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Supply chain network equilibrium with strategic financial hedging using futures

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ABSTRACT

In this paper, we develop a network equilibrium model for supply chain networks with strategic financial hedging. We consider multiple competing firms that purchase multiple materials and parts to manufacture their products. The supply chain firms' procurement activities are exposed to commodity price risk and exchange rate risk. The firms can use futures contracts to hedge the risks. Our research studies the equilibrium of the entire network where each firm optimizes its own operation and hedging decisions. We use variational inequality theory to formulate the equilibrium model, and provide qualitative properties. We provide analytical results for a special case with duopolistic competition, and use simulations to study an oligopolistic case. The analytical and simulation studies reveals interesting managerial insights.

1. Introduction

Supply chains today have become increasingly complex and global, which have made firms at different stages of supply chains more and more vulnerable to various risk factors. Understanding and managing these risks as well as their impacts on supply chain operations and profitability have become a business imperative for many companies. Therefore, supply chain risk management has drawn increasing attentions from both academians and practitioners.

In this research we focus on using futures to hedge foreign exchange risk and commodity price risk in supply chains. A survey by Scott (2009) showed that foreign exchange risk was ranked as the second most important risk factor by the risk management executives of 500 global companies. The fluctuations of currency values can cause significant loss to firms that are engaged in global trades. For example, in January 2015, the chief executive officer of Procter & Gamble warned that the appreciating value of dollar would result in a 5% reduction of the company 2015 sales and a 12% reduction in profit (Narvaez, 2015). For another example, in 2016 the British companies rushed to hedging their foreign exchange risk to protect themselves from the growing "Brexit" risk (Nag, 2016). Moreover, supply chains are also affected by the commodity price risk directly by the raw material prices and indirectly by the energy and transportation cost (Zsidisin, Hartley, & Gaudenzi, 2016). For example, in 2011 the consumer production company, Kimberly-Clark, suffered sales and profit decline partially due to the increasing wood pulp price (Zsidisin & Hartley, 2012). Commodity prices can be very volatile in the global market. For instance, from August 2003 to March 2004, soybean prices increased by 74% from \$237 to \$413, and then dropped to \$256 within the next two years (Zsidisin & Hartley, 2012). For another instance, from April 2010 to April 2011, the price of silver tripled in the commodity market (Zsidisin & Hartley, 2012).

According to a study of over 7000 nonfinancial firms from 50 countries, about 60% of the surveyed firms have conducted some form of hedging using financial derivatives (Bartram, Brown, & Fehle, 2009). Our research focuses on the use of futures to hedge foreign exchange and commodity price risks. In the world largest futures and option market, CME Group, (Chicago Mercantile Exchange and Chicago Board of Trade), futures are the dominant form of derivative contract for foreign exchange rates and commodity prices. For example, as of October 2017, for foreign exchange rate, the ADV (average daily volume) of futures is 832,165 and the ADV of options is 78,688; for metals, the ADV of futures is 506,049 and the ADV of options is 47,462; and for commodities and alternative investment, the ADV of futures is 1,116,089 and the ADV of options is 249,468 (CME Group, 2017). CME also provides detailed guides for businesses at different stages of supply chains to engage in financial hedging.

Our paper falls in the research stream of supply chain risk hedging, which uses two main approaches: operational hedging

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and financial hedging (see, e.g., Van Mieghem, 2003). Operational hedging uses operational and processing flexibility to mitigate supply chain risks. Such operational flexibility may be incorporated in various supply chain decisions, such as, facility locations, multisourcing, subcontracting, etc. For example, Huchzermeier and Cohen (1996) investigated the value of operational flexibility under exchange rate risk. Kazaz, Dada, and Moskowitz (2005) studied the selection of production policies with the option of postponing allocation decisions under foreign exchange risk. Goh, Lim, and Meng (2007) studied a multi-stage supply chain network with foreign exchange risk, demand and supply risk, and disruption risk using a stochastic model.

Financial hedging, on the other hand, uses financial markets and financial instruments, such as, forward contracts, futures and options, to counterbalance various risk factors. Financial hedging has long been studied in the area of finance. For a detailed and complete review of financial derivatives and financial hedging strategies, we refer the audience to the textbook by Hull (2002). Studies in the literature have also investigated the relationships between operational hedging and financial hedging, and the strategies to integrate the two approaches. Mello, Parsons, and Triantis (1995) studied a multinational firm that had production sourcing flexibility and financial hedging tools to mitigate foreign exchange risk. Ding, Dong, and Kouvelis (2007) also investigated the integration of financial hedging and operational hedging for an international firm. The firm in the study was allowed to hedge foreign exchange risk by optimally using financial options and/or delaying production allocation at different markets. A mean-variance approach was used to model the risk-averse behavior of the firm. Van Mieghem (2003) discussed the literature regarding capacity management under uncertainty. The paper reviewed capacity investment models, and compared financial and operational hedging methods. Caldentey and Haugh (2009) designed a Stackelberg game to study the flexible supply chain contracts between a producer and a retailer with and without financial hedging. The authors showed that the producer preferred the flexible contract with hedging while the preference of the retailer depended on the model parameters. Hommel (2003) used a real option approach to show that operational hedging created through operational flexibility provided a strategic complement to any financial hedging based on variance minimization. Moreover, operational flexibility also affected the composition of the financial hedging portfolio. Chowdhry and Howe (1999) studied the conditions under which multinational firms would reduce risk exposure by operational hedging. The paper found that firms would use operational hedging only when they were exposed to both exchange rate risk and demand risk. The paper also discussed the plant location and capacity decisions under different conditions of demand and foreign exchange rate. In addition, the authors showed that the firms could execute the optimal financial hedging policy with call and put and forward contracts. Chen, Li, and Wang (2014) used the mean-variance approach to study a firm's capacity planning problem with financial hedging when it had potential suppliers in multiple low-cost countries and was exposed to foreign exchange risk and demand uncertainty. The paper investigated the benefits of hedging strategies in different scenarios. The authors also found that the financial and operational hedging could interact each other to maximize the firm utility. Chod, Rudi, and Van Mieghem (2010) studied a firm capacity investment under demand uncertainty. The paper considered two risk factors: mismatch between capacity and demand and profit variability. It showed that operational flexibility could mitigate the mismatch risk while financial hedging could mitigate profit variability. The paper showed that whether operational flexibility and financial hedging could be complements or substitutes depended on the type of the operational flexibility. Zhao and Huchzermeier (2015) reviewed the literature, and proposed a risk management framework for the integration of operations-financial interface models. The paper also investigated the conditions under which operations and finance should be integrated, and presented categorizations of operational and financial hedging. In addition, the authors discussed the connections between relationship analysis and approach choice. Zhao and Huchzermeier (2017) studied a multinational corporation that could use production switching, capacity reshoring, and financial hedging to mitigate supply-demand mismatches and exchange rate risk. The authors decomposed operations and finance to optimize the mean-conditional value-at-risk. The paper found that the financial and operational hedging could be complements in optimizing the profit and risk. In addition, the paper showed that the two hedging methods were substitutes in risk reduction, and coordination was important to minimize the substitution effects. Park, Kazaz, and Webster (2017) investigated how a firm could mitigate global economic risk through production hedging, pricing, and financial hedging under exchange rate and demand uncertainty. Their model assumed that the firm maximized the expected profit with the consideration of a value-at-risk constraint. Gamba and Triantis (2014) studied a dynamic integrated risk management strategy consisting of liquidity management, financial hedging, and operational flexibility, and analyzed the impacts of different components and their interactions on risk management. Caldentey and Haugh (2006) developed an optimal control model that allowed a firm to dynamically optimize the operation policy and hedging strategy based on financial markets.

Note that these studies in the literature typically focused on the optimal strategies of a single firm or a single supply chain consisting of a firm and its supplier. Our research differs from the above studies in that our model is based on a network equilibrium approach that allows multiple heterogenous supply chains with or without financial hedging instruments to compete in a noncooperative manner. Our approach allows one to investigate the interaction between supply chain decisions and financial hedging decisions in large and realistic competitive markets. In particular, the contributions of our research to the literature are as follows:

1. Our study is the first attempt to model the financial hedging decisions and operation decisions of heterogenous firms in large scale supply chain networks. We prove the existence of the equilibrium solution to the general network model. We also prove the monotonicity of the model which guarantees that the algorithm used in the paper converges to the equilibrium solution.
2. For a special case of the model we provide and discuss closed-form analytical results regarding how volatility and the basis risk affect two competing supply chain firms' profitability and risks, which have not been reported in the literature.
3. Our general network model provides a flexible, realistic and powerful approach which can be applied to study large-scale real-world problems. Our simulation case studies based on the general network model discovered findings that could not be revealed by the analytical results.
4. Our study generates interesting managerial insights which can help managers and policy-makers better understand the connections between financial hedging and other activities in supply chain networks in a competitive environment.

In this paper the supply chain firms' risk averse behaviors are modeled using the classic mean-variance approach. The mean-variance framework was originally developed by the Nobel Laureate Harry Markowitz in his seminal work of portfolio selection, which has become part of the foundation of the modern finance theory. The mean-variance approach provides very good approximation to a variety of utility functions and nonnormal probability distributions, and suggests "good recommendations" that result in close-to-maximal utility (Kroll, Levy, & Markowitz, 1984; Levy & Markowitz, 1979; Van Mieghem, 2007). Therefore, the mean-variance framework has also been widely adapted by the researchers in the area of operations and supply chain management to model the risk averse behaviors of decision makers under uncertain environment. Hodder (1984), Hodder and Jucker (1985), and Hodder and Dincer (1986) applied the mean-variance approach in the study of facility location problems. Chen and Federgruen (2000) developed the efficient frontier for the single echelon inventory problem based on the mean-variance approach. Liu and Nagurney (2011) applied the mean-variance framework to study the operational hedging against foreign exchange rate risk with competition. The mean-variance approach has also been utilized in many other areas, such as, supply chain coordination (Gan, Sethi, & Yan, 2004; Choi, Li, Yan, & Chui, 2008), the newsvendor problem (Choi, Li, & Yan, 2001; Wu, Li, Wang, & Cheng, 2009), and return policies (Lau & Lau, 1999; Tsay, 2002). For a review of models using the mean-variance approach in supply chain risk management, see Chiu and Choi (2016).

In particular, in the literature of supply chain and financial hedging variance is a widely accepted and commonly used risk measure. For example, Tekin and zekici (2015) developed a mean-variance approach to the newsvendor model, which considered the optimization of both the ordering policy and the financial portfolio. Goel and Tanrisever (2017) studied the optimal financial hedging and procurement policies of a firm that purchased an input commodity to produce an output commodity. The paper also used variance and covariance to model the risk. For more research regarding supply chain and financial hedging which used variance or volatility to model the risk, see Chen et al. (2014), Chod et al. (2010), Ding et al. (2007), Gamba and Triantis (2014), Hommel (2003), Kouvelis, Li, and Ding (2013), Sayin, Karaesmen, and zekici (2014), Sun, Wissel, and Jackson (2016), and Sun, Chen, Ren, and Liu (2017). Since our study is in the same research stream, we follow these studies in the literature to use variance as the risk measure. Our future research plans to extend the model to incorporate other risk measures, such as, value-at-risk and conditional value-at-risk.

Next, we briefly recall the variational inequality theory based on which our model is developed. The variational inequality problem with finite dimensions, VI $(F, \mathcal{K})$, is to find a vector $X^* \in \mathcal{K} \subset R^n$, such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{1}$$

where F is a given continuous function from $\mathcal{K}$ to R^n, $\mathcal{K}$ is a closed and convex set, and $< \cdot, \cdot >$ denotes the inner product in n-dimensional Euclidean space.

The variation inequality theory has a close connection to many other mathematical programming problems including complementarity problems, fixed point problems, and optimization problems. Nagurney (1999) provides a detailed discussion regarding variational inequality theory and its applications in different areas. One of the advantages of the variational inequality theory is that it allows one to naturally integrate equilibrium and optimization problems to model and analyze large and complex real-world problems in many fields, such as, transportation, supply chains, finance, and electric power (see, for example, Cruz & Wakolbinger, 2008; Dong, Zhang, Yan, & Nagurney, 2005; Liu & Nagurney, 2009; & Nagurney & Ke, 2006). In particular, in the field of supply chain management, the variational inequality theory has been used to model the competition and cooperation of companies at different stages of a supply chain network. For example, Cruz, Nagurney, and Wakolbinger (2006) used the variational inequality approach to study social networks and global supply chains with financial engineering and risk management. For an example of applying variational inequality models in large-scale problems using real-world data see Liu and Nagurney (2009).

This paper is organized as follows: In Section 2, we develop the supply chain network model with financial hedging and competition. We model the behaviors of multiple supply chain firms in competitive markets, derive the conditions governing the equilibrium of the network, and provide the variational inequality formulation. We then discuss qualitative properties of the model in Section 3. In Section 4, we analyze special cases of the model and provide closed-form solutions and analytical results. In Section 5, we apply the model to conduct simulation case studies. Section 6 summarizes managerial insights. Section 7 presents conclusions.

