

## ~~A New PSO-RBF Model for Groundwater Quality Assessment~~

~~Zhang Bi<sup>a</sup>, Wang Jiayang and Zhang Sulan~~

~~Chengdu University of Information Technology, Chengdu 610041, China~~

~~<sup>a</sup>wjj@cuit.edu.cn~~

**Keywords:** ~~radial basis function; particle swarm optimization; evaluation model; underground water~~

**Abstract.** There are three adjustable parameters in the radial basis function, the center of the basis function  $c_j$ , the width parameter  $\sigma$  and the output unit weight  $w_j$ . Through optimization the parameters of the radial basis function by Particle swarm optimization algorithm, a neural network model of underground water is generated, which is used to study the grade of underground water in the ten monitoring points of the black dragon hole. By applying the PSO-RBF model to underground water assessment in the ten monitoring points of the black dragon hole, the results of this evaluation, which correspond with the real conditions, are basically in accord with those obtained by other evaluation methods, and also show the practicability to groundwater quality assessment.

### Introduction

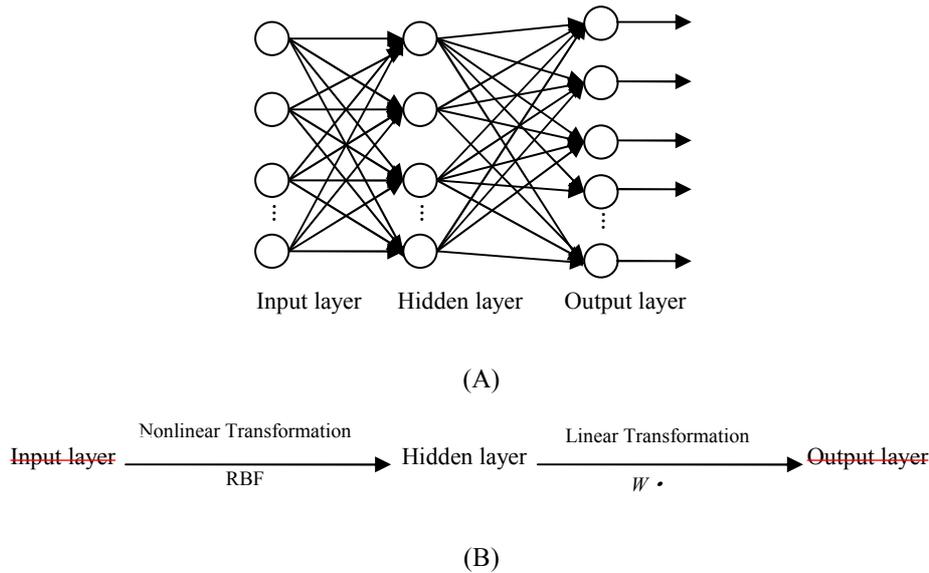
Radial Basis Function (RBF) neural network is a feed-forward artificial neural network<sup>[1,2]</sup>, which is based on the locality that the human brain neurons response to the outside world. RBF is of high computing speed and strong nonlinear mapping capability and can approximate an arbitrary nonlinear function with any precision. More importantly, the radial basis function (RBF) network provides a new and more effective method for training. Since it can avoid complicated and prolix calculations, its training speed is much faster than that of the BP network<sup>[3]</sup>. Thus it has been widely used in many fields.

The RBF network contains three adjustable parameters, the center of the basis function  $c_j$ , the width parameter  $\sigma$  and the output unit weight  $w_j$ . Each node of the output layer is weight estimation. The value of the output unit weight  $w_j$  can be optimized and determined by the Particle Swarm Optimization (PSO) algorithm. So the hybrid PSO-RBF model was put forward in the article.

RBF model has been widely used in many fields, such as the evaluation of underground water quality. The main task of groundwater quality evaluation is estimate the comprehensive levels of the underground water quality through the established mathematical model, based on the evaluation indexes of underground water and the water quality assessment standards. In the recent years, some methods such as principal component analysis, fuzzy analysis, gray cluster method, matter element analysis model and artificial neural networks<sup>[4-6]</sup> are mainly applied to comprehensively the evaluation of the underground water quality. Each method has its advantages and disadvantages. fuzzy analysis, gray cluster method and matter element analysis model need structure many utility functions, which lack of standardization in the function design, and have larger randomness and subjectivity. So the article adopted the hybrid PSO-RBF model to evaluate the levels of underground water quality.

