# A New Unsynchronized Two-Terminals Fault Location Method on Series Compensated Lines 

Mustafa Kizilcay, Member, IEEE, and Piergiovanni La Seta, Student Member, IEEE


#### Abstract

This paper deals with the fault location on high voltage series compensated lines. A new approach to evaluate the fault distance is proposed. The distributed-parameter line equations are preferred to describe the overhead lines. The algorithm uses two-terminal measurements, but those data do not require synchronization. No information on the fault resistance value and on the fault type is necessary.

Numerical tests with time-domain sampled measurements are carried out. The simulations are performed using EMTP-ATP. Results for various fault resistance and fault distance values, and fault type conditions, are reported.


Keywords - Distance protection, series compensated lines, EMTP, fault location, line equations.

## I. Introduction

Identification of a fault and accurate estimation of the fault distance become increasingly important for high power transmission lines in the deregulated electricity market.

The possible approaches to the digital fault location problem using moderately low sampling rate of measured data, on which this paper will be focused, can be divided into two main groups, namely the algorithms using data from one terminal of a transmission line, and the other using data from both terminals. The former is superior in the economical point of view because it requires no data transfer along long distances and no data synchronization [11]. The latter is superior in the accuracy of fault location point of view, but requires a data transfer system.

Regarding to the line description for fault location algorithms it can be distinguished between different methods. For example, one of these approaches can be the use of lumped line impedances and capacitances. Alternatively, the use of complete line differential equations allows a higher accuracy in the line modelling [1], [2]. In particular, the lossless representation allows to solve the line equations directly in the time domain. Considering more realistically the frequency dependence of line resistance and inductance, mainly due to the skin effect [7], the line equations are solved in frequency domain using the modal theory. Line equations that consider distributed line parameters are utilized for the line representation in this work.

[^0]The present paper deals with the fault location estimation on series compensated lines. The compensation stage is generally consisted of capacitor banks and protective devices, like MOV, that prevent overstressing of the capacitors during faults. The non-linearity of such equipment is well known, and the accuracy in its model strongly affects the accuracy of the fault distance evaluation.

If the voltage drop across the line is expressed as a function of the distance, the presence of a compensation stage along the line introduces a discontinuity at the compensation location. Due to this discontinuity a problem for the fault location algorithm arises to assume that the fault location is either in front of or behind the compensation point, thus providing two solutions to the problem.

A fault location algorithm for series compensated lines has to be supported by a criterion to decide which one of the two estimated solutions is the correct one. Different criteria were proposed in the literature; in [5] and [9] a criterion based on the estimated fault resistance has been suggested.

For the new method based on two-terminal measurements no assumption is to be made regarding the fault resistance value and the fault type. The proposed approach is general, and based only upon known electrical properties of the transmission line.

## II. Line Equations

As mentioned in the Introduction, the overhead line is modelled using distributed-parameter line equations. This way, unbalance conditions and coupling between phases of transmission lines and circuits of parallel lines can be taken into account.

The conductors' electrical properties can be expressed defining resistive, inductive and capacitive elements as quantities per unit length. These terms are called respectively $R^{\prime}, L^{\prime}$ and $C^{\prime}$. The conductance term $G^{\prime}$, to take into account dissipative effects like Corona and others, will be neglected. The equivalent of an infinitesimal section of a single-phase line is shown in Fig. 1.


Fig. 1. An infinitesimal section of a single-phase homogeneous line.

Let us consider an infinitesimal section of a multi-phase homogenous line, and denote with $\mathbf{R}^{\prime}, \mathbf{L}^{\prime}$ and $\mathbf{C}^{\prime}$ the coupled and symmetric line resistance, inductance and capacitance matrices. The time domain voltage and current equations for such a line section are as follows:

$$
\begin{gather*}
\frac{\partial \mathbf{V}(x, t)}{\partial x}=\mathbf{R}^{\prime} \mathbf{I}(x, t)+\mathbf{L}^{\prime} \frac{\partial \mathbf{I}(x, t)}{\partial t}  \tag{1}\\
\frac{\partial \mathbf{I}(x, t)}{\partial x}=\mathbf{C}^{\prime} \frac{\partial \mathbf{V}(x, t)}{\partial t} \tag{2}
\end{gather*}
$$

