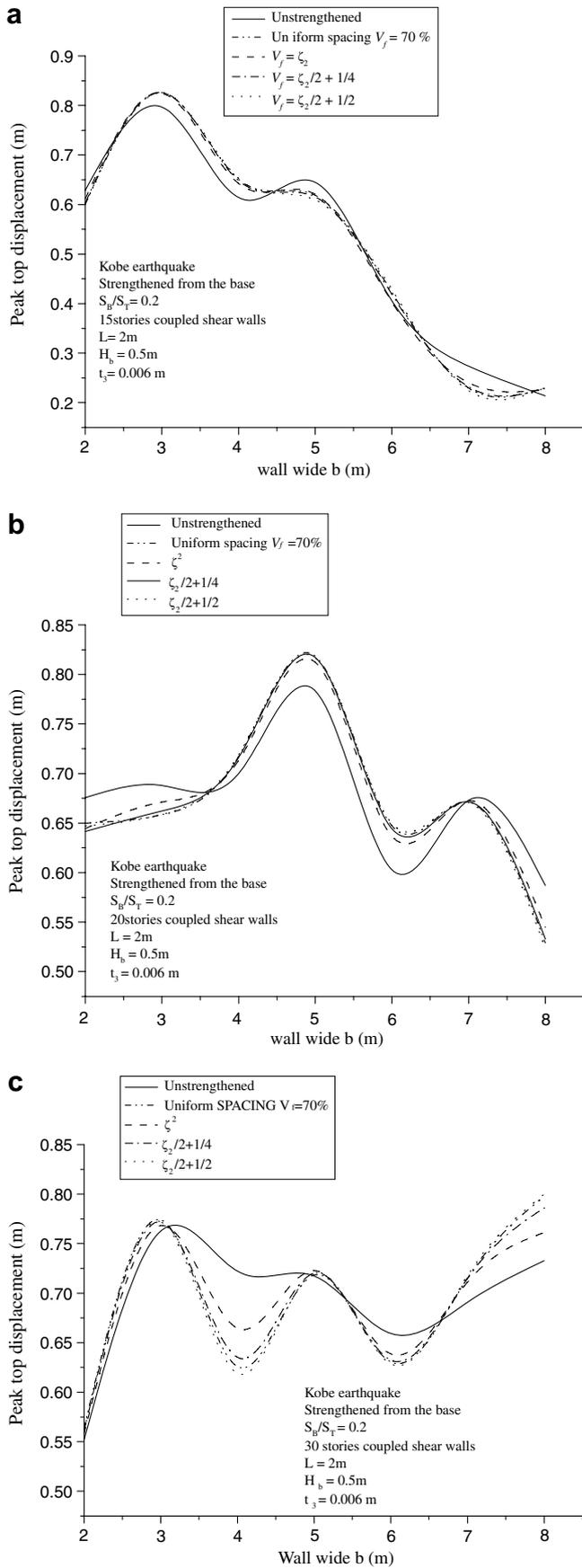


Fig. 12. Typical plot of peak top response versus b: (a) 15 stories; (b) 20 stories and (c) 30 stories.

