



## Application of MCDM methods for flotation machine selection

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### ABSTRACT

MCDM methods can be used in many areas when decision making process is weighed on the basis of many influential parameters. Results of this study show that MCDM methods, such as TOPSIS and VIKOR, can also be applied for flotation machine selection.

Five flotation machines were evaluated by three experts using ten criteria divided in three groups: constructional, economical and technical and the results of their evaluation were the basis for application of VIKOR and TOPSIS methods. In the first step of the selection the results obtained with these two MCDM methods were different: machine  $A_4$  was seen as the best alternative by TOPSIS, while VIKOR recognized machine  $A_3$  as the best alternative. In order to determine which of these two machines was really the most appropriate one, the machines were re-evaluated based only on two groups of criteria: constructional and economical. The results of the additional evaluation showed that the machine  $A_4$  was the best ranked by both methods.

### 1. Introduction

Flotation is a separation technique used in many industrial areas (Prakash et al., 2018), starting from mineral concentration (Sokolovic et al., 2006; Fuerstenau et al., 2007; Chelgani et al., 2015; Espiritu and Waters, 2018; Farrokhpay et al., 2018), wastewaters treatment and purification (Rubio et al., 2002), up to ink flotation in waste paper recycling (Vashisth et al., 2011; Husovska, 2013) and separation of various waste plastics (Fraunholz, 2004; Negari et al., 2018), etc. There are many parameters that have influence on efficiency of flotation, such as chemical and mineralogical composition of ore, state of the surface of the mineral particles, pulp density, pH value, particle size, reagent type and dosage, and many others (Wills and Finch, 2016).

Flotation machines also have a huge impact because they are devices in which all elementary processes of flotation take place, starting from collision of air bubbles with mineral particles, through their attachment to hydrophobic particles and the formation of the “mineral particle - air bubble” aggregate, levitation or sputtering aggregate “mineral particle - air bubble” on the surface of the pulp and the formation of mineralized foam until it is removed into a concentrate (Mesa and Brito-Parada, 2019). Since the tasks of the flotation machine are very complex, they must meet the following requirements: uniform dispersion of air bubbles, maintenance of all particles suspended in order to achieve the best conditions for the collision of air bubbles and

mineral particles, enablement of continued existence of undisturbed boundary between the pulp and the foam layer, maintenance of a constant level of the pulp and allow of the process unfolding in the continuous mode (Nelson and Leliński, 2000; Cilek and Yilmazer, 2003; Cilek, 2009).

Bearing all this in mind, choosing the adequate flotation machine is very important, but also a very complex process, precisely due to large number of factors having an impact: size and the shape of the tank, construction of aeration and agitation systems, etc., so the planner who is making the decision about the most appropriate machine for the facility has a difficult task (Kondratev and Lavrinenko, 2008; Jameson, 2010; Yianatos and Díaz, 2011).

In recent years a large number of methods for decision making have been applied for solving similar problems. From the beginning of the 1970s up to now, a significant number of multiple criteria decision-making (MCDM) methods, as well as their extensions, have been introduced. A summary overview of these methods, as well as their applications, was considered by Kahraman (2008), Zavadskas and Turskis (2011) and Zavadskas et al. (2014).

All these MCDM methods can be used to solve a wide variety of problems from different areas. Compared to some other areas, the application of MCDM methods in solving problems related to mineral processing is still in the early stages. Mineral processing as a scientific area is very suitable for application of MCDM methods because of large

