A Distributed Slack Bus Model and Its Impact on Distribution System Application Techniques

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Abstract—**Distribution system operating environments are changing rapidly. For example, with the steady and significant increase in dispersed generation expected, planning and operating application techniques must also change. This paper re-evaluates the single slack bus assumption typically employed in steady-state distribution power flow solvers. Specifically, a distributed slack bus model based on the concept of generator domains will be discussed and integrated into a power flow solver. Improved representation and allocation of distribution system losses to multiple generators is expected to impact other application techniques such as capacitor placement, service restoration, etc. In addition economic impacts of dispersed generation are significant and simulation results will be presented to highlight this impact.**

I. INTRODUCTION

Dispersed generation is increasing rapidly. It is expected that 20% or higher of the power supply in some distribution systems will be provided by dispersed generation. To accommodate these changes, new distribution power flow analysis tools for planning and operating techniques need to be developed. As a consequence, distribution application tools such as capacitor placement and network reconfiguration need to be revisited. In addition, a fair pricing scheme for distributed generators (DGs) then becomes an interesting and important problem in the power market.

This paper aims to demonstrate the applications of a network based distributed slack bus model based on iterative participation factors [1]. The model is embedded in an unbalanced power flow solver [1,2], and the participation factors quantify the amount of real power output from the DG that is contributed to loss. This allows for the development of pricing models based upon the load and loss contributions of each source. In this work, simulation results will include comparisons of the network based distributed slack bus model with that of a single slack bus model and a model with participation factors based upon generator capacities.

Previous works employing a distributed slack bus model using participation factors were applied solely in transmission systems. In [3, 4], the participation factors are related to the characteristics of turbines on each generator bus and load allocation. In [5], the authors applied participation factors using combined cost and reliability criteria in power flow for fair pricing. In [6], the author provides a method of choosing participation factors based on the scheduled generator outputs. While these models are novel in transmission system for various reasons they could not be directly applied in distribution systems.

In [7, 8], the concept of generator domains and provide a method to distinguish the load and loss contribution of each generator for transmission systems was introduced. We utilize this concept to calculate participation factors in distribution systems. Namely, the concept of assigning physical generator domains is expanded to three-phase unbalanced distribution systems. This captures the effects of unbalanced network parameters, loads and generator locations. Using this information, we define participation factors for each distributed generator and the substation.

A summary of the model and power flow equations is presented in Section II. Based on the new power flow and iterative participation factors, relevant applications are then presented in Section III. Simulation results are reported in Section IV.

II. DISTRIBUTED SLACK BUS MODEL

A network-based distributed slack bus model is presented here [1,2] which uses iterative participation factors for the substation and participating DGs whose real power outputs can be adjusted. The concept of generator domains [7] was extended to unbalanced power systems and used to determine iterative participation factors in multi-phase, unbalanced systems.

A. Participation Factors

Participation factors are proposed to quantify and distribute the system real power loss to network sources based upon the network topology, load distribution and source capacities. Each participating generator and the substation are assigned a participation factor, K_i, calculated as follows:

$$K_i = \frac{P_{Gi}^{loss}}{P_{Loss}} \qquad i = 0,1,2,\cdots m \qquad (1)$$

where: $$\sum_{i=0}^{m} K_i = 1 \qquad (2)$$

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$$P_{Gi}^{loss} = P_{Gi}^{loss,a} + P_{Gi}^{loss,b} + P_{Gi}^{loss,c} \quad (3)$$

and

0	the substation index
m	the number of participating DGs in the system
P_{Loss}	the total real power loss in the system
P_{Gi}^{loss}	the loss associated with generator i
$P_{Gi}^{loss,p}$	the loss associated with generator i, phase p

We note that the sum of all participation factors is one and, for unbalanced distribution systems, the loss associated with each generator must be calculated for each phase. This will result in multi-phase generator domains.

