

# Reliability-network-equivalent approach to distribution-system-reliability evaluation

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**Abstract:** The paper presents a reliability-network-equivalent approach to distribution-system-reliability assessment. In this technique, a general feeder is defined and a simple set of equations is utilised. The basic general feeder equations and the reliability network equivalent provide a practical technique for evaluating the reliability of complex radial distribution systems. The procedure is illustrated by application to a relatively simple but practical system example.

## 1 Introduction

The main thrust of power-system-reliability evaluation over the past few decades has been concentrated on generation and transmission, with relatively little effort applied to the distribution domain, particularly low-voltage distribution systems. The basic reason for this is that generation and transmission systems are capital intensive and their inadequacy can have widespread catastrophic consequences for both society and its environment. The contribution of distribution systems to overall customer unreliability is, however, quite significant. Utility statistics show that distribution-system failures account for approximately 80% of the average customer interruptions [1]. The reliability of an individual customer load point is very dependent on the topology, design and operation of the local-distribution system.

The analytical techniques required for distribution-system-reliability evaluation are highly developed. Many of the published concepts and techniques are presented and summarised in [2]. Conventional techniques for distribution-system-reliability evaluation are generally based on failure-mode-and-effect analysis (FMEA) [2–4]. This is an inductive approach which systematically details, on a component-by-component basis, all possible failure modes and identifies their resulting effects on the system. Possible failure events or malfunctions of each component in the distribution system are identified and analysed to determine the effect on surrounding load points. A final list of failure events is formed to

evaluate the basic load-point indexes. The FMEA technique has been used to evaluate a wide range of radial-distribution systems. In systems with complicated configurations and a wide variety of components and element-operating modes, the list of basic-failure events can become lengthy and can include thousands of basic-failure events. This requires considerable analysis when the FMEA technique is used. It is therefore difficult to use FMEA directly to evaluate a complex radial-distribution system. A reliability-network-equivalent approach is introduced in this paper to simplify the analytical process. The main principle in this approach is that an equivalent element can be used to replace a portion of the distribution network and therefore decompose a large distribution system into a series of simpler distribution systems. This is a novel approach to distribution-system evaluation which provides a repetitive and sequential process to evaluate the individual load-point-reliability indexes.

## 2 Definition of a general feeder

Fig. 1 shows a simple radial-distribution system consisting of transformers, transmission lines, breakers, fuses and disconnect switches. Associated disconnect switches and transmission lines such as s1 and l2 are designated as a main section. The main sections deliver energy to the different power-supply points. An individual load point is normally connected to a power-supply point through a transformer, fuse and lateral transmission line. A combination such as f1, t2 and l5 is called a lateral section. The lateral transmission line may be on either the high- or low-voltage side of the transformer.

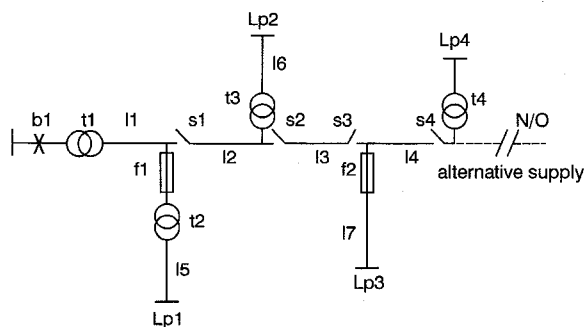


Fig. 1 Simple distribution system

t = transformer  
l = transmission line  
f = fuse  
s = disconnect  
b = breaker  
Lp = load point

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A simple distribution system is usually represented by a general feeder which consists of  $n$  main sections,  $n$  lateral sections and a series component, as shown in Fig. 2. In this feeder,  $S_i$ ,  $L_i$ ,  $M_i$  and  $L_{pi}$  represent series component  $i$ , lateral section  $i$ , main section  $i$  and load point  $i$ , respectively.  $L_i$  could be a transmission line, a line with a fuse or a line with a fuse and a transformer.  $M_i$  can be a line, a line with one disconnect switch or a line with disconnect switches on both ends.

