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Technical Note

Numerical model for unsaturated sandy soils under cyclic loading: Application to liquefaction

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Abstract

This paper includes a presentation of a numerical model for the description of the behavior of unsaturated sandy soils under cyclic loading with particular attention for liquefaction. The model is developed within the framework of the theory of Biot and the formulation of Coussy. It is also based on laboratory observations. The formulation leads to a simplified model, which allows also to deal with saturated soils. The cyclic elastoplastic constitutive model ''MODSOL'' is used to describe the cyclic behavior of the soil. The paper presents an analysis of the influence of the soil saturation on the response of a sandy soil to both monotonic and cyclic undrained loading paths. It shows that the decrease in the saturation degree of a sandy soil leads to an important increase in their resistance to liquefaction.

Keywords: Coupled; Liquefaction; Numerical model; Cyclic; Seismic; Unsaturated soils

1. Introduction

Soil liquefaction constitutes a major cause of damage induced by earthquakes. It results from the interaction between the liquid and solid phases in porous media. This phenomenon may occur in both saturated and unsaturated soils; it results from a reduction in the effective mean stress, which leads to a deterioration in both the strength and stiffness of soils. Previous researches in liquefaction concerned mainly saturated soils. Since unsaturated soils are frequently encountered in geotechnical engineering [1], it is of major interest to investigate the soil liquefaction under partially saturated conditions. Triaxial cyclic tests on unsaturated Hostun sand indicated the possibility of liquefaction of partially saturated soils [2] and showed an increase in the resistance to liquefaction of soils with the decrease in the saturation degree.

The hydro-mechanic coupled theory founded by Refs. [3,4] together with the concept of effective stress proposed by Ref. [5] were used to study the liquefaction phenomena in saturated soils [6,7]. This paper presents a numerical model for the description of the response of unsaturated soils to cyclic loading. This model is based on new developments in the area of unsaturated soils [1,8–10] and on the generalization of the concept of effective stress to unsaturated conditions [11]. After a presentation of this model, the paper presents a study of the influence of the saturation degree on the response of a sandy soil to monotonic and cyclic undrained triaxial loading; this study allows the investigation of the influence of the soil saturation on the liquefaction of partially saturated sandy soils.

2. Numerical model for unsaturated sandy soil

The main difference between saturated and unsaturated soil lies in the existence of soil suction, which is defined as the difference between the pore air pressure and the pore water pressure. It is an important parameter for unsaturated soils. However, for sandy soils, because of the big diameter of soil grains; the soil suction is so small that it can be neglected. Fig. 1 shows the water retention curve for Hostun sand. It can be observed that the soil suction remains low over a very wide range of water saturation

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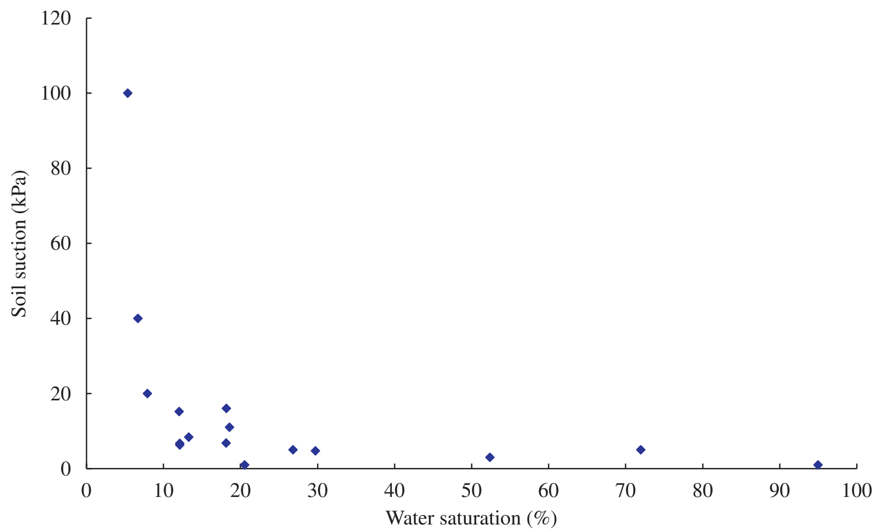

Fig. 1. Water retention curve for Hostun sand.

degree: suction remains lower than 5 kPa when the degree of water saturation is higher than 20%. This value is so small that it can be neglected. In the following, the soil suction is neglected, which means that the pore water pressure is equal to the pore air pressure ($p_a = p_w$). Under this condition, the effective stress concept of Terzaghi will be used.