B. Generator Domains

In [7], the concept of single-phase generator domains was introduced to associate portions of a transmission network, their losses and loads, to different generators. Also, the domain of a generator was defined as the set of buses and branches whose power is supplied by the generator. In distribution systems, since the loads are unbalanced, the buses and branch flows supplied by the same generator may be different across phases. Thus, in [1], the definitions from [7] were extended to unbalanced systems and for each generator, and a given P_{Gi},

$$P_{Gi} = P_{Gi}^{load} + P_{Gi}^{loss} \qquad i = 0,1,2,\cdots m \quad (4)$$

where:

$$P_{Gi}^{load} = P_{Gi}^{load,a} + P_{Gi}^{load,b} + P_{Gi}^{load,c} \quad (5)$$

and

P_{Gi}^{load}	the load associated with generator i
$P_{Gi}^{load,p}$	the load associated with generator i, phase p

Domains vary for each phase and are assigned based on:

- positive power flow direction per phase
- proportionality of commons (areas assigned to more than one source).

B.1) Positive Power Flow Direction

The positive power flow direction will be used to assign a directed graph onto the distribution system. For two directly connected buses, bus i, phase p and bus j, phase p

- If $\mathrm{Re}(V_i^p I_{ij}^{p*}) - \mathrm{Re}(V_j^p I_{ij}^{p*}) > 0$, we state that positive real power flows from bus i to bus j over phase p;
- If $\mathrm{Im}(V_i^p I_{ij}^{p*}) - \mathrm{Im}(V_j^p I_{ij}^{p*}) > 0$, we define that positive reactive power flows from bus i to bus j over phase p.

where

V_i^p : the voltage on bus i in phase p

I_{ij}^p : the current from bus i to bus j over phase p

The positive real power flows and positive reactive power flows may be different. In this work, we are interested in the real power slack and the positive real power flow directions are used. The concept of a common for unbalanced systems is now discussed.

B.2) Generator Commons

The loss on a branch or the load on a single node may be supplied by many sources; therefore, the domains of different generators intersect in this phase and they have the branch or load in common. Therefore, the definition of a generator common is modified to be a set of contiguous nodes and branches by phase, whose power is supplied by the same generators. The proportion of loss and loads supplied by different sources to a common is assumed to be the same as the proportion of the positive real power injected by the sources to this common.

The proportion of loads and losses of a common are then assigned to the corresponding generator domains, and the load and loss contributed by each source can be found [7]. As such, an iterative participation factor can be developed by embedding the determination of K within an unbalanced power flow solver.

C. Distribution power flow equations

For each iteration of power flow, the participation factors can be determined and the total real power outputs of individual sources including the substation and participating DGs can be expressed as:

$$P_{Gi} = P_{Gi}^{load} + K_i P_{Loss} \qquad i = 0,1,2\cdots m \quad (6)$$

Since the total system real power loss P_{Loss} is unknown and varies according to the slack distribution, an additional equation at the substation is be used:

$$(P_{G0}^{load} + K_0 P_{Loss}) - \sum_{p=a}^{c} P_{D0}^p = \sum_{p=a}^{c} P_0^p \quad (7)$$

where:

P_{D0}^p	the real power load on phase p of the substation bus
P_0^p	the real power flow equation on Bus 0, phase p

Also, if the participating DGs are assumed to be voltage source inverters (VSI) connections [8], DG buses are modeled as a new type of $P|V|$ buses, which provides balanced three-phase voltage outputs and adjustable real power inputs. In such a case, there is only one unknown at each DG, the voltage phase angle θ_i^a, and the real power balance equation is required. For the substation bus and m generator buses, $i = 0,1,2,\cdots m$:

$$f_{Pi} = (P_{Gi}^{load} + K_i P_{Loss}) - \sum_{p=a}^{c} P_{Di}^p - \sum_{p=a}^{c} P_i^p = 0 \quad (8)$$

For *n-m* load buses:

$$\begin{aligned} f_{Pi}^p &= -P_{Di}^p - P_i^p = 0 \\ f_{Qi}^p &= -Q_{Di}^p - Q_i^p = 0 \end{aligned} \qquad i = m+1, m+2, \cdots, n \quad (9)$$

The above equations can solved with a Newton-Raphson solver updating the participation factors at each iteration [2].