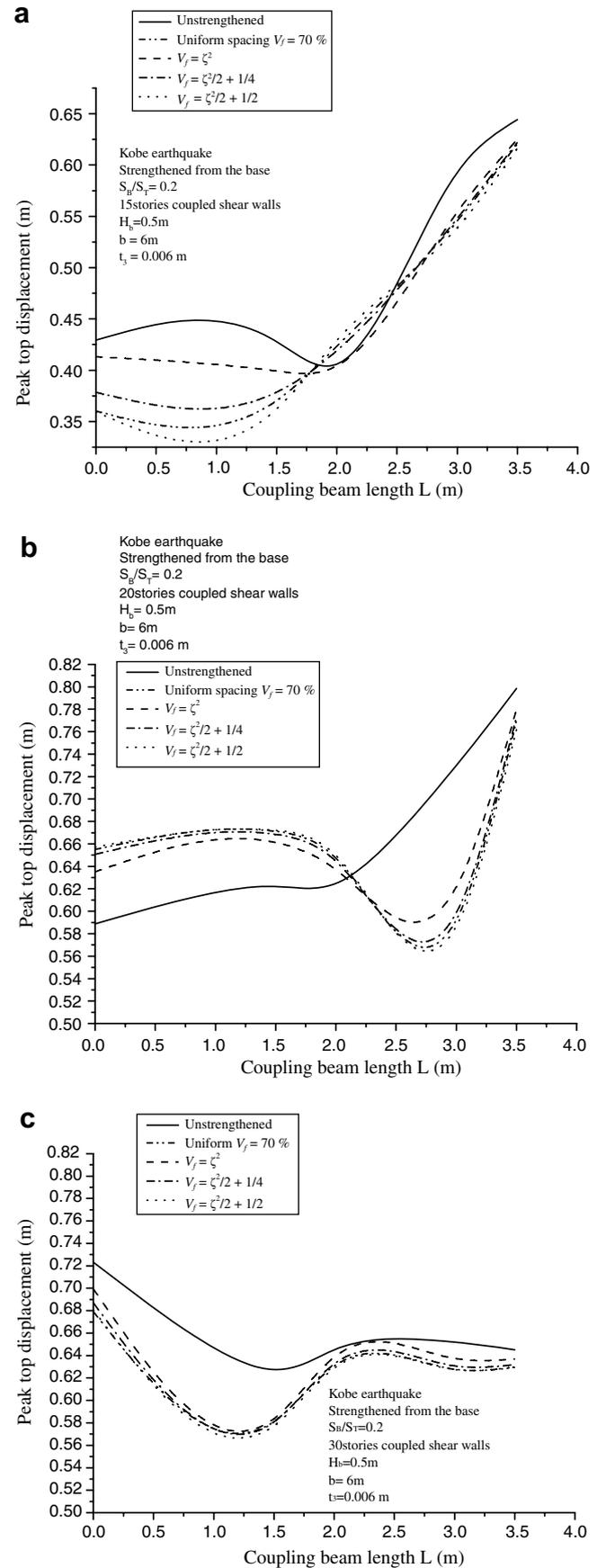


Fig. 13. Typical plot of peak top response versus L, (a) 15 stories; (b) 20 stories and (c) 30 stories.

The composite material investigated, consisting of graphite with an epoxy matrix. Material properties are given in Table 1.

Three different distributions of fibers were considered, each characterized by variations of the fibers volume fraction V_f , as given in Table 2. As can be seen from Table 2, these fibers distribution fall into V_f of 75% and 100%. Table 2 also gives $(V_f)_{av}$, which is the average value of V_f for each distribution. That is

$$(V_f)_{av} = \int_0^1 V_f(\xi) d\xi \quad (14)$$

The material properties of the reinforced concrete and the adhesives adopted in this study are respectively:

$$E_y^{(1)} = 30.0 \text{ GPa}, \quad \nu_{xy}^{(1)} = 0.18$$

$$E_y^{(2)} = 3.0 \text{ GPa}, \quad \nu_{xy}^{(2)} = 0.35.$$

And the mass of each storey is taken as 120 000 kg.

The results for all models of the strengthened coupled shear walls structure under each of the three earthquake records are presented below. Results plotted in Fig. 8 displayed good performance of the distribution such as $V_f = 1/4 + \xi^2/2$ with an average reduction of 13.6%. On the other hand, the $V_f = \xi^2$ distribution affect the top deflection with an average reduction of 11%. The highest reduction has been achieved by the $V_f = 1/2 + \xi^2/2$ distribution with an average reduction of 16%.

In term of the location of the bonded area. As shown in Fig. 9 the best performance with an average reduction of 16% has been achieved by composite plates located at the base. The bonded plates located at the center and the top of the structures affect moderately the top deflection with an average reduction of 2.33% and 0.1%, respectively.

Analyzing the effect of the thickness of the bonded composite plates (see Fig. 10), it could be observed that a larger thickness lead to small deflection, with an average reduction on the top deflection of 21.5% for composites bonded plates with $t_3 = 0.01$ m. The second highest reduction has been achieved by bonded composite plates with $t_3 = 0.006$ m with an average reduction of 16%. This was followed by a structure with $t_3 = 0.003$ m with an average reduction of 8%.

In term of efficiency of the bonded composite sheets under a variety of earthquake loadings, the significantly best performance has been achieved under El Centro earthquake with an average reduction 15.4%. The second best performance with an average reduction of 8.7% has been displayed under the Hachinohe earthquake. In the case of the Kobe earthquake record, the efficiency of the bonded composite plates has been slightly lower with a reduction of 6.7%.

4.3. Parametric investigations

The seismic response of three examples coupled shear walls having 15, 20 and 30 stories level are analyzed to

investigate the effect of the geometric characteristics of coupled shear walls under Kobe earthquake Fig. 6c. The geometric parameters of coupled shear walls are presented in Fig. 7a; H_b varies from 0.2 to 1.5 m, b from 2 to 8 m and L from 0.0 to 4.0 m. Results presented illustrated in Figs. 11–13 demonstrate the feasibility of using composite plates having variable fiber spacing to mitigate the adverse seismic response of RC coupled shear walls buildings structures. As the natural frequencies of these structural models range from 0.125 Hz to 1.897 Hz and were often within the frequency range of dominant modes of Kobe earthquake considered in this study (0.29–1.12 Hz), this study treated resonant vibration. It was probably due to this reason that there were no particular trends in the responses under the seismic loading.

The figures show that the seismic response of coupled shear walls structures strengthened with variable fibers spacing that is $V_f = 1/4 + \xi^2/2$ is practically the same to that with uniform spacing. We found that 58% of fibers are needed to have the same effect when we vary the fibers spacing, which is more effective from economic point of view. The outcome of this study is to find an efficiency concept, both in new designs and retrofitting RC coupled shear walls high-rise buildings.

5. Conclusions

An efficient seismic analysis method for the analysis of RC coupled shear walls structures strengthened with thin composites plates having variable fibers spacing was proposed in this study by using the mixed finite element method. In this study, a typical 20-storey RC strengthened coupled shear walls was analyzed under three different earthquakes. Significant improvement in the seismic deflections was observed when the fibers are clustering near the wall edges. This study can be extended to provide an efficiency concept in the field of strengthening RC coupled shear walls buildings in some earthquake-prone countries such as Algeria.

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