2. The supply chain model with financial hedging using futures contracts

We now develop the network equilibrium model of the supply chain network with financial hedging. We define $\phi \in \Phi$ as the scenarios of uncertainty. The uncertainty space Φ consists of four factors: the exchange rate spot prices at stage 2, the exchange rate futures prices at stage 2, the commodity spot prices at stage 2, and the commodity futures prices at stage 2. We assume that there are $j = 1, \ldots, J$ supply chain firms. We use n, $n = 1, \ldots, N$ to denote the foreign countries, use i, $i = 1, \ldots, I$ to denote component/part. In addition, the commodity materials are denoted by m, $m = 1, \ldots, M$. In Tables 1 and 2 we provide the definitions of the decision variables and model parameters used in our model. The equilibrium solution variables are indicated by "*".

2.1. Multi-criteria decision-making behavior the supply chain firms

We now discuss the behavior of the supply chain firms under exchange rate risk and commodity price risk. A simple numerical example of financial hedging using futures is provided in Appendix. Our model considers the decision process in two time stages with J supply chain firms, and M demand markets. Each firm faces two types of decisions: supply chain decisions and financial hedging decisions. Fig. 1 depicts the timeline of the model with supply chain and financial activities occurring at each time stage.

We first describe the supply chain decision process of the firms. In time stage 1 each firm plans the production, and orders parts it needs from foreign suppliers. The parts will be delivered later in the second stage. In time stage 2 the firms decide the quantities of commodity materials they need to purchase from the spot market, and manufacture the product. In the second stage the firms also need to pay the foreign suppliers after they receive the parts. Finally, the firms sell the products in demand markets. Note that our model allows the supply chain firms to adjust its production level at stage 2 so that the actual production may not be equal to the planned production. For example, if in the second stage the actual commodity material price turns out to be very high, which significantly increases the cost of the product. The rising cost will cause the final product price to increase, which will lower the demand in the market. In this case, the firms may produce less than planned. This additional operational flexibility can help the firms better deal with the risk.

Due to the uncertainties of foreign exchange rates and commodity prices the firms may choose to use financial markets to hedge such risks. We now discuss the financial hedging decisions of the firms. Since the firms order from foreign suppliers in stage 1 and pay for the parts in stage 2 they are exposed to the foreign exchange risk. We allow the firms to use exchange rate futures to hedge this risk. In particular, in stage 1 when the firms place orders to foreign suppliers they can purchase/long certain amount of exchange rate futures. In stage 2, when the firms receive the parts they pay the foreign suppliers and sell the exchange rate futures.

Table 1
Decision variables for the global supply chain network with strategic financial hedging model.

Notation	Definition
X_j	NI-dimensional vector associated with firm j; $j=1,\ldots,J$ with components: $\{x_{jni}; n=1,\ldots,N, \quad i=1,\ldots,I\}$ denoting the quantity of part i ordered from country n. We group all X_js vector X.
Y_j	M-dimensional vector associated with firm j; $m=1,\ldots,M$ with components: $\{y_{jm}; m=1,\ldots,M\}$ denoting the quantity of material m firm j expects to purchase from the spot market at stage 2. We group all Y_js vector Y.
W_j	N-dimensional vector associated with firm j; $j=1,\ldots,J$ with components: $\{w_{jn}; n=1,\ldots,N\}$ denoting the quantity of exchange rate hedging contract n purchased by firm j. We group all W_js vector W.
U_j	M-dimensional vector associated with firm j; $j=1,\ldots,J$ with components: $\{u_{jm}; m=1,\ldots,M\}$ denoting the quantity of commodity hedging contract m purchased by firm j. We group all U_js vector U.
$s_{jk\phi}$	the supply from firm j in market k in scenarios ϕ. We group the $s_{jk\phi}$s of firm j in scenario ϕ into the vector $S_{j\phi}$, the $s_{jk\phi}$s in scenario ϕ into the vector S_ϕ, and group all S_ϕs into the vector S.
$s^{all}_{k\phi}$	the total supply of all firms in market m, that is, $\sum_{j=1}^{J} s_{jk\phi}$.
$\hat{y}_{jm\phi}$	the actual quantity of material m firm j purchased from the spot market in scenario ϕ at stage 2. We group the $\hat{y}_{jm\phi}$ s of firm j in scenario ϕ into the vector $\hat{Y}_{j\phi}$, group all $\hat{Y}_{j\phi}$ s in scenario ϕ into the vector $\hat{Y}_\phi$, and group all $\hat{y}_{jm\phi}$ s into the vector $\hat{Y}$.
$z_{jni\phi}$	the unused quantity of part i firm j has after stage 2 in scenario ϕ. We group the $z_{jni\phi}$s of firm j in scenario ϕ into the vector $Z_{j\phi}$, the $z_{jni\phi}$s in scenario ϕ into the vector Z_ϕ, and group all Z_ϕs into the vector Z.

Table 2
Parameters for the supply chain network with financial hedging model.

Notation	Definition
α_j	risk averse factor of firm j
β_{1ji}	The amount of part i needed by firm j to manufacture one unit of its product
β_{2jm}	The amount of commodity material m needed by firm j to manufacture one unit of its product
$\rho_k(s^{all}_{k\phi})$	the inverse demand function at demand market k
P^1_ϕ	the vector of imported part prices in U.S. dollars in scenario ϕ with component for part i from country n denoted by $p^1_{\phi ni}$
P^2_ϕ	the vector of commodity prices at spot markets in scenario ϕ with component for material m denoted by $p^2_{\phi m}$
CAP_j	the production capacity of firm j
$c_{jni}(x_{jni})$	the transaction and transportation cost of firm j in purchasing part i from country n
$c_{jm}(\hat{y}_{jm\phi})$	the transaction and transportation cost incurred by firm j in purchasing commodity m from the spot market
$c_{jk}(s_{jk\phi})$	the transaction and transportation cost between firm j and demand market k
$c_j(S_{j\phi})$	production cost of firm j
$h_{jn}(w_{jn})$	the cost incurred by firm j in financial hedging for exchange rate risk of country n
$h_{jm}(u_{jm})$	the cost incurred by firm j in financial hedging for commodity m
η^0_n	The current exchange rate futures price of country n in stage 1
$\eta_{n\phi}$	The exchange rate futures price of country n in stage 2 in scenario ϕ
θ^0_m	The current futures price of commodity m in stage 1
$\theta_{m\phi}$	The futures price of commodity m in stage 2 in scenario ϕ
γ_{ji}	firm j's discount factor for the value of unused part i at the end of the planning horizon
V	the $(NI+2M+N)\times(NI+2M+N)$ dimensional variance-covariance matrix of the foreign exchange rates, $p^1_{\phi ni}$, commodity spot prices, $p^2_{\phi m}$, foreign exchange futures prices, $\eta_{n\phi}$, and commodity futures prices, $\theta_{m\phi}$.
$B_{x_{jni}}$	The upper bound for x_{jni}
$B_{u_{jm}}$	The upper bound for u_{jm}
$B_{s_{jl\phi}}$	The upper bound for $s_{jk\phi}$
$B_{z_{jni\phi}}$	The upper bound for $s_{jni\phi}$

In addition, due to the volatility of commodity prices each firm can also choose to purchase/long certain amount of commodity futures in stage 1 when it plans for the production. In stage 2 the firm sells the futures when it purchases the commodity materials from the spot market.

We use the classic mean-variance framework to model the supply chain firms optimization problem where they maximize expected profits and minimize the foreign exchange and commodity price risks. Each supply chain firm's decision-making process can be formulated as a two-stage stochastic programming problem (see, e.g., Barbarosoglu & Arda, 2004; Dupacova, 1996). In particular, each supply chain firm needs to determine its optimal part order quantities and futures contract quantities in stage 1, and its optimal responses in each scenario of the foreign exchange rate and commodity price in stage 2. Therefore, each firm maximizes its expected profit and minimizes its risk. The equilibrium state of the network is where the optimality conditions of all supply chain firms are satisfied simultaneously so that no firm can be better off by unilaterally changing his decisions.

The optimality conditions of all supply chain firms can be formulated as a finite-dimensional variational inequality problem. Next, we first model the supply chain firms' decision-making problem as a two-stage stochastic programming problem. We will then develop the variational inequality formulation that governs the equilibrium state of all the supply chain firms.

2.2. The supply chain firms' optimization problems

In the first stage, the firms need to determine their part purchase quantities, x_{jni}, from foreign suppliers. Note that these parts are delivered and paid later in stage 2. In stage 1, they also need to purchase/long the exchange rate futures contracts to hedge the foreign exchange risk. The quantities of the foreign exchange rate futures, w_{jn}s, are determined using the entire optimization model. In addition, in stage 1, the firms can purchase/long the commodity futures contracts. In particular, in order to determine the quantity of the commodity futures, u_{jm}s, each firm needs to estimate, y_{jm}s, the expected quantities of the commodity materials it will need in stage 2. The quantities of commodity futures are also determined using the entire optimization model.

In stage 2, the actual exchange rate and commodity price scenario $\phi \in \Phi$ is revealed. The firms pay the suppliers and sell

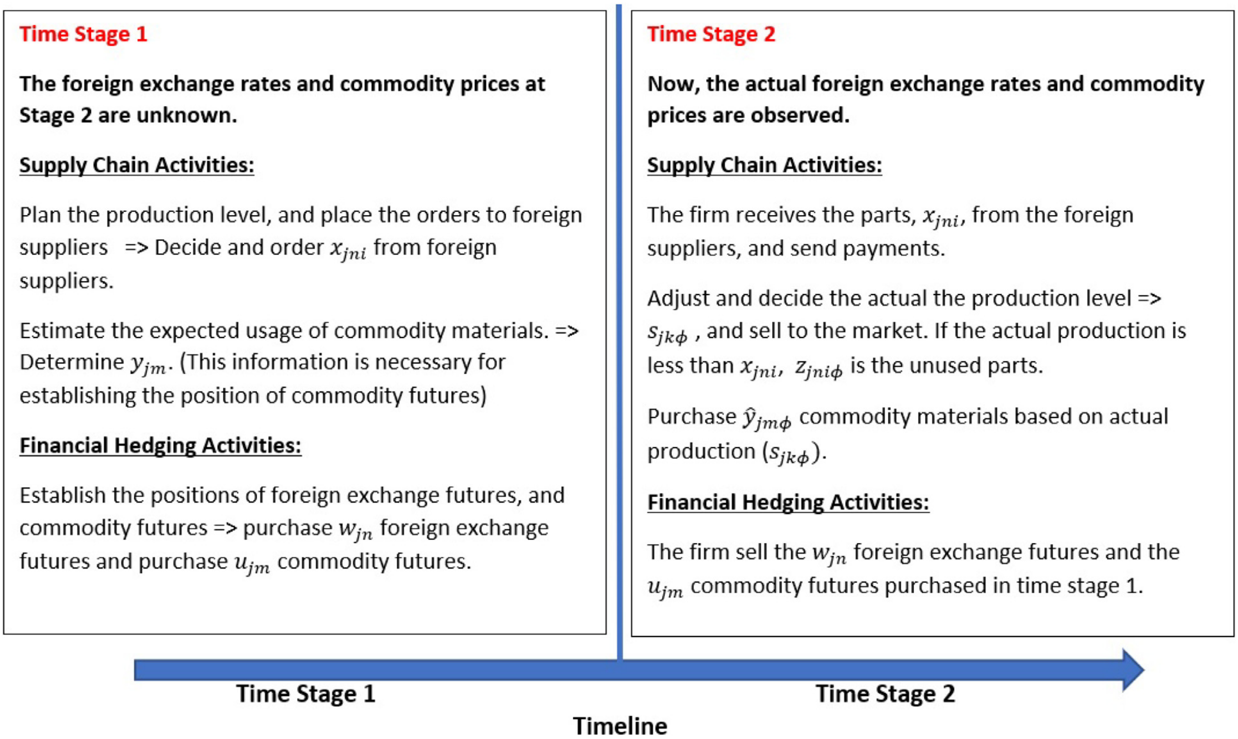

Fig. 1. Timeline of the supply chain and financial hedging activities.

the foreign exchange rate futures after the parts are delivered from the foreign suppliers. The firms also purchase commodity materials from the spot market, and sell the commodity futures. The quantities of the commodity materials, $\hat{y}_{jm\phi}$s, are determined based on the actual production quantities of the product, $s_{jk\phi}$s, in stage 2. Note that since the part orders were placed in stage 1 before scenario ϕ is known and the actual production quantity is determined, the quantity of the parts ordered may be more than the quantity required by production. The unused parts, $z_{jni\phi}$, can be held in the inventory for the future. However, the value of the unused parts is discounted due to the inventory cost.