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number of parameters that influence the final outcome of the process. In recent years some of the researchers have recognized positive aspects of the application of MCDM methods for solving different problems in mineral processing. Safari et al. (2010), used MCDM method, Analytic Hierarchy Process (AHP), for the selection of location for mineral processing plant. All proposed locations were ranked using eight criteria, in order to pick the best location for the plant. Bakhtavar and Lotfian (2017), used a similar approach, fuzzy AHP and grey MCDM, for ranking six locations for mineral processing plant according to six criteria. Stirbanovic et al. (2013), applied Rough Set Theory (RST) for choosing location for flotation tailings dump. Kostovic and Gligoric (2015), dealt with a problem of a collector selection in the flotation of lead–zinc sulfide ore by using MCDM method, TOPSIS. They ranked three different collectors (KEX, NaPX and KIBX) from two suppliers in three dosages, obtaining a sum of 18 alternatives. The ranking of collectors was done using 9 criteria. Baral et al. (2014), also used TOPSIS along with graphical methods (line graph and spider diagrams) to rank the various candidate alternatives for optimization of leaching parameters for the extraction of rare earth metals. Selection of grinding and flotation circuits was also an issue for researches Stanujkic et al. (2013a, 2013b), Stanujkic et al. (2014), and Zavadskas et al. (2016), who used different MCDM methods such as WASPAS, MOORA and grey compromise programming. Yavuz (2016), used TOPSIS method for selection of open-pit truck. The results of this research presented in the paper showed that MCDM can be applied for mining equipment selection. Alpay and Iphar (2018) used fuzzy TOPSIS and fuzzy VIKOR methods for selection of the most convenient hydraulic excavator for a magnesite mine. Analytical Hierarchy Process (AHP) method was used for selection of the best primary crusher by Rahimdel and Ataei (2014). The selection was done between gyratory, double toggle jaw, single toggle jaw, high speed roll crusher, low speed sizer, impactor, hammer mill and feeder breaker crushers. Capacity, feed size, product size, rock compressive strength, abrasion index and mobility of crusher were considered as criteria for selection. Samanta et al. (2002), also considered AHP method for opencast mining equipment selection. Sitorus et al. (2018), gave a detailed view of MCDM methods applications for solving various problems in mining and mineral processing.

All these examples show that MCDM methods are very applicable for solving various problems in mining and mineral processing, but no examples for flotation machine selection were found. That is the reason why MCDM approach for the selection of flotation machines is proposed in this paper. MCDM methods TOPSIS and VIKOR were used for ranking five flotation machines based on ten criteria.

## 2. Methodology

Flotation machines from mineral processing equipment manufacturers, FLSmidth: DO-70 RT and DO-100 RT (FLSmidth, 2008) and METSO: RCS 70, RCS 100 and RCS 130 (Metso, 2017), were evaluated by using VIKOR and TOPSIS methods. These two MCDM methods were selected because they were successfully used for solving many different decision-making problems, and were also used for mining equipment selection by Bazzazi et al. (2008, 2009, 2011), Lashgari et al. (2012), Yazdani-Chamzini (2014) and Alpay and Iphar (2018).

Machines were marked  $A_1$  to  $A_5$ , in order to avoid favoring certain manufacturer. Also, it is important to emphasize that this study relates to the selection of a flotation machine for rougher flotation of sulphide copper ore from the porphyry bearing. Prior to the selection of machines, criteria used for selection were chosen and their significance was determined.

### 2.1. Criteria

The first step in every MCDM method is the selection of criteria that would be used, as well as the determination of their significance. In the professional and scientific literature, several methods for determining

the significance of the criteria have been proposed, and as some of the most prominent the following could be mentioned: AHP (Satty, 1980), SWARA (Kersuliene et al., 2010) and BWM (Rezaei, 2015). However, the possibility of directly assigning the significance of criteria, as well as the sub-criteria, based on the opinions of experts involved in evaluation should not be neglected. In such approach the following condition should be also satisfied:

$$\sum_{j=1}^n w_j = 1 \tag{1}$$

where  $w_j$  denotes the weight, or significance, of criterion  $j$ , and  $n$  denotes the number of criteria.

