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Robust adaptive multiple models based fuzzy control of nonlinear systems

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ABSTRACT

A new robust adaptive multiple models based fuzzy control scheme for a class of unknown nonlinear systems is proposed in this paper. The nonlinear system is expressed by using the Takagi–Sugeno (T–S) method, and some identification adaptive T–S models along with their corresponding controllers, are used in order to control efficiently the unknown system. The modeling error that is produced due to the use of the T–S plant model can cause instability problems if it is not taken into account in the adaptation rules. In this paper, in order to solve this problem, we design a control scheme that is based on updating rules that utilize the σ-modification method. Every T–S controller is updated indirectly by using the robust updating rules and the final control signal is determined by using a performance index and a switching rule. By using the Lyapunov stability theory it is shown that σ-modification based rules can ensure the robustness of the system and define a bound for the steady state identification error. The main objectives of the robust controller are: (i) to ensure that the real plant system will remain stable despite the existence of modeling errors and (ii) to ensure that the real plant will track with a high accuracy the state trajectory of a given reference model. The effectiveness of the proposed method is demonstrated by computer simulations on a well known benchmark problem.

1. Introduction

Robustness issues are very crucial in control systems design, especially in cases when fuzzy or neural networks (NNs) theory tools are used to mathematically express an unknown nonlinear plant [1] along with adaptive control techniques which are used to control the plant. The necessity for using fuzzy or NNs theory in system modeling is imperative when the system's nonlinearities impose difficulties to the controller design procedure. One of the most popular fuzzy models is the T–S formulation [2]. The main advantage of this method is that it uses linear submodels which are fuzzy blended and finally produce the nonlinear fuzzy model. These linear models are easily controlled by using linear control techniques and finally another fuzzy blending of the sub-controllers produces the final nonlinear controller of the system. Another characteristic of T–S representation is the “universal function approximation” property which offers the possibility to approximate any nonlinear function to any degree of accuracy [3]. Although T–S method is very effective when describing a nonlinear system, there is a very important factor that should be taken into account when someone studies a controller. The system to be controlled is rarely free of unmodeled dynamics and unknown nonlinearities. Any approximation error associated with the T–S modeling inaccuracies is called modeling error and might raise robustness issues especially when adaptive control techniques are used [4–6]. In [6], it was shown that a combination of modeling approximation errors and adaptive control techniques cannot ensure stability if the adaptive methods are not taking into account the modeling error. These stability problems could be surpassed by utilizing some robust adaptive control methods including injection of small signals to make the regressor of the adaptive law persistently exciting or modifications to the adaptive laws by using leakage, parameter projection, dead-zones etc. A number of these methods has been embodied in the following fuzzy control schemes: in [7] authors used a robustifying term in the control signal to cope with unmodeled dynamics and bounded disturbances, in [8] authors used σ-modification along with backstepping design and and in [9] authors also used σ-modification in a direct adaptive fuzzy control scheme.

Adaptive control combines an online parameter estimator with a control law in order to control classes of plants whose parameters are unknown or highly uncertain. Adaptive control methods can be classified into two main categories according to the way the unknown parameter estimator is combined with the control law. In the first category, referred to as “direct” adaptive control,

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the system is parameterized in terms of the desired controller parameters and afterwards these control parameters are estimated directly without intermediate calculations involving system parameters estimates. On the other hand, in "indirect" adaptive control the system parameters are estimated online and then used to calculate the controller parameters assuming that this is the "true" plant. In general, direct adaptive control is suitable to be used when the plant can be expressed in a parameterized form involving only the control parameters and indirect adaptive control is suitable when the estimated plant is controllable and observable or at least stabilizable and detectable. A lot of new intelligent (Fuzzy Systems, NNs, etc.) adaptive control methods have been developed during the last decade [10–24] in order to deal with nonlinear systems which are characterized by unknown or highly uncertain parameters.