The goal of each supply chain firm is to maximize the expected profit and minimize the cost variance due to the exchange rate and commodity price risk. We now formulate the optimization problem faced by supply chain firm j; $j = 1, \dots, J$, as follows:

$$MAX \quad -\sum_{n=1}^{N}(\eta_n^0 w_{jn} + h_{jn}(w_{jn})) - \sum_{m=1}^{M}(\theta_m^0 u_{jm} + h_{jm}(u_{jm}))$$
$$-\alpha_j R(X_j, Y_j, W_j, U_j) + E[(G_{j\phi}(X_{jni}, \hat{Y}_{j\phi}, S_{j\phi}, Z_{j\phi})] \tag{2}$$

subject to:

$$y_{jm} = E[\hat{y}_{jm\phi}], \quad m = 1, \dots, M \tag{3}$$

$$\begin{aligned} &B_{x_{jni}} \geq x_{jni} \geq 0, \quad n = 1, \dots, N, \quad i = 1, \dots, I, \\ &B_{w_{jn}} \geq w_{jn} \geq 0, \quad n = 1, \dots, N, \\ &B_{u_{jm}} \geq u_{jm} \geq 0, \quad m = 1, \dots, M, \quad B_{y_{jm}} \geq y_{jm} \geq 0, \quad m = 1, \dots, M, \\ &B_{s_{jk}} \geq s_{jk\phi} \geq 0, \quad k = 1, \dots, K, \quad B_{\hat{y}_{jm}} \geq \hat{y}_{jm\phi} \geq 0, \quad m = 1, \dots, M, \\ &B_{z_{jni}} \geq z_{jni\phi} \geq 0, \quad n = 1, \dots, N, i = 1, \dots, I. \end{aligned} \tag{4}$$

where $R(X_j, Y_j, W_j, U_j) \equiv [X_j^T, Y_j^T, W_j^T, U_j^T]V[X_j^T, Y_j^T, W_j^T, U_j^T]^T$.

Constraint (3) defines y_{jm} as the expected commodity material quantity firm j needs in stage 2. Constraint (4) indicates that the decision variables are non-negative and bounded. The first two terms in the objective function stand for the cost of exchange rate futures and the transaction cost of trading these futures, respectively. The third and the fourth terms represent the cost of commodity price futures and the associated transaction cost, respectively. The fifth term of the objective function is the risk function of foreign exchange rates and commodity prices, which is represented by the variance of the total cost of the materials and parts as well as the associated futures contracts. The last term represents the expected value of the net income of the supply chain firm in period 2, $G_{j\phi}(X_{jni}, \hat{Y}_{j\phi}, S_{j\phi}, Z_{j\phi})$, over all scenarios.

Note that each supply chain firm tries to maximize the expected profit and minimize the cost variance due to the exchange rate and commodity price risk. In the first stage, each firm plans the production, optimize the expected profit, and establish the positions in futures contracts. These behaviors are mapped in the optimization problem (2). Futures here are used to reduce the value of the cost variations.

In particular, $G_{j\phi}(X_{jni}, \hat{Y}_{j\phi}, S_{j\phi}, Z_{j\phi})$ represents the following subproblem of supply chain firm j in scenario ϕ:

$$MAX \quad G_{j\phi} = \sum_{k=1}^{K}(\rho_k(s_{k\phi}^{all})s_{jk\phi})$$
$$+\sum_{n=1}^{N}\eta_{n\phi}w_{jn} + \sum_{m=1}^{M}\theta_{m\phi}u_{jm} - c_j(S_{j\phi}) - \sum_{m=1}^{M}p_{m\phi}^2\hat{y}_{jm\phi}$$
$$-\sum_{n=1}^{N}\sum_{i=1}^{I}p_{ni\phi}^1 x_{jni} + \sum_{n=1}^{N}\sum_{i=1}^{I}\gamma_{ji}p_{ni\phi}^1 z_{jni\phi} \tag{5}$$

subject to:

$$\beta_{1ji}\sum_{k=1}^{K}s_{jk\phi} + \sum_{n=1}^{N}z_{jni\phi} \leq \sum_{n=1}^{N}x_{jni}, \quad i = 1, \dots, I, \tag{6}$$

$$z_{jni\phi} \leq x_{jni}, \quad i = 1, \dots, I, \tag{7}$$

$$\beta_{2jm}\sum_{k=1}^{K}s_{jk\phi} \leq \hat{y}_{jm\phi}, \quad m = 1, \dots, M. \tag{8}$$

$$\sum_{k=1}^{K}s_{jk\phi} \leq CAP_j, \tag{9}$$

$$B_{s_{jk}} \geq s_{jk\phi} \geq 0, \quad k = 1, \ldots, K, \quad B_{\hat{y}_{jm}} \geq \hat{y}_{jm\phi} \geq 0, \quad m = 1, \ldots, M,$$
$$B_{z_{jni}} \geq z_{jni\phi} \geq 0, \quad n = 1, \ldots, N, i = 1, \ldots, I. \tag{10}$$

Note that since the supply chain firms have placed the orders of the parts from foreign suppliers in the first period thus cannot change x_{jni}s in the second period. Thus, x_{jni}s do not depend on the individual scenario ϕ. The first term of the objective function (5) represents the sum of supply chain firm j's sales revenues from all demand markets. The product price of firm j in market k, $\rho_k(s^{all}_{k\phi})$, depends on $s^{all}_{k\phi}$, the total supplies of all firms in market k. The second and third terms in (5) represent the sales revenues from the foreign exchange rate futures and the commodity futures, respectively. The fourth term represents the cost of manufacturing the products. The fifth term in (5) represents the total cost of buying the commodity materials from the spot market. Note that the commodity spot price, $p^2_{jm\phi}$, depends on the uncertainty scenario ϕ which is unknown in stage 1. After ϕ is revealed in stage 2 $p^2_{jm\phi}$ is considered as a constant. The sixth term in (5) represents the total payout to the foreign suppliers. Note that the part unit price, $p^1_{jni\phi}$, depends on the uncertainty scenario ϕ which is unknown in stage 1. After ϕ is revealed in stage 2 $p^1_{jni\phi}$ becomes a constant. The last term in (5) represents the ending value of unused parts where γ_j is the discount factor.

Constraint (6) indicates that the total quantity of purchased parts is greater than or equal to the quantity of used parts plus that of unused parts. Constraint (7) ensures that the quantity of unused parts purchased from a supplier cannot exceed the total quantity of the parts purchased from the same supplier. Constraint (8), in turn, reflects that the quantity of commodity materials used in production is less than or equal to the total quantity purchased from the spot markets. Constraint (9) represents the production capacity limit. Constraint (10) indicates that the decision variables are non-negative and bounded.

Note that in the second stage after the four uncertain factors, exchange rates, exchange futures prices, commodity spot prices, and commodity futures prices, are revealed, there will be no uncertainty in the scenario. The problem becomes a classic Cournot game. Now, each firm optimizes the actual production decision $s_{jk\phi}$, commodity purchase, $\hat{y}_{jm\phi}$, and unused inventory, $z_{jni\phi}$, given the revealed prices and delivered parts.

Using a standard transformation in the stochastic programming theory firm j's two-stage stochastic programming problem can be rewritten as the following maximization problem (see Birge & Louveaux, 2011):

$$MAX - \sum_{n=1}^{N}(\eta^0_n w_{jn} + h_{jn}(w_{jn})) - \sum_{m=1}^{M}(\theta^0_m u_{jm} + h_{jm}(u_{jm}))$$
$$-\alpha_j R(X_j, Y_j, W_j, U_j) + \sum_{\phi\in\Phi} f(\phi)\Bigg[\sum_{k=1}^{K}(\rho_k(s^{all}_{k\phi})s_{jk\phi})$$
$$+\sum_{n=1}^{N}\eta_{n\phi}w_{jn} + \sum_{m=1}^{M}\theta_{m\phi}u_{jm} - c_j(S_{j\phi}) - \sum_{m=1}^{M}p^2_{m\phi}\hat{y}_{jm\phi}$$
$$-\sum_{n=1}^{N}\sum_{i=1}^{I}p^1_{ni\phi}x_{jni} + \sum_{n=1}^{N}\sum_{i=1}^{I}\gamma_{ji}p^1_{ni\phi}z_{jni\phi}\Bigg] \tag{11}$$

subject to:

$$y_{jm} = \sum_{\phi\in\Phi} f(\phi)\hat{y}_{jm\phi}, \quad i = m, \ldots, M \tag{12}$$

$$\beta_{1ji}\sum_{k=1}^{K}s_{jk\phi} + \sum_{n=1}^{N}z_{jni\phi} \leq \sum_{n=1}^{N}x_{jni}, \quad i = 1, \ldots, I, \quad \phi \in \Phi, \tag{13}$$

$$z_{jni\phi} \leq x_{jni}, \quad i = 1, \ldots, I, \quad \phi \in \Phi, \tag{14}$$

$$\beta_{2jm}\sum_{k=1}^{K}s_{jk\phi} \leq \hat{y}_{jm\phi}, \quad m = 1, \ldots, M, \quad \phi \in \Phi, \tag{15}$$

$$\sum_{k=1}^{K}s_{jk\phi} \leq CAP_j, \quad \phi \in \Phi, \tag{16}$$

$$B_{x_{jni}} \geq x_{jni} \geq 0, \quad n = 1, \ldots, N, \quad i = 1, \ldots, I,$$
$$B_{w_{jn}} \geq w_{jn} \geq 0, \quad n = 1, \ldots, N,$$
$$B_{u_{jm}} \geq u_{jm} \geq 0, \quad m = 1, \ldots, M,$$
$$B_{y_{jm}} \geq y_{jm} \geq 0, \quad m = 1, \ldots, M,$$
$$B_{s_{jk}} \geq s_{jk\phi} \geq 0, \quad k = 1, \ldots, K,$$
$$B_{\hat{y}_{jm}} \geq \hat{y}_{jm\phi} \geq 0, \quad m = 1, \ldots, M,$$
$$B_{z_{jni}} \geq z_{jni\phi} \geq 0, \quad n = 1, \ldots, N, i = 1, \ldots, I. \tag{17}$$

2.3. The equilibrium of the supply chain network

Definition 1 (The Equilibrium of the Supply Chain Network with Financial Hedging)**.** The equilibrium state of the entire supply chain network is one where the optimality conditions for all firms simultaneously hold, so that no supply chain firm can be better off by unilaterally altering its decisions.

Theorem 1 (Variational Inequality Formulation)**.** *Suppose that the manufacturing cost function and hedging costs for each supply chain firm are continuously differentiable and convex (hence, they could be linear) and that the inverse demand functions are continuously differentiable and decreasing functions of the supply. In addition, suppose that the supply chain firms compete in a noncooperative manner. The equilibrium state of the entire supply chain network can be expressed as the following variational inequality (cf. Bazaraa, Sherali, & Shetty, 1993; Gabay & Moulin, 1980; Nagurney, 1999): Determine* $(W^*, U^*, X^*, Y^*, \hat{Y}^*, S^*) \in \mathcal{K}^1$ *satisfying:*

$$\sum_{j=1}^{J}\sum_{n=1}^{N}\left[\eta^0_n + \frac{\partial h_{jn}(w^*_{jn})}{\partial w_{jn}} + \alpha_j\frac{\partial R(X^*_j, Y^*_j, W^*_j, U^*_j)}{\partial w_{jn}} - \sum_{\phi\in\Phi} f(\phi)\eta_{n\phi}\right]$$
$$\times\left[w_{jn} - w_{jn}\right] + \sum_{j=1}^{J}\sum_{m=1}^{M}\left[\theta^0_m + \frac{\partial h_{jm}(u^*_{jm})}{\partial u_{jm}}\right.$$
$$\left.+\alpha_j\frac{\partial R(X^*_j, Y^*_j, W^*_j, U^*_j)}{\partial u_{jm}} - \sum_{\phi\in\Phi} f(\phi)\theta_{m\phi}\right] \times \left[u_{jm} - u^*_{jm}\right]$$
$$+\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{i=1}^{I}\left[\alpha_j\frac{\partial R(X^*_j, Y^*_j, W^*_j, U^*_j)}{\partial x_{jni}} + \sum_{\phi\in\Phi} f(\phi)p^1_{jni\phi}\right]$$
$$\times\left[x_{jni} - x^*_{jni}\right] + \sum_{j=1}^{J}\sum_{m=1}^{M}\alpha_j\frac{\partial R(X^*_j, Y^*_j, W^*_j, U^*_j)}{\partial y_{jm}} \times \left[y_{jm} - y^*_{jm}\right]$$
$$+\sum_{\phi\in\Phi}\sum_{j=1}^{J}\sum_{m=1}^{M} f(\phi)p^2_{m\phi} \times \left[\hat{y}_{jm\phi} - \hat{y}^*_{jm\phi}\right]$$
$$-\sum_{\phi\in\Phi}\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{i=1}^{I} f(\phi)\gamma_{ji}p^1_{ni\phi} \times \left[z_{jni\phi} - z^*_{jni\phi}\right]$$
$$-\sum_{\phi\in\Phi}\sum_{j=1}^{J}\sum_{k=1}^{K} f(\phi)\left[\rho_k(s^{all*}_{k\phi}) + \frac{\partial\rho_k(s^{all*}_{k\phi})}{\partial s_{jk\phi}}s^*_{jk\phi} - \frac{\partial c_j(S^*_{j\phi})}{s_{jk\phi}}\right]$$
$$\times\left[s_{jk\phi} - s^*_{jk\phi}\right] \geq 0,$$
$$\forall (W, U, X, Y, \hat{Y}, Z, S) \in \mathcal{K}^1, \tag{18}$$

where $\mathcal{K}^1 \equiv ((W, U, X, Y, \hat{Y}, Z, S)|(W, U, X, Y, \hat{Y}, Z, S) \in R_+^{JN(1+I)+2JM+|\Phi|J(NI+M+K)}$, *and (12) to (17) hold)*

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Proof. The theorem is derived based on the standard variational inequality theory (see Bazaraa et al., 1993; Gabay & Moulin, 1980; Nagurney, 1999). □

Note that in our model, in the second stage after the four uncertain factors, exchange rates, exchange futures prices, commodity spot prices, and commodity futures prices, are observed, there will be no uncertainty in the scenario. The problem becomes a classic Cournot game. So, $s_{jk\phi}$ and $\hat{y}_{jm\phi}$ in this specific scenario ϕ can be determined precisely. In the equilibrium solution in Theorem 1, the vectors of of $\hat{Y}$ and S simply consist of the list of the equilibrium solution in every scenario after the uncertain factors have been observed.