In cases of evaluation where sub-criteria are also used, the final weight of the sub-criteria is determined in the following way:

$$w_{jl} = w_j \cdot w_l \tag{2}$$

where  $w_{jl}$  denotes the recalculated weight of criterion  $j$ , and  $w_l$  denotes the weight of sub-criterion  $l$  obtained on the basis of the expert opinion.

### 2.2. TOPSIS method

TOPSIS method, or Technique for Order Preference by Similarity to an Ideal Solution, was proposed by Hwang and Yoon (1981). The TOPSIS method is based on the idea that the best alternative should have the shortest distance from the ideal point and the farthest distance from the anti-ideal point in Euclidean space. The relative distance of each alternative from the ideal point  $d_i^+$  is determined as follows:

$$d_i^+ = \left\{ \sum_{j=1}^n (w_j(r_{ij} - r_j^+))^2 \right\}^{1/2} \tag{3}$$

where  $w_j$  denotes the weight of criterion  $j$ ,  $r_{ij}$  denotes normalized performance rating of alternative  $i$  in relation to criterion  $j$ ,  $r_j^+$  denotes  $j$ -th coordinate of the ideal point, and  $n$  denotes the number of criteria.

Similarly, the relative distance from the anti-ideal point  $d_i^-$  is determined as follows:

$$d_i^- = \left\{ \sum_{j=1}^n (w_j(r_{ij} - r_j^-))^2 \right\}^{1/2} \tag{4}$$

where  $r_j^-$  denotes  $j$ -th coordinate of the anti-ideal point.

After that, in the next step, the relative distance  $C_i$  of the alternative  $i$  to the ideal solution is determined as follows:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{5}$$

According to the TOPSIS method, the alternative with the highest value of  $C_i$  is at the same time the best alternative.

The ideal and anti-ideal points in TOPSIS method are determined as follows:

$$A^+ = \{r_1^+, r_2^+, \dots, r_n^+\} = \{(\max_i r_{ij} | j \in \Omega_{\max}), (\min_i r_{ij} | j \in \Omega_{\min})\} \tag{6}$$

$$A^- = \{r_1^-, r_2^-, \dots, r_n^-\} = \{(\min_i r_{ij} | j \in \Omega_{\max}), (\max_i r_{ij} | j \in \Omega_{\min})\} \tag{7}$$

where  $\Omega_{\max}$  and  $\Omega_{\min}$  denote set of the benefit and cost criteria, respectively.

Finally, it should be noted that TOPSIS method uses vector normalization procedure, as follows:

$$r_{ij} = \frac{x_{ij}}{(\sum_{i=1}^m x_{ij}^2)^{1/2}} \tag{8}$$

where  $x_{ij}$  denotes the performance rating of alternative  $i$  in the relation to the criterion  $j$ , and  $m$  denotes the number of alternatives.

### 2.3. VIKOR method

VIKOR method was proposed by Opricovic (1998), and it can be also mentioned as a prominent and often used MCDM method. VIKOR means Multicriteria Optimization and Compromise Solution (VIsekriterijumska optimizacija i KOmpromisno Resenje in Serbian).

The VIKOR method is based on the idea of ideal and compromise solution. The best alternative in this method is determined on the basis of the overall ranking index  $Q_i$ , which is determined as follows:

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)} \quad (9)$$

where  $S_i$  and  $R_i$  denote the average and the worst group score of alternative  $i$ , respectively,  $S^* = \min_i S_i$ ,  $S^- = \max_i S_i$ ,  $R^* = \min_i R_i$ ,  $R^- = \max_i R_i$ , and  $v$  represents a significance of the strategy, which value is usually set to be 0.5.