In [10], the authors developed a method for controlling uncertain nonlinear multiinput–multioutput (MIMO) discrete-time systems. This method which uses a number of subsystems to compose the MIMO system and high order NNs to approximate the desired controllers ensures that the output errors converge to a compact set and that the number of the adjustable parameters is highly reduced. A method that is based on the modeling of uncertainties and external disturbances by using fuzzy logic systems and the backstepping technique was presented in [11] where the objective was to control a MIMO nonlinear system and finally it was proven that the robustness to dynamic uncertainties and external disturbances is improved. In [12], the authors ensured the robustness by using T–S systems to approximate the unknown functions of an uncertain MIMO system, developing at the same time an adaptive control scheme that utilizes "dynamic-surface" control and "minimal learning parameters" techniques'. In [13], the authors presented a method for the control of a class of uncertain single-input/single-output (SISO) nonlinear strict-feedback systems. More specifically they utilized fuzzy logic to approximate the desired control signals and then an adaptive fuzzy controller was constructed via backstepping ensuring the boundedness of all the signals and minimizing the computation burden due to the fact that only one adaptive controller needs to be updated online. An adaptive fuzzy output-feedback dynamic surface control design with prescribed performance was investigated in [14] for a class of uncertain SISO nonlinear systems in strict-feedback form ensuring that the dynamic errors converge to a predefined arbitrarily small residual set for all times and overcoming the problem of 'explosion of complexity' that appears in other adaptive control approaches. A similar method for MIMO systems was presented in [15]. Authors in [16] presented an adaptive fuzzy output tracking control approach for a class of SISO switched nonlinear systems with completed unknown nonlinear functions, unmeasured states, unmodeled dynamics and dynamic disturbances. The importance of this work lies in the fact that this scheme can be successful without the restrictive condition that all the states of the controlled systems are available, incorporates the dynamical signal into the average dwell time period and finally the unmodeled dynamics are taken into account.

Unlike the adaptive control methodologies used in the aforementioned papers there is a very common case where the adaptive fuzzy model approximates the plant which is finally used to produce the control signal employing the certainty equivalence principle, i.e. indirect adaptive control methods [17–22]. In [17,18], the authors proposed a stable control scheme for T–S Models (i.e. not for the nonlinear plant) that is based on a T–S adaptive model whose parameters are used by an adaptive controller at every step in order to form the control signal. In [19], the authors proposed an indirect adaptive control scheme for discrete-time uncertain nonlinear systems by using a T–S adaptive model and under the assumption that the T–S model represents accurately the real plant (i.e. there is no modeling error). The same assumption was made in [20] where the authors proposed an adaptive control scheme for discrete-time state-space T–S fuzzy systems with general relative degree. In [23,24], the authors enhanced these adaptive control methods by introducing the multiple adaptive T–S identification models control scheme architecture which promises a better performance than traditional single model methods. These multiple adaptive T–S identification models form the basis for deriving the corresponding fuzzy controllers which are designed to control a class of dynamical fuzzy systems. The aforementioned control schemes appearing in [17–20,22–24] are designed to be very effective when there is not any modeling error, that is, when the fuzzy model describes the plant accurately. Although, it has been experimentally shown that they perform satisfactorily well when they are applied in the real plant [17,24], due to modeling errors, the stability analysis which is made only for the fuzzy model cannot guarantee that the real nonlinear plant will remain stable. Also, multiple models based control has been shown to perform better in complex systems that change with time or their parameters are unknown. In addition, it has to be noted that the vast majority of the bibliography in multiple models adaptive control concerns linear plants or special classes of nonlinear plants [25–32] thus the capacity and the usefulness of the multiple models methods have not be extensively explored, especially in difficult nonlinear control problems where fuzzy theory techniques could provide powerful tools for this kind of problems.

The aim of this paper is to address both issues of robustness and nonlinearity. It introduces a new robust adaptive control scheme for nonlinear systems which uses multiple adaptive T–S models and takes into account the modeling error which is added to the fuzzy model of the nonlinear system. The parameters of the identification models are updated using the σ-modification method and a stability analysis based on Lyapunov theory ensures the robustness of the controller. A performance index and a switching rule define the control signal at every time instant. The steady state identification and reference model errors, which are unavoidable due to the negative effect of the modeling error, can be diminished by choosing the appropriate values for the learning rates in the updating rules and by using enough fuzzy rules in order to reduce the modeling error.

Compared with the existing results, the main contributions of the proposed method are as follows: (i) unlike the other works [17–20,22–24], where the modeling error is neglected, the proposed control scheme makes a substantial improvement by taking into account the modeling error of the T–S models and providing a new Theorem which ensures the robustness of the controller; (ii) multiple models robust fuzzy control is applied to nonlinear systems extending the results of [25–32] which are using mainly linear or special nonlinear-plants and (iii) it provides a new framework in which these three powerful tools (i.e. fuzzy modeling, multiple models and robust control) can be combined in many new ways in order to face the difficult nonlinear control problems.

The rest of the paper is organized as follows: In Section 2, the mathematical expression of the nonlinear plant is given and its corresponding T–S model is constructed. In Section 3, the multiple models based controller architecture is described and the switching mechanism is given. The T–S identification models and the fuzzy controller expressions are given in Section 4. In Section 5, a robustness analysis based on the σ-modification adaptation rules is given. Simulation studies results are given in Section 6 and finally the conclusions of this work are given in Section 7.

