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ScienceDirect
Journal of Sound and Vibration 319 (2009) 463–475
JOURNAL OF SOUND AND VIBRATION
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Contact-impact analysis of geared rotor systems

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Received 14 March 2006; received in revised form 27 April 2008; accepted 13 May 2008
Handling Editor: L.G. Tham
Available online 7 July 2008

Abstract

In this paper, a finite element formulation, used to analyze the contact-impact behavior of geared rotor systems coupled with the rotational, lateral, and axial vibrations between gears at high rotational speeds, has been developed. A gear impact element to model the contact-impact behavior between gears has been developed and its numerical method is discussed. A relative displacement measurement idea has been proposed to measure vibration parameter for contrast experiment in high rotational geared system. The equations of motion are derived and solved iteratively during each time increment until the unbalanced force decrease to an acceptable tolerance level. Based on the proposed method, an analysis program, GEARS, has been developed. The contact-impact behavior of geared rotor systems is analyzed especially under high rotational speed condition as numerical examples, which are demonstrated to show the effectiveness of the proposed method.

1. Introduction

Several studies on gear dynamics have been made by the finite element method (FEM). Stress on the teeth of gears, produced on the spur gears by static loading, has been examined [1]. A two-dimensional FEM model of the teeth of spur gears has been built, and the stress of the gear has been calculated at given dynamic loads [2]. In addition, impact loading to a three-dimensional model of gears has also been developed [3]. A multitooth finite element model was used to analyze the contact of bevel gears [4]. On the other hand, several analyses have been conducted on the dynamic behaviors of geared rotor systems using the FEM by many authors [5–9]. Furthermore, the dynamic behavior of rotor-gear-bearing systems is also simulated using modal synthesis [10]. A model of geared rotor systems on flexible bearings is built and the coupling vibrations between the torsional and the transverse vibrations of the gears are considered [11]. A model of mesh stiffness to consider elasticity coupling between the gear teeth is used to calculate the dynamic forces in spur gears [12]. An analytical model for a helical geared system is developed to consider the coupling vibration between axial,

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2008.05.023

Nomenclature

B	backlash
B_0	initial backlash
$\mathbf{C}_B$	damping coefficient of bearing
C_g	damping coefficient of gears
$\mathbf{e}$	relative displacement between the shaft and the housing
e_p, e_g	pitch error
E_p, E_g	irregularities of gears
f_d	damping force gears
f_i	impact force of gears
f_m	frequency of the motor alternator
$\mathbf{F}_d$	damping vector of gears
$\mathbf{F}_i$	impact force vector of gears
h_i	damping coefficient
k	spring coefficient of gears
k_1, k_2	linear spring coefficients of bearing in axial direction
k_x	spring function in axial direction of bearing
$\mathbf{k}_B$	spring stiffness function of bearing
$\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$	mass, damping, and stiffness matrices
$\mathbf{n}$	unit normal vector on the contact face of the gears
$\mathbf{q}$	generalized coordinate vector
r_g, r_g	radii of gears
S	relative displacement between gears
S_p, S_g	rotational displacement of gears
t	time
T_0	initial motor torque
T_m	motor torque
$\mathbf{U}_p$, $\mathbf{U}_g$	motion displacement vectors of gears
[ ]	diagonal matrix

Greek letters

ΔB	differentiation of backlash
θ_p, θ_g	rotation angles of gears
λ_i	eigenvalue
$\boldsymbol{\varphi}_i$	normalized modal vector
$\boldsymbol{\Phi}$	modal matrix
ϖ_i	natural angular frequency of mode i

Subscripts

g	gear
p	pinion

Superscripts

b	all of motions except the torsional motion
r	torsional motion
s	housing

lateral, and torsional directions [13]. A dynamic model for multibody systems and a gear element to take into account the time variant stiffness is developed [14]. An elemental property matrix of the shaft element is used to analyze the torsional vibration of gear-branched systems [15]. Recently, simulation of dynamical behavior of a speed-increase gearbox is developed [16], but the backlash is not taken into account in the contact element.

However, the analysis of geared rotor systems using FEM to describe the contact-impact behavior with coupling vibration (six dofs) of gear-rotor-housing between bevel gears has been little published.

An electric disk grinder used in various industries is a typical geared rotor system. A backlash exists between the teeth of bevel gears of an electric disk grinder because contact-impact behaviors such as forward impact, rebound, and backward impact occur between the gears at high rotational speeds. The axial, lateral, and torsional vibrations occur in the shafts due to the impact. These vibrations also cause the housing to vibrate through the bearings. Thus, a complicated coupled contact-impact behavior exists between gears, housing, and shafts.