A mathematical way to solve this equation system in the time domain would lead to non-homogenous wave equations, whose non-homogeneous term would be in particular a term depending mainly on the resistance of the conductors. The best way is to solve this problem in the frequency domain by applying the Fourier theorem and reducing the time and space depending functions (voltage and current) to only space dependent functions, for a constant frequency. Since line equations will be evaluated at a constant frequency for fault location, $\omega=2 \pi f$ will be omitted in the further analysis. The equations in frequency domain can be expressed now as follows:

$$
\begin{equation*}
\frac{d \mathbf{V}(x)}{d x}=\mathbf{Z}^{\prime} \mathbf{I}(x) \quad \frac{d \mathbf{I}(x)}{d x}=\mathbf{Y}^{\prime} \mathbf{V}(x) \tag{3}
\end{equation*}
$$

where:

$$
\mathbf{Z}^{\prime}=\mathbf{R}^{\prime}+j \omega \mathbf{L}^{\prime} \quad \mathbf{Y}^{\prime}=j \omega \mathbf{C}^{\prime}
$$

Differentiating both equations (3) according to the $x$ variable, and rearranging the terms, one can easily obtain:

$$
\begin{align*}
& \frac{d^{2} \mathbf{V}(x)}{d x^{2}}=\mathbf{Z}^{\prime} \mathbf{Y}^{\prime} \mathbf{V}(x)  \tag{4a}\\
& \frac{d^{2} \mathbf{I}(x)}{d x^{2}}=\mathbf{Y}^{\prime} \mathbf{Z}^{\prime} \mathbf{I}(x) \tag{4b}
\end{align*}
$$

The matrices $\mathbf{Z}^{\prime}$ and $\mathbf{Y}^{\prime}$ are not diagonal, and this means that the solution of the equations (4a) and (4b) is not straightforward. The differential equation set that describes threephase single- or multi-circuit lines can be solved in an efficient way with the help of modal theory [2] [3]. In modal domain a number of single-phase equivalent uncoupled circuits is represented, each of which being a mode that can be dealt with independently from others modes.

Although modal theory is well known, basic equations will be given below for completeness. In general, the matrices $\mathbf{Z}$, and $\mathbf{Y}^{\prime}$, are to be assumed symmetric. Using two transformation matrices, $\mathbf{T}_{\mathbf{i}}$ for currents and $\mathbf{T}_{\mathrm{v}}$, for voltages:

$$
\begin{equation*}
\mathbf{V}=\mathbf{T}_{\mathrm{v}} \mathbf{V}_{\mathrm{m}} \quad \mathbf{I}=\mathbf{T}_{\mathbf{i}} \mathbf{I}_{\mathrm{m}} \tag{5}
\end{equation*}
$$

the line equations can be transformed from phase coordinates to the modal domain. By using the eigenvalue theory to determine the transformation matrices, it results:

$$
\begin{align*}
& \frac{d^{2} \mathbf{V}_{m}}{d x^{2}}=\mathbf{T}_{\mathrm{v}}^{-1} \mathbf{Z}^{\prime} \mathbf{Y}^{\prime} \mathbf{T}_{\mathbf{v}} \mathbf{V}_{\mathrm{m}}=\boldsymbol{\Gamma}^{2} \mathbf{V}_{\mathrm{m}}  \tag{6a}\\
& \frac{d^{2} \mathbf{I}_{\mathrm{m}}}{d x^{2}}=\mathbf{T}_{\mathrm{i}}^{-1} \mathbf{Y}^{\prime} \mathbf{Z}^{\prime} \mathbf{T}_{\mathbf{i}} \mathbf{I}_{\mathrm{m}}=\boldsymbol{\Gamma}^{2} \mathbf{I}_{\mathrm{m}} \tag{6b}
\end{align*}
$$

The differential equations result to be decoupled through the square of matrix $\Gamma$, that is the modal propagation matrix. So it is possible to find a solution to the equation system (6) by solving a number of simple second order equations. In a matrix form, this general solution can be expressed as follows:

$$
\begin{gather*}
\mathbf{V}_{\mathrm{m}}(x)=\mathbf{C}_{1} e^{-\Gamma x}+\mathbf{C}_{2} e^{\mathrm{\Gamma} x}  \tag{7a}\\
\mathbf{I}_{\mathrm{m}}(x)=\mathbf{Y}_{\mathrm{Cm}}\left[\mathbf{C}_{1} e^{-\boldsymbol{\Gamma} x}-\mathbf{C}_{2} e^{\mathrm{\Gamma} x}\right] \tag{7b}
\end{gather*}
$$