3. Mathematical formulation

The multiphase theory of Coussy is used for the elaboration of the state equation of partially saturated sandy soils. According to this theory, the state equations for the soil and liquid phases can be expressed as follows [9,10]:

$$\delta\sigma_{ij} = C_{ijkl}\delta\varepsilon_{kl} - \left(\frac{\delta m}{\rho}\right)_j M_{jk} b_k I, \tag{1}$$

$$\delta p_i = -M_{ij}\left\{ b_j \delta\varepsilon_v - \left(\frac{\delta m}{\rho}\right)_j \right\}, \tag{2}$$

where σ and ε denote the stress and strain tensors, respectively; m_j and p_j stand for fluid mass content per unit volume and the pressure of fluid j, respectively; C is the undrained elastic matrix; M_{jk} and b_k are the Biot moduli and Biot coefficient, respectively; ε_v designates the volumetric deformation of the skeleton, ρ_j is the density of fluid j; I is the second-order unit tensor. The summation convention over repeated indices is used.

The unsaturated soil can be considered as the superposition of the soil skeleton, the pore-water (index w) and the pore-air (index a). Since the pore-air and pore-water occupy the total porous space, the porosity can be divided into two parts, as follows:

$$n = n_w + n_a. \tag{3}$$

The saturation of each fluid is defined as:

$$S_w = \frac{n_w}{n}, \qquad S_a = \frac{n_a}{n}. \tag{4}$$

The Biot coefficient of each phase for unsaturated soil can be assumed to associate with its saturation:

$$b_i = bS_i, \qquad b = \sum b_i = b(S_a + S_w), \tag{5}$$

b is the Biot coefficient for saturated porous media. This coefficient is equal to unit for sandy soils, because of the incompressibility of the soil grains. According to Eqs. (1) and (2), the state equations for partially saturated sandy soils can be expressed in the incremental form:

$$\delta\sigma_{ij} = \lambda\delta\varepsilon_{kk}\delta_{ij} + 2\mu\delta\varepsilon_{ij} - \left\{ \frac{\delta m_w}{\rho_w} \frac{\delta m_a}{\rho_a} \right\} \times \begin{bmatrix} M_{ww} & M_{wa} \\ M_{aw} & M_{aa} \end{bmatrix} \begin{Bmatrix} S_w \\ S_a \end{Bmatrix} \delta_{ij}, \tag{6}$$

$$\begin{Bmatrix} \delta p_w \\ \delta p_a \end{Bmatrix} = \begin{bmatrix} M_{ww} & M_{wa} \\ M_{aw} & M_{aa} \end{bmatrix} \left(-\begin{Bmatrix} S_w \\ S_a \end{Bmatrix} \delta\varepsilon_{kk} + \begin{Bmatrix} \dfrac{\delta m_w}{\rho_w} \\ \dfrac{\delta m_a}{\rho_a} \end{Bmatrix} \right), \tag{7}$$

δ_{ij} is the Kronecker's delta. λ and μ are the undrained elastic parameters.

The stress partition can be expressed as follows [10]:

$$\delta\sigma_m = (1 - n)\delta\sigma_m^s - n_j \delta p_j, \tag{8}$$

$\delta\sigma_m$ is the incremental mean stress; n_j is the porosity of fluid j; $\delta\sigma_m^s$ is the incremental mean stress of the matrix, which can be expressed as:

$$\delta\sigma_m^s = K_s\, \delta\varepsilon_v^s, \tag{9}$$

K_s is the matrix bulk modulus, while $\delta\varepsilon_v^s$ denotes the incremental volumetric deformation of matrix.

The incremental volumetric deformation of the matrix is linked to the incremental deformation of the skeleton and the variation of the porosity [9]:

$$(1-n)\delta\varepsilon_v^s = (1-n)\,\delta\varepsilon_v - \delta n, \tag{10}$$

$\delta n = n_0 - n$ is the variation of the porosity, n_0 is the initial porosity.