The objective of this study is to develop a finite element formulation and its numerical calculation method, which is used to analyze the contact-impact behavior of geared rotor systems at high rotational speed, to demonstrate the nonlinear vibration mechanism of geared rotor system at high rotational speed, such as grinder, etc. And a new gear-element is developed, in order to describe the nonlinear contact-impact behavior of bevel gears, which use relative displacement–force pattern to solve dynamic behavior of bevel geared rotor system efficiently. Moreover, an analysis program, GEARS, is developed to simulate the nonlinear dynamic behavior of geared rotor system since power switch on.

2. Formulation for the analysis of contact-impact behavior of geared rotor systems

2.1. System components

An electric disk grinder consists of the following main components: an armature shaft, housing and bearings, a pair of bevel gears, a brush, a fan, and a grindstone (Fig. 1). A dynamics model, which includes the above components, is developed. The motor torque T_m, which is generated by armature, and depends on the time and rotational speed of the armature, is given by

$$T_m = T_0(1 - \cos(4\pi f_m t)) \tag{1}$$

where the generating power of motor T_0 is a function of the rotational speed of motor, f_m is the frequency of the motor alternator, and t is the time.

2.2. Contact-impact model of gear

The gear, assumed to be a rigid disk with six degrees of freedom (axial and lateral displacements, and axial and lateral rotations). In the equation, the rotational displacement of pinion S_p and the rotation displacement of the gear S_g can be written as follows:

$$S_p = r_p\theta_p(0 \leqslant \theta_p \leqslant 2\pi) \tag{2}$$

$$S_g = r_g\theta_g(0 \leqslant \theta_g \leqslant 2\pi) \tag{3}$$

where r_p and r_g are the radii of the pinion and the gear, respectively, and θ_p and θ_g are the rotation angles of the pinion and the gear, respectively. In order to express the contact-impact behavior between gears, the elastic action of gears is described by the relationship between the relative displacement, S, and impact force, f_i, of the gears. The relative displacement, S, between gears can be expressed as follows:

$$S = (S_p + E_p(\theta_p)) - (S_g + E_g(\theta_g)) \tag{4}$$

where E_p and E_g are the total irregularities of the pinion and gear teeth, respectively, and can be expressed as the error of the tooth pitch on the circumference:

$$E_p(\theta_p) = e_p\theta_p/2\pi \tag{5}$$

$$E_g(\theta_g) = e_g\theta_g/2\pi \tag{6}$$

The pitch error e_p, e_g (e_p, $e_g \leqslant 90\,\mu\text{m}$) between the teeth is based on statistical data from previous measurement experiments.

Backlash B and displacement S are used to describe the contact-impact behavior between the gears. In this study, the contact-impact behavior is considered as follows (Fig. 2):

(i) $S \geqslant B$, the pinion impacts the gear on the forward side of the pinion tooth, and this is called forward impact (Fig. 2b).

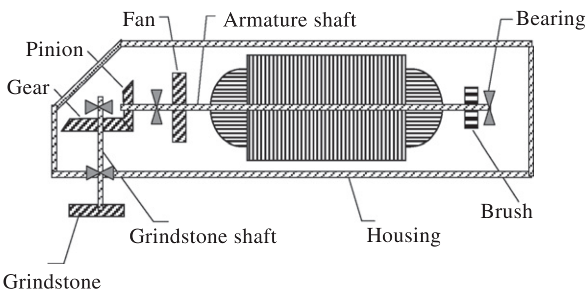


Fig. 1. Structure of an electric disk grinder.