The average score  $S_i$  and the worst group score  $R_i$  for each alternative are determined as follows:

$$S_i = \sum_j w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-), \text{ and} \quad (10)$$

$$R_i = \max_j [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)] \quad (11)$$

where  $p$  is a parameter;  $p \in [1, \infty)$  and  $x_j^*$  and  $x_j^-$  are determined as follows:

$$x_j^* = \begin{cases} \max_j x_{ij} & j \in \Omega_{\max} \\ \min_j x_{ij} & j \in \Omega_{\min} \end{cases} \quad (12)$$

$$x_j^- = \begin{cases} \min_j x_{ij} & j \in \Omega_{\max} \\ \max_j x_{ij} & j \in \Omega_{\min} \end{cases} \quad (13)$$

## 3. Results and discussions

### 3.1. Criteria for flotation machine selection

As it was said earlier, flotation machines are very important in the flotation process because all elementary processes take place in them. For that reason, choosing an adequate flotation machine is one of the things that is essential for receiving the best results of the flotation process.

When choosing a flotation machine, a large number of parameters should be considered. All these parameters can generally be divided into three groups: constructional, economical and technical. In the first group there are such parameters as the size and the shape of the machine, the volume or the capacity of the machine, the construction of agitation and aeration system and number of the machines. The economical parameters include: investments (the price of the machine unit, transportation costs and installment costs), terms of payment and maintenance and operating costs. In the third group of parameters, technical parameters, there are: warranty period, delivery time and maintenance conditions.

All these parameters are important for selection of the flotation machine but they do not have the same importance. Therefore, every parameter was given appropriate weight which shows a significance of the parameter.

In this approach, three experts performed the direct assignation of the weights of the criteria, as well as the sub-criteria, based on their experience. Using the Delphi method, after several iterations, the values shown in Table 1 were obtained.

### 3.2. Flotation machine selection

In this session a numerical illustration is given in order to present

the proposed methodology.

At the beginning of evaluation, three experts determined the ratings of alternatives in relation to the select set of sub-criteria. As in the case of determining the weight of the criteria, the Delphi method was also used here. For evaluation, 1–9 scale, shown in Table 2, was used.

The ratings of five alternatives in relation to the selected set of sub-criteria are shown in Table 3.

The normalized ratings determined by using Eq. (8), are shown in Table 4.

The optimization directions of the criteria are also shown in Table 4, whereby the abbreviation min denotes cost criteria; i.e. the lower is better, and abbreviation max denotes the benefit criteria, i.e. the higher is the better.

In next step, on the basis of values from normalized decision-making matrix, the ideal and anti-ideal points are determined. The ideal point and anti-ideal point, determined by Eqs. (6) and (7), are shown in Table 5.

On the basis of data from Tables 4 and 5, the relative distances of each alternative from the ideal and anti-ideal point are calculated using Eqs. (3) and (4). Finally, the relative distance of each alternative to the ideal solution is calculated using Eq. (5).

The above mentioned distances from the ideal and anti-ideal point, as well as the relative distance of each alternative to the ideal solution are shown in Table 6. Table 6 shows the ranking order of considered alternatives obtained by using TOPSIS method as well.

As it can be seen from the Table 6, the best ranked alternative based on the TOPSIS method is alternative denoted as  $A_4$ .

The similar evaluation is performed by using VIKOR method. After determining  $x_j^*$  and  $x_j^-$ , shown in Table 7, by using Eqs. (12) and (13), the average and the worst group score for each alternative, obtained by Eqs. (10) and (11), are shown in Table 8. The overall ranking index  $Q_i$  obtained by Eq. (9), as well as the ranking order of considered alternatives are also shown in Table 8.

As it can be seen from Table 8, the best ranked alternative based on the VIKOR method is alternative denoted as  $A_3$ . Tables 6 and 8 also show that the results obtained by TOPSIS and VIKOR methods are not the same. Such cases in which different MCDM methods do not give the same best-placed alternative are very rare, as well as the studies that address this problem. For example: Stanujkic et al. (2013), Kolios et al. (2016) and Mulliner et al. (2016), can be mentioned.