The incremental constitutive equation of the saturating fluid may be expressed as:

$$\delta p_j = K_j \frac{\delta\rho_j}{\rho_j} \quad j = (\mathrm{a}, \mathrm{w}). \tag{11}$$

Generally, the pore-air is considered as an ideal gas, so the modulus of compressibility of the pore-air is associated with the absolute pore-air pressure:

$$K_a = \bar{p}_a = p_a + p_{at}, \tag{12}$$

where $\bar{p}_a$ is the absolute pore-air pressure, and p_a is the measured pore-air pressure, p_{at} is the atmospheric pressure. While for the pore-water, the modulus of compressibility is about 2000 MPa, compared with the compressibility of pore-air, the linear property is assumed for the pore-water.

From Eqs. (8)–(12), the incremental macroscopic mean stress can be expressed as follows:

$$\delta\sigma_m = (1-n)K_s\,\delta\varepsilon_v - n_j K_j \frac{\delta\rho_j}{\rho_j} - K_s\,\delta n. \tag{13}$$

The conservation of the mass is given as:

$$\delta m_j = J\rho_j n_j - \rho_{0j} n_{0j}. \tag{14}$$

Together with the infinitesimal transformation:

$$J \approx 1 + \delta\varepsilon_v. \tag{15}$$

Combination of Eqs. (13)–(15) allows the derivation of the following expression for the variation of the porosity:

$$\delta n = \sum_{j=a,w} \delta n_j = \sum_{j=a,w} \left(\frac{\delta m_j}{\rho_j} - \frac{\delta\rho_j}{\rho_j} n_j - n_j\,\delta\varepsilon_v \right). \tag{16}$$

In soil mechanics, particularly for sandy soil, the soil grain is generally supposed incompressible. We suppose that in the sandy soil, the fluids are composed of an ideal air and pure water. The combination of Eqs. (10), (11) and (16) leads to:

$$\delta\varepsilon_v = \frac{\delta m_a}{\rho_a} + \frac{\delta m_w}{\rho_w} - \frac{n_a}{K_a}\,\delta p_a - \frac{n_w}{K_w}\,\delta p_w. \tag{17}$$

Eq. (16) together with the assumption ($\delta p_a = \delta p_w$) allows the derivation of the following expression for the pore-air and pore-water pressures:

$$\delta p_a = \delta p_w = M' \frac{\delta m_a}{\rho_a} + M' \frac{\delta m_w}{\rho_w} - M'\,\delta\varepsilon_v, \tag{18}$$

M' stands for the Biot's modulus for unsaturated sandy soils:

$$\frac{1}{M'} = \frac{n_a}{K_a} + \frac{n_w}{K_w} = \frac{n(1-S_w)}{p_w + p_{a0}} + \frac{nS_w}{K_w}. \tag{19}$$

Comparison of Eqs. (7) and (18) yields:

$$M_{aa} = M_{aw} = M_{wa} = M_{ww} = M'. \tag{20}$$

Neglecting the air flux, the constitutive equation is governed by the following expressions:

$$\delta\sigma = C : \delta\varepsilon - M' \frac{\delta m_w}{\rho_w} I,$$

$$\delta p_w = \delta p_a = -M'\,\delta\varepsilon_v + M' \frac{\delta m_w}{\rho_w}. \tag{21}$$

It is of interest to indicate that, in Eq. (19), if the water saturation is unit, that means the soil is saturated, the formulation of Zienkiwicz for saturated soil is recovered. It also means that this formulation can be used for both saturated and unsaturated problems.

4. Variation of the water saturation

The water saturation in the constitutive equation should be determined. Generally, for unsaturated soils, the water saturation is determined by the water retention curve, however for sandy soils, the ideal gas law could be used instead of the water retention curve. The assumption of zero air flux which is used for the formulation of the model will be used to determine the water saturation. Neglecting the air-flux, all the air will rest in its original place. We consider an unsaturated soil specimen, with a initial volume V_0 and an initial measured pore pressure p_{0w}. The initial porosity is n_0 and the initial water saturation is S_w^0. So the initial pore-air and pore-water volume are:

$$V_a^0 = n_0(1-S_w^0)V_0, \qquad V_w^0 = n_0 S_w^0 V_0. \tag{22}$$

After a small perturbation, under drained or undrained conditions, the assumption of the soil grains incompressibility leads to the following expressions:

$$\delta V = \varepsilon_v V_0 = (n - n_0)V_0, \tag{23}$$

$$\delta V = \delta V_a + \delta V_w. \tag{24}$$

According to the ideal gas law, the variation of the pore-air volume is equal to:

$$\delta V_a = \left(\frac{p_{0w} + p_{a0}}{p_w + p_{a0}} - 1 \right) V_a^0. \tag{25}$$