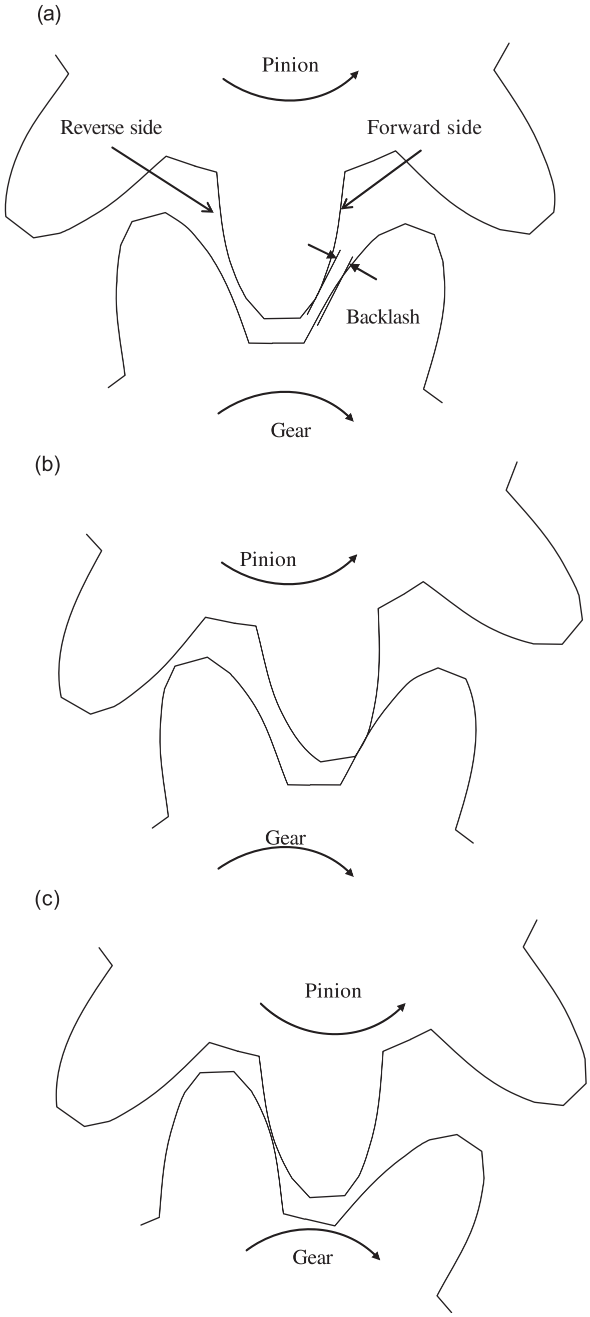


Fig. 2. The contact classification of gears: (a) a pair of gears, (b) forward impact and (c) backward impact.

(ii) $S \leqslant -B$, the gear impacts the pinion on reverse side of the pinion tooth, and this is called reverse impact (Fig. 2c).
(iii) $B > S > -B$, the pinion does not contact the gear.

Since shafts vibrate in the axial and lateral direction, it is considered that backlash B changes dynamically, and it can be written as

$$B = B_0 + \Delta B \tag{7}$$

where B_0 is the initial backlash in the respective direction on the circumference of the gears. ΔB is the change value of backlash:

$$\Delta B = (\mathbf{U}_p - \mathbf{U}_g) \cdot \mathbf{n}_B \tag{8}$$

where $\mathbf{U}_p$ and $\mathbf{U}_g$ are the motion displacement vectors of the pinion and the gear. $\mathbf{n}_B$ is normal vector of the teeth surface at the pitch circle. The backlash B and ΔB are calculated automatically in the analysis program.

2.3. Gear impact element

The impact between the gears and the elastic deformation of the gears has been considered. In order to describe the elastic deformation, a history of the relationship between relative displacement S and the impact force f_i, has been considered as a nonlinear spring characteristic (Fig. 3). It can be got from static load experiment of gear or static contact FEM analysis of gear .In the numerical analysis the function can be used of a linear function approximately instead of the nonlinear characteristic [17]:

$$f_i = \begin{cases} k(S - B), & S \geqslant B \\ k(S + B), & S < B \end{cases} \tag{9}$$

where k is *spring* coefficient.

It is very complicate problem when the gears contact at high rotational speed. Because there are grace and dust membrane between the teeth, a complicated unknown damp phenomenon occurs when the gears contact each other. Since the motion energy is larger than the damp energy when gears contact each other between the teeth clearly at high rotational speed. Here, we use damp coefficient C_g to describe the damp force simply:

$$f_d = C_g \dot{S} \tag{10}$$

where $\dot{S}$ is velocity of S.

The impact force f_i and the damping force f_d are considered as action in the normal direction at the contact face of the gears. The impact force vector $\mathbf{F}_i$ and damping vector $\mathbf{F}_d$ can be written as

$$\mathbf{F}_i = f_i \mathbf{n} \tag{11}$$

$$\mathbf{F}_d = f_d \mathbf{n} \tag{12}$$

where $\mathbf{n}$ is a unit normal vector on the contact face of the gears. It depends on the shape of the gear tooth. Here, the impact force vector $\mathbf{F}_i$ and damping force vector $\mathbf{F}_d$ between the bevel gear act in the x, y, and z directions of the local coordinate system (Fig. 4). Therefore, the shafts receive the force from the axial, lateral, and torsional directions. The gear element is described by the nonlinear impact force $\mathbf{F}_i$, which depends on the backlash B and the nodal displacements $\mathbf{U}_p$ and $\mathbf{U}_g$ on node of the gears. Since the gear element cannot be

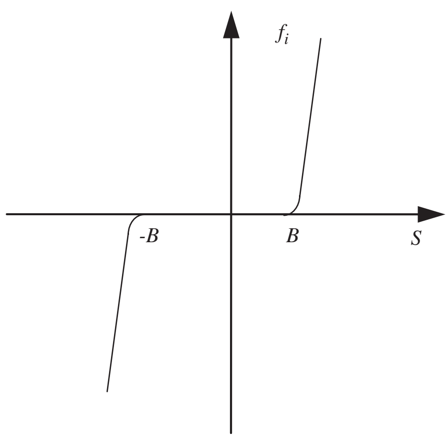


Fig. 3. Relationship between impact force f_i and relative displacement S.