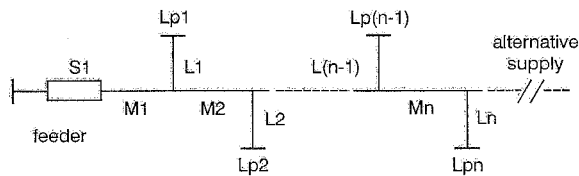


Fig. 2 General feeder

### 3 Basic formulas for a general feeder

Based on the element data and the configuration of the general feeder, a set of general formulas for calculating the three basic load-point indexes of load-point failure rate  $\lambda_j$ , average outage duration  $r_j$  and average annual outage time  $U_j$  for load point  $j$  of a general feeder is as follows:

$$\lambda_j = \lambda_{sj} + \sum_{i=1}^n \lambda_{ij} + \sum_{k=1}^n p_{kj} \lambda_{kj} \quad (1)$$

$$U_j = \lambda_{sj} r_{sj} + \sum_{i=1}^n \lambda_{ij} r_{ij} + \sum_{k=1}^n p_{kj} \lambda_{kj} r_{kj} \quad (2)$$

$$r_j = \frac{U_j}{\lambda_j} \quad (3)$$

where  $p_{kj}$  is the control parameter of lateral section  $k$  which depends on the fuse-operating model. It can be 0 or 1 corresponding to no fuse or a 100% reliable fuse, respectively, and a value between 0 and 1 for a fuse which has a probability of unsuccessful operation of  $p_{kj}$ . The parameters  $\lambda_{ij}$ ,  $\lambda_{kj}$  and  $\lambda_{sj}$  are the failure rates of the main section  $i$ , lateral section  $k$  and series element  $s$ , respectively, and  $r_{ij}$ ,  $r_{kj}$  and  $r_{sj}$  are the outage durations (switching time or repair time) for the three elements, respectively. Eqns. 1–3 do not include the effects of overlapping failures of elements in a radial configuration. It is assumed that these effects are negligible.

The  $r_{ij}$ ,  $r_{kj}$  and  $r_{sj}$  data have different values for different load points when different alternative supply operating modes are used and disconnect switches are installed in different locations on the feeder. This is illustrated in the following three cases.

#### 3.1 Case 1: no alternative supply

In this case,  $r_{sj}$  is the repair time of the series element  $s$  and  $r_{ij}$  is the switching time for those load points which can be isolated by disconnection from the failure of main section  $i$  or the repair time for those load points which cannot be isolated from a failure of the main section  $i$ . In this case,  $r_{kj}$  is the switching time for those load points which can be isolated by disconnection from a failure on a lateral section  $k$  or the repair time for those load points which cannot be isolated from a failure on a lateral section  $k$ .

#### 3.2 Case 2: 100% reliable alternative supply

In this case,  $r_{ij}$  and  $r_{kj}$  take the same values as in case 1. The parameter  $r_{sj}$  is the switching time for those load points which are isolated from the failure of a series element by disconnection or the repair time for those load points not isolated from the failure of a series element  $s$ .

#### 3.3 Case 3: alternative supply with availability $p_a$

In this case,  $r_{ij}$  is the repair time ( $r_1$ ) for those load points not isolated by disconnection from the failure of main section  $i$ , the switching time ( $r_2$ ) for those load points supplied by the main supply and isolated from the failure of the main section  $i$  or  $r_2 p_a + (1 - p_a)r_1$  for those load points supplied by an alternative supply and isolated from the failure of the main section  $i$ . The parameter  $r_{kj}$  is the repair time  $r_1$  for those load points not isolated by disconnection from the failure of lateral section  $k$ , the switching time  $r_2$  for those load points supplied by the main supply and isolated from the failure of lateral section  $k$  or  $r_2 p_a + (1 - p_a)r_1$  for those load points supplied by an alternative supply and isolated from the failure of a lateral section  $k$ . The parameter  $r_{sj}$  is the same as in case 2.