It should be noted that the difference between the first and second placed alternatives is very small in the case of ranking based on the TOPSIS method, i.e.  $A_4 = 0.740$  versus  $A_3 = 0.724$ . Also, based on VIKOR method these two machines are best ranked but only in reverse order, machine  $A_3$  was ranked as the best alternative while  $A_4$  machine is the second best. Ranking order of other machines was the same for both methods.

In order to see the impact that individual group of criteria have on ranking order of the machines, partial rankings were conducted by assigning weight 1 to one criterion group, while the other two had weight zero. The results of the partial ranking of the alternatives according to individual group of criteria, Constructional, Economical and Technical, respectively are shown in Tables 8 and 9.

As can be seen from Tables 9 and 10:

- The TOPSIS and VIKOR methods gave the same ranking order of alternatives in relation to the first and the second group of criteria,
- TOPSIS and VIKOR methods did not give the same ranking order in relation to the third group of criteria,
- Alternative  $A_3$  is the best ranked in relation to the first and third group of criteria, while the alternative  $A_4$  is the best ranked in relation to the second group of criteria.
- It is also notable that the first and second group of criteria rank machine  $A_4$  as the first or second best alternative, while considering only third group of criteria this machine is ranked fourth, in both TOPSIS and VIKOR.

**Table 1**  
The weights of criteria and sub-criteria by means of a direct assignation obtained from 3 DMs.

Criteria	Criteria weights	Sub-criteria	Sub-criteria weights	Calculated sub-criteria weights
C <sub>1</sub> Constructional parameters	0.35	C <sub>11</sub> - The size and the shape of the machine	0.20	0.070
		C <sub>12</sub> - The volume or the capacity of the machine	0.20	0.070
		C <sub>13</sub> - The construction of agitation and aeration system	0.20	0.070
		C <sub>14</sub> - The number of the machines (quantity)	0.40	0.140
C <sub>2</sub> Economical parameters	0.40	C <sub>21</sub> - Investments	0.50	0.200
		C <sub>22</sub> - Terms of payment and maintenance	0.20	0.080
		C <sub>23</sub> - Operating costs	0.30	0.120
C <sub>3</sub> Technical parameters	0.25	C <sub>31</sub> - Warranty period	0.50	0.125
		C <sub>32</sub> - Delivery time	0.20	0.050
		C <sub>33</sub> - Maintenance conditions	0.30	0.075

**Table 2**  
Evaluation scale used for ranking flotation machines.

Value	Meaning
9	Excellent
7	Very good
5	Good
3	Satisfactory
1	Sufficient

**Table 3**  
The ratings of evaluated alternatives with respect to each criterion.

Alternative	C <sub>1</sub>				C <sub>2</sub>			C <sub>3</sub>		
	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	C <sub>14</sub>	C <sub>21</sub>	C <sub>22</sub>	C <sub>23</sub>	C <sub>31</sub>	C <sub>32</sub>	C <sub>33</sub>
A <sub>1</sub>	3	7	4	4	6	4	6	8	5	8
A <sub>2</sub>	4	6	5	5	5	5	5	8	6	9
A <sub>3</sub>	6	4	5	6	4	5	5	9	7	9
A <sub>4</sub>	5	6	6	5	3	6	4	7	8	9
A <sub>5</sub>	2	8	3	4	6	3	6	7	7	8

Based on the above, it is obvious that alternatives denoted as A<sub>3</sub> and A<sub>4</sub> are candidates for the best-ranked alternative, but in this specific case it is difficult to determine which one is really the best-ranked one, or better said, the most appropriate machine for this study.