It is worth noting that when a solution satisfies variational inequality (18), then all supply chain firms simultaneously reach the optimality conditions. We can rewrite the variational inequality formulation (18) in the standard form as follows: Determine $X^* \in \mathcal{K}$ satisfying

$$\left\langle F(X^*)^T, X - X^* \right\rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{19}$$

where $X \equiv (W, U, X, Y, \hat{Y}, Z, S)^T$, $\mathcal{K} \equiv \mathcal{K}^1$,

$$F(X) \equiv (F^W_{jn}, F^U_{jm}, F^X_{jni}, F^Y_{jm}, F^{\hat{Y}}_{jm\phi}, Y^Z_{jni\phi}, Y^S_{jk\phi}), \tag{20}$$

with the functional terms $(F^W_{jn}, F^U_{jm}, F^X_{jni}, F^Y_{jm}, F^{\hat{Y}}_{jm\phi}, Y^Z_{jni\phi}, Y^S_{jk\phi})$ preceding the multiplication signs in (18). Here $< \cdot, \cdot >$ denotes the inner product in $\mathcal{H}$-dimensional Euclidean space where $\mathcal{H} = JN(1+I) + 2JM + |\Phi| J(NI + M + K)$.

3. Existence of the equilibrium solution to the general model

We now discuss some important qualitative properties of the model. In particular, we first present conditions under which an equilibrium solution to the model exists. We will also prove the monotonicity of our model, which means that the Jacobian matrix of the function F(x) given by (20) is positive semidefinite. It is worth noting that that the role of monotonicity in variational inequalities is similar to that of convexity in optimization problems. It is also critical for the model since it is necessary for the algorithm used in this research to converge to the equilibrium solution.

Theorem 2 (Existence). *Variational inequality (16) has a solution if all transaction cost functions, manufacturing cost functions, hedging cost functions, and inverse demand functions at all demand markets are continuously differentiable.*

Proof. Given that all decision variables are bounded, the feasible set of (18) is compact, and nonempty. Since $F(X)$ in (18) is continuous variational inequality (18) has a solution(cf. Nagurney 1999). □

We now discuss monotonicity conditions of the function F that enters variational inequality (18). The monotonicity property is important for establishing convergence of the computational method used for the model.

Theorem 3 (Monotonicity). *Suppose that the inverse demand functions at all demand markets are decreasing, concave (including linear), and continuously differentiable. In addition, suppose that the manufacturing cost functions, transaction cost functions, and hedging cost functions in the model are continuously differentiable and convex (including linear). Then the variational inequality formulation for the supply chain network with financial hedging is monotone, that is,*

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, X' \neq X''. \tag{21}$$

Proof. First, we can write the Jacobian matrix of $F(X)$ that enters (20) as

$$Jacobian = \begin{pmatrix} \Omega_{wuxy} & 0 & 0 \\ 0 & \Omega_{\hat{y}z} & 0 \\ 0 & 0 & \Omega_s \end{pmatrix}, \tag{22}$$

where Ω_{wuxy} is the $JN(1+I) + 2JM \times JN(1+I) + 2JM$ submatrix associated with F^W_{jn}, F^U_{jm}, F^X_{jni}, and F^Y_{jm}, $i = 1, \ldots, I$, $m = 1, \ldots, M$, and $j = 1, \ldots, J$; $\Omega_{\hat{y}z}$ is the $J(NI+M)|\Phi| \times J(NI+M)|\Phi|$ submatrix associated with $F^{\hat{Y}}_{jm\phi}$ and $F^Z_{jni\phi}$, $j = 1, \ldots, J$, $i = 1, \ldots, I$, $m = 1, \ldots, M$, and $\phi \in \Phi$. Ω_s is the $JK|\Phi| \times JK|\Phi|$ submatrix associated with $F^S_{jk\phi}$, $j = 1, \ldots, J$, $k = 1, \ldots, K$, and $\phi \in \Phi$.

First, since the transaction cost functions and hedging cost functions are continuously differentiable and convex, and the covariance matrix is positive semidefinite, Ω_{wuxy} is positive semidefinite. In addition, since $F^{\hat{Y}}_{jm\phi}$ and $F^Z_{jni\phi}$ are constant $\Omega_{\hat{y}z}$ is a zero matrix.

Now, we prove that submatrix, Ω_s, is also a positive semidefinite. We first rewrite Ω_s as

$$\Omega_s = \Omega_s^c + \Omega_s^\rho \tag{23}$$

where Ω_s^c corresponds to $\frac{\partial c_j(S^*_{j\phi})}{s_{jk\phi}}$ in $F^S_{jk\phi}$, and Ω_s^ρ corresponds to $-[\rho_k(s^{all*}_{k\phi}) + \frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s_{jk\phi}} s^*_{jk\phi}]$ in $F^S_{jk\phi}$. Since $c_j(S^*_{j\phi})$ is convex Ω_s^c is positive semidefinite.

Now, we prove that Ω_s^ρ is also positive semidefinite. Ω_s^ρ can be written as

$$\Omega_s^\rho = \begin{pmatrix} f(\phi_1)A^1_{\phi_1} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 \\ \vdots & \cdots & f(\phi)A^k_\phi & \cdots & \vdots \\ 0 & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & f(\phi_{|\Phi|})A^K_{\phi_{|\Phi|}} \end{pmatrix}, \tag{24}$$

where A^k_ϕ is a $J \times J$ submatrix is associated with $-[\rho_k(s^{all*}_{k\phi}) + \frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s_{k\phi}} s^*_{jk\phi}]$ in $F^S_{jk\phi}$, for demand market k and in scenario ϕ.

Note that the element at row j and column l of A^k_ϕ is $-2\frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} - \frac{\partial^2 \rho_k(s^{all*}_{k\phi})}{\partial {s^{all}_{k\phi}}^2} \times s_{jk\phi}$ if l is equal to j, and $-\frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} - \frac{\partial^2 \rho_k(s^{all*}_{k\phi})}{\partial {s^{all}_{k\phi}}^2} \times s_{jk\phi}$ if l is not equal to j. Therefore, we can decompose A^k_ϕ into three different matrices, $A^k_\phi = A^{k1}_\phi + A^{k2}_\phi + A^{k3}_\phi$, where

$$A^{k1}_\phi = -\begin{pmatrix} \frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 \\ \vdots & \cdots & \frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} & \cdots & \vdots \\ 0 & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} \end{pmatrix}, \tag{25}$$

$$A^{k2}_\phi = -\frac{\partial \rho_k(s^{all*}_{k\phi})}{\partial s^{all}_{k\phi}} \begin{pmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \ddots & \cdots & \cdots & 1 \\ \vdots & \cdots & 1 & \cdots & \vdots \\ 1 & \cdots & \cdots & \ddots & 1 \\ 1 & \cdots & \cdots & 1 & 1 \end{pmatrix}, \tag{26}$$

and

$$A_{\phi}^{k3} = -\frac{\partial^2 \rho_k(s_{k\phi}^{all*})}{\partial {s_{k\phi}^{all}}^2} \times \begin{pmatrix} s_{1k\phi} & s_{1k\phi} & \cdots & \cdots & s_{1k\phi} \\ s_{2k\phi} & \ddots & \cdots & \cdots & s_{2k\phi} \\ \vdots & \cdots & s_{jk\phi} & \cdots & \vdots \\ s_{(J-1)k\phi} & \cdots & \cdots & \ddots & s_{(J-1)k\phi} \\ s_{Jk\phi} & \cdots & \cdots & s_{Jk\phi} & s_{Jk\phi} \end{pmatrix}. \quad (27)$$

Since the inverse demand function $\rho_k(s_{k\phi}^{all})$ is a decreasing function of $s_{k\phi}^{all}$, $-\frac{\partial \rho_k(s_{k\phi}^{all})}{\partial s_{k\phi}^{all}}$ is nonnegative. Thus, A_{ϕ}^{k1} is positive semidefinite. In addition, the only non-zero eigenvalue of matrix A_{ϕ}^{k2} is equal to $-J\frac{\partial \rho_k(s_{k\phi}^{all*})}{\partial s_{k\phi}^{all}}$. Since $-\frac{\partial \rho_k(s_{k\phi}^{all*})}{\partial s_{k\phi}^{all}}$ is positive, A_{ϕ}^{k2} is a positive semidefinite matrix. The matrix, A_{ϕ}^{k3}, has only one non-zero eigenvalue which is equal to $-\frac{\partial^2 \rho_k(s_{k\phi}^{all*})}{\partial {s_{k\phi}^{all}}^2} s_{k\phi}^{all}$. Since $\rho_k(s_{k\phi}^{all})$ is concave, $-\frac{\partial^2 \rho_k(s_{k\phi}^{all*})}{\partial {s_{k\phi}^{all}}^2} s_{k\phi}^{all} \geq 0$. So, A_{ϕ}^{k3} is also positive semidefinite. Now, we have shown that A_{ϕ}^{m1}, A_{ϕ}^{m2}, and A_{ϕ}^{m3} are positive semidefinite, which implies that A_{ϕ}^{k} and Ω_s are both positive semidefinite.

Therefore, the three submatrix of the Jacobian, Ω_{wuxy}, $\Omega_{\hat{y}z}$, Ω_s are positive semidefinite, the Jacobian matrix of the model is positive semidefinite, which implies that the variational inequality formulation of the supply chain network is monotone. □

In Section 5 we use the modified projection method (see Nagurney, 1999) for the computation of the model. The algorithm converges to a solution if $F(X)$ is monotone and Lipschitz continuous, and a solution exists, which is the case for our model.

4. Analytical results for special cases

In this section, we investigate two special cases of the model where we present the closed-form solutions of the equilibrium and provide managerial insights. In particular, we focus on two supply chain firms who are exposed to the foreign exchange risk. We simplify the general model in order to focus on the volatility and basis risk which also helps keep the problem analytically tractable. In particular, we assume that each firm has one foreign supplier and there is one demand market. We also assume that the actual production quantity s is always equal to the planned production/order quantity x. Note that the impact of this assumption is that the firms lose some operational flexibility and reply more on financial hedging. We also assume that the transaction cost for the futures contracts is zero, and the production cost is linear. We focus on two decision variables, the production/order quantity x and the foreign exchange hedging quantity w.

It is worth nothing that although in this section we focus on the foreign exchange risk, under the same assumptions the analytical results for commodity price risk are the same.

We compare two cases where in the first case both firms can hedge using futures while in the second case only firm one can use futures. We will provide analytical solutions and discuss the results.

Special Case 1: Both firms can hedge using futures contracts

We now assume that both firms A and B are capable of financial hedging using futures contracts. In this special case, the optimization models of Firm A and Firm B can be simplified as follows:

Firm A:

$$MAX - \eta^0 w_A - \alpha_A R(x_A, w_A) + \rho(x_A + x_B)x_A - c_A x_A + \sum_{\phi\in\Phi} f(\phi)[\eta_\phi w_A - p_\phi^1 x_A] \quad (28)$$

Firm B:

$$MAX \; - \eta^0 w_B - \alpha_B R(x_B, w_B) + \rho(x_A + x_B)x_B - c_B x_B + \sum_{\phi\in\Phi} f(\phi)[\eta_\phi w_B - p_\phi^1 x_B] \quad (29)$$

Since we focus on the impacts of foreign exchange risk and basis risk, we control all the other financial and cost factors of the two firms. In particular, we assume that the expected return of the futures contracts is zero, that is, $\eta^0 = \sum_{\phi\in\Phi} \eta_\phi$. We also assume that the two firms have the same unit production cost and, and let c to represent the sum of the unit cost and the expected unit purchase cost for the two firms, that is, $c = \sum_{\phi\in\Phi} p_\phi^1 + c_A = \sum_{\phi\in\Phi} p_\phi^1 + c_B$. In addition, we assume the inverse demand function is linear, and takes the form: $\rho(x_A + x_B) = a - b(x_A + x_B)$. We also assume that the two firms have the same risk averse factor, α. Now, the optimization problems of the two firms can be expressed as the follows:

Firm A:

$$MAX \quad \rho(x_A + x_B)x_A - cx_A - \alpha \times (x_A, w_A)\begin{pmatrix} \sigma_p^2 & \sigma_{p\eta} \\ \sigma_{p\eta} & \sigma_\eta^2 \end{pmatrix}(x_A, w_A)^T \quad (30)$$

where σ_p^2, σ_η^2, and $\sigma_{p\eta}$ represent the variance of the exchange rate, the variance of the futures price, and the covariance between the two factors, respectively. Firm B's optimization problem can be rewritten in a symmetric manner.