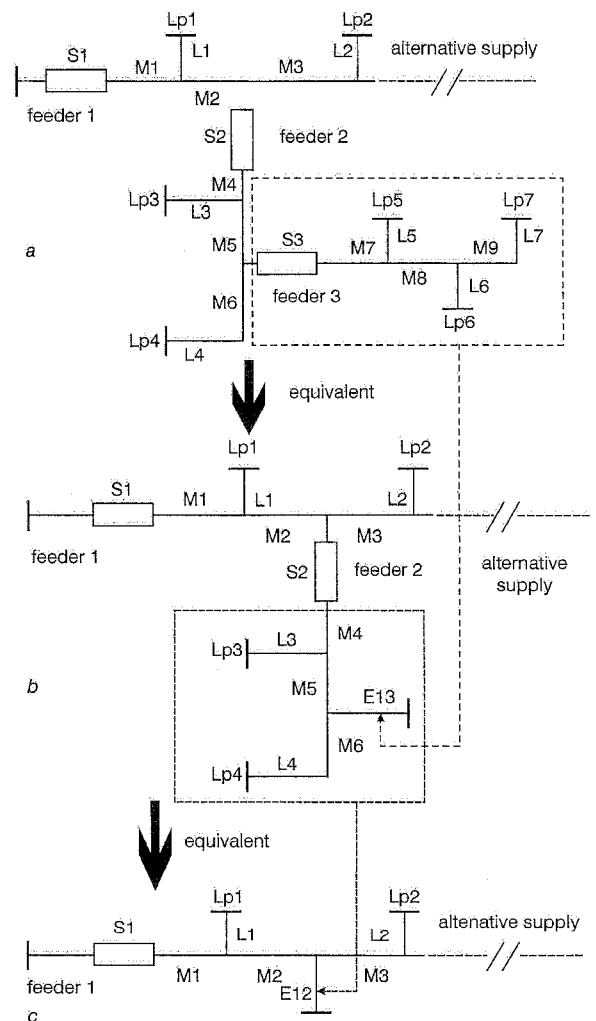


Fig. 3 Reliability-network equivalent

### 4 Network-reliability equivalent

A practical distribution system is usually a relatively complex configuration which consists of a main feeder and subfeeders as shown in Fig. 3. The main feeder is

connected to a bus station. A subfeeder is a feeder connected such as feeder 2 and feeder 3 in Fig. 3. The three basic equations presented above cannot be used directly to evaluate the reliability indexes of this system. The reliability-network-equivalent approach, however, provides a practical technique to solve this problem. The basic concepts in this approach can be illustrated using the distribution system shown in Fig. 3. The original configuration is given in Fig. 3a and successive equivalents are shown in Figs. 3b and c. The procedure involves the development of equivalent lateral sections and associated series sections.

#### 4.1 Equivalent lateral sections

The failure of an element in feeder 3 will affect load points not only in feeder 3 but also in feeders 1 and 2. The effect of feeder 3 on feeders 1 and 2 is similar to the effect of a lateral section on feeder 2. Feeder 3 can be replaced by the equivalent lateral section (El 3) shown in Fig. 3b. The equivalent must include the effect of the failures of all elements in feeder 3. The equivalent lateral section (El 2) of feeder 2 can then be developed as shown in Fig. 3c. The contributions of the failures of different elements to parameters of an equivalent lateral section will depend on the location of the disconnect switches. The reliability parameters of an equivalent lateral section can be divided into two groups and obtained using the following equations:

$$\lambda_{e1} = \sum_{i=1}^m \lambda_i \quad (4)$$

$$U_{e1} = \sum_{i=1}^m \lambda_i r_i \quad (5)$$

$$r_{e1} = \frac{U_{e1}}{\lambda_{e1}} \quad (6)$$

$$\lambda_{e2} = \sum_{i=1}^n \lambda_i \quad (7)$$

$$U_{e2} = \sum_{i=1}^n \lambda_i r_i \quad (8)$$

$$r_{e2} = \frac{U_{e2}}{\lambda_{e2}} \quad (9)$$

where  $\lambda_{e1}$  and  $r_{e1}$  are the total failure rate and restoration time of the failed components which are not isolated by disconnects in the subfeeder and  $m$  is the total number of these elements. The parameters  $\lambda_{e2}$  and  $r_{e2}$  are the total equivalent failure rate and the switching time of those failed elements which can be isolated by disconnects in the branch and  $n$  is the total number of these elements.

#### 4.2 Equivalent series component

Using successive network equivalents, the system is reduced to a general distribution system in the form shown in Fig. 3c. Only feeder 1 remains in the system. The basic equations, eqns. 1–3, can now be used to evaluate the load-point indexes of feeder 1. On the other hand, the failure of elements in feeder 1 also affect the load points in feeders 2 and 3. These effects are equivalent to those of a series element S2 in feeder 2. Feeder 2 becomes a general distribution system after the equivalent series element is calculated. The load-point indexes of feeder 2 and the parameters of the equivalent series element S3 are then calculated in the

same way as with feeder 1. Finally, the load-point indexes of feeder 3 are evaluated. The reliability parameters of a equivalent series component can be calculated using the method used for the load-point indexes. The only difference is that the equivalent parameters should be divided into two groups. The effect of one group on the load points of a subfeeder is independent of the alternative supplies in subfeeders; the effect of the other group depends on the alternative supplies in the subfeeders.

The simplification in computation provided by the proposed method can be illustrated using Fig. 3a. In this distribution system, there are seven load points and 19 elements. Using the standard FMEA approach,  $19 \times 7 = 133$  calculations are required as all load points are checked for each element failure. Using the reliability-network-equivalent approach, however,  $7 + 7 + 7 = 21$  calculations are required to find the equivalent lateral sections and  $7 \times 3 + 7 \times 3 + 7 \times 3 = 63$  calculations to find the load-point indexes, giving a total of 84. This is 63% of the required FMEA calculation. This is a simple network. If there are more elements in each subfeeder, the savings can be quite substantial. In addition, all the network must be searched for each element failure to find the affected load points in a standard FMEA. The search procedure, for the affected load points outside a feeder, for element failures in the feeder is the same. This search procedure requires considerable computer time. Using the reliability network equivalent, no repeat searches are required, with an attendant saving in computer time.

### 5 Procedure for calculating reliability indexes

The procedure described in Section 4 for calculating the reliability indexes in a complex distribution system using the reliability-network-equivalent approach can be summarised by two protocols.

A bottom-up process is used to search all the subfeeders and to replace them by corresponding equivalent lateral sections. As shown in Fig. 3, the equivalent lateral section El 3 is found first, followed by El 2. The system then is reduced to a general distribution system.

Following the bottom-up process, a top-down procedure is then used to evaluate the load-point indexes of each feeder and equivalent series components for the corresponding subfeeders until all the load-point indexes of feeders and subfeeders are evaluated. As noted in Section 4, the series element represents the impact on the subfeeders of failures in the higher-order feeders. Referring to Fig. 3, the load-point indexes in feeder 1 and the equivalent series element S2 for feeder 2 are calculated first, followed by the load-point indexes in feeder 2 and S3. The load-point indexes in feeder 3 are finally calculated. After all the individual load-point indexes are calculated, the final step is to obtain the feeder and system indexes. The example presented in Fig. 3a considers a single alternative supply. The procedure can be extended, however, to consider more than one supply to a general feeder.