2. Problem formulation

In this section, the problem formulation and some mathematical expressions which make the problem more tractable are given.

Consider the following nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $f: R^n \to R^n$ and $g: R^n \to R^n$ are two unknown nonlinear functions. Assuming that $C(x,u) = f(x) + g(x)u$ on a compact set $X \times U$ is an affine continuous function with $C(0,0) = 0$ and $f(x)$ is continuously differentiable on X, then the nonlinear system (1) can be approximated to any degree of accuracy by using a T–S model [33,34] which consists of the following fuzzy rules:

Rule i : IF $x_1(t)$ is M_1^i and $x_2(t)$ is M_2^i and …and $x_n(t)$ is M_n^i,
THEN $\dot{x}(t) = A_i x(t) + B_i u(t) + \epsilon_i$

where $i = 1, \ldots, l$ is the number of fuzzy rules, $M_p^i, p = 1, \ldots, n$ are the fuzzy sets, $x(t) \in R^n$ is the state vector, $u(t) \in R$ is the input vector, $A_i^{n \times n}, B_i^{n \times 1}$ are the state and input matrices respectively which are considered to be unknown and ϵ_i is the modeling error for every rule. The matrices A_i, B_i are of the following form:

$$A_i = \begin{bmatrix} 0 & & & \\ 0 & & \mathbf{I}_{(n-1)} & \\ \vdots & & & \\ \alpha_n^i & \alpha_{n-1}^i & \ldots & \alpha_1^i \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b^i \end{bmatrix}$$

where $\mathbf{I}_{(n-1)}$ is an $(n-1) \times (n-1)$ identity matrix.

Using the above fuzzy rules, the final fuzzy model for a pair of $x(t), u(t)$ is given as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{l} h_i(x(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{l} h_i(x(t))} + \epsilon_f \tag{2}$$

where $h_i(x(t)) = \prod_{p=1}^{n} M_p^i(x_p(t)) \geq 0$ and $M_p^i(x_p(t))$ is the grade of membership of $x_p(t)$ in $M_p{}^i$ for all $i = 1, \ldots, l$, $p = 1, \ldots, n$, $\epsilon_f \in R^n$ is the modeling error due to the fuzzy modeling and $\| \epsilon_f \| < d$, i.e. the modeling error is bounded. The main objective of this paper is the design of a robust controller that will be able to make the real plant follow a reference model which is given as follows:

$$\dot{x}_m = A_d x_m \tag{3}$$

where $x_m \in R^n$ is the state vector of the desired reference model and $A_d^{n \times n}$ is a stable matrix in companion form. Eq. (2) can be expressed in the state space parametric model (SSPM) [35] form as follows:

$$\dot{x}(t) = A_d x(t) + \frac{\sum_{i=1}^{l} h_i(x(t))((A_i - A_d)x(t) + B_i u(t))}{\sum_{i=1}^{l} h_i(x(t))} + \epsilon_f \tag{4}$$

Due to the fact that the parameters matrices A_i, B_i are unknown, an estimation model must be used in order to design the control signal based on the certainty equivalence approach. The series parallel model (SPM) [35] can be expressed as follows:

$$\dot{\hat{x}}(t) = A_d \hat{x}(t) + \frac{\sum_{i=1}^{l} h_i(x(t))((\hat{A}_i - A_d)x(t) + \hat{B}_i u(t))}{\sum_{i=1}^{l} h_i(x(t))} \tag{5}$$

where $\hat{x}, \hat{A}_i, \hat{B}_i$ denote the estimations of $x(t), A_i$ and B_i respectively.

Based on the above modeling, a control architecture that uses multiple T–S identification models is described in the following section.

3. Controller architecture: adaptive T–S multiple models and switching mechanism

In this section, the main parts of the controller architecture are described and depicted schematically. Moreover, the performance index and the employed switching rule are described. The proper combinations of these three tools ensure the stability and satisfactory performance of the closed-loop system.