The modal characteristic impedance matrix $\mathbf{Z}_{\mathbf{C m}}$ (and the modal characteristic admittance matrix $\mathbf{Y}_{\mathbf{C m}}$ ) can be expressed as:

$$
\begin{equation*}
\mathbf{Z}_{\mathrm{Cm}}=\mathbf{Y}_{\mathrm{Cm}}^{-1}=\left(\mathbf{Z}_{\mathrm{m}} \mathbf{Y}_{\mathrm{m}}^{-1}\right)^{1 / 2} \tag{8}
\end{equation*}
$$

where:

$$
\mathbf{Z}_{\mathrm{m}}=\mathbf{T}_{\mathrm{v}}^{-1} \mathbf{Z} \mathbf{T}_{\mathrm{i}} \quad ; \quad \mathbf{Y}_{\mathrm{m}}=\mathbf{T}_{\mathrm{i}}^{-1} \mathbf{Y} \mathbf{T}_{\mathrm{v}}
$$

If the line were considered as lossless, it would be easy to find a solution directly in the time domain, because the equation set (7) for each mode would have a purely imaginary exponential term, so that those equations would be transformed from frequency to time domain by simply introducing a time delay. In the present work, the losses and the frequency dependence of the line parameters are taken into account.

For a line with a length $l$, the two wave coefficient vectors $\mathbf{C}_{1}$ and $\mathbf{C}_{2}$ can be found from the equations (7) by evaluating at $x=0$ and $x=l$, corresponding to the two terminals of the line $K$ and $M$, respectively. Substitution of the boundary conditions in the equations (7) results in:

$$
\begin{align*}
& \mathbf{V}_{\mathrm{Mm}}=\frac{1}{2}\left(e^{-\Gamma l}+e^{\Gamma l}\right) \mathbf{V}_{\mathrm{Km}}+\frac{1}{2}\left(e^{-\Gamma l}-e^{\mathrm{\Gamma} l}\right) \mathbf{Z}_{\mathrm{Cm}} \mathbf{I}_{\mathrm{Km}}  \tag{9a}\\
& \mathbf{I}_{\mathrm{Mm}}=\frac{1}{2} \mathbf{Y}_{\mathrm{Cm}}\left(e^{-\mathrm{\Gamma} l}-e^{\mathrm{\Gamma} l}\right) \mathbf{V}_{\mathrm{Km}}+\frac{1}{2}\left(e^{-\Gamma l}+e^{\Gamma}\right) \mathbf{I}_{\mathrm{Km}} \tag{9b}
\end{align*}
$$

## III. FaUlt Location Method

The main aspects of the proposed approach are summarized in this section briefly. For this purpose it is worth to refer to the line configuration shown in Fig. 2 for the following considerations.

In Fig. 2 a fault on a series compensated high voltage overhead transmission line is depicted. The two terminals of the line are identified by the nodes $K$ and $M$, respectively. The line is connected to the power network, represented by Thevenin equivalents at both terminals consisting of voltage sources and source impedances $Z_{K}$ and $Z_{M}$. The capacitor banks equipped with MOV protection devices are placed at a known distance $h$ from the node $K$. Let us denote the two buses immediately


Fig. 2. Single-line diagram of a series compensated line with a fault at distance $d$.


Fig. 3. Equivalent circuit of the series compensated line and source networks expressed in modal quantities.
before and after the capacitor banks as $P$ and $S$, respectively. In Fig. 2 the fault location is supposed to be after the compensation stage, at a distance $d$ from node $K$. The total line length is $l$.

In Fig. 3 the representation of the system in Fig. 2 in modal quantities is shown. Furthermore, in Fig. 3 the distances are referred to the fault location point " 0 ", although unknown.

## A. MOV and equivalent impedance

The MOV is one of the suitable devices to protect the capacitor banks used for series compensation. The necessary equipment protection has from the protective relaying point of view the drawback that the used devices with nonlinear characteristic have their impact on the fault location estimation and line tripping.