Since the objective function consists of the quadratic function of $\rho(x_A + x_B)x_A = ax_A - bx_A^2 - bx_A x_B$ and the covariance matrix, $-\alpha \times (x_A, w_A)\begin{pmatrix} \sigma_p^2 & \sigma_{p\eta} \\ \sigma_{p\eta} & \sigma_\eta^2 \end{pmatrix}(x_A, w_A)^T$, the Hessian Matrix is negative semi-definite. Therefore, we can obtain the equilibrium solution of the model by solving the first order conditions of the two firms. The closed-form equilibrium solutions of the two firms as follow:

Firm A:

$$x_A^* = \frac{\sigma_\eta^2(a-c)}{-2\alpha{\sigma_{p\eta}}^2 + 3b\sigma_\eta^2 + 2\alpha\sigma_\eta^2\sigma_p^2} \quad (31)$$

$$w_A^* = \frac{-\sigma_{p\eta}(a-c)}{-2\alpha{\sigma_{p\eta}}^2 + 3b\sigma_\eta^2 + 2\alpha\sigma_\eta^2\sigma_p^2} \quad (32)$$

Since firm A and firm B are identical the solution of firm B is the same as that of firm A. Based on the equilibrium solution we can obtain the profits and risks of the two firms as follows:

Firm A:

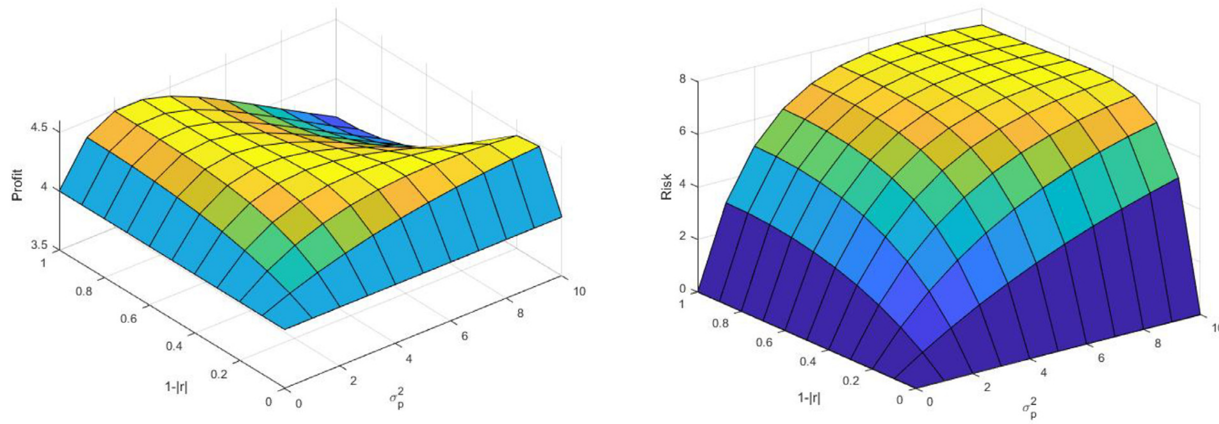
$$Profit_A{}^* = \frac{(a-c)^2(-2\alpha\sigma_p^2(1-r_{p\eta}^2) + b)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2) + 3b)^2} \quad (33)$$

$$Risk_A{}^* = \frac{-\sigma_p^2(r_{p\eta}^2 - 1)(a-c)^2}{(-2\alpha\sigma_p^2 r_{p\eta}^2 + 3b + 2\alpha\sigma_p^2)^2} \quad (34)$$

where, $r_{p\eta} = \frac{\sigma_{p\eta}}{\sigma_p\sigma_\eta}$, represents the correlation efficient between the spot price and the future price. Since firm A and firm B are identical their profits and risks are equal.

Next, we will present propositions that analyze the impacts of the exchange rate volatility and basis risk on the firms' profit, risk, and market price paid by consumers. In our analysis, the exchange rate volatility is measured by σ_p^2, and the basis risk is measured by $1 - |r|$.

Note that σ_p^2 represents the magnitude of the foreign exchange risk while $1 - |r_{p\eta}|$ represents how much the foreign exchange



(a) Profit of the Firms (b) Risk of the Frims

Fig. 2. Profit and risk of the firms in Case 1.

risk can be financially hedged. Throughout this section, we assume that $r_{p\eta} \neq 0$.

Proposition 1. *As the exchange rate volatility σ_p^2 increases the profits of the two firms will increase if $v_1 > 0$ and the profits will decrease if $v_1 < 0$, where:*

$$v_1 \equiv (1 - r_{p\eta}^2)(b - 2\alpha\sigma_p^2(1 - r_{p\eta}^2)). \tag{35}$$

Proof. See Appendix. □

Proposition 2. *As the basis risk $1 - |r_{p\eta}|$ increases the profits of the two firms will increase if $v_2 > 0$ and the profits will decrease if $v_2 < 0$, where:*

$$v_2 \equiv b - 2\alpha\sigma_p^2(1 - r_{p\eta}^2). \tag{36}$$

Proof. See Appendix. □

Proposition 3. *As the exchange rate volatility σ_p^2 increases the risks of the two firms will increase if $v_3 > 0$ and the profits will decrease if $v_3 < 0$, where:*

$$v_3 \equiv (1 - r_{p\eta}^2)(3b - 2\alpha\sigma_p^2(1 - r_{p\eta}^2)). \tag{37}$$

Proof. See Appendix. □

Proposition 4. *: As the basis risk $1 - |r_{p\eta}|$ increases the risks of the two firms will increase if $v_4 > 0$ and the profits will decrease if $v_4 < 0$, where:*

$$v_4 \equiv r_{p\eta}(3b - 2\alpha\sigma_p^2(1 - r_{p\eta}^2)). \tag{38}$$

Proof. See Appendix. □

We also constructed Fig. 2a and 2b to demonstrate Propositions 1–4 based on the parameter values: $a = 10$, $b = 1$, $c = 4$, and $\alpha = 0.2$. Note that Fig. 1a and 1b do not presents a general depiction of the impacts, but a visualization of such impacts under the specified parameters.

Discussion

First, we can see that Propositions 1, 2 and Fig. 2a show complex relationships between the risk factors and the firms' profit. We can see that at very low levels of exchange rate volatility and basis risk, when these risk factors increase the profits of the two firms also increase. The economic explanation is as follows. In general, in a duopolistic game due to the competition, the two firms tend to produce more and receive lower market price than in a non-competitive setting. Thus, due to the direct competition, the profits of the two firms in this case are lower than the profits they could make without competition. Now, as the risk factors slightly increase from very low levels, due to the fact that the existence of the basis risk prevents the firms from perfectly hedging the foreign exchange risk, the two firms will also have to reduce production and supply levels to lower risk. The reduced supplies in the market *unintentionally* alleviate the intensity of the competition, and increase the price of the products, which results in higher profits of the two firms. However, as the values of the risk factors increase more, such effect is outweighed by the further reduction of productions which leads to decreasing profits.

Second, Propositions 3–4 and Fig. 2b present the impacts of the foreign exchange volatility and the basis risk on the risk of the firms. We can see that at the low and medium risk levels, as the risk factors increase the firms' risks also increase. At the high risk levels, as the risk factors rise the firms' risks stay flat or slight decrease. This indicates that when the risk factors increase from high levels the firms further reduce production and sacrifice profits to control the risk.

Proposition 5. *As the exchange rate volatility σ_p^2 increases the market price for consumers will increase.*

Proof. See Appendix. □

Proposition 6. *As the basis risk $1 - |r_{p\eta}|$ increases the market price for consumers will increase.*

Proof. See Appendix. □

Although the impacts of σ_p^2 and $1 - |r_{p\eta}|$ on the profit and risk are complicated, Propositions 5 and 6 very clearly show that these risks always increase the price paid by the consumers. Therefore, well-functioning financial hedging instruments and futures markets will benefit consumers by lowering the prices.

Special Case 2: Only one firm can hedge using futures contracts

In this case, we assume that firm A is capable of hedging using futures contracts while firm B is not able to conduct financial hedging. In this scenario, the optimization models of firm A and firm B can be simplified as follows:

Firm A:

$$MAX \quad \rho(x_A + x_B)x_A - cx_A - \alpha \times (x_A, w_A)\begin{pmatrix} \sigma_p^2 & \sigma_{p\eta} \\ \sigma_{p\eta} & \sigma_\eta^2 \end{pmatrix}(x_A, w_A)^T \tag{39}$$

Firm B:

$$MAX \quad \rho(x_A + x_B)x_B - cx_B - \alpha \times \sigma_p^2 x_B^2. \tag{40}$$

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where σ_p^2, σ_η^2, and $\sigma_{p\eta}$ represent the variance of the exchange rate, the variance of the futures price, and the covariance between the two factors, respectively. Similar to special case 1 the Hessian matrix for each firm is negative semi-definite. Therefore, we can find the solution of the model by solving the first order conditions. The closed-form equilibrium solutions of the two firms are shown as follows:

Firm A:

$$x_A^* = \frac{\sigma_\eta^2(a-c)(b+2\alpha\sigma_p^2)}{-4\alpha^2{\sigma_{p\eta}}^2\sigma_p^2+4\sigma_\eta^2\alpha^2\sigma_p^4-4\alpha b{\sigma_{p\eta}}^2+8\sigma_\eta^2\alpha b\sigma_p^2+3\sigma_\eta^2 b^2} \tag{41}$$

$$w_A^* = \frac{-\sigma_{p\eta}(a-c)(b+2\alpha\sigma_p^2)}{-4\alpha^2{\sigma_{p\eta}}^2\sigma_p^2+4\sigma_\eta^2\alpha^2\sigma_p^4-4\alpha b{\sigma_{p\eta}}^2+8\sigma_\eta^2\alpha b\sigma_p^2+3\sigma_\eta^2 b^2}. \tag{42}$$

Firm B:

$$x_B^* = \frac{(a-c)(-2\alpha{\sigma_{p\eta}}^2+b\sigma_\eta^2+2\alpha\sigma_\eta^2\sigma_p^2)}{-4\alpha^2{\sigma_{p\eta}}^2\sigma_p^2+4\sigma_\eta^2\alpha^2\sigma_p^4-4\alpha b{\sigma_{p\eta}}^2+8\sigma_\eta^2\alpha b\sigma_p^2+3\sigma_\eta^2 b^2}. \tag{43}$$

Note that since firm B has no financial hedging capability it does not have w_B. Based on the equilibrium solution we can obtain the profits and risks of the two firms as follows:

Firm A:

$$Profit_A{}^* = \frac{(a-c)^2(b+2\alpha\sigma_p^2)^2(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{44}$$

$$Risk_A{}^* = \frac{\sigma_p^2(1-r_{p\eta}^2)(a-c)^2(b+2\alpha\sigma_p^2)^2}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{45}$$

Firm B:

$$Profit_B{}^* = \frac{(a-c)^2(b+2\alpha\sigma_p^2)(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)^2}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{46}$$

$$Risk_B{}^* = \frac{\sigma_p^2(a-c)^2(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)^2}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{47}$$

where, $r=\frac{\sigma_{p\eta}}{\sigma_p\sigma_\eta}$, represents the correlation efficient between the spot price and the future price.

Next, we will present propositions that analyze the impacts of the exchange rate volatility and basis risk on the firms' profit, risk, and market price paid by consumers.

Proposition 7. *As the exchange rate volatility σ_p^2 increases the profit of firm A will increase if $\upsilon_5>0$ and the profits will decrease if $\upsilon_5<0$; and the profit of firm B will increase if $\upsilon_6>0$ and the profits will decrease if $\upsilon_6<0$, where:*

$$\upsilon_5 \equiv 2\alpha b^2\sigma_p^2(1+r_{p\eta}^2)(1-r_{p\eta}^2)+b^3(1+r_{p\eta}^2)-8\alpha^3\sigma_p^6(r_{p\eta}^2-1)^2 \\ -4\alpha^2 b\sigma_p^4(r_{p\eta}^2-1)^2 \tag{48}$$

$$\upsilon_6 \equiv 4\alpha b^3\sigma_p^2(r_{p\eta}^4-3r_{p\eta}^2+1)+b^4-16\alpha^4\sigma_p^8(1-r_{p\eta}^2)^3 \\ -16\alpha^3 b\sigma_p^6(1-r_{p\eta}^2)^2-12\alpha^2b^2\sigma_p^4r_{p\eta}^2(1-r_{p\eta}^2)(1-2r_{p\eta}^2) \tag{49}$$

Proof. See Appendix. □

Proposition 8. *As the basis risk $1-|r_{p\eta}|$ increases the profit of firm A will decrease; and the profit of firm B will increase.*

Proof. See Appendix. □

Proposition 9. *As the exchange rate volatility σ_p^2 increases the risk of firm A will increase if $\upsilon_7>0$ and the risk will decrease if $\upsilon_7<0$; and the risk of firm B will increase if $\upsilon_8>0$ and the risk will decrease if $\upsilon_8<0$, where:*

$$\upsilon_7 \equiv (1-r_{p\eta}^2)[-8\alpha^3\sigma_p^6(1-r_{p\eta}^2)+4\alpha^2b\sigma_p^4(r_{p\eta}^2+1) \\ +4\alpha b^2\sigma_p^2r_{p\eta}^2+10\alpha b^2\sigma_p^2+3b^3] \tag{50}$$

$$\upsilon_8 \equiv 12\alpha^2b^2\sigma_p^4(3r_{p\eta}^2-2)(r_{p\eta}^2-1)+4\alpha b^3\sigma_p^2(4-5r_{p\eta}^2)+3b^4 \\ -16\alpha^4\sigma_p^8(1-r_{p\eta}^2)^3-16\alpha^3b\sigma_p^6r_{p\eta}^2(1-r_{p\eta}^2)^2 \tag{51}$$

Proof. See Appendix. □

Proposition 10. *As the basis risk $1-|r_{p\eta}|$ increases the risk of firm B will always increase; and the risk of firm A will increase if $\upsilon_9>0$ and the risk will decrease if $\upsilon_9<0$, where:*

$$\upsilon_9 \equiv 4\alpha^2\sigma_p^4(r_{p\eta}^2-1)+4\alpha b\sigma_p^2r_{p\eta}^2+3b^2. \tag{52}$$

Proof. See Appendix. □

We also constructed Fig. 3a–3d to demonstrate Propositions 7–10 based on the same parameter values as in special case 1. Note that these Figures do not presents a general depiction of the impacts, but a visualization of such impacts under the specified parameters.