Combination of Eqs. (23) and (25) allows the determination of the current pore-water and pore-air volumes as follows:

$$V_w = V_w^0 + \delta V - \mathrm{d}V_a, \qquad V_a = V_a^0 + \delta V_a. \tag{26}$$

The current water saturation is equal to:

$$S_w = \frac{V_w}{V_w + V_a}. \tag{27}$$

Combination of Eqs. (22)–(27) leads to the following expression for the current water saturation:

$$S_w = \frac{n_0 S_w^0 + \varepsilon_v - [((p_{0w} + p_{a0})/(p_w + p_{a0})) - 1]n_0(1 - S_w^0)}{n_0 + \varepsilon_v}. \tag{28}$$

5. Influence of saturation degree on the response of a sand to undrained loading

The numerical model presented in the previous section is used to study the influence of the soil saturation on the response of a sandy soil to both monotonic and cyclic undrained triaxial loading. The sand behavior is described suing the cyclic constitutive model MODSOL [12–14]. A brief presentation of this constitutive equation is first presented. It will be followed by a presentation of the results of analyses.

5.1. Presentation of the constitutive equation "MODSOL"

The model is formulated using the effective stress tensor σ'. The elastic part of the model is governed by nonlinear elasticity. The Shear modulus and the bulk modulus are determined according to the following expressions:

$$K = K_0\left(\frac{p'}{p_r}\right)^N A(p', q), \qquad G = G_0\left(\frac{p'}{p_r}\right)^N, \tag{29}$$

$$A(p', q) = \left[1 - \frac{1}{9}\left(\frac{1-v_0}{1-2v_0}\right)N\left(\frac{q}{p'}\right)^N\right]^{-1}, \tag{30}$$

p_r is a reference pressure, N is a constitutive parameters, while p' and q stand for the mean effective stress and the deviatoric stress, respectively.

The constitutive relation uses two loading surfaces. The first one is the limit surface, which describes the soil behavior under monotonic loading:

$$f_m = q - M_f p' R_m. \tag{31}$$

The hardening function R_m depends on the plastic deviatoric strains (ε_d^p):

$$R_m = \frac{\varepsilon_d^p}{b + \varepsilon_d^p}, \tag{32}$$

b is a model parameter.

The constitutive relation takes into consideration the variation of the soil resistance in the deviatoric plane using the following expression for M_f:

$$M_f = \frac{6 \sin \varphi}{3 - \sin \varphi \sin 3\theta},$$

φ and θ denote the internal friction angle and the Lode's angle. Non-associated flow rule is used in the model. The gradient of plastic potential is given by

$$\frac{\partial g_m}{\partial p} = \frac{\exp(-\alpha_0 \varepsilon_d^p)}{M_c p'}\left(M_c - \frac{q}{p'}\right), \qquad \frac{\partial g_m}{\partial q} = \frac{1}{M_c p'}, \tag{33}$$

with

$$M_c = \frac{6 \sin \varphi_{cv}}{3 - \sin \varphi_{cv} \sin 3\theta}, \tag{34}$$

α_0 is a constitutive parameter and φ_{cv} stands for the characteristic angle.

The cyclic loading surface is assumed to be conic in the stress domain. Its expression is given by

$$f_c = q^l - p^l R_c \tag{35}$$

with

$$q^l = \sqrt{s_{ij}^l s_{ij}^l}, \quad s_{ij}^l = \sigma_{ij} - p^l \alpha_{ij}, \qquad p^l = \alpha_{ij}\sigma_{ij}, \quad R_c = \frac{\varepsilon_{dc}^p}{b + \varepsilon_{dc}^p}, \tag{36}$$

R_c controls the isotropic hardening of the cyclic loading surface; ε_{dc}^p denotes the plastic deviatoric deformation associated to the cyclic loading, which should be initialized at each loading inversion. The unit tensor α_{ij} denotes the axis of the cyclic loading surface; its evolution is governed by

$$d\alpha_{ij} = d\lambda H_{ij}, \qquad H_{ij} = c_c(1 - AF)R_c s_{ij}^l, \tag{37}$$