The presence of an MOV makes the compensation block to be nonlinear. It is possible to represent the compensation stage as an equivalent series impedance at operation frequency (50 Hz ). In this equivalent representation, both the resistance and the reactance is dependent on the amplitude of the power frequency current component flowing into the block capacitanceMOV. This representation at power frequency is sufficient, because for the fault location only power frequency components of measured currents and voltages will be taken into consideration.

The $V-I$ characteristic of the installed MOVs should be known. Fig. 4 shows the typical equivalent impedance for the block capacitance-MOV.

## B. Equations to determine the fault location

The three line sections shown in Fig. 3 are represented by the distributed-parameter line model in modal domain. The line section between nodes $K$ and $P$ of length $h$ is described in modal quantities by applying equation set (9):

$$
\begin{align*}
& \mathbf{V}_{\mathrm{Pm}}=\frac{1}{2}\left(e^{-\Gamma h}+e^{\Gamma h}\right) \mathbf{V}_{\mathrm{Km}}+\frac{1}{2}\left(e^{-\Gamma h}-e^{\Gamma h}\right) \mathbf{Z}_{\mathrm{Cm}} \mathbf{I}_{\mathrm{Km}}  \tag{10a}\\
& \mathbf{I}_{\mathrm{Pm}}=\frac{1}{2} \mathbf{Y}_{\mathrm{Cm}}\left(e^{-\Gamma h}-e^{\Gamma h}\right) \mathbf{V}_{\mathrm{Km}}+\frac{1}{2}\left(e^{-\Gamma h}+e^{\Gamma h}\right) \mathbf{I}_{\mathrm{Km}} \tag{10b}
\end{align*}
$$

where $e^{\Gamma h}$ is the diagonal matrix of modal propagation coefficients, $\mathbf{Z}_{\mathbf{C m}}$ and $\mathbf{Y}_{\mathbf{C m}}$ are modal characteristic impedance and modal characteristic admittance matrices, respectively.


Fig. 4. Typical current-dependent characteristics of the impedance of the block capacitance-MOV

The presence of a MOV makes the compensation block to be nonlinear. However, as mentioned, it is possible to adopt an equivalent current-dependent series impedance representation. The diagonal impedance matrix of capacitors and MOV is denoted as $\mathbf{Z}_{\text {MOV }}$, whereas its modal representation is:

$$
\begin{equation*}
\mathbf{Z}_{\mathrm{mov}_{\mathrm{m}}}=\mathbf{T}_{\mathrm{v}}^{-1} \mathbf{Z}_{\mathrm{Mov}} \mathbf{T}_{\mathrm{i}} \tag{11}
\end{equation*}
$$

Assuming that the fault is behind the compensation stage, i.e. between nodes $S$ and $M$ in Fig. 3, the modal voltages and currents immediately after the compensation location (indicated in Fig. 3 for a single mode as $V_{S m}$ and $I_{S m}$ ) can be determined independent from the unknown fault distance $d$. The non-linear equivalent impedance of the compensation stage is taken into account and the matrix $\mathbf{Z}_{\mathbf{M O V m}}$ is used to calculate the voltage drop to express the voltages at node $S$. The current vector $\mathbf{I}_{\mathbf{S m}}$ is equal to the current vector $\mathbf{I}_{\mathbf{P m}}$.

$$
\begin{equation*}
\mathbf{V}_{\mathrm{Sm}}=\mathbf{V}_{\mathrm{Pm}}-\mathbf{Z}_{\mathrm{MOVm}} \mathbf{I}_{\mathrm{Pm}} \tag{12a}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathbf{I}_{\mathrm{Sm}}=\mathbf{I}_{\mathrm{Pm}} \tag{12b}
\end{equation*}
$$

Buses $S$ and $M$ can now be considered as the first and second nodes to evaluate the fault location. A numerical algorithm is developed to obtain the fault distance. The proposed method expresses the voltage and current phasors at the fault location, denoted as node $N$, as functions of the distance.