Discussion

First, Proposition 8 and Fig. 3a and 3b indicate that as the financial hedging instrument becomes more effective and the basis risk decreases, the competitive advantage of firm A becomes stronger which always increases firm A's profit and decreases firm B's profit. Proposition 7 and Fig. 3 a show that the increasing volatility will amplify firm A's competitive advantage, and significantly increase firm A's profit when the basis risk is low, and may reduce firm A's profit when basis risk is high. Proposition 7 and Fig. 2b, in turn, show that the increasing volatility will almost always decrease firm B's profit. Only in some extreme situations where the volatility level is low and the basis risk is high, firm B's profit can slightly increase due to the alleviated competition as we discussed in special case 1.

Second, Fig. 3c and 3d show that in most situations as the volatility increases, the risks of both firm A and firm B will increase. Proposition 9 provided the closed-form thresholds of the impacts. Proposition 10 and Fig. 3c and 3d shows that in most situations increasing basis risk increases the risks of both firms.

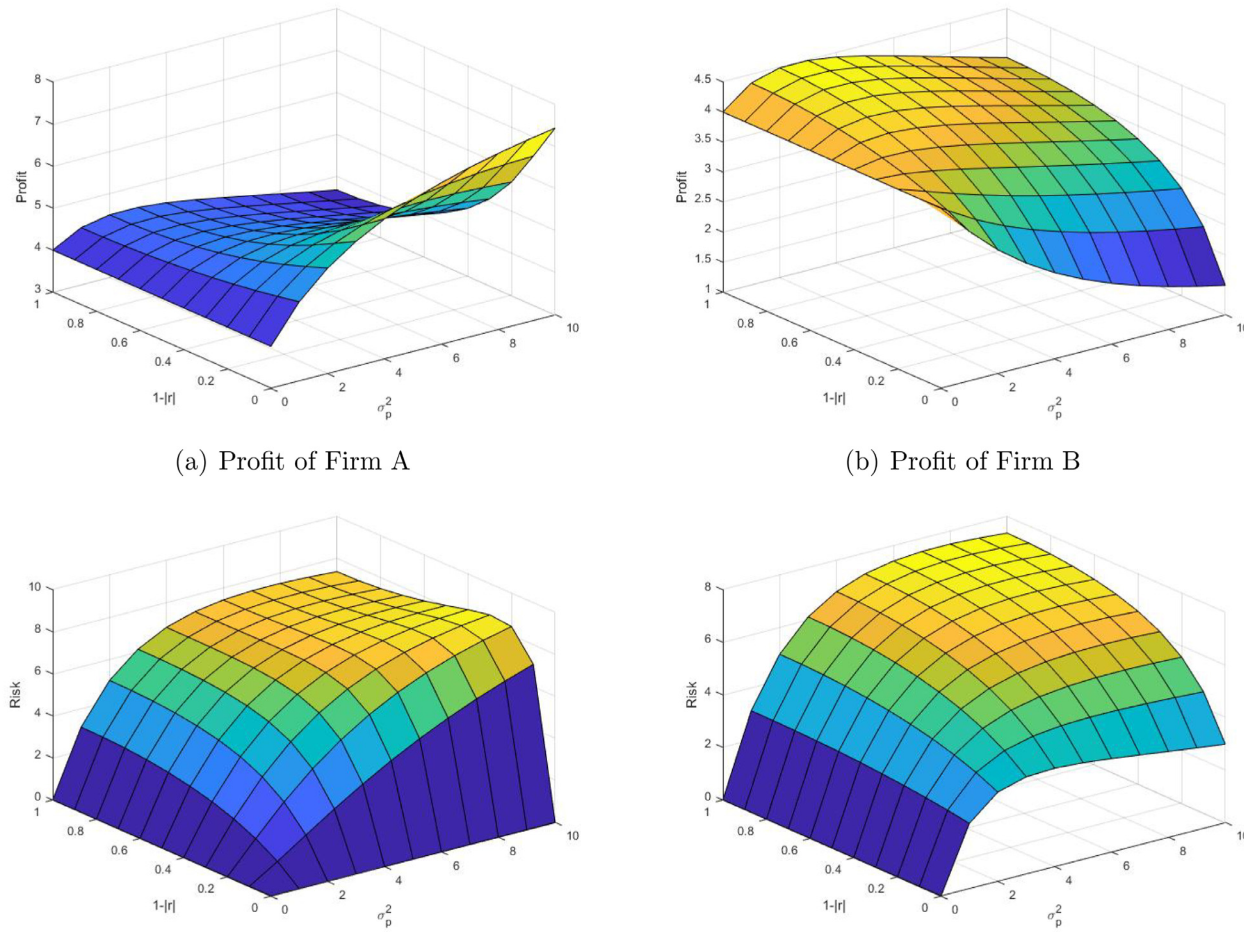
It is noteworthy that both Proposition 10 and Fig. 3d indicate that increasing basis risk will always increase firm B's risk which doesn't not even use financial hedging. This indirect impact of the basis risk is due to the competition in the market. When the basis risk increases financial hedging becomes less effective. Therefore, firm A has to reduce the production to contain the risk, which allows firm B to increase production to take more market share. As a result, the risk exposure of firm B also increases.

Proposition 11. *Firm A always has higher profit than firm B.*

Proof. See Appendix. □

Although the impacts of σ_p^2 and $1-|r_{p\eta}|$ on the profit and risk are very complicated, Proposition 11 confirms that the financial hedging using futures always provides firm A a competitive advantage against firm B.

Proposition 12. *As the exchange rate volatility σ_p^2 increases the market price for consumers will increase.*



(a) Profit of Firm A (b) Profit of Firm B

(c) Risk of the Firm A (d) Risk of the Firm B

Fig. 3. Profit and Risk of the Firms in Case 2.

Proof. See Appendix. □

Proposition 13. *As the basis risk* $1-|r_{p\eta}|$ *increases the market price for consumers will increase.*

Proof. See Appendix. □

Similar to special case 1, Propositions 5 and 6 demonstrate that higher exchange rate volatility and the basis risk always hurt consumers by increasing product prices.

5. Simulation case studies

We demonstrate our model using two simulation case studies that focuses on a oligopoly setting with five supply chain firms ($J=5$) and one demand market ($K=1$). The first simulation case study focuses on the impact of the exchange rate and commodity price volatility; and the second case study focuses on the impact of the basis risk, the effectiveness of financial hedging instruments.

The simulation case studies extend and complement the results based on the closed-form solutions of the special cases. In particular, the analytical results are only based on two firms while our simulation studies can investigate a more realistic setting with multiple heterogenous firms, which discovers interesting results that are not revealed by the analytical solutions of the special cases. For example, our simulation results show that the impacts of volatility and basis risk also significantly depend on the proportion of the firms practicing financial hedging.

Note that our model is capable of handling even larger networks with multiple firms, multiple countries and suppliers, multiple markets, and multiple commodity materials. In the simulation study we focus on the issue we attempt to investigate while keeping other factors simple and controlled.

Simulation Case Study 1

In Example 1, we apply our model to study the impacts of foreign exchange and commodity price volatilities on the decisions, profitability, and risks of five (J = 5) heterogeneous supply chain firms. In particular, we consider two types of firms: Type A firms are able to use futures for financial hedging; and type B firms cannot conduct financial hedging.

We consider one commodity material ($M=1$) and one foreign country ($N=1$) where there is one supplier ($I=1$). Since in this example we focus on the impact of financial hedging on the supply chains decisions, profits, and risks of the firms, we assume that the five supply chain firms have the same production cost parameters.

We assume that the covariance matrix of the foreign exchange rate, the exchange rate futures price, the commodity price, and the commodity futures price takes the following form:

$$Covariance\ Matrix = \begin{pmatrix} \sigma^2 & 0.8\sigma^2 & 0 & 0 \\ 0.8\sigma^2 & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0.8\sigma^2 \\ 0 & 0 & 0.8\sigma^2 & \sigma^2 \end{pmatrix}, \quad (53)$$

Note that the futures price of the exchange rate (commodity price) is not perfectly correlated with the exchange rate (commodity price) due to the basis risk.

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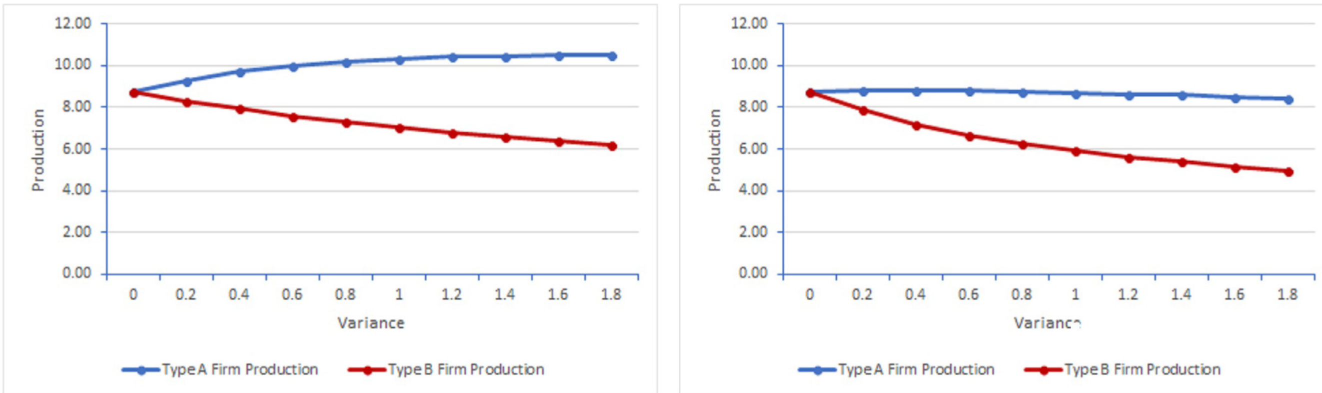
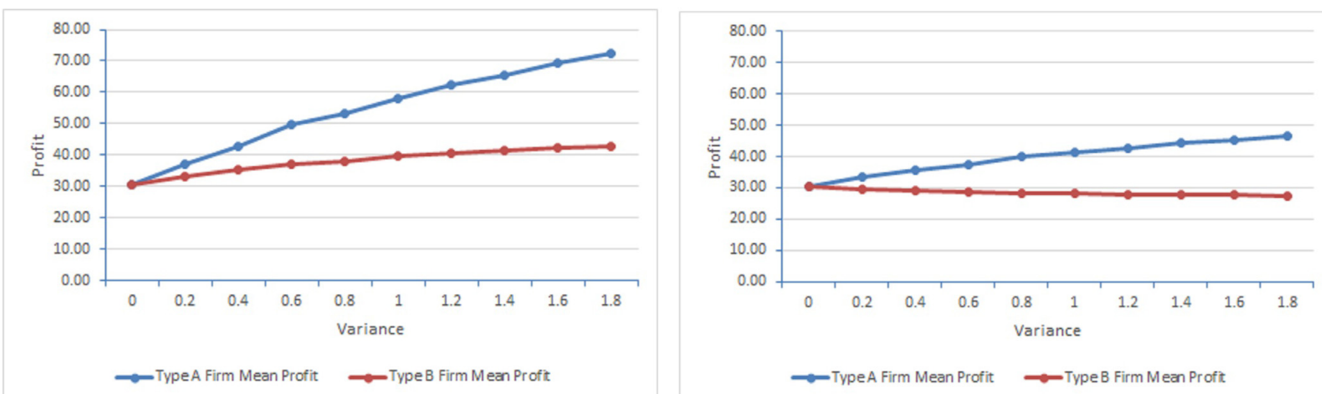


(a) Production in Case 1.1 (b) Production in Case 1.2

Fig. 4. Case 1: Production Level.



(a) Profit in Case 1.1 (b) Profit in Case 1.2

Fig. 5. Case 1: Profits.

Table 3
The model parameters of example 1.