### Radial Basis Function Neural Network Model

**The Structure of RBF Network.** Radial basis function neural network is a three-layer feedforward network consisting of an input layer, a hidden layer and an output layer, as shown in Fig. 1(A). The input layer is composed of the signal source nodes. The second layer is the hidden layer, whose nodes are constructed by the radialized functions as the Gaussian function. The number of the hidden layer nodes is determined by the needs of the problem. The third layer is the output layer, which is used to response to the input mode. The transformation from the input space to hidden layer space is non-linear, while from the hidden layer space to the output layer space is linear, as shown in Fig. 1(B).



~~Fig 1. Structure of RBF network~~

The transformation in the hidden layer is RBF function, which is a local distribution center of the radial symmetry of the non-negative non-linear attenuation function.

**The Basic Idea of RBF Network.** According to the mode theory (recognition), the nonlinear separable problem in the low-dimensional space can be mapped into a high dimension space and transformed into linear-separable one. In the RBF network, input information mapped into the hidden layer is nonlinear (because the function of the hidden layer is non-linear), while that from the hidden layer to the output layer is linear. The output unit can be regarded as a single -layer perceptron (SLP). In this way, as long as the number of hidden units (the number of dimensionality in high dimensional space) and its action function are chosen reasonably, the original problem can be mapped into a linearly separable problem, which can be resolved by a linear unit (an output node) finally<sup>[1]</sup>.

In the RBF network, the center vector is the mean of the class. The number of training sample indexes corresponds to the number of input nodes. The grade number of training samples is the number of the hidden layer. The calculation of the node  $i$  in the hidden layer uses the Gaussian function as shown in equation (1).

$$\varphi(x) = \exp\left(-\frac{\|x - C\|^2}{2\sigma^2}\right) \quad (\sigma > 0, \quad x > 0) \quad (1)$$

Where  $C$  is the centre of the kernel function.  $\|x - C\|$  is the Euclidean distance between any point  $x$  in space to the center  $C$ .  $\sigma$  is the width parameter of the function, which controls radial range of function. The calculation of output neurons uses a simple linear transfer function as follows,

$$y_i = \sum_{i=1}^m w_i R_i(x) \quad (2)$$

Where  $w_i$  is the connection weight.  $R_i$  is the action function of the  $i$ -th neuron in the hidden layer.

### The Hybrid Pso-Rbf Model

The network learning process can be divided into two steps. First, the center of the basis function and the width parameters need to be determined. The second step is the learning of the weight. In this case, the RBF network contains three adjustable parameters, the center of the basis function  $c_j$ , the width parameter  $\sigma$  and the output unit weight  $w_j$ . Each node of the output layer is weight estimation. The value of the output unit weight  $w_j$  can be optimized and determined by the particle swarm optimization (PSO). The objective function adopted is shown in equation (3),

$$Q = \frac{1}{K} \sum_{k=1}^K (y_k - y_{k0})^2 \tag{3}$$

Where,  $y_k$  is the output value of the  $k$ -th sample and  $y_{k0}$  is the target output value of the  $k$ -th sample.  $K$  is the total number of samples.

The value of  $w_j$  is adjusted by training the RBF network repeatedly until the final output error meets the precision requirement. Then the output weights  $w_j$  of the optimum RBF structure are obtained.

### Case Study

The hybrid PSO-RBF model was used to evaluate the levels of underground water quality in the ten monitoring points of the black dragon hole.

The evaluation index system of underground water is shown in table 1<sup>[6]</sup>, which including six evaluation indexes: the total hardness, sulphate, chloride, nitrate, dissolved solids and fluoride. The measured value of ten monitoring points of the black dragon hole were shown in table 2.

~~Table 1 The national evaluation index system of underground water unit: mg·L<sup>-1</sup>~~

Grade	Total hardness	Sulphate	Chloride	Nitrate	Dissolved solids	Fluoride	Target value	Fitted values	Relative error(%)
I	150	50	50	2	300	0.5	0.1	0.0992	0.8
II	300	150	150	5	500	0.8	0.2	0.2091	4.55
III	450	250	250	20	1000	1	0.3	0.2863	4.57
IV	550	350	350	30	2000	2	0.4	0.4151	3.775
V	650	400	450	40	3000	3	0.5	0.5084	1.68

~~Table 2 The measured value (unit: mg · L<sup>-1</sup>) and the assessment results of the ten monitoring points of the black dragon hole~~