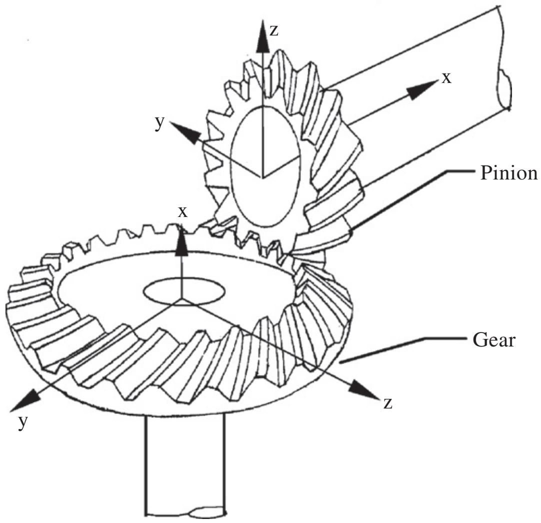


Fig. 4. The local coordinate system of gears.

usually expressed as the element matrix, nodal force vectors, which are equivalent to the impact force vector $\mathbf{F}_i$ and damping force vector $\mathbf{F}_d$, are given as element nodal force vectors on the node of the gears. A gear element, which is a function of the relationship between the relative displacement *S,* impact force f_i (Fig. 3), and damping force f_d, has been developed.

2.4. Model of housing and bearing

The dynamic behavior of the bearings is described by the contact force $\mathbf{f}_B$ and damping force $\mathbf{f}_c$, which comprise three dofs (axial and lateral direction):

$$\mathbf{f}_B = \mathbf{k}_B(\mathbf{e})\mathbf{e} \tag{13}$$

where $\mathbf{e}$ is the relative displacement between the shaft and the housing. $\mathbf{k}_B$ is a spring stiffness function, which is derived from the loading–displacement experiment. The nonlinear spring characteristic in the axial direction is shown in Fig. 5a where, f_{Bx} denotes the contact force and e_x denotes the relative displacement at the axis of the bearing. In the numerical analysis, the spring function k_x in the axial direction is described by the approximating the linear spring coefficients k_1 and k_2 (Fig. 5b):

$$k_x(e_x) = \begin{cases} k_1 & 0 \leqslant |e_x| \leqslant a \\ k_2 & |e_x| > a \end{cases} \tag{14}$$

Based on the experiment, the spring stiffness function is approximated as linear in the lateral direction:

$$\mathbf{f}_c = \mathbf{C}_B\dot{\mathbf{e}} \tag{15}$$

where $\dot{\mathbf{e}}$ is the velocity of $\mathbf{e}$ and $\mathbf{C}_B$ is the coefficient of damping.

Shafts are modeled using beam element (Fig. 6). Since the torsion stiffness of the shafts in the *x* direction is high, a small number of motion nodes are enough to express the torsional motion. Thus, a large number of nodes on shafts rotating in *x* direction are not necessary as in the axial and lateral directions. Here, the motion equation of the beam element *e* is divided into rotational motion (rotation angles θ_x^i, θ_x^j in *x* direction) and equation of motions in other directions (displacements *u*, *v*, *w* in *x*, *y*, and *z* directions, and rotation angles θ_y and θ_z in the y and z direction, respectively) (Fig. 6). It can be written as follows:

$$\mathbf{M}_e^r\ddot{\mathbf{U}}_e^r + \mathbf{C}_e^r\dot{\mathbf{U}}_e^r + \mathbf{K}_e^r\mathbf{U}_e^r = \mathbf{F}_e^r \tag{16}$$

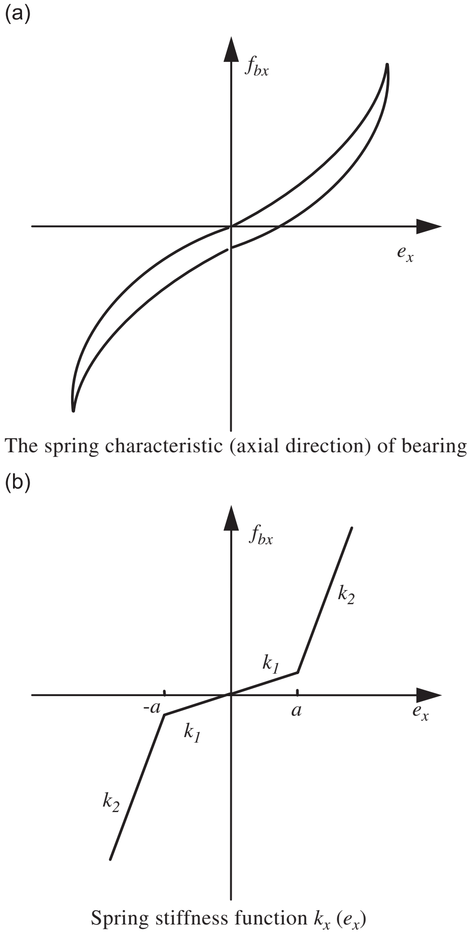


Fig. 5. Nonlinear spring stiffness function of the bearing.