3.1. T–S multiple identification models

The main part of the control scheme which is depicted in Fig. 1 consists of a bank of T–S identification models. The role of these models is to approximate the behavior of the real plant which is unknown. The models bank contains N T–S identification adaptive models $\{\mathcal{M}_k\}_{k=1}^N$ of the plant, which are operating in parallel. Every identification model is connected with its own adaptive controller and the certainty equivalence approach is used for the adaptation of the controllers' parameters. The objective of the control scheme is to drive the reference model tracking error, $e_m = x - x_m$, very close to zero. The SPM formulation (5) is used to describe all the identification models $\{\mathcal{M}_k\}_{k=1}^N$ whose initial parameters estimations are different. The uncertain parameters of A_i, B_i are denoted as $\mathcal{E}_{AB} \in \Xi \subset R^s$, where Ξ is a compact space indicating the region of all the possible parameters values combinations and s is equal to the number of the unknown parameters. The critical point here is that the initial estimations are not picked randomly but they are distributed uniformly over a lattice in Ξ. Although N controllers are used in the proposed scheme, only one of them defines the control signal u, which is finally applied to all the T–S models and the real plant too. A state estimate $\hat{x}_k$ is produced for every model. The identification error which defines how "close" the models are to the real plant is given as $e_k = x - \hat{x}_k$. A feedback linearization controller $\mathcal{C}_k$ with an output u_k corresponds to identification model $\mathcal{M}_k$. The controller's $\mathcal{C}_k$ signal is

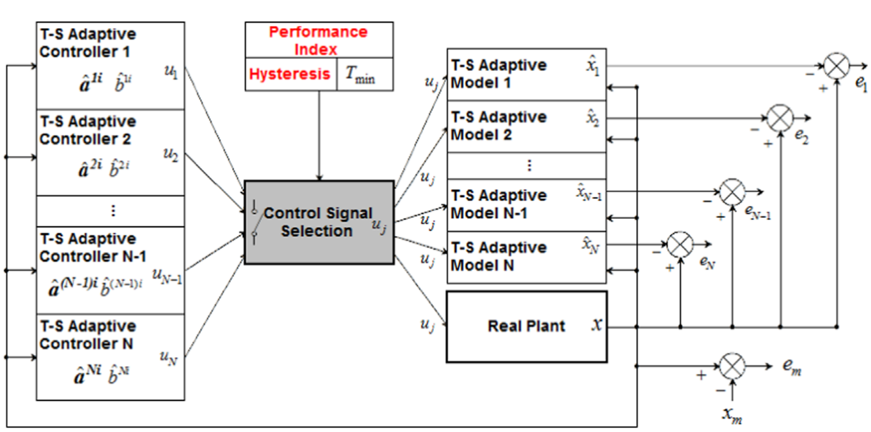

Fig. 1. The multiple T–S estimation models switching control scheme.

designed so that when applied to the corresponding T–S plant $\mathcal{M}_k$, the output is given by a state equation identical to that of the reference model (3). At every time instant, the appropriate controller is chosen by using a switching rule, which is based on a specific cost criterion J_k. The switching mechanism is described in the following subsection.

3.2. Performance index and switching rule

The switching scheme, which is defined by the performance index and the switching rule, is very important for the stability and the performance of the system. The performance index that is used in this paper has the following form:

$$J_k(t) = a_c e_k^2(t) + b_c \int_0^t e^{-\lambda(t-\tau)} e_k^2(\tau)\, d\tau \tag{6}$$

where a_c, b_c are design parameters and λ is a forgetting factor which determines the memory of the index and ensures boundedness of $J_k(t)$ for bounded e_k. It is obvious that this expression embodies both past and instantaneous values of the state identification errors. Based on this performance index, the switching rule selects at every time instant the most appropriate controller. An additive hysteresis constant h [36], and a T_{min} [37] are used because they are essential for the stability of the system. The switching rule is given as follows: If $J_j(t) = \min_{k \in \Lambda}\{J_k(t)\}$, $\Lambda = \{1, \ldots, N\}$, and $J_j(t) + h \le J_{cr}(t)$ is valid at least for the last evaluation of the performance index in the time interval $[t, t+T_{min}]$ then the model $\mathcal{M}_j$ is chosen to be tuned according to rules that will be described in the next sections. The corresponding controller $\mathcal{C}_j$ of $\mathcal{M}_j$ is calculated and being exploited using the certainty equivalence approach. It is this controller's signal that is applied on the real plant and all the identification models. Here, $J_{cr}(t)$ is the index of the current active T–S model $\mathcal{M}_{cr}$. It has to be noted that the algorithm step is not bigger than T_{min} and thus there will be more than one evaluations in the time interval $[t, t+T_{min}]$. For example if T_{min} is equal to three algorithm steps then the inequality $J_j(t) + h \le J_{cr}(t)$ should be valid at least during the third step in order to change the controller. If the aforementioned inequality is not valid, the controller $\mathcal{C}_{cr}$ remains active, meaning that it is the ideal controller for the time instant $t + T_{min}$. Note that $\mathcal{M}_j$, i.e. the model with the minimum performance index, may change during the evaluations in the time interval $[t, t+T_{min}]$. The above procedure is repeated at every step. The notations $\mathcal{C}_j$ and $\mathcal{M}_j$ will be used for the dominant controller and the dominant T–S model respectively in the following sections.