The voltage at fault location and the first equation to calculate the fault currents can be determined, taking into account that the section between nodes $S$ and $N$ has a length $d-h$ :

$$
\begin{align*}
\mathbf{V}_{\mathrm{N} \mathrm{~m}}(d)= & \frac{1}{2}\left(e^{-\mathrm{r}(d-h)}+e^{\mathrm{r}(d-h)}\right) \mathbf{V}_{\mathrm{Sm}} \\
& +\frac{1}{2}\left(e^{-\mathrm{r}(d-h)}-e^{\mathrm{T}(d-h)}\right) \mathbf{Z}_{\mathrm{Cm}} \mathbf{I}_{\mathrm{Sm}}  \tag{13a}\\
\mathbf{I}_{\mathrm{Nm} 1}(d)= & \frac{1}{2} \mathbf{Y}_{\mathrm{Cm}}\left(e^{-\mathrm{r}(d-h)}-e^{\mathrm{r}(d-h)}\right) \mathbf{V}_{\mathrm{Sm}}  \tag{13b}\\
& +\frac{1}{2}\left(e^{-\mathrm{r}(d-h)}+e^{\mathrm{r}(d-h)}\right) \mathbf{I}_{\mathrm{sm}}
\end{align*}
$$

The second equation to calculate the fault currents can be determined considering the phase currents measured at node M , expressed in modal domain and referred to the fault location:

$$
\begin{align*}
\mathbf{I}_{\mathrm{Nm} 2}(d)= & \frac{1}{2} \mathbf{Y}_{\mathrm{Cm}}\left(e^{\mathrm{r}(l-d)}-e^{-\mathrm{r}(l-d)}\right) \mathbf{V}_{\mathrm{Mm}} \\
& +\frac{1}{2}\left(e^{\mathrm{r}(l-d)}+e^{-\mathrm{r}(l-d)}\right) \mathbf{I}_{\mathrm{Mm}} \tag{14}
\end{align*}
$$

The fault distance-dependent modal current flowing into the fault resistance $R_{F}$ will be:

$$
\begin{equation*}
\mathbf{I}_{\mathrm{Nm}}(d)=\mathbf{I}_{\mathrm{Nm} 1}(d)-\mathbf{I}_{\mathrm{N} \mathrm{~m} 2}(d) \tag{15}
\end{equation*}
$$

Under the assumption that the fault impedance is purely resistive, i.e. $R_{F}$, for real values of the distance the phase angle of fault voltage $V_{\mathrm{Nm}}$ equals to the phase angle of the fault current $I_{\mathrm{Nm}}$ only at the fault location. The crossing point identifies a solution candidate. On this basic principle is the solution algorithm for the fault location estimation based, which was proposed by the authors in [12] and adapted for the case of series compensated lines.

The case of a fault in front of the compensation stage, i.e. in the first part of the line, can be treated equally only by inverting the current phasors signs and looking into the system depicted in Fig. 3 from node $M$ to node $K$. Thus the second candidate of solution for $d$ is obtained. To identify the correct one, i.e. correct fault location, a new criterion is developed.

## C. Use of unsynchronized measured data

In [11] a fault location method is described that uses unsynchronized measured data at two-ends of a line. The solution of the synchronization angle transcendent equation in [11] is provided through the Newton-Raphson iterative method. In this work a different solution method is followed, when the measured data at the two line terminals are unsynchronized as in [11]. The alternative solution method is based on the fact that the proposed fault location algorithm is itself iterative.

Assuming that the modal voltages and currents measured at nodes $K$ and $M$ are unsynchronized, and denoting the synchronization angle between the corresponding phasors as $\delta$, the modal vectors at node $K$ follows:

$$
\begin{align*}
& \mathbf{V}_{\mathrm{Km}}^{(\mathrm{s})}=\mathbf{V}_{\mathrm{Km}} e^{j \delta}  \tag{16a}\\
& \mathbf{I}_{\mathrm{Km}}^{(\mathrm{s})}=\mathbf{I}_{\mathrm{Km}} e^{j \delta} \tag{16a}
\end{align*}
$$

As a consequence, the equations for the fault location estimation have to be rewritten. Introducing the angle $\delta$, the modal voltages and currents at fault location $N$ given in equations (13a) and (15) are expressed as:

$$
\begin{gather*}
\mathbf{V}_{\mathrm{Nm}}^{(\mathrm{s})}(d)=\mathbf{V}_{\mathrm{Nm}}(d) e^{j \delta}  \tag{17}\\
\mathbf{I}_{\mathrm{Nm}}^{(\mathrm{s})}(d)=\mathbf{I}_{\mathrm{N} \mathrm{~m} 1}(d) e^{j \delta}-\mathbf{I}_{\mathrm{N} \mathrm{~m} 2}(d) \tag{18}
\end{gather*}
$$