### 6 Program and system analysis

A general program for calculating the load-point and system-reliability indexes of a complex radial-distribution system has been developed using the network-reliability-equivalent technique. The program can be used to calculate the indexes for different main-section

configurations containing no disconnects, one disconnect or two disconnects on the main sections and different fuse-operating models on the lateral sections. The following illustrates an application to a practical test system known as the RBTS [5], which contains five local-distribution systems. Fig. 4 shows one of these systems. Each system segment consists of a mixture of components. The disconnects, fuses and alternative supplies can operate in the different modes described above. The data used in these studies are given in [5]. The existing disconnect switches are shown in Fig. 4, but additional switches can be added at any location. System analysis has been carried out for three different operating conditions. The detailed procedure followed in the reliability-network-equivalent approach is illustrated in case 1.

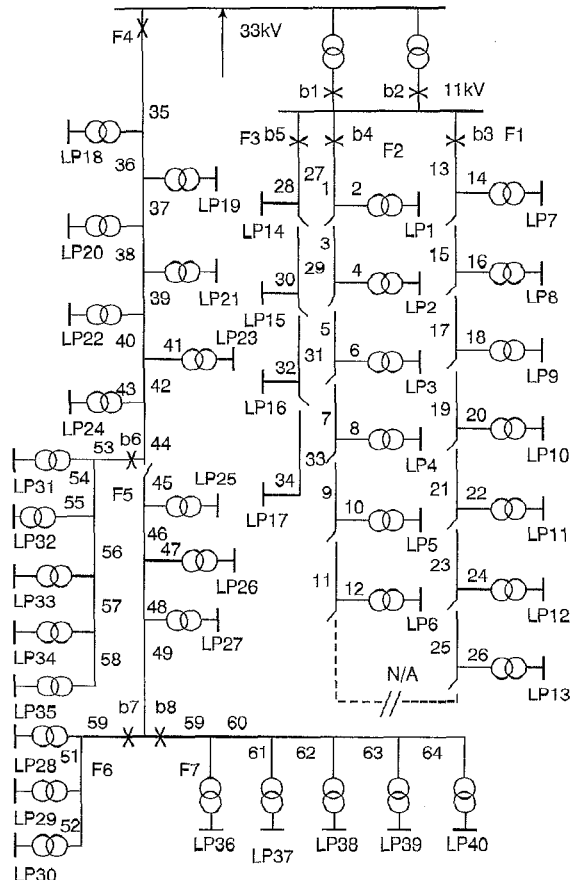


Fig. 4 Distribution system of RBTS

**Case 1:** To illustrate the reliability-network-equivalent approach in a general sense, breakers 6, 7 and 8 are assumed to be 80% reliable with no alternative supply to main feeder 4. The detailed analysis is as follows. There are three subfeeders in this system. The first step is to find the equivalent lateral sections of feeders 5, 6 and 7. The equivalent-lateral-section parameters for the three feeders are:

For feeder 5:

$$\begin{aligned} \lambda_{e51} &= 0.8645 \text{ (occ/year)} \\ U_{e51} &= 4.3225 \text{ (h/year)} \\ r_{e51} &= 5.0 \text{ (h)} \\ \lambda_{e52} &= 0 \text{ (occ/year)} \\ U_{e52} &= 0 \text{ (h/year)} \\ r_{e52} &= 0 \text{ (h)} \end{aligned}$$

For feeder 6:

$$\lambda_{e61} = 0.5525 \text{ (occ/year)}$$

$$U_{e61} = 2.7625 \text{ (h/year)}$$

$$r_{e61} = 5.0 \text{ (h)}$$

$$\lambda_{e62} = 0 \text{ (occ/year)}$$

$$U_{e62} = 0 \text{ (h/year)}$$

$$r_{e62} = 0 \text{ (h)}$$

For feeder 7:

$$\lambda_{e71} = 0.8385 \text{ (occ/year)}$$

$$U_{e71} = 4.1925 \text{ (h/year)}$$

$$r_{e71} = 5.0 \text{ (h)}$$

$$\lambda_{e72} = 0 \text{ (occ/year)}$$

$$U_{e72} = 0 \text{ (h/year)}$$

$$r_{e72} = 0 \text{ (h)}$$

After the equivalent lateral sections of feeders 5, 6 and 7 have been found, feeder 4 becomes a general feeder.