This multiple slack bus power flow tool can then be applied to study and re-evaluate different distribution application functions.

III. APPLICATIONS

This new power flow with a network-based distributed slack bus model for unbalanced distribution systems can be applied to:

- judge proper DG installation sizes and locations for DG placement
- provide a guide for DG planners to schedule DGs' outputs to service desired amounts of loads
- develop economic indicators of distribution losses for utilities/DG owners in power market
- affect other distribution application techniques

Embedding the distributed slack bus power flow analysis within distribution applications may yield significantly different placement and control commands for dispersed generators, capacitors and network switches. For example, capacitor settings and locations and switch operations for network reconfiguration may be revised as their problem formulations typically focus on loss reduction. Service restoration schemes will also be affected as they are often formulated in terms of power delivered to the loads. Also, by distributing losses, we can more realistically quantify the economic impacts of each generator on the system. In the following section, simulation results will focus on these economic impacts.

IV. SIMULATIONS

A 20-bus test distribution system (Figure 1) is used for our simulations. In this system, all loads are constant *PQ* loads and the total system loads are 6.0451MW and 3.2724 Mvar. In the following examples, two cases will be investigated:

- Case 1: the DG is installed on Bus 3
- Case 2: the DG is installed on Bus 4

Each case will assume one DG exists to service 1,500kW of load. We note testing of the network-based distributed slack bus model on larger systems with varying numbers of DGs was reported in [2].

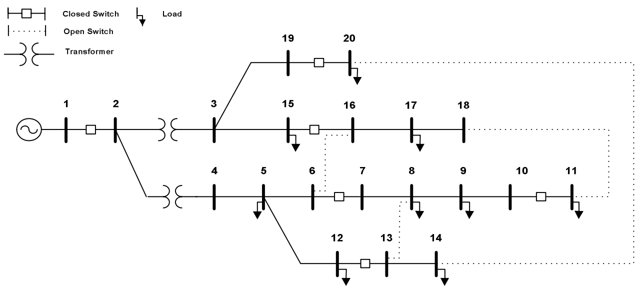

Figure 1. A 20-bus test distribution system

In each case, simulation results from three-phase power flows with three different distributed slack bus models are compared. The modeling process and the advantages and disadvantages of each are now discussed:

- *Three-phase power flow with a single slack bus* [9] assumes the substation has participation factor 1, absorbing all system loss; and the DG has participation factor 0. These participation factors do not reflect the fact that each source contributes to the load and loss at the same time.
- *Three-phase power flow with distributed slack bus assigned by capacities* considers that all sources absorb part of the loss proportional to their scheduled real power outputs [4]. In this model, a DG has the same participation factor regardless of its location in the system. This model does not include the network parameters which affect the loss contributions.
- *Three-phase power flow with distributed slack bus assigned by generator domains* [11] is presented in this paper. In this model, the participation factors for DGs are different at different locations. The participation factors reflect the network parameters and represent loss contributions for each source.

The power flow with a single slack bus is used as a benchmark for comparisons in the following cases.

Case 1: one DG on Bus 3 to Service 1,500 kW load

In this case, one DG is installed on Bus 3 to service 1,500 kW load. Real power outputs of the DG in the table below are the sum of three per-phase real powers, which are not equal in unbalanced systems. Simulation and comparison results for this case are shown in Table 1.