As it can be seen from (17) and (18), it is not possible to have a solution for $d$ unless the synchronization angle $\delta$ is determined. As suggested in [11], if it is not possible to derive directly an equation for $\delta$, two iterative cycles can be used alternately, the first one to find $\delta$, assuming a constant estimated fault distance, and the second one to evaluate the fault distance, assuming constant synchronization angle that was determined in the previous iteration.

The modal voltages at node $N$ to be obtained by introducing the measured data at nodes $K$ and $M$ according to (9a), can be used to determine the synchronization angle. The modal voltages at node $N$ due to measured data at node $K$, which take into account the angle $\delta$, are given in (17). The modal voltages at node $N$ due to measured data at node $M$ follow:

$$
\begin{align*}
\mathbf{V}_{\mathrm{Nm}}^{\mathrm{M}}(d)= & \frac{1}{2}\left(e^{\mathrm{\Gamma}(l-d)}+e^{-\mathrm{\Gamma}(l-d)}\right) \mathbf{V}_{\mathrm{Mm}} \\
& +\frac{1}{2}\left(e^{\mathrm{\Gamma}(l-d)}-e^{-\mathrm{\Gamma}(l-d)}\right) \mathbf{Z}_{\mathrm{Cm}} \mathbf{I}_{\mathrm{Mm}} \tag{19}
\end{align*}
$$

The modal voltages at node N from (17) and (19) have to be obviously the same, independently from the parameter $d$. Thus:

$$
\begin{equation*}
\mathbf{V}_{\mathrm{Nm}}^{(\mathrm{s})}(d)=\mathbf{V}_{\mathrm{Nm}}(d) e^{j \delta}=\mathbf{V}_{\mathrm{Nm}}^{\mathrm{M}}(d) \tag{20}
\end{equation*}
$$

Since the equality in (20) has to be valid for each modal component, to determine $\delta$ the use of the first modal component only is sufficient:

$$
\begin{equation*}
e^{j \delta}=\frac{V_{\mathrm{N} 1}^{\mathrm{M}}(d)}{V_{\mathrm{N} 1}(d)} \tag{21}
\end{equation*}
$$

Being normally the angle $\delta$ small, an approximation for the Euler formula can be used

$$
e^{j \delta}=\cos \delta+j \sin \delta \cong 1+j \delta
$$

so that the incremental angle variation of $\delta$ at each iteration $i$ results in:

$$
\begin{align*}
& \Delta \delta=-j\left[\frac{V_{\mathrm{N} 1}^{\mathrm{M}}(d)}{V_{\mathrm{N} 1}(d)}-1\right]  \tag{22}\\
& \rightarrow \quad \delta_{i+1}=\delta_{i}+\Delta \delta
\end{align*}
$$

For each iteration of the fault location algorithm, an iterative calculation to determine the synchronization angle is performed. A solution for $\delta$ is obtained at the end of this second iterative cycle, when $|\Delta \delta|$ as in (22) is smaller than a predefined threshold (e.g. $10^{-4}$ ).

## D. Criterion for the selection of the correct solution

One of the problems in the fault location estimation on series compensated lines is that the presence of the compensation stage makes it difficult to determine whether the fault position is in front of or behind the compensation itself. In such case, usually the fault distance is estimated under both assumptions, and the correct solution is chosen with the help of a selection criterion.