The solution of this problem could be found in reassigning new weights to criteria, adding some new criteria or sub-criteria or reducing some of the criteria or sub-criteria. The reassigning of new weights would be inconsistent with the previous analysis, since it would alter the relative weights between the sub-criteria. Similarly, adding new criteria or sub-criteria was also deemed inconsistent. In this case, an attempt was made to reduce the number of criteria by eliminating the influence of one of the criteria groups and recalculating the weights of other criteria and sub-criteria, thus accomplishing an additional analysis, or an analysis “what if” for the case of similar values of technical parameters. In other words, the selection would be made only by using criteria with higher impact based on their assigned weights. According to the original scenario, all three criteria groups, constructional,

**Table 4**  
The normalized ratings of the alternatives with respect to each criterion.

Alternative	C <sub>1</sub>				C <sub>2</sub>			C <sub>3</sub>		
	C <sub>11</sub> min	C <sub>12</sub> max	C <sub>13</sub> max	C <sub>14</sub> min	C <sub>21</sub> max	C <sub>22</sub> min	C <sub>23</sub> max	C <sub>31</sub> min	C <sub>32</sub> max	C <sub>33</sub> min
A <sub>1</sub>	0.316	0.494	0.380	0.368	0.543	0.380	0.511	0.457	0.335	0.415
A <sub>2</sub>	0.422	0.423	0.475	0.460	0.453	0.475	0.426	0.457	0.402	0.467
A <sub>3</sub>	0.632	0.282	0.475	0.552	0.362	0.475	0.426	0.514	0.469	0.467
A <sub>4</sub>	0.527	0.423	0.569	0.460	0.272	0.569	0.341	0.400	0.536	0.467
A <sub>5</sub>	0.211	0.564	0.285	0.368	0.543	0.285	0.511	0.400	0.469	0.415

economical, and technical, were deemed important, with weights, respectively 0.35, 0.40, and 0.25. The detailed analysis, by which the recalculation could be attempted by eliminating each of the criteria, is mathematically sound, but not justifiable because of the actual meaning of the criteria. While the Warranty period (sub-criteria C<sub>31</sub>), Delivery time (sub-criteria C<sub>32</sub>), and Maintenance conditions (sub-criteria C<sub>33</sub>) are important in selecting the flotation machine, both constructional and economical parameters were considered by experts to be more important, which is reflected in the original weights’ values. Therefore, the influence of the criteria that was assessed to have the smallest weight (hence, the least impact) was selected for elimination.

Even though this is not an established practice in MCDM approaches, some similar cases in which a reduction in the number of criteria is considered can be found in literature, such as: Thoai (2012), Chakraborty and Chatterjee (2013), Liu et al. (2015) and Karabasevic et al. (2017).

In order to determine what will happen when the influence of the technical parameters is omitted by setting the value of C<sub>3</sub> to zero, additional analysis was done. Criteria C<sub>3</sub> were chosen because they have the lowest weight compared to C<sub>1</sub> and C<sub>2</sub>, and in accordance to that they should also have the lowest impact on result of analysis.

In order to continue to fulfill the condition (1) it was necessary to assign new weights to criteria C<sub>1</sub> and C<sub>2</sub> and to recalculate sub-criteria weights, as shown in Table 11.

The essential data obtained by recalculation using TOPSIS and VIKOR methods are shown in Tables 12 and 13.

As can be seen from Tables 12 and 13, the same ranking order of alternatives is obtained by using TOPSIS and VIKOR methods, where the alternative denoted A<sub>4</sub> is selected as the most appropriate, as it is summarized in Table 14.

It is necessary to mention that more precise MCDM models for flotation machine selection can be formed using a greater number of criteria, as well as sub-criteria. In most cases proposed model with 3 groups of criteria or 10 sub-criteria should give an appropriate selection. However, in some extreme cases, when a selection cannot be done with a whole set of criteria, like in previously considered example, an adequate selection can also be made on the basis of a smaller number of criteria or sub-criteria.