$d\lambda$ is the plastic multiplier and c_c is a constitutive parameter. A and F are determined as follows:

$$A = 1 \quad \text{if } s_{ij}s_{ij}^l \geqslant 0, \qquad A = -1 \quad \text{if } s_{ij}s_{ij}^l \geqslant 0, \qquad F = \frac{q}{M_f p' R_m} \quad (F \leqslant 1). \tag{38}$$

The gradient of the cyclic plastic potential is given by

$$\text{if } \frac{\partial g_m}{\partial \sigma'_{ij}} d\sigma'_{ij} \geqslant 0, \quad \frac{\partial g_c}{\partial \sigma'_{ij}} = \frac{B \exp(-\alpha_0 \varepsilon_d^p)}{3M_c p'}\left(M_c - \frac{q}{p'}\right)\delta_{ij} + \frac{3}{2M_c p'}\frac{s_{ij}}{q},$$
$$\text{if } \frac{\partial g_m}{\partial \sigma'_{ij}} d\sigma'_{ij} < 0, \quad \frac{\partial g_c}{\partial \sigma'_{ij}} = \frac{B \exp(-\alpha_0 \varepsilon_d^p)}{3M_c p^l}\left(M_c - \frac{3\sqrt{j_2^l}}{p^l}\right)\alpha_{ij} + \frac{3}{2M_c p^l}\frac{s_{ij}^l}{\sqrt{j_2^l}}. \tag{39}$$

Table 1
Constitutive parameters for Hostun sand [12]

G_0 (100 kPa)	K_0 (100 kPa)	N	φ (°)	φ_{cv} (°)	b	α_0	c_c	b_c
76.9	166.7	0.5	33	30	0.001	12	0.005	500