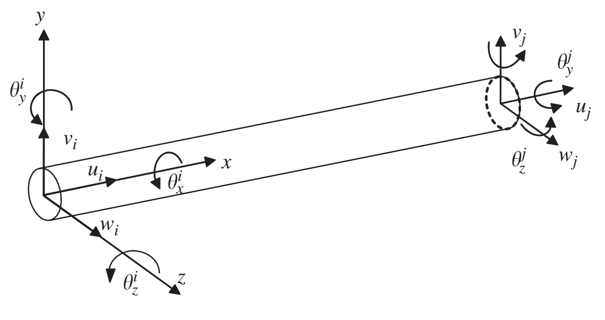


Fig. 6. A beam element.

where **M**, **C**, and **K** are the mass, damping, and stiffness matrices, respectively. Superscript r denotes the torsional motion of the beam. $\mathbf{U}_e^r$ is the nodal vector of the rotation angles:

$$\mathbf{U}_e^r = [\,\theta_x^i \quad \theta_x^j\,] \tag{17}$$

where subscripts i and j denote nodes i and j:

$$\mathbf{M}_e^b\ddot{\mathbf{U}}_e^b + \mathbf{C}_e^b\dot{\mathbf{U}}_e^b + \mathbf{K}_e^b\mathbf{U}_e^b = \mathbf{F}_e^b \tag{18}$$

where superscript b denotes motions in other directions except the torsional motion, and nodal vector

$$\mathbf{U}_e^b = [u_i \quad v_i \quad w_i \quad \theta_y^i \quad \theta_z^i \quad u_j \quad v_j \quad w_j \quad \theta_y^j \quad \theta_z^j] \tag{19}$$

2.5. Numerical method

A finite element model of an electric disk grinder used to analyze the contact-impact behavior of a geared rotor system has been proposed in this paper. It consists of the following elements: (a) a gear impact element for contact-impact behavior between gears, (b) beam elements for shafts, (c) shell elements for housing, (d) nonlinear spring and damping elements for bearings, (e) damping elements for loss in torque of brush and fan, and (f) a mass element for grindstone.

Considering the motion equations of the elements mentioned above, the equations to solve the contact-impact behavior between the gears of the geared rotor system can be written as follows:

$$\mathbf{M}^r\ddot{\mathbf{U}}^r + \mathbf{C}^r\dot{\mathbf{U}}^r + \mathbf{K}^r\mathbf{U}^r = \mathbf{F}^r(\mathbf{U}_p, \mathbf{U}_g, \dot{\mathbf{U}}_p, \dot{\mathbf{U}}_g) + T_m \tag{20}$$

$$\mathbf{M}^b\ddot{\mathbf{U}}^b + \mathbf{C}^b\dot{\mathbf{U}}^b + \mathbf{K}^b\mathbf{U}^b = \mathbf{F}^b(\mathbf{U}_p, \mathbf{U}_g, \dot{\mathbf{U}}_p, \dot{\mathbf{U}}_g, \mathbf{e}, \dot{\mathbf{e}}) \tag{21}$$

$$\mathbf{M}^s\ddot{\mathbf{U}}^s + \mathbf{C}^s\dot{\mathbf{U}}^s + \mathbf{K}^s\mathbf{U}^s = \mathbf{F}^s(\mathbf{e}, \dot{\mathbf{e}}) \tag{22}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively. Superscript s denotes the housing. $\mathbf{U}$ is the nodal vector and $\dot{\mathbf{U}}$ denotes the velocity vector. $\mathbf{F}_r$, $\mathbf{F}_b$, and $\mathbf{F}_s$ are nonlinear nodal loading force vectors. $\mathbf{F}_r$ and $\mathbf{F}_b$ include the impact force vector $\mathbf{F}_i$ and the damping force $\mathbf{F}_d$, which depend on the displacement between nodes where the gears contact, and contact force $\mathbf{f}_b$ and damping force $\mathbf{f}_c$ between the housing and the gears.