**Table 5**  
The ideal and anti-ideal point of the considered alternatives.

Alternative	C <sub>1</sub>				C <sub>2</sub>			C <sub>3</sub>		
	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	C <sub>14</sub>	C <sub>21</sub>	C <sub>22</sub>	C <sub>23</sub>	C <sub>31</sub>	C <sub>32</sub>	C <sub>33</sub>
A <sup>+</sup>	0.632	0.282	0.569	0.552	0.272	0.569	0.341	0.514	0.335	0.467
A <sup>-</sup>	0.211	0.564	0.285	0.368	0.543	0.285	0.511	0.400	0.536	0.415

**Table 6**  
Ranking results obtained on the basis of TOPSIS method.

Alternative	d <sub>i</sub> <sup>+</sup>	d <sub>i</sub> <sup>-</sup>	S <sub>i</sub>	Rank
A <sub>1</sub>	0.072	0.018	0.202	4
A <sub>2</sub>	0.045	0.038	0.455	3
A <sub>3</sub>	0.024	0.063	0.724	2
A <sub>4</sub>	0.025	0.071	0.740	1
A <sub>5</sub>	0.080	0.003	0.040	5

**Table 7**  
Values of the best and worst performance ratings of alternatives in relation to the criteria.

Alternative	C <sub>1</sub>				C <sub>2</sub>			C <sub>3</sub>		
	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	C <sub>14</sub>	C <sub>21</sub>	C <sub>22</sub>	C <sub>23</sub>	C <sub>31</sub>	C <sub>32</sub>	C <sub>33</sub>
x <sub>j</sub> <sup>+</sup>	6	4	6	6	3	6	4	9	5	9
x <sub>j</sub> <sup>-</sup>	2	8	3	4	6	3	6	7	8	8

**Table 8**  
Ranking results obtained on the basis of VIKOR method.

Alternative	S <sub>i</sub>	R <sub>i</sub>	Q <sub>i</sub>	Rank
A <sub>1</sub>	0.803	0.200	0.883	4
A <sub>2</sub>	0.463	0.133	0.413	3
A <sub>3</sub>	0.210	0.067	0.000	1
A <sub>4</sub>	0.298	0.125	0.275	2
A <sub>5</sub>	0.983	0.200	1.000	5
	S <sup>+</sup> = 0.210	R <sup>+</sup> = 0.067		
	S <sup>-</sup> = 0.983	R <sup>-</sup> = 0.200		

**Table 9**  
Partial ranking results achieved using the TOPSIS method.

Group of criteria	I		II		III	
	S <sub>i</sub>	Rank	S <sub>i</sub>	Rank	S <sub>i</sub>	Rank
A <sub>1</sub>	0.220	4	0.112	4	0.603	2
A <sub>2</sub>	0.528	3	0.401	3	0.572	3
A <sub>3</sub>	0.873	1	0.647	2	0.694	1
A <sub>4</sub>	0.655	2	1.000	1	0.182	4
A <sub>5</sub>	0.000	5	0.000	5	0.171	5

**Table 10**  
Partial ranking results achieved using the VIKOR method.

Group of criteria	I		II		III	
	Q <sub>i</sub>	Rank	Q <sub>i</sub>	Rank	Q <sub>i</sub>	Rank
A <sub>1</sub>	0.911	4	0.967	4	0.488	3
A <sub>2</sub>	0.414	3	0.608	3	0.274	2
A <sub>3</sub>	0.000	1	0.358	2	0.000	1
A <sub>4</sub>	0.352	2	0.000	1	0.854	4
A <sub>5</sub>	1.000	5	1.000	5	1.000	5

**Table 11**  
The recalculated weights of criteria and sub-criteria after criteria reduction.

Criteria	Criteria weight	New criteria weight	Sub-criteria	Sub-criteria weights	Recalculated sub-criteria weights
C <sub>1</sub>	0.35	0.47	C <sub>11</sub>	0.20	0.094
			C <sub>12</sub>	0.20	0.094
			C <sub>13</sub>	0.20	0.094
			C <sub>14</sub>	0.40	0.188
C <sub>2</sub>	0.40	0.53	C <sub>21</sub>	0.50	0.265
			C <sub>22</sub>	0.20	0.106
			C <sub>23</sub>	0.30	0.159
			C <sub>31</sub>	0.50	0.000
C <sub>3</sub>	0.25	0.00	C <sub>32</sub>	0.20	0.000
			C <sub>33</sub>	0.30	0.000