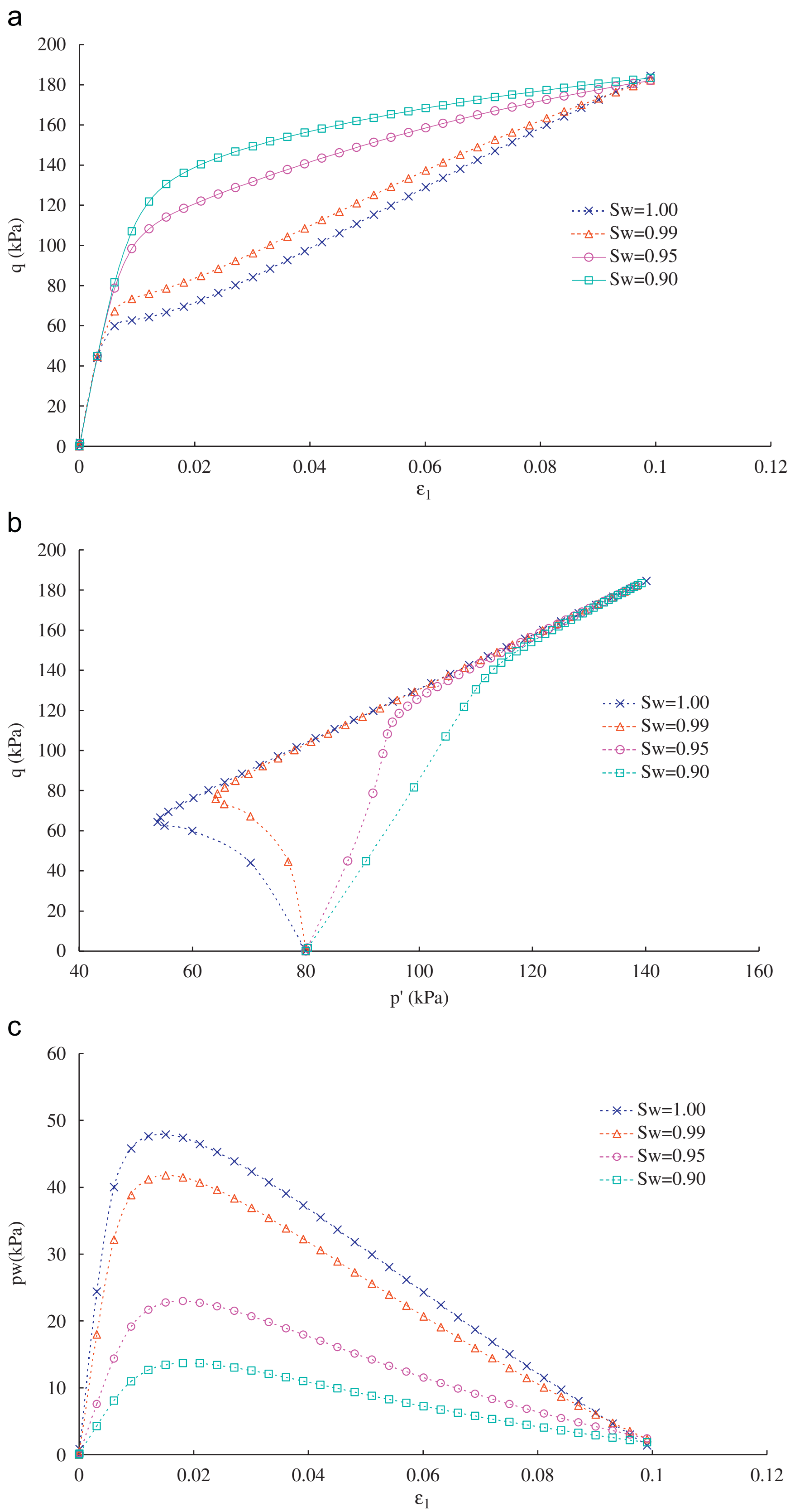

Fig. 2. Influence of initial water saturation on the undrained response of loose sand in the (a) (ε_1–q) and (b) (p'–q), and (c) (ε_1–p_w) planes.