In the present work, a precise selection criterion is proposed. Once the fault location algorithm provides a candidate of solution, called $\bar{d}$, the assumption that the fault is in front of or behind the compensation stage can be verified by comparing the modal fault voltages for the distance $\bar{d}$ using the data measured at node $K$, i.e. $\mathbf{V}_{\mathrm{Nm}}$ as expressed in (13a), and the data measured at node $M$, i.e. $\mathbf{V}_{\mathrm{Nm}}^{\mathrm{M}}$ as expressed in (19). The scalar index $\varepsilon_{S}$ is so introduced:

$$
\begin{equation*}
\varepsilon_{S}=\frac{\sum_{m=1}^{3}\left|V_{\mathrm{N} m}(\bar{d})-V_{\mathrm{N} m}^{\mathrm{M}}(\bar{d})\right|}{V_{n o r m}} \tag{23}
\end{equation*}
$$

This equation calculates the sum of the magnitude differences of these voltages, which are normalized by $V_{\text {norm }}$. The nominal voltage of the line can be chosen as $V_{\text {norm }}$. This index is calculated twice with the assumption that either the fault is in front of or behind the compensation stage. Theoretically, at the correct fault location, the index $\varepsilon_{S}$ should be zero. Since in practice the solution is strongly dependent on the correct evaluation of the equivalent impedance of the block capaci-tance-MOV, $\varepsilon_{S}$ will be not zero. The correct solution will be the one with smaller $\varepsilon_{S}$.

## IV. Numerical Examples and Results

The simulations of the faulted line in steady-state conditions are performed using the electrical network simulation program EMTP-ATP [10]. The line is physically described and, the electrical distributed parameters model is obtained using built-in supporting program Line Constants.

The line is a three-phase, two conductor-bundled overhead line, with two ground wires " 0 ", in a horizontal configuration. In Fig. 5 the line configuration is depicted.

The $380-\mathrm{kV}$ line is 250 km long, and it is supplied from both sides by two source networks at 380 kV (line-to-line rms value), whose positive-sequence source impedances are:

$$
\begin{aligned}
& Z_{K}=(1.334+\mathrm{j} 13.34) \Omega \\
& Z_{M}=(1.6+\mathrm{j} 16.01) \Omega
\end{aligned}
$$



Fig. 5. 380-kV overhead line with two ground wires and 2-bundle phase wires
Voltage phase displacement angles are $\delta_{K}=0^{\circ}$ and $\delta_{M}=-15^{\circ}$, respectively. The required matrices of the series impedances and shunt admittances per length (see (6)) are computed by LINE CONSTANTS routine at power frequency.

The compensation stage is located in the middle of the line, 125 km far from both line terminals. A two-phase-to-ground fault (A-B-G) is assumed at a distance $d=175 \mathrm{~km}$ from node $K$ behind the compensation stage. The fault resistance is assumed to be $5 \Omega$. The sampling rate of measured data is selected as 2 kHz . Fig. 6 shows the estimated distance using Fourier analysis as a function of time for the two possibilities that the fault is in front of or behind the compensation stage referring to node $K$.


Fig. 6. Estimated fault distances for the assumption of the fault in front of (a) and behind (b) the compensation stage

The index $\varepsilon_{S}$ calculated for the two possibilities of fault location are:

$$
\begin{aligned}
& \varepsilon_{S}=1.245 \quad \text { (fault in front of the compensation stage) } \\
& \varepsilon_{S}=0.023 \text { (fault behind the compensation stage). }
\end{aligned}
$$

This result clearly indicates that the fault seen from node $K$ is behind the compensation stage. Accordingly, the calculated distance of the fault is obtained from Fig. 6b, which shows the estimated fault distance as $d=177 \mathrm{~km}$ with an error of $1.14 \%$. The fault resistance for the correct solution is estimated as $R_{\mathrm{F}}=4.897 \Omega$.

In Table I the results of various fault location tests on a three-phase series compensated faulted line are provided. In the first column the simulated fault location, resistance, type of the fault and location of the compensation stage are specified. The first six tests refer to a compensation stage in the middle of the line, the remaining two tests refer to a compensation stage at 80 km measured from the terminal $K$ of the line ( $32 \%$ of the total length). Estimated fault location and index $\varepsilon_{S}$ assuming that the fault is in front of and behind the compensation stage, respectively, are given in the next columns. The estimated fault resistance for the correct solution is provided in the last column. The correct fault locations in Table I are indicated by bold-face. The iterative process to synchronize measured data at the two terminals of the line has been proved to be effective in all the cases, so the fault location algorithm is tested in this example using synchronized data.