4. T–S identification models and fuzzy controller design

The fuzzy controller design is based on the T–S identification models. The mathematical expressions of the basic elements of the proposed architecture are given below. Every T–S identification model $\mathcal{M}_k$ is described by the following fuzzy rules:

T–S Identification Model $\mathcal{M}_k$

Rule i:

IF $x_1(t)$ is M_1^{ki} and $x_2(t)$ is M_2^{ki} and…and $x_n(t)$ is M_n^{ki}

THEN $\dot{\hat{x}}_k(t) = A_d \hat{x}_k(t) + (\hat{A}_{ki} - A_d)x(t) + \hat{B}_{ki} u(t)$

where $k \in \Lambda = \{1, \ldots N\}$ and $i = 1, \ldots, l$. The final form of every T–S model is given by the following equation:

$$\dot{\hat{x}}_k(t) = A_d \hat{x}_k(t) + \frac{\sum_{i=1}^{l} h_{ki}(x)((\hat{A}_{ki} - A_d)x(t) + \hat{B}_{ki} u(t))}{\sum_{i=1}^{l} h_{ki}(x)} \tag{7}$$

where $h_{ki}(x) = \prod_{p=1}^{n} M_p^{ki}(x_p(t)) \ge 0$ and $M_p^{ki}(x_p(t))$ is the grade of membership of $x_p(t)$ in $M_p{}^{ki}$, $k \in \Lambda$, $i = 1, \ldots, l$ and $p = 1, \ldots, n$. Also, $M_p^{ki}(x_p(t)) = M_p^{i}(x_p(t))$ and $h_{ki}(x) = h_i(x)$ for all k, i, p. The matrices of all the T–S models are of the following form:

$$\hat{A}_{ki} = \begin{bmatrix} 0 & & \mathbf{I}_{(n-1)} & \\ 0 & & & \\ \vdots & & & \\ \hat{a}_n^{ki} & \hat{a}_{n-1}^{ki} & \cdots & \hat{a}_1^{ki} \end{bmatrix}, \quad \hat{B}_{ki} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \hat{b}^{ki} \end{bmatrix}$$

Using a feedback linearization technique, and supposing that $\mathcal{M}_j$ is the dominant T–S model, the control signal for the plant is identical to the control signal of the controller $\mathcal{C}_j$ and is given by the following equation:

$$u(t) = u_j(t) = \frac{\sum_{i=1}^{l} h_{ji}(x)(\mathbf{a}^d - \hat{\mathbf{a}}^{ji})^T x(t)}{\sum_{i=1}^{l} h_{ji}(x)\hat{b}^{ji}} \tag{8}$$

where

$$(\hat{\mathbf{a}}^{ji})^T = \left[\hat{a}_n^{ji}\ \hat{a}_{n-1}^{ji} \cdots \hat{a}_2^{ji}\hat{a}_1^{ji}\right], \quad (\mathbf{a}^d)^T = \left[a_n^d\ a_{n-1}^d \cdots a_2^d\ a_1^d\right]$$

are the nth rows of the estimated state and reference matrices respectively.

Applying the control input $u(t)$ to $\mathcal{M}_j$ and taking into account that $h_{ji}(x) = h_{ki}(x) = h_i$ we obtain

$$\dot{\hat{x}}_j(t) = A_d \hat{x}_j(t) + \frac{1}{\sum_{i=1}^{l} h_i}\left(\sum_{i=1}^{l} h_i\left((\hat{A}_{ji} - A_d)x(t) + \hat{B}_{ji}\frac{\sum_{i=1}^{l} h_i(\mathbf{a}^d - \hat{\mathbf{a}}^{ji})^T x(t)}{\sum_{i=1}^{l} h_i \hat{b}^{ji}}\right)\right)$$

$$= A_d\hat{x}_j(t) + \frac{1}{\sum_{i=1}^{l} h_i}\left\{\begin{bmatrix} 0 & & \mathbf{I}_{(\mathbf{n-1})} & \\ 0 & & & \\ \vdots & & & \\ \sum_{i=1}^{l} h_i\hat{a}_n^{ji} & \sum_{i=1}^{l} h_i\hat{a}_{n-1}^{ji} & \cdots & \sum_{i=1}^{l} h_i\hat{a}_1^{ji} \end{bmatrix}\right.$$

$$- \begin{bmatrix} 0 & & \mathbf{I}_{(\mathbf{n-1})} & \\ 0 & & & \\ \vdots & & & \\ \sum_{i=1}^{l} h_i a_n^d & \sum_{i=1}^{l} h_i a_{n-1}^d & \cdots & \sum_{i=1}^{l} h_i a_1^d \end{bmatrix}$$