Table 1. DG on Bus 3 to service 1,500kW load

		A Single Slack	Dist. Slack Gen. Cap.	Dist. Slack Gen. Dom.
Participation Factor K for the DG		0	0.2481	0.0139
DG Outputs P_G^{out} (kW)		1500.0	1555.6	1503.1
Total Sys. Loss P_{Loss}^{sys} (kW)		224.23	224.21	224.23
Load Contri. (kW)	$P_G^{load} = P_G^{out} - KP_{loss}^{sys}$	1500.0	1500.0	1500.0
Loss Contri. (kW)	$P_G^{loss} = KP_{Loss}^{sys}$	0	55.6	3.1
Diff. in Loss Contri (kW)	$\Delta P_G^{loss} = P_G^{loss} - P_{G,Bench}^{loss}$	0	55.6	3.1
Diff. Values in Loss (USD/year)	$\Delta P_G^{loss} \cdot 8760 \cdot 0.065$	0	31,658.64	1,765.14

From Table 1 row 1, we can observe that the participation factors using different distributed slack bus models vary significantly. From rows 2-4, the DG has been allowed to participate in absorbing slack and, consequently, has different real power output in order to service the same 1,500 kW load. The difference between P_G^{out} and P_G^{load} is the loss contribution P_G^{loss}, which also equals KP_{Loss}^{sys}.

Rows 5 and 6 show differences in loss considering a single bus model as the benchmark $P_{G,Bench}^{loss}$. To quantify the economic differences over one year (8760 hours,) the cost of \$0.065/kWh was used. Then, the dollar value of the differences in loss per year compared to the benchmark are \$31,658.64 (24.8% of total system loss cost) for slack assigned by capacities and \$1,765.14 (1.3% of total system loss cost) for slack assigned by generator domains.

Case 2: one DG on Bus 4 to Service 1,500 kW load

In this case, one DG is installed on Bus 4 to service 1,500 kW load. Simulation and comparison results for this case are shown in Table 2.

Table 2 DG on Bus 4 to service 1500kW load

		A Single Slack	Dist. Slack Gen. Cap.	Dist. Slack Gen. Dom.
Participation Factor *K* for the DG		0	0.2481	0.3250
DG Outputs P_G^{out} (kW)		1500.0	1551.3	1567.2
Total Sys. Loss P_{Loss}^{sys} (kW)		206.97	206.78	206.72
Load Contri. (kW)	$P_G^{load} = P_G^{out} - KP_{loss}^{sys}$	1500.0	1500.0	1500.0
Loss Contri. (kW)	$P_G^{loss} = KP_{Loss}^{sys}$	0	51.3	67.2
Diff. in Loss Contri. (kW)	$\Delta P_G^{loss} = P_G^{loss} - P_{G,Bench}^{loss}$	0	51.3	67.2
Diff. Values in Loss (USD/year)	$\Delta P_G^{loss} \cdot 8760 \cdot 0.065$	0	29,210.22	38,263.68

From Table 2, we still observe that the different models have different participation factors. In this case, while the DG services the same amount of load as in Case 1, the bus 4 location produces a significantly different participation factor. Again in Case 2, different loss contributions to service the same amount of load result, and the dollar values in loss per year are \$29,210.22 for slack assigned by capacities and \$38,263.68 slack assigned by generator domains, which are 24.8% and 32.5% of total system loss cost, respectively.

In addition, following the methodology of [7], for each case we can determine generator domains from the single slack bus power flow to allocate loads and losses to DGs. For Case 1, post-processing the generator domains from the single slack bus power flow solution produced no significant difference in P_G^{load} and P_G^{loss}. However, in Case 2, the kW contributions toward loss were approximately 5% different. Thus iteratively determining the generator domains yield different results depending on location.

We summarize comments and observations from the above simulations and analysis as follows:

- participation factors assigned using capacities versus generator domains vary significantly,
- DG locations should be represented by the participation factors; and the proposed iterative participation factor model reflects network parameters and provides for this,
- participation factors obtained by the proposed model can be applied to determine economic indicators to the loss and load contributions of each DG.

V. CONCLUSIONS

This paper presents a network based distributed slack bus model with iterative participation factors, for unbalanced distribution systems with DGs. The model achieves the purpose of distributing the system loss to multiple sources during power flow calculations. The impacts on several distribution system applications were discussed. Simulation results focusing on the economic costs of distributing the slack with DGs have been presented.

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