The next step is to calculate the load-point indexes in feeder 4 and the equivalent series elements of feeder 4. The parameters of the equivalent series components for feeders 5, 6 and 7 are as follows:

For feeder 5:

$$\lambda_{es5} = 2.7703 \text{ (occ/year)}$$

$$U_{es5} = 10.9566 \text{ (h/year)}$$

$$r_{es5} = 3.95824 \text{ (h)}$$

For feeder 6:

$$\lambda_{es6} = 2.7911 \text{ (occ/year)}$$

$$U_{es6} = 13.9555 \text{ (h/year)}$$

$$r_{es6} = 5.0 \text{ (h)}$$

For feeder 7:

$$\lambda_{es7} = 3.0199 \text{ (occ/year)}$$

$$U_{es7} = 15.0995 \text{ (h/year)}$$

$$r_{es7} = 5.0 \text{ (h)}$$

After the equivalent series elements for feeders 5, 6 and 7 have been found, the load-point indexes can be calculated. Table 1 shows a representative sample of the load-point-reliability indexes.

Table 1: Load-point indexes: case 1

Load point (i)	Failure rate (occ/year)	Outage duration (h)	Unavailability (h/year)
1	0.3303	2.4716	0.8163
10	0.3595	2.2434	0.8065
20	3.4769	4.1915	14.5735
25	3.4769	5.0216	17.4595
30	3.3586	5.0223	16.8680
35	3.6498	4.2298	15.4380
40	3.8734	5.0194	19.4420

**Case 2:** In this case, breakers 6, 7 and 8 are assumed to be 100% reliable and no alternative supply is available to feeder 4.

**Case 3:** Breakers 6, 7 and 8 are assumed to be 80% reliable and alternative supply is available to feeder 4 at the point between the two breakers in F6 and F7.

The system indexes for feeder 4 can be evaluated using the load-point indexes. The system indexes for the three cases are shown in Table 2.

It can be seen by comparing the results of case 2 with those of case 1 that the probability of successful operation of breakers 6, 7 and 8 is important for the reliability of the whole distribution system. Comparing the results of case 1 and case 3, it can be seen that the reliability of the overall system is greatly increased by

**Table 2: System indexes for cases 1, 2 and 3**

Case	1	2	3
SAIFI (int./cus. year)	1.6365	1.0065	1.6365
SAIDI (hr./cus. year)	6.9695	3.8197	4.8478
CAIDI (hr./cus. int.)	4.2588	3.7949	2.9623
ASAI	0.9992	0.9996	0.9995
ASUI	0.0008	0.0004	0.0005
ENS (MWh/year)	83.9738	48.3691	57.8922
AENS (kWh/Cus. year)	0.0286	0.0165	0.0197

providing the alternative supply in feeder 4. These conclusions can obviously be determined by other techniques such as the standard FMEA approach. The reliability-network-equivalent method is a novel approach to this problem which uses a repetitive and sequential process to evaluate the individual load point and subsequently the overall system indexes.

## 7 Conclusion

This paper illustrates a practical technique for complex-radial-distribution-system reliability evaluation. A general feeder is defined and a set of basic equations is

developed based on a general-feeder concept. A complex radial-distribution system is reduced to a series of general feeders using reliability network equivalents. Basic equations are used to calculate the individual load-point indexes. The reliability-network-equivalent method provides a simplified approach to the reliability evaluation of complex distribution systems. Reliability evaluations for several practical test distribution systems have shown this technique to be superior to the conventional FMEA approach. This method avoids the required procedure of finding the failure modes and their effect on the individual load points, and results in a significant reduction in computer solution time.

## 8 References

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