Monitoring point	The total hardness	Sulphate	Chloride	Nitrate	Dissolved solids	Fluoride	PSO-RBF model		fuzzy mathematics model <sup>[6]</sup>
							Evaluation index	Grade	
Wuan industrial areas in Xinxing Pipes	330.2	63.9	13.5	3.66	275	0.1	0.1143	1	1
Ci mountain town	282.7	22.8	11	3.44	287	0.1	0.0905	1	1
Gu town	291.3	103.9	19.5	5.36	398	0.3	0.1319	2	2
Erli mountain waterworks	396.3	109.3	25.5	6.03	468	0.3	0.1527	2	2
Party school waterworks	400	114.7	21.5	3.76	457	0.2	0.1503	2	2
No. 5 well of Yang									
Jiaopu water sources	385.2	120.1	19.2	4.25	431	0.3	0.1524	2	2
No. 1 well of Handan power plant	298.7	15.6	14.5	3.37	296	0.3	0.0981	1	1
Fengfeng mine in Sun village	334.5	63.9	17	4.58	319	0.3	0.1234	2	1
Mining bureau waterworks	397.5	71.4	25.5	4.01	463	0.2	0.1369	2	2
Helong hole spring	381.5	110.3	18	3.83	379	0.3	0.1477	2	2

The five grade criterion of groundwater evaluation system shown in table 1 is set as the training sample of RBF network. The structure of the RBF network is 6-5-1. The normalized formula is as follows:

$$x_i = c_i / c_{i\max} \quad (4)$$

Parameters of PSO algorithm were set as: population  $m=30$ , dimension  $D=2$ ,  $c_1=c_2=2$ , maximum iterations  $T=10000$ , inertia weights:  $(w_{\max} - w_{\min}) \cdot \frac{t}{10000}$ ,  $w_{\max}=1.4$ ,  $w_{\min}=1.4$ .

PSO algorithm was used to optimize the weight values  $w_j$ , when  $Q_{\min} = 1.14 \times 10^{-4}$ ,  $w_1=-0.6695$ ,  $w_2=1.0584$ ,  $w_3=-0.3219$ ,  $w_4=-0.2241$ ,  $w_5=0.6876$ .

The monitoring value of the ten monitoring points in the black dragon hole is shown in table 1. Each index data was generated into the RBF model to get the evaluation index value and the corresponding grade, after the normalization calculation by equation (4). Results were shown in table 2.

The results of the attribute recognition method<sup>[6]</sup> were shown in table 2 in order to compare to the results of PSO-RBF. From the table it can be seen that the evaluation result is correspond with the real conditions, are basically in accord with those obtained by other evaluation methods, and also show the practicability to groundwater quality assessment.

## Conclusion

There are three adjustable parameters in the radial basis function, the center of the basis function  $c_j$ , the width parameter and the output unit weight  $w_j$ . Through optimization the parameters of the radial basis function by Particle swarm optimization algorithm, a neural network model of underground water is generated. By applying the PSO-RBF model to underground water assessment in the ten monitoring points of the black dragon hole, the results of this evaluation, which correspond with the real conditions, are basically in accord with those obtained by other evaluation methods, and also show the practicability to groundwater quality assessment.

## Acknowledgements

~~This work was financially supported by the Science and Technology Plan of Sichuan Province (11RKX0113) and the National Natural Science Foundation(51179110, 41101542).~~

## References

- ~~[1] V. David Sánchez A. Searching for a solution to the automatic RBF network design problem. *Neurocomputing*, Vol. 42 (2002) p. 147-170.~~
- ~~[2] D. Du, K. Li, M. Fei. A fast multi-output RBF neural network construction method. *Neurocomputing*, Vol. 73 (2010) p. 2196-2202.~~
- ~~[3] Wuxing L, Tse P W, Guicai Z and Tielin S. Classification of gear faults using cumulants and the radial basis function network. *Mechanical Systems and Signal Processing*, Vol. 18 (2004) p. 381-389.~~
- ~~[4] BOKAR H, TANG Jie, LIN Nian-feng. Groundwater quality and contamination index mapping in Changchun city, China. *Chinese Geographical Science*, Vol. 14(1) (2004) p. 63-70~~
- ~~[5] HONG Yoon-seok, MICHAEL R. Intelligent characterization and diagnosis of the groundwater quality in an urban fracture-rock aquifer using an artificial neural network. *Urban Water*, Vol. 3(3) (2001) p. 193-204~~
- ~~[6] Liu Bin, Zhou Yujuan, Yi Qinghua. Application of fuzzy mathematics in the comprehensive appraisal of the quality of underground water of spring basin of Heilongdong. *Journal of Hebei Institute of Architectural Science and Technology*, Vol. 23(1) (2006) p. 8-10.~~