$$\left. + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \sum_{i=1}^{l} h_i \end{bmatrix} \frac{1}{\sum_{i=1}^{l} h_i}\left[\sum_{i=1}^{l} h_i(a_n^d - \hat{a}_n^{ji})\ \sum_{i=1}^{l} h_i(a_{n-1}^d - \hat{a}_{n-1}^{ji}) \cdots \sum_{i=1}^{l} h_i(a_1^d - \hat{a}_1^{ji})\right]\right\} x(t)$$

$$= A_d\hat{x}_j(t)$$

$$+ \frac{1}{\sum_{i=1}^{l} h_i}\left\{\begin{bmatrix} 0 & & \mathbf{0}_{(\mathbf{n-1})} & \\ 0 & & & \\ \vdots & & & \\ \sum_{i=1}^{l} h_i(\hat{a}_n^{ji} - a_n^d) & \sum_{i=1}^{l} h_i(\hat{a}_{n-1}^{ji} - a_{n-1}^d) & \cdots & \sum_{i=1}^{l} h_i(\hat{a}_1^{ji} - a_1^d) \end{bmatrix}\right.$$

$$\left. + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}\left[\sum_{i=1}^{l} h_i(a_n^d - \hat{a}_n^{ji})\ \sum_{i=1}^{l} h_i(a_{n-1}^d - \hat{a}_{n-1}^{ji}) \cdots \sum_{i=1}^{l} h_i(a_1^d - \hat{a}_1^{ji})\right]\right\} x(t).$$

where $\mathbf{0}_{(\mathbf{n-1})}$ is a $(n-1)\times(n-1)$ zero matrix.

From the above equations it follows that

$$\dot{\hat{x}}_j(t) = A_d \hat{x}_j(t) \tag{9}$$

From (9) it is obvious that when $u_j(t)$ is applied to $\mathcal{M}_j$, this model is linearized and has an identical behavior to that of the desired reference model (3). When the numerator of the control signal (8) equals zero, then the control signal will not be able to control the system. In this case, the Next Best Controller Logic (NBCL) [24] is used ensuring a nonzero control signal.

In the following section the adaptation rules for the T–S models are derived based on Lyapunov stability analysis.

5. Adaptation rules and robust stability analysis

In this section, the main objective is to formulate appropriate adaptation rules for the parameters of the adaptive models that will lead the system to the desired behavior ensuring at the same time its stability. The identification error for every T–S model – as already mentioned in Section 3 – is given by the following equation:

$$e_k = x - \hat{x}_k \tag{10}$$

The error e_k is equal to the difference between the state of the plant and the state of the T–S model $\mathcal{M}_k$. The time derivative of the identification error e_k is given by the following equation:

$$\begin{aligned}
\dot{e}_k &= \dot{x} - \dot{\hat{x}}_k = A_d e_k - \frac{\sum_{i=1}^{l} h_i \tilde{A}_{ki}}{\sum_{i=1}^{l} h_i} x - \frac{\sum_{i=1}^{l} h_i \tilde{B}_{ki}}{\sum_{i=1}^{l} h_i} u + \epsilon_f \\
&= A_d e_k - \frac{\sum_{i=1}^{l} h_i \begin{bmatrix} 0 & & \\ 0 & \mathbf{0}_{(\mathbf{n-1})} & \\ \vdots & & \\ \tilde{a}_n^{ki} & \tilde{a}_{n-1}^{ki} & \cdots & \tilde{a}_1^{ki} \end{bmatrix}}{\sum_{i=1}^{l} h_i} x \\
&\quad - \frac{\sum_{i=1}^{l} h_i \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \tilde{b}^{ki} \end{bmatrix}}{\sum_{i=1}^{l} h_i} u + \epsilon_f = A_d e_k - \frac{\sum_{i=1}^{l} h_i \begin{bmatrix} \mathbf{0}_{\mathbf{n}\times(\mathbf{n-1})} & \tilde{\mathbf{a}}^{ki} \end{bmatrix}^T}{\sum_{i=1}^{l} h_i} x \\
&\quad - \frac{\sum_{i=1}^{l} h_i \begin{bmatrix} 0 & 0 & \cdots \tilde{b}^{ki} \end{bmatrix}^T}{\sum_{i=1}^{l} h_i} u + \epsilon_f
\end{aligned} \tag{11}$$

where $\tilde{A}_{ki} = \hat{A}_{ki} - A_i$, $\tilde{B}_{ki} = \hat{B}_{ki} - B_i$, $\tilde{a}_p^{ki} = \hat{a}_p^{ki} - a_p^i$, $\tilde{b}^{ki} = \hat{b}^{ki} - b^i$, $\tilde{\mathbf{a}}^{ki} = \left[\tilde{a}_n^{ki}\ \tilde{a}_{n-1}^{ki}\ \cdots\ \tilde{a}_1^{ki}\right]^T$ is a $n \times 1$ vector, $\mathbf{0}_{\mathbf{n}\times(\mathbf{n-1})}$ is a zero matrix of dimension $n \times (n-1)$, $p = 1, \ldots, n$ and ϵ_f is the modeling error. Here, ${a_p}^i$refer to the elements of the last row of A_i and $\tilde{\mathbf{a}}^{ki}$ refer to the elements of the last row of $\tilde{A}_{ki}$.