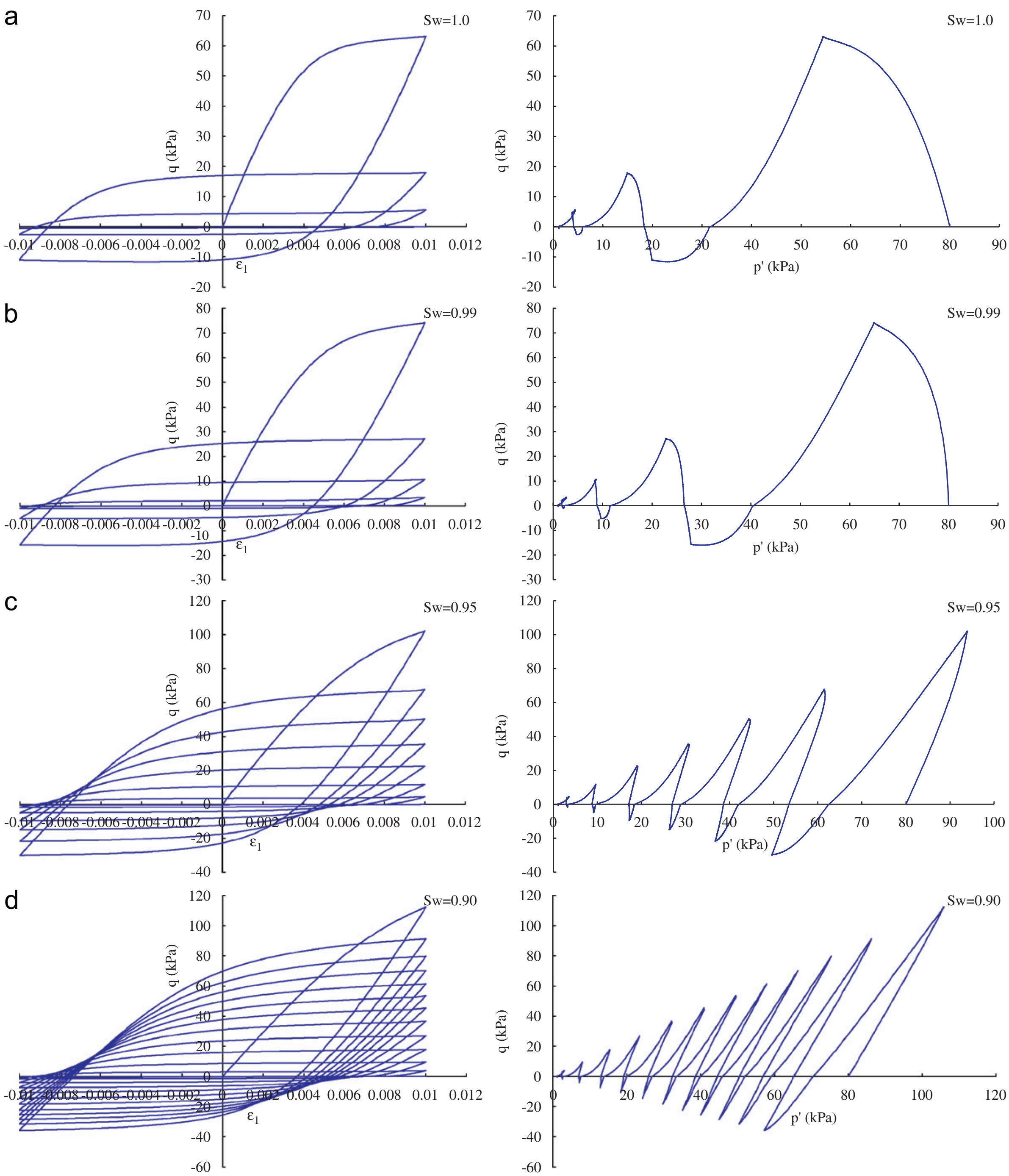

Fig. 3. Influence of water saturation on the undrained cyclic response of sand, S_w: (a) 100%, (b) 99%, (c) 95%, and (d) 90%.

With $B = \exp(-b_c|\varepsilon_v^p|)$, ε_v^p is the plastic volumetric deformation, b_c is a constitutive parameter which controls the plastic deformation under cyclic loading.

5.2. Results of analyses

Analyses are conducted for loose Hostun sand using the constitutive parameters determined by Ref. [12] (Table 1). A sand specimen is considered, with different values of the initial water saturation ($S_w = 100\%$, 99%, 95% and 90%). The initial pore pressure is supposed equal to zero. The confining pressure is 80 kPa.

Fig. 2 shows the response of the soil specimen to triaxial undrained monotonous loading. It can be observed that the soil saturation highly affects the soil response. Fig. 2c shows that the decrease in the saturation degree leads to a decrease in the induced pore-water pressure. This result is due to the influence of the degree of saturation on the

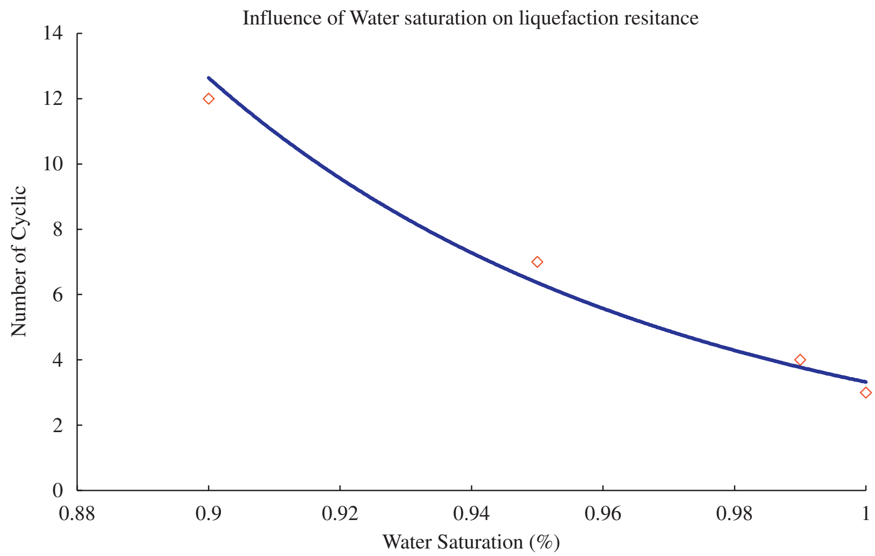

Fig. 4. Influence of initial water saturation on the undrained cyclic response of sand.

compressibility of the fluid mixture: the decrease in the saturation degree leads to an increase in the compressibility of the pore-fluid, and consequently to a decrease in the rate of the excess pore pressure generation. The water saturation degree affects qualitatively the soil response in the (q, ε_1) and (q, p') planes. The response of the full saturated soil ($S_w = 1.0$) shows first a decrease in the effective mean stress (because of the contracting phase), which is followed by an increase in p' because of the dilating phase (Fig. 2b); a quasi stabilization of the stress deviator is observed in the zone of transition from the contracting phase to the dilating one (Fig. 2a). The decrease in S_w induces gradually a shift of the stress path in the (q, p') plane towards the left and to the disappearing of the quasi-stabilization phase of q; for $S_w = 0.95$, a continuous increase in p' is observed with a regular variation in q.