TABLE I:
Fault Location and Fault Resistance Estimation for a three-phase Single Circuit Series Compensated Line

| Specification: <br> fault distance (km), <br> resistance $(\Omega)$, type, <br> compensation stage <br> distance (km) | Estimated results |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | Location <br> $1[\mathrm{~km}]$ | $\varepsilon_{\mathrm{S} 1}$ | Location <br> $2[\mathrm{~km}]$ | $\varepsilon_{\mathrm{S} 2}$ | Resistance <br> $[\Omega]$ |
| 50,10, A-G, 125 | $\mathbf{5 1 . 3}$ | 0.027 | 115.5 | 0.636 | 10.511 |
| 75,5, A-B, 125 | $\mathbf{7 2 . 9}$ | 0.024 | 192 | 0.901 | 5.097 |
| 175,5, A-B-G, 125 | 43.6 | 1.245 | $\mathbf{1 7 7}$ | 0.023 | 4.897 |
| 110,10, A-C-G, 125 | $\mathbf{1 0 8 . 2}$ | 0.019 | 234 | 0.919 | 10.082 |
| 225,10, B-G, 125 | 158.6 | 0.853 | $\mathbf{2 2 6 . 5}$ | 0.02 | 10.274 |
| 20,5, A-C, 125 | $\mathbf{1 9 . 2}$ | 0.02 | 93.9 | 0.879 | 5.21 |
| 50,5, A-G, 80 | $\mathbf{5 0 . 8}$ | 0.037 | 117.8 | 0.713 | 5.4 |
| 225,10, A-C-G, 80 | 145.2 | 1.157 | $\mathbf{2 2 6 . 8}$ | 0.028 | 10.349 |

## V. Conclusions

This paper deals with fault location on series compensated lines. The developed method uses line equations taking into consideration the distributed nature of line impedances and capacitances in contrary to former methods that describe the line by a lumped series impedance.

The presence of a non-linear element along the line, such as the series capacitance and its protective device MOV, does not allow linear representation such as the Thevenin method. For that reason a two-terminal fault location algorithm is proposed.

The fault distance is determined in a general way using modal theory. The developed method does not need any knowledge of fault resistance and fault type. Only line parameters and the impedances of source networks interfacing the line at both ends are required. The compensation stage with the protection device can be represented by currentdependent equivalent impedance at power frequency.

The solution algorithm proposed by the same authors in [12] is adapted to series compensated lines in this paper. Unsynchronized measured data from the two line terminals are taken into consideration in the iterative solution method. Two candidates of a solution for the fault distance are obtained, assuming that the fault is in front of and behind the compensation stage. To select the correct solution, a new criterion is introduced.

Several tests performed using numerical fault simulations show that the method is accurate under idealized condition, i.e. influence of instrument transformers is not taken into account. Effects of instrument transformers and appropriate signal processing of measured quantities will be the subject of future work.

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## VII. BIographies



Mustafa Kizilcay (M'94) was born in Bursa, Turkey in 1955. He received the B.Sc. degree from Middle East Technical University of Ankara in 1979, Dipl.-ing. degree and Ph.D. degree from University of Hanover, Germany in 1985 and 1991. From 1991 until 1994, he was as System Analyst with Lahmeyer International in Frankfurt, Germany. 1994-2004 he has been professor for Power Systems at Osnabrueck University of Applied Sciences, Germany. In 2004 he received a chair at the University of Siegen, Germany, for electrical power systems. Dr. Kizilcay is winner of literature prize of Power Engineering Society of German Electroengineers Association (ETG-VDE) in 1994. His research fields are power system analysis, digital simulation of power system transients and dynamics, and digital protection. He is a member of IEEE, CIGRE, VDE and VDI in Germany


Piergiovanni La Seta was born in Reggio Calabria, Italy, in 1976. He received the degree in Management Engineering from University of Calabria, Italy in 2001. He worked as scientific collaborator at Osnabrück University of Applied Sciences, Germany, on fault location on EHV transmission lines. Since 2003 he is Ph.D. student at the Technische Universität Dresden, Germany. His research interests concern electric machine drive, power converters, HV transmission lines fault studies, and innovative stability studies on local energy systems.


[^0]:    M. Kizilcay is with the University of Siegen, D-57068 Siegen, Germany (e-mail kizilcay@ieee.org)
    P. La Seta is with the Technische Universität Dresden, D-01063 Dresden, Germany (e-mail laseta@ieeh.et.tu-dresden.de)