Consider the following functions as Lyapunov function candidates:

$$V_k(e_k, \tilde{\mathbf{a}}^{ki}, \tilde{b}^{ki}) = e_k^T P e_k + \sum_{i=1}^{l} h_i \frac{(\tilde{\mathbf{a}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i r^k} + \sum_{i=1}^{l} h_i \frac{(\tilde{b}^{ki})^2}{\sum_{i=1}^{l} h_i r^k} \tag{12}$$

where $r^k > 0$ is a design constant, $V_k \geq 0$, and $P = P^T > 0$ is the solution of the Lyapunov equation:

$$A_d^T P + P A_d = -\mathbb{I}_n \tag{13}$$

The following inequality can be obtained from (12):

$$V_k(e_k, \tilde{\mathbf{a}}^{ki}, \tilde{b}^{ki}) \leq \lambda_{max}(P) \| e_k \|^2 + \sum_{i=1}^{l} h_i \frac{\| \tilde{\mathbf{a}}^{ki} \|^2}{\sum_{i=1}^{l} h_i r^k} + \sum_{i=1}^{l} h_i \frac{| \tilde{b}^{ki} |^2}{\sum_{i=1}^{l} h_i r^k} \tag{14}$$

The time derivative of V_k is given as follows:

$$\begin{aligned}
\dot{V}_k &= \dot{e}_k^T P e_k + e_k^T P \dot{e}_k + 2 \sum_{i=1}^{l} h_i \frac{(\dot{\tilde{\mathbf{a}}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i r^k} + 2 \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \dot{\tilde{b}}^{ki}}{\sum_{i=1}^{l} h_i r^k} \\
&= e_k^T A_d^T P e_k + e_k^T P A_d e_k + 2 \sum_{i=1}^{l} h_i \frac{(\dot{\tilde{\mathbf{a}}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i r^k} + 2 \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \dot{\tilde{b}}^{ki}}{\sum_{i=1}^{l} h_i r^k} \\
&\quad - \frac{\sum_{i=1}^{l} h_i x^T \left[\mathbf{0}_{\mathbf{n}\times(\mathbf{n-1})}\ \tilde{\mathbf{a}}^{ki}\right]}{\sum_{i=1}^{l} h_i} P e_k + \epsilon_f^T P e_k \\
&\quad - e_k^T P \frac{\sum_{i=1}^{l} h_i \left[\mathbf{0}_{\mathbf{n}\times(\mathbf{n-1})}\ \tilde{\mathbf{a}}^{ki}\right]^T x}{\sum_{i=1}^{l} h_i} + e_k^T P \epsilon_f \\
&\quad - \frac{\sum_{i=1}^{l} h_i u^T \left[0\ 0\ \cdots\ \tilde{b}^{ki}\right]}{\sum_{i=1}^{l} h_i} P e_k - e_k^T P \frac{\sum_{i=1}^{l} h_i \left[0\ 0\ \cdots\ \tilde{b}^{ki}\right]^T u}{\sum_{i=1}^{l} h_i}
\end{aligned}$$

Utilizing known matrix properties and after some mathematical manipulations the time derivative of V_k is given by the following equation:

$$\begin{aligned}
\dot{V}_k &= -e_k^T e_k + \epsilon_f^T P e_k + e_k^T P \epsilon_f + 2 \sum_{i=1}^{l} h_i \frac{(\dot{\tilde{\mathbf{a}}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i r^k} + 2 \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \dot{\tilde{b}}^{ki}}{\sum_{i=1}^{l} h_i r^k} \\
&\quad - 2 \frac{\sum_{i=1}^{l} h_i P_s^T e_k x^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i} - 2 \frac{\sum_{i=1}^{l} h_i \tilde{b}^{ki} P_s^T e_k u}{\sum_{i=1}^{l} h_i}
\end{aligned} \tag{15}$$

where $P_s \in R^{n \times 1}$ is the nth column of P.