Notation	Value
α_j	$0.1, \forall j$
β_{1ji}	$1\ \forall i, j$
β_{2jm}	$1\ \forall j, m$
$\rho_k(s_{k\phi}^{all})$	$\rho_k(s_{k\phi}^{all}) = 30 - 0.5 s_{k\phi}^{all}, \forall k$
CAP_j	$15, \forall j$
$c_j(S_{j\phi})$	$c_j(S_{j\phi}) = 1 * S_{j\phi}, \forall j$
η_n^0	$4, \forall n$
θ_m^0	$4, \forall m$
γ_{ji}	$0.8, \forall i, j$

We consider two sub-cases: 1. There are one type A firm and four type B firms; 2. there are four type A firms and one type B firm. For each case we change σ^2 from 0 to 0.9.

At each level of σ^2, we use Monte Carlo simulation to generate 1000 scenarios ($|\Phi| = 1000$) where $p_{jni\phi}^1$, $\eta_{n\phi}$, $p_{m\phi}^2$, $\theta_{m\phi}$ follows a multivariate normal distribution with mean of [4,4,4,4] and covariance matrix defined by (28). We then solve the equilibrium solution of model for each level of σ^2.

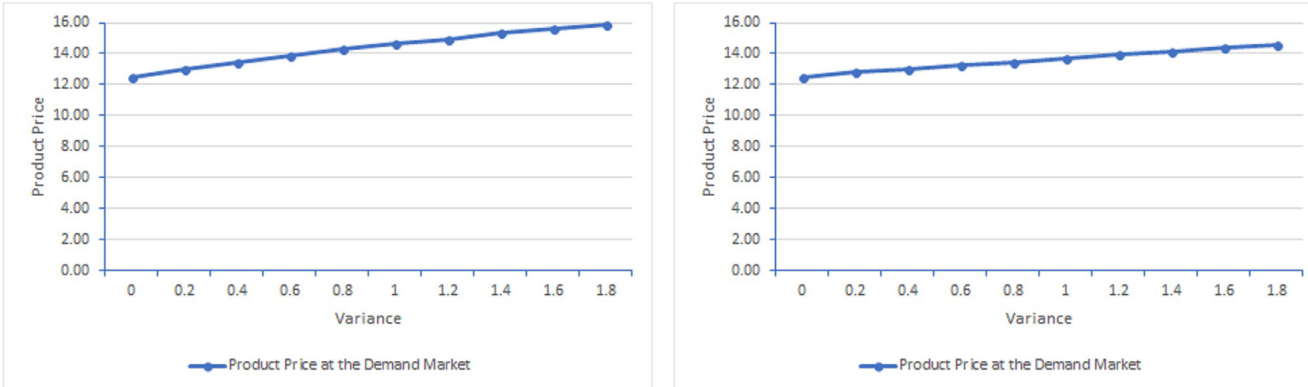
The parameters of Example 1 are specified in Table 3. The unspecified parameters are equal to zero.

Discussion

Figs. 4–6 presents the results of case 1. First, in Fig. 5 we can see that in both sub-cases and at each volatility level, the profit of type A firms is always higher than that of type B firms, which is consistent with analytical results of the special case. In addition, Fig. 6 shows that in both subcases as the volatility increases the market product price increases which hurts consumers.

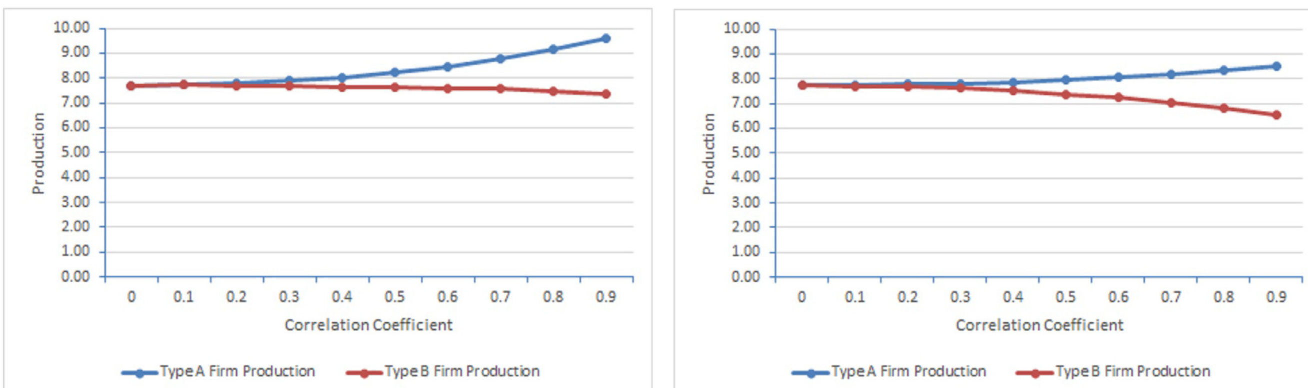
The simulation case study also provides unique insights. In Fig. 4a (case 1.1) where there is only one type A firm, the production level of the type A firm will increase when the foreign exchange rate and commodity price volatility increases while in Fig. 4b (case 1.2) where four out of five firms are type A firms the type A firm's production decreases as the volatility increases. Such results demonstrate that the mix of the heterogenous firms affects the optimal strategy. In case 1.1 when the proportion of type A firm is low, the type A firm can make the most of its competitive advantage against the type B firms, expand its market share, and boost its profit. However, in case 1.2 when the majority of the firms are able to use financial hedging, the competitive advantage becomes smaller and financial hedging itself is not sufficient to control all the risk. As a result, the four type A firms now have to also reduce their production levels to help manage the risk.

Fig. 5a and 5b also show that the profits of type A firm increase as the volatility increases which is consistent with Fig. 2a in the analytical results when the basis risk, $1 - |r|$, is relatively low (correlation coefficient $= 0.8$ in cases 1.1 and 1.2). The profit of type B firms, however, exhibits different trends in Fig. 5a and 5b. In case 1.1 (Fig. 5a) where there is only one type A firm the profit of type B firms increases as the volatility increases while in case 1.2 (Fig. 5b) the profit of type B firm decreases as the volatility increases. This is because in case 1.2 when the majority of the firms can use financial hedging the type B firm struggles more due to the amplified competitive disadvantage.



(a) Price in Case 1.1 (b) Price in Case 1.2

Fig. 6. Case 1: Market product price.



(a) Production in Case 2.1 (b) Production in Case 2.2

Fig. 7. Case 2: Production level.

The results in this case study demonstrate that the proportion of the firms practicing financial hedging greatly affects the impacts of foreign exchange rate and commodity price volatilities on the production strategies and profits of the firms. A different mix of the firms can lead to very different results.

Simulation Case Study 2

Simulation case study 2 investigates how the basis risk affects the decisions, profits and risks of the supply chain firms. Similar to the first simulation case study, we assume that in the first subcase there is one type A firm and four type B firms, and in the second subcase there are four type A firms and one type B firm. The parameters of the models in this case are identical to those of simulation case study 1 except that the covariance matrix is now defined as follows:

$$Covariance\ Matrix = \begin{pmatrix} 0.8 & 0.8*r & 0 & 0 \\ 0.8*r & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0.8*r \\ 0 & 0 & 0.8*r & 0.8 \end{pmatrix}, \quad (54)$$

We change r from 0 to 0.9, and at each covariance level solve the equilibrium model. Note that as the correlation coefficient between the spot price and the futures price, r, increases the basis risk decreases, which indicates that the financial hedging instrument is more effective.

Discussion

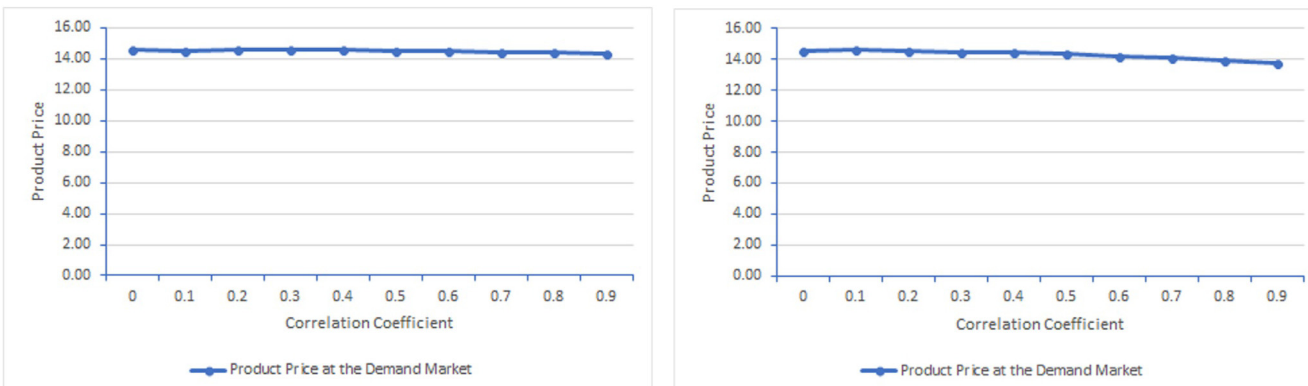
Figs. 7–9 present the results of simulation case 2. First, from Fig. 7 we can see that the in both subcases and each correlation coefficient level, the production level of type A firm is always higher than that of type B firm. Fig. 8 also shows that the profit of type A firm is always higher that of type B firm. In addition, Fig. 9 shows that in both subcases, the increase of the correlation coefficient between the spot prices and future prices will lower the product price paid by the consumers. These findings, in general, are consistent with the results based on the analytical solutions of the special cases.

The simulation studies also generated unique insights. From Fig. 8 we can see that while the profit of type B firms will always decrease as the correlation coefficient increases, the profit of type A firm exhibits different trends in two subcases. In particular, in subcase 1 where only one firm is type A firm the profit of the type A firm will increase as the correlation coefficient increases. In subcase 2 where there are four type A firms, the profit of type A firms will decrease as the correlation coefficient increases. Note that the increasing correlation coefficient, on one hand, will strengthen the competitive advantage of type A firms against type B firms, on the other hand, will also increase the competition intensity among type A firms. So, our results demonstrate that whether such advantage can be translated into higher profit depends on the mix of the firms in the industry.

Moreover, case 2 suggests a potential paradox for the industry, that is, a more effective financial hedging tool may result in an equilibrium where every firm makes less profit. From Fig. 8a and 8b we can see that in each subcase and at each level of the correlation coefficient, type A firms have higher profit than type B firms. The profit gap also increases as the financial hedging instrument has better correlation coefficient. Now, suppose that at

(a) Profit in Case 2.1 (b) Profit in Case 2.2

Fig. 8. Case 2: Profits.



(a) Price in Case 2.1 (b) Price in Case 2.2

Fig. 9. Case 2: Market Product Price.

the beginning, the financial hedging tool is completely ineffective (correlation coefficient=0), and firms in the industry do not use financial hedging, which is similar to Case 2.1. From Fig. 8a we can see that the profit of these firms is a little above 40 when the correlation coefficient is close to zero. If the financial hedging instrument is improved and its correlation coefficient becomes higher and higher. The increasing profit gap between type A and type B firms will attract more type B firms to start to use financial hedging and become type A firms. Therefore, the industry becomes similar to case 2.2 where the majority of the firms are type A firms. Now, suppose that the financial hedging instrument has become very effective and its correlation coefficient is 0.9. From Fig. 8b we can see that the profit of type A firm is around 40 and the profit of type B firm is only around 30. Therefore, every firm is now making less profit than before. In summary, in this case, the increasing effectiveness of the financial hedging tool attracted more firms to use it, which intensified the competition of the industry, and resulted in an unfavorable equilibrium for companies the industry. From Fig. 9, we can see that the consumers gained from this process by paying lower price.

6. Managerial insights

We now discuss and summarize some of the interesting managerial insights generated from our analytical results and simulation case studies.

Our results indicate that if the volatility risk is increasing from an elevated level, the profits of the firms with financial hedging capacity are likely to decline. However, if volatility risk is increasing from a relatively low level, the profits of the firms with financial hedging capability are likely to increase.

In addition, we find that more effective financial hedging instrument that has lower basis risk in general has a positive impact on the profits of the firms using financial hedging. However, when two firms with financial hedging compete with each other and the volatility is at low level, more effective hedging instrument can actually intensify the competition by allowing the firms producing more and supplying more to the market, which results in lower market price and lower profits for both firms. It is also interesting to see that increasing basis risk will always increase the risk of the firm that does not even use finance hedging. The indirect effect is passed through the market competition with the other firm that performs financial hedging. We proved that rising volatility and basis risk will always increase the market price of the product, which hurts consumers. Therefore, a well-functioning and efficient futures market will always benefit consumers.

Our simulation results show that the prevalent industry practice is also a very important factor. The mix of the firms with and without financial hedging can sometimes have a deciding impact on how the volatility risk and basis risk effect the profitabilities of the firms.

Our simulation case studies also reveal a potential paradoxical phenomenon. An increasingly effective hedging instrument may attract more and more firms to use financial hedging. The increasing number of firms using financial hedging can intensify the market competition, lower the product price, and finally result in an unfavorable equilibrium for the industry where no firm makes higher profit than before. Note that the findings from the simulation studies are based on multiple heterogenous firms, which is hard to be discovered through results based on closed-form solutions.