Since the housing is a shell structure, a shell element has been used to model it. Moreover, since the housing is a complicated curved-surface shell structure, a large number of elements are needed for the modeling. In order to effectively solve the dynamic behavior of housing, a modal method was applied to solve the motion equation of housing. The modal equation of the housing can be rewritten as follows:

$$\ddot{\mathbf{q}} + 2[h_i\varpi_i]\dot{q} + [\lambda_i]q = \mathbf{\Phi}^{\mathrm{T}}\mathbf{F}^s(\mathbf{e}, \dot{\mathbf{e}}) \tag{23}$$

where $\mathbf{q} = (\mathbf{q}_1 \quad \mathbf{q}_2 \quad \cdots \quad \mathbf{q}_n)$ is generalized coordinate vector, $\mathbf{\Phi} = (\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \ldots \quad \boldsymbol{\varphi}_n)$ is the modal matrix, and $\boldsymbol{\varphi}_i$ is the normalized modal vector. h_i is the damping coefficient. λ_i is eigenvalue, and ϖ_i is the natural angular frequency of mode i. The symbol [ ] denotes a diagonal matrix.

The motion equations of the geared rotor systems at time t+Δt can be expressed as

$$\mathbf{M}^r\ddot{\mathbf{U}}^r_{t+\Delta t} + \mathbf{C}^r\dot{\mathbf{U}}^r_{t+\Delta t} + \mathbf{K}^r\mathbf{U}^r_{t+\Delta t} = \mathbf{F}^r_{t+\Delta t}(\mathbf{U}_{p,t+\Delta t}, \mathbf{U}_{g,t+\Delta t}, \dot{\mathbf{U}}_{p,t+\Delta t}, \dot{\mathbf{U}}_{g,t+\Delta t}) + T_{m,t+\Delta t} \tag{24}$$

$$\mathbf{M}^b\ddot{\mathbf{U}}^b_{t+\Delta t} + \mathbf{C}^b\dot{\mathbf{U}}^b_{t+\Delta} + \mathbf{K}^b\mathbf{U}^b_{t+\Delta} = \mathbf{F}^b_{t+\Delta t}(\mathbf{U}_{p,t+\Delta t}, \mathbf{U}_{g,t+\Delta}, \dot{\mathbf{U}}_{p,t+\Delta}, \dot{\mathbf{U}}_{g,t+\Delta t}, \mathbf{e}_{t+\Delta t}, \dot{\mathbf{e}}_{t+\Delta t}) \tag{25}$$

$$\ddot{\mathbf{q}}_{t+\Delta t} + 2[h_i\varpi_i]\dot{\mathbf{q}}_{t+\Delta t} + [\lambda_i]q_{t+\Delta t} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{F}^s_{t+\Delta t}(\mathbf{e}, \dot{\mathbf{e}}) \tag{26}$$

Here, we assumed that the solutions of Eqs. (24)–(26) are known at time t. However, the solutions of the equations at t+Δt remain to be determined. A time integration scheme, Newmark-constant-average-acceleration method is used to solve the equation at time t+Δt.

3. Experiment

A relative displacement measurement idea has been proposed to measure vibration parameters for contrast experiment in high rotational geared system. In order to solve the contact-impact phenomenon between gears, which is considered to be one of main factors in the vibration of a disk grinder, the relative angle between the gears and the acceleration of the housing is measured; as shown in Fig. 7. In the experiment, the disk grinder is freely suspended. An acceleration sensor is attached to the housing just over the fan. Two magnetic pickup

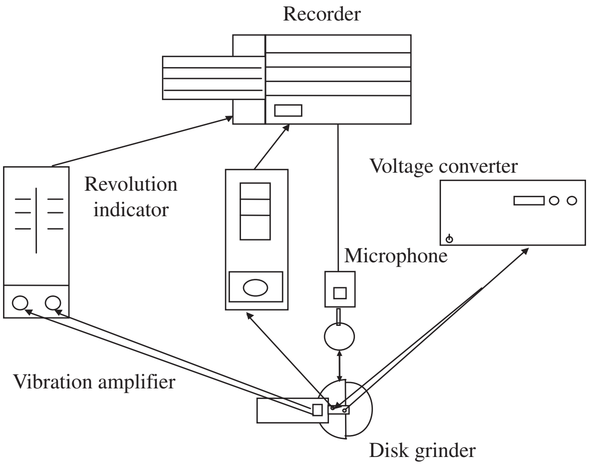


Fig. 7. The experiment for measuring the relative displacement S.

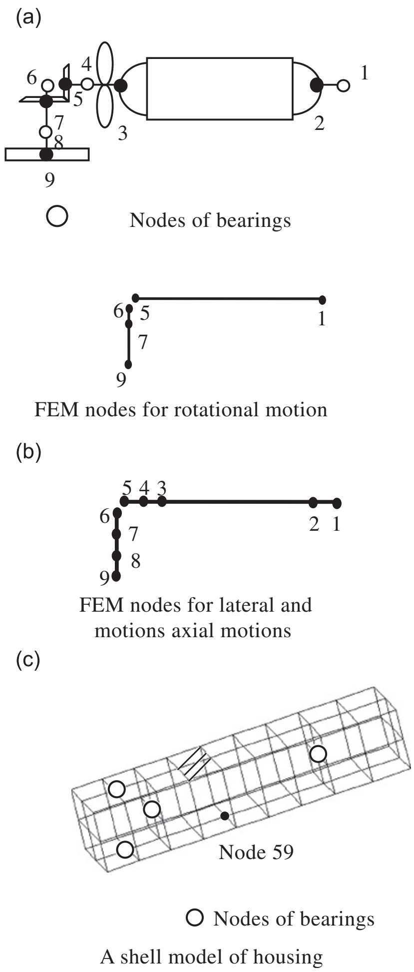


Fig. 8. FEM model for the electric disk grinder.