**Table 12**  
Ranking results obtained on the basis of TOPSIS method after criteria reduction.

Alternative	d <sub>i</sub> <sup>+</sup>	d <sub>i</sub> <sup>-</sup>	S <sub>i</sub>	Rank
A <sub>1</sub>	0.181	0.092	0.339	4
A <sub>2</sub>	0.147	0.120	0.449	3
A <sub>3</sub>	0.122	0.148	0.548	2
A <sub>4</sub>	0.101	0.150	0.598	1
A <sub>5</sub>	0.187	0.082	0.306	5

**Table 13**  
Ranking results obtained on the basis of VIKOR method after criteria reduction.

Alternative	S <sub>i</sub>	R <sub>i</sub>	Q <sub>i</sub>	Rank
A <sub>1</sub>	0.887	0.267	0.932	4
A <sub>2</sub>	0.511	0.178	0.458	3
A <sub>3</sub>	0.236	0.089	0.043	2
A <sub>4</sub>	0.163	0.094	0.013	1
A <sub>5</sub>	1.000	0.267	1.000	5
	S <sup>+</sup> = 0.163	R <sup>+</sup> = 0.089		
	S <sup>-</sup> = 1.000	R <sup>-</sup> = 0.267		

**Table 14**  
Comparative results obtained by TOPSIS and VIKOR methods after criteria reduction.

Alternative	TOPSIS		VIKOR	
	S <sub>i</sub>	Rank	Q <sub>i</sub>	Rank
A <sub>1</sub>	0.339	4	0.932	4
A <sub>2</sub>	0.449	3	0.458	3
A <sub>3</sub>	0.548	2	0.043	2
A <sub>4</sub>	0.598	1	0.013	1
A <sub>5</sub>	0.306	5	1.000	5

**4. Conclusion**

Selection of the appropriate flotation machine is very difficult and also very responsible task, because it has a long term influence on the flotation process. There are many parameters that have a significant role, so the process designer has to take them all into consideration.

MCDM methods can simplify decision making process, but also there are certain things that should be taken into account.

Results of the study given in this paper, show that MCDM methods, VIKOR and TOPSIS, can be successfully used for selection of flotation machines. Five flotation machines from two flotation machine suppliers were evaluated.

The first step is the selection of the criteria that would be used and which must be carefully chosen because of the existence of too many criteria, but also insufficient number of criteria can lead to an inappropriate selection.

Weight of the criteria is also important because it can have an impact on the selection process. Weight can be determined either subjectively by decision maker or using some mathematical methods. In this study direct assignment approach of the weights of the criteria was used. Three experts in mineral processing equipment design were consulted and asked to give weight to each criterion. Weights of criteria, as well as sub-criteria, were determined by using Delphi method and direct assignment method.

Using two methods for solving the same problem can sometimes lead to a different outcome. It is not very common for MCDM methods to have disagreement regarding the final result, but it can happen. In this case, VIKOR and TOPSIS had different alternative for best flotation machine. When something like this happens it is difficult to tell which method is right, or which machine is the most appropriate for this study. In order to determine the best alternative, additional analysis was needed, so a repeated evaluation by changing the number of criteria, i.e. by setting the weight of one group of criteria to zero was done.

Re-evaluation of flotation machines by changing weight of criteria  $C_3$  to zero gave the same outcome. Flotation machine denoted as  $A_4$  was selected as the best choice for considered example, but it should not be forgotten that it is just slightly better than the machine denoted as  $A_3$ . Under some other conditions, such as the application of machine for some other type of ore or different constraints, the ranking order of considered alternatives would probably be different.

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