7. Conclusions

This paper studied the financial hedging decisions and operation decisions of supply chain firms under competition. In particular, we developed a variational inequality model that considers multiple supply chain firms with multiple foreign suppliers and commodity materials. We allowed the supply chain firms to use futures contracts to hedge various risk factors. We proved important qualitative properties for the model. We provided analytical results for a special case with duopolistic competition, and used simulations to study an oligopolistic case. We also discussed the managerial insights generated by the analytical and simulation studies. One limitation of the model is that it only considered futures as the financial hedging instruments. Future studies may extend the model to consider both futures and options. In addition, future research can also incorporate downside-risk measures, such as, value-at-risk and conditional value-at-risk into the decision-making.

Appendix

Proof of Proposition 1. We take the partial derivative of (33) with respect to σ_p^2, and obtain:

$$\frac{\partial Profit_A^*}{\partial \sigma_p^2} = \frac{2\alpha(1-r_{p\eta}^2)(a-c)^2(-2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(2\alpha\sigma_p^2(1-r_{p\eta})+3b)^3} \tag{55}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $v_1 \equiv (1-r_{p\eta}^2)(b-2\alpha\sigma_p^2(1-r_{p\eta}^2)) > 0$. □

Proof of Proposition 2. We take the partial derivative of (33) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Profit_A^*}{\partial r_{p\eta}} = \frac{-4\alpha\sigma_p^2 r_{p\eta}(a-c)^2(-2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)^3} \tag{56}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} > 0$ and $v_2 \equiv (b-2\alpha\sigma_p^2(1-r_{p\eta}^2)) < 0$, or if $r_{p\eta} < 0$ and $v_2 \equiv (b-2\alpha\sigma_p^2(1-r_{p\eta}^2)) > 0$. Therefore, when $1-|r_{p\eta}|$ increases the profit increases if $r_{p\eta} \neq 0$ and $v_2 > 0$. □

Proof of Proposition 3. We take the partial derivative of (34) with respect to σ_p^2, and obtain:

$$\frac{\partial Risk_A^*}{\partial \sigma_p^2} = \frac{(1-r_{p\eta}^2)(a-c)^2(-2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)^3} \tag{57}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $v_3 \equiv (1-r_{p\eta}^2)(3b-2\alpha\sigma_p^2(1-r_{p\eta}^2)) > 0$. □

Proof of Proposition 4. We take the partial derivative of (34) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Risk_A^*}{\partial r_{p\eta}} = \frac{-2\sigma_p^2 r_{p\eta}(a-c)^2(2\alpha\sigma_p^2 r_{p\eta}^2+3b-2\alpha\sigma_p^2)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)^3} \tag{58}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} > 0$ and $v_4 \equiv (3b-2\alpha\sigma_p^2(1-r_{p\eta}^2)) < 0$, or if $r_{p\eta} < 0$ and $v_4 > 0$. Therefore, when $1-|r_{p\eta}|$ increases the profit increases if $r_{p\eta} \neq 0$ and $v_4 > 0$. □

Proof of Proposition 5. Based on the equilibrium solution, we obtain the market price as follows:

$$\rho^* = \frac{a-2b(a\sigma_\eta^2-c\sigma_\eta^2)}{-2\alpha\sigma_{p\eta}{}^2+3b\sigma_\eta^2+2\alpha\sigma_\eta^2\sigma_p^2} \tag{59}$$

We take the partial derivative of the price with respect to σ_p^2, and obtain:

$$\frac{\partial \rho^*}{\partial \sigma_p^2} = \frac{4\alpha b(1-r_{p\eta}^2)(a-c)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)^2} \tag{60}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is always nonnegative. □

Proof of Proposition 6. We take the partial derivative of the price (see the proof of Proposition 5) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial \rho^*}{\partial r_{p\eta}} = \frac{-8\alpha b\sigma_p^2 r_{p\eta}(a-c)}{(2\alpha\sigma_p^2(1-r_{p\eta}^2)+3b)^2} \tag{61}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is less than zero if $r_{p\eta} > 0$ and greater than zero if $r_{p\eta} < 0$. Therefore, when $1-|r_{p\eta}|$ increases the price increases. □

Proof of Proposition 7. For firm A we take the partial derivative of (44) with respect to σ_p^2, and obtain:

$$\frac{\partial Profit_A^*}{\partial \sigma_p^2} = \frac{2\alpha(a-c)^2(b+2\alpha\sigma_p^2) * v_5}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{62}$$

where

$$v_5 \equiv -8\alpha^3\sigma_p^6(r_{p\eta}^2-1)^2-4\alpha^2 b\sigma_p^4(r_{p\eta}^2-1)^2 +2\alpha b^2\sigma_p^2(1+r_{p\eta}^2)(1-r_{p\eta}^2)+b^3(1+r_{p\eta}^2) \tag{63}$$

Since $a > c$ the partial derivative is greater than zero if $v_5 > 0$.

For firm B we take the partial derivative of (46) with respect to σ_p^2, and obtain:

$$\frac{\partial Profit_B^*}{\partial \sigma_p^2} = \frac{2\alpha(a-c)^2 v_6}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{64}$$

where

$$v_6 \equiv 4\alpha b^3\sigma_p^2(r_{p\eta}^4-3r_{p\eta}^2+1)+b^4-16\alpha^4\sigma_p^8(1-r_{p\eta}^2)^3 - 16\alpha^3 b\sigma_p^6(1-r_{p\eta}^2)^2-12\alpha^2 b^2\sigma_p^4 r_{p\eta}^2(1-r_{p\eta}^2)(1-2r_{p\eta}^2) \tag{65}$$

Since $a > c$ the partial derivative is greater than zero if $v_6 > 0$. □

Proof of Proposition 8. For firm A we take the partial derivative of (44) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Profit_A^*}{\partial r_{p\eta}} = \frac{4\alpha\sigma_p^2 r_{p\eta}(a-c)^2(b+2\alpha\sigma_p^2)^2(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)^2}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{66}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} > 0$ and less than zero if if $r_{p\eta} < 0$. Therefore, when $1-|r_{p\eta}|$ increases the profit increases.

For firm B we take the partial derivative of (46) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Profit_B^*}{\partial r_{p\eta}} = \frac{-8\alpha b\sigma_p^2 r_{p\eta}(a-c)^2(b+2\alpha\sigma_p^2)^2(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{67}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} < 0$ and less than zero if $r_{p\eta} > 0$. Therefore, when $1-|r_{p\eta}|$ increases the profit decreases. □

Proof of Proposition 9. For firm A we take the partial derivative of (45) with respect to σ_p^2, and obtain:

$$\frac{\partial Risk_A^*}{\partial \sigma_p^2} = \frac{(a-c)^2(b+2\alpha\sigma_p^2)\upsilon_7}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{68}$$

where

$$\upsilon_7 \equiv (1-r_{p\eta}^2)(-8\alpha^3\sigma_p^6(1-r_{p\eta}^2)+4\alpha^2 b\sigma_p^4(r_{p\eta}^2+1) +4\alpha b^2\sigma_p^2 r_{p\eta}^2+10\alpha b^2\sigma_p^2+3b^3) \tag{69}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $\upsilon_7 > 0$.

For firm B we take the partial derivative of (47) with respect to σ_p^2, and obtain:

$$\frac{\partial Risk_B^*}{\partial \sigma_p^2} = \frac{(a-c)^2\upsilon_8}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{70}$$

where

$$\upsilon_8 \equiv 12\alpha^2 b^2\sigma_p^4(3r_{p\eta}^2-2)(r_{p\eta}^2-1)+4\alpha b^3\sigma_p^2(4-5r_{p\eta}^2)+3b^4 -16\alpha^4\sigma_p^8(1-r_{p\eta}^2)^3-16\alpha^3 b\sigma_p^6 r_{p\eta}^2(1-r_{p\eta}^2)^2. \tag{71}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $\upsilon_8 > 0$. □

Proof of Proposition 10. For firm A we take the partial derivative of (45) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Risk_A^*}{\partial r_{p\eta}} = \frac{-2\sigma_p^2 r_{p\eta}(b+2\alpha\sigma_p^2)^2\upsilon_9}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{72}$$

where

$$\upsilon_9 \equiv 4\alpha^2\sigma_p^4(r_{p\eta}^2-1)+4\alpha b\sigma_p^2 r_{p\eta}^2+3b^2. \tag{73}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} > 0$ and $\upsilon_9 < 0$, or if $r_{p\eta} < 0$ and $\upsilon_9 > 0$. Therefore, when $1-|r_{p\eta}|$ increases the profit increases if $r_{p\eta} \neq 0$ and $\upsilon_9 > 0$.

For firm B we take the partial derivative of (47) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial Risk_A^*}{\partial r_{p\eta}} = \frac{-8\alpha b\sigma_p^4 r_{p\eta}(a-c)^2(b+2\alpha\sigma_p^2)(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^3} \tag{74}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is greater than zero if $r_{p\eta} < 0$ and less than zero if $r_{p\eta} > 0$. Therefore, when $1-|r_{p\eta}|$ increases firm B's risk increases if $r_{p\eta} \neq 0$. □

Proof of Proposition 11. We subtract firm B's profit from firm A's profit and obtain:

$$Profit_A - Profit_B = \frac{2\alpha\sigma_p^2 r_{p\eta}^2(a-c)^2(b+2\alpha\sigma_p^2)(2\alpha\sigma_p^2(1-r_{p\eta}^2)+b)}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{75}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the difference is greater than zero. □

Proof of Proposition 12. Based on the equilibrium solution, we obtain the market price as follows:

$$\rho^* = \frac{ab^2\sigma_\eta^2+2b^2c\sigma_\eta^2-2a\alpha b\sigma_{p\eta}{}^2-2\alpha bc\sigma_{p\eta}{}^2-4a\alpha^2\sigma_{p\eta}{}^2\sigma_p^2+4a\alpha^2\sigma_\eta^2\sigma_p^4+4a\alpha b\sigma_\eta^2\sigma_p^2+4\alpha bc\sigma_\eta^2\sigma_p^2}{-4\alpha^2\sigma_{p\eta}{}^2\sigma_p^2+4\sigma_\eta^2\alpha^2\sigma_p^4-4\alpha b\sigma_{p\eta}{}^2+8\sigma_\eta^2\alpha b\sigma_p^2+3\sigma_\eta^2 b^2} \tag{76}$$

We take the partial derivative of the price with respect to σ_p^2, and obtain:

$$\frac{\partial\rho^*}{\partial\sigma_p^2} = \frac{2\alpha b(a-c)(4\alpha^2\sigma_p^4((r_{p\eta}^2-1)^2+1-r_{p\eta}^2)+8\alpha b\sigma_p^2(1-r_{p\eta}^2)+b^2(2-r_{p\eta}^2))}{(-4\alpha^2\sigma_p^4 r_{p\eta}^2+4\alpha^2\sigma_p^4-4\alpha b\sigma_p^2 r_{p\eta}^2+8\alpha b\sigma_p^2+3b^2)^2} \tag{77}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is always nonnegative. □

Proof of Proposition 13. We take the partial derivative of the price (see the proof of Proposition 12) with respect to $r_{p\eta}$, and obtain:

$$\frac{\partial\rho^*}{\partial r_{p\eta}} = \frac{-4\alpha b\sigma_p^2 r_{p\eta}(a-c)(b+2\alpha\sigma_p^2)^2}{(4\alpha^2\sigma_p^4(1-r_{p\eta}^2)+4\alpha b\sigma_p^2(1-r_{p\eta}^2)+4\alpha b\sigma_p^2+3b^2)^2} \tag{78}$$

Since $a > c$ and $r_{p\eta}$ is the correlation coefficient which is between 0 and 1, the partial derivative is less than zero if $r_{p\eta} > 0$ and greater than zero if $r_{p\eta} < 0$. Therefore, when $1-|r_{p\eta}|$ increases the price increases. □

A Simple Example of Financial Hedging using Futures Contracts

Futures contracts have been widely used by supply chain firms including suppliers and manufacturers to hedge various risks, such as, foreign exchange risk and commodity price risk. We now use a simplified example to briefly explain how futures contracts can be used to hedge foreign exchange risk. Suppose that a manufacturing firm in the U.S. orders some parts from a supplier located in country X. The parts will be delivered in eight months after which the payment will be made in country X's currency. The total price of the order is 1 million in country X's currency. Suppose that the current exchange rate of country X's currency to U.S. dollar is 2 to 1, which means that current cost of the order is 500,000 U.S. dollars. If after eight months the exchange rate rises to 1.5 to 1, the manufacturer needs to pay 1 million/1.5 = 666,666.67 U.S. dollars. In order to hedge such risk the U.S. firm can long 1 million country X's currency futures now which allows the firm to buy 1 million country X's currency for $500,000 in eight months. In eight months after the parts are delivered the U.S. firm sells the futures contract and receives $666,666.67–$500,000 = $166,666.67. The firm can then add this additional revenue from the futures market to $500,000, and pay the supplier $666,666.67. Note that the increase of the value of the futures contract offsets the extra cost due to the exchange rate change. On the other hand, if the exchange rate moves in favor of the manufacturer the gain will also be offset by the loss from the futures contract. In realty, however, the price of the futures usually is not exactly equal to the exchange rate at the spot market, which is called the basis risk. Due to the basis risk the exchange risk cannot be fully hedged, and the effectiveness depends on how closely the price of a futures contract and the spot market price are correlated. The commodity price risk can be hedged in a similar manner, which is also subject to the basis risk.

For a complete discussion and review of futures contracts and hedging strategies we refer the audience to the book by Hull (2002).

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