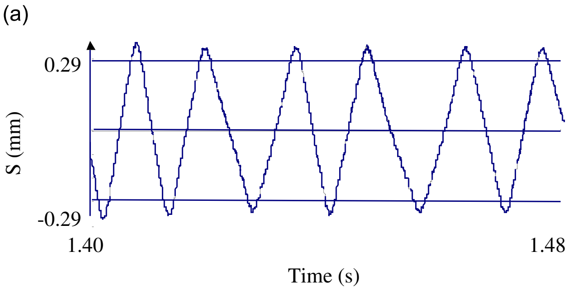
sensors are set on the surface of close to the gears and are connected to channel 1 (the pinion) and channel 2 (the gear) of the voltage converter. The relative angle between the gears is measured from the differences of the phases. Additionally, for comparison with numerical results, this value is converted into relative displacement.

4. Application and numerical results

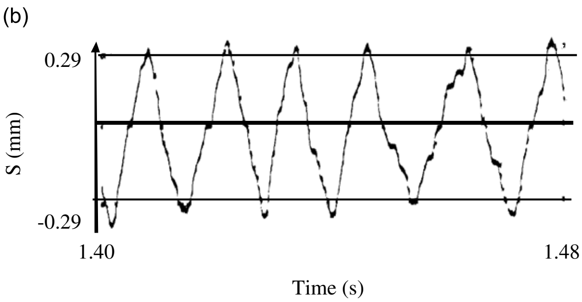
Based on the analysis method mentioned above, an analysis program, GEARS, which is used to analyze the contact-impact behavior of geared rotor systems, has been developed. GEARS can simulate dynamic motions of the gears, shafts, and housing instantly. Furthermore, it has the element library of gear, beam, shell, damping, and mass elements. An FEM model of an electric disk grinder with an armature shaft 15.5 cm long and 1.0 cm in diameter, and a grindstone shaft 5.9 cm long and 1.0 cm in diameter is illustrated in Fig. 8. The pinion and the gear consist of 14 and 35 teeth, respectively. The spring coefficient of gear $k = 680{,}000\,\mathrm{N/m}$, which is got from static load experiment of gear provided by Hitachi Koki Co. Ltd., and damp coefficient C_g is set as 0. In this study, one element is used for rotational motion at each shaft. Apart from this, to express the other motions except for torsional motion of the shaft, four beam elements are used for the armature shaft, three beam elements are used for the shaft, and 88 shell elements are used for the housing.

The relative displacement S between the bevel gears from time $t = 1.40$ to 1.48 s is shown in Fig. 9. Figs. 9(a) and (c) show numerical and the experimental results, respectively. The upper and lower levels show the initial backlash B_0 (upper: 0.29 mm, lower: -0.29 mm). If S exceeds upper and lower levels, it shows that the forward and reverse impacts occur between bevel gears, respectively. It is interesting to note that the actual contact time of gears is extremely short. The numerical results are in good agreement with the experimental values. The impact force between gears is shown in Fig. 10.

The rotational motion of the pinion about the x direction is shown in Fig. 11. It is understood that a torsional vibration of shaft occurs due to the impact between the gears. The axial vibrations also occur at the armature shaft due to the impact force (Fig. 12). As shown in Fig. 13, after the electric disk grinder goes into stable rotational situation, the maximum lateral vibration acceleration of the housing is approximately $72.1\,\mathrm{m/s^2}$.



Numerical result



Experimental result

Fig. 9. Relative displacement S.

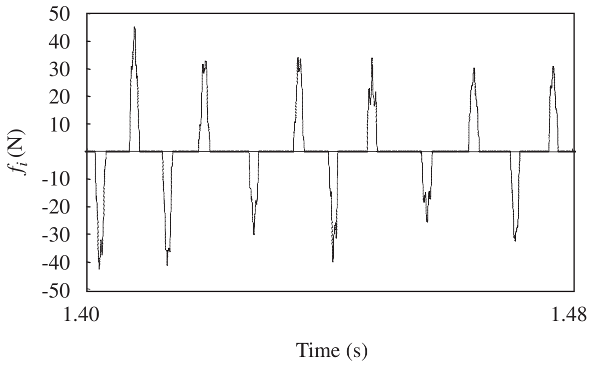


Fig. 10. Impact force f_i.