Fig. 3 shows the influence of the saturation degree on the specimen response to a stain-controlled cyclic undrained triaxial loading ($-0.01 \leqslant \varepsilon_1 \leqslant 0.01$). For the full-saturated soil ($S_w = 1.0$), the cyclic loading induces a rapid decrease in the effective mean stress; the liquefaction (p' close to zero) is observed after three cycles. For the specimen with $S_w = 0.90$, the rate of decrease in the effective mean stress is lower than that obtained with the full saturated soil; liquefaction is obtained after 12 cycles. This is due to the high compressibility of the unsaturated pore-fluid compared to the saturated one. The high compressibility of the porous fluid mixture reduces the rate of increase in the pore pressure as observed for monotonous loading and consequently to an in increase in the soil resistance to liquefaction. Numerical simulations were also conducted for $S_w = 0.99$ and 0.95. Fig. 4 summarizes the influence of the initial saturation degree on the soil liquefaction; it can be observed that the decrease S_w leads to an important increase in the soil resistance to liquefaction.

6. Conclusion

This paper includes the presentation of a simple numerical model for the description of the response of unsaturated sandy soils to cyclic loading with a particular interest in liquefaction. The model is based on the theory of Biot and the formulation of Coussy together with the hypothesis of low suction, zero air flux and immiscible interaction between pore-water and pore-air in sandy soils. This model is similar to that of U-P model and thus it is very easy in using and integrating into finite element program. Numerical simulations conducted with this model showed that the soil saturation degree largely affects the resistance of sandy soils to liquefaction. The presence of pore-air reduces the excess pore pressure generation rate. And the induced partial saturation could mitigate liquefaction.

References

[1] Fredlund DG, Rahardjo H. Soil mechanics for unsaturated soils. Wiley-Interscience ISBN-10: 047185008X; 1993.

[2] Bouferra R. Etude en laboratoire de la liquéfaction des sols. Thèse de doctorat, Université des Sciences et Technologies de Lille; 2000.

[3] Biot MA. General theory of three-dimensional consolidation. J Appl Phys 1941;12:155–64.

[4] Biot MA. Mechanics of deformation and acoustic propagation in porous dissipation media. J Acoust Soc Am 1962;34:1254–64.

[5] Terzaghi K. The shearing resistance of saturated soils and the angle between the planes of shear. In: Proceedings of the 1st international conference on soil mechanics and foundation engineering, vol. 1, 1936. p. 54–6.

[6] Zienkiewicz OC, Chang CT, Bettess P. Drained, undrained, consolidating and dynamic behaviour assumptions in soil. Geotechnique 1980;30(4):385–95.

[7] Zienkiewicz OC, Shiomi T. Dynamic behaviour of saturated porous media; the generalized Biot formulation and its numerical solution. Int J Numer Anal Methods Geomech 1984;8:71–96.

[8] Fredlund DG. Unsaturated soil mechanics in engineering practice. J Geotech Geoenviron Eng ASCE 2006;132(3):286–321.
[9] Coussy O. Poro mechanics. Wiley; 2004.
[10] Coussy O. Mécanique des milieux poreux. Paris: Technip; 1991. 437pp.
[11] Bishop AW. The principle of effective stress. Teknisk Ukeblad 1959;39:859–63.
[12] Khroshavan AK. Problèmes de sols saturés sous chargement dynamique: modèle cyclique pour les sols et validation sur des essais en centrifugeuse. Thèse de doctorat, Université des Sciences et Technologies de Lille; 1995.
[13] Ousta, R. Etude du comportement sismique des micropieux. Thèse de doctorat, Université des Sciences et Techniques de Lille; 1998.
[14] Ousta R, Shahrour I. Three-dimensional analysis of the seismic behaviour of micropiles used in the reinforcement of saturated soils. Int J Numer Anal Methods Geomech 2001;25:183–96.