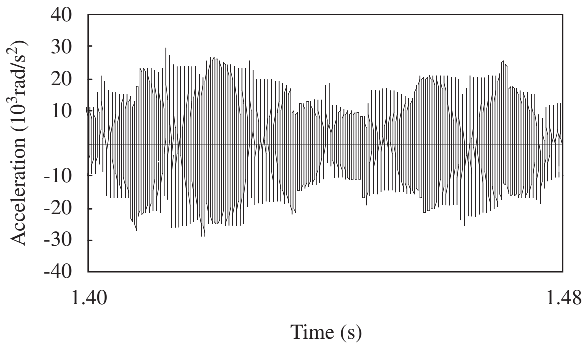


Fig. 11. Rotational acceleration of the pinion.

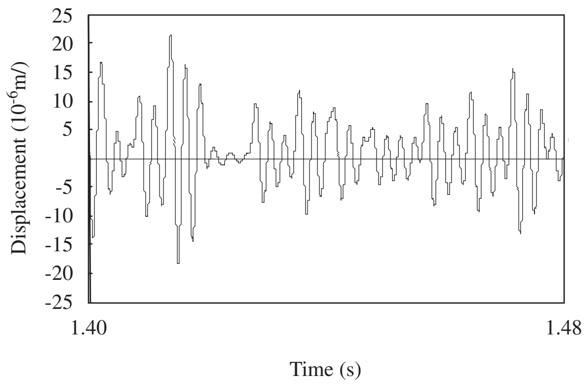


Fig. 12. Axial displacement of the armature shaft.

The numerical results of relative displacement S and the impact force f_i from time $t = 0$ to 2 s are shown in Figs. 14 and 15. It can be considered that after switching on the motor, the gears experience strong contact and impact behavior. The maximum value of the impact force f_i is approximately 240 N and it reduces to approximately 30 N at the state of a stable rotation.

5. Conclusion

In this study, a finite element formulation, which is used to resolve the contact-impact behavior of geared rotor systems during high-speed rotation, is given. A gear impact element, which expresses the contact

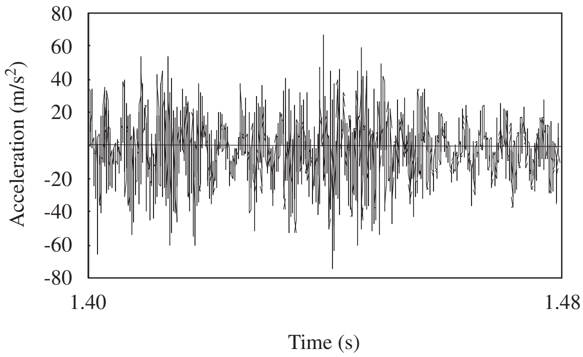


Fig. 13. Acceleration of the housing.

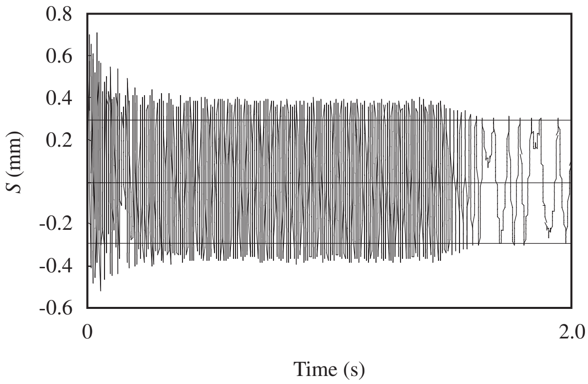


Fig. 14. The history of relative displacement S.

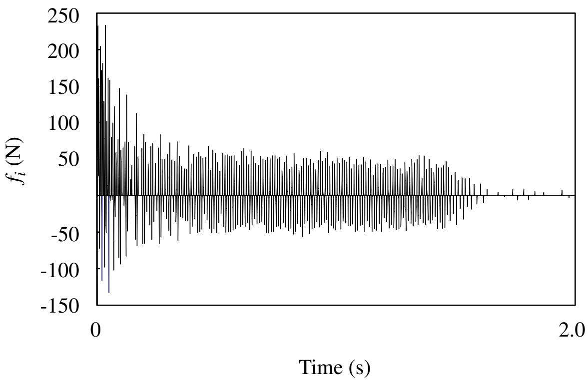


Fig. 15. The history of impact force.

behavior between gears, is developed. The validity of this formulation is confirmed through its application to an actual electric disk grinder and comparison with experimental results. The nonlinear vibration mechanism of the geared rotor system is demonstrated. It is concluded that this approach can be used to analyze the complicated contact-impact behavior of various products during high-speed rotation and can also be used to design low-vibration and low-noise geared rotor systems.

Acknowledgment

The authors would like to thank Hitachi Koki Co. Ltd. for support of this research.

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