The main objective at this point is to choose the appropriate values for the parameters vectors updating laws $(\dot{\tilde{\mathbf{a}}}^{ki})^T$, $\dot{\tilde{b}}^{ki}$ in order to make the time derivative of V_k negative. The adaptation rules are given as follows:

$$\begin{aligned}
(\dot{\tilde{\mathbf{a}}}^{ki})^T &= (\dot{\hat{\mathbf{a}}}^{ki})^T = r^k P_s^T e_k x^T - r^k \sigma (\hat{\mathbf{a}}^{ki})^T \\
\dot{\tilde{b}}^{ki} &= \dot{\hat{b}}^{ki} = r^k P_s^T e_k u - r^k \sigma \hat{b}^{ki}
\end{aligned} \tag{16}$$

where σ is a small design parameter and the adaptation rules follow the so-called σ-modification approach of the adaptive control literature [6]. Applying the values of (16) in (15), $\dot{V}_k$ takes the following form:

$$\begin{aligned}
\dot{V}_k &= -e_k^T e_k + \epsilon_f^T P e_k + e_k^T P \epsilon_f - 2 \sum_{i=1}^{l} h_i \frac{\sigma (\hat{\mathbf{a}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i} - 2 \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \sigma \hat{b}^{ki}}{\sum_{i=1}^{l} h_i} \\
&= -e_k^T e_k + \epsilon_f^T P e_k + e_k^T P \epsilon_f - 2\sigma \sum_{i=1}^{l} h_i \frac{(\tilde{\mathbf{a}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i} - 2\sigma \sum_{i=1}^{l} h_i \frac{(\mathbf{a}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i} \\
&\quad - 2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \tilde{b}^{ki}}{\sum_{i=1}^{l} h_i} - 2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} b^{ki}}{\sum_{i=1}^{l} h_i}
\end{aligned} \tag{17}$$

From (17), it follows that

$$\begin{aligned}
\dot{V}_k &\leq -\| e_k \|^2 + 2d \| P \| \| e_k \| - 2\sigma \sum_{i=1}^{l} h_i \frac{(\tilde{\mathbf{a}}^{ki})^T \tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l} h_i} + 2\sigma \sum_{i=1}^{l} h_i \frac{\| \mathbf{a}^{ki} \| \| \tilde{\mathbf{a}}^{ki} \|}{\sum_{i=1}^{l} h_i} \\
&\quad - 2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki} \tilde{b}^{ki}}{\sum_{i=1}^{l} h_i} + 2\sigma \sum_{i=1}^{l} h_i \frac{| \tilde{b}^{ki} | | b^{ki} |}{\sum_{i=1}^{l} h_i} \leq -\frac{\| e_k \|^2}{2} + 2d^2 \| P \|^2 \\
&\quad - \sigma \sum_{i=1}^{l} h_i \frac{\| \tilde{\mathbf{a}}^{ki} \|^2}{\sum_{i=1}^{l} h_i} + \sigma \sum_{i=1}^{l} h_i \frac{\| \mathbf{a}^{ki} \|^2}{\sum_{i=1}^{l} h_i} - \sigma \sum_{i=1}^{l} h_i \frac{| \tilde{b}^{ki} |^2}{\sum_{i=1}^{l} h_i} \\
&\quad + \sigma \sum_{i=1}^{l} h_i \frac{| b^{ki} |^2}{\sum_{i=1}^{l} h_i}
\end{aligned} \tag{18}$$

Using the inequality (14) it is obvious that

$$-\sigma\sum_{i=1}^{l}h_i\frac{\|\tilde{\mathbf{a}}^{ki}\|^2}{\sum_{i=1}^{l}h_i}-\sigma\sum_{i=1}^{l}h_i\frac{|\tilde{b}^{ki}|^2}{\sum_{i=1}^{l}h_i}\le-\sigma r^kV_k+\sigma r^k\lambda_{(max)}(P)\|e_k\|^2 \tag{19}$$

From (18) and (19) it follows that

$$\dot{V}_k\le(\sigma r^k\lambda_{(max)}(P)-(1/2))\|e_k\|^2-\sigma r^kV_k+2d^2\|P\|^2$$
$$+\sigma\sum_{i=1}^{l}h_i\frac{\|\mathbf{a}^{ki}\|^2}{\sum_{i=1}^{l}h_i}+\sigma\sum_{i=1}^{l}h_i\frac{|b^{ki}|^2}{\sum_{i=1}^{l}h_i} \tag{20}$$

The main objective at this point is to ensure that $\dot{V}_k\le0$ for certain circumstances. In order to achieve this goal we pick the appropriate values for σ, P and r^k so that inequalities (21) and (22) hold,

$$\lambda_{max}(P)\le\frac{1}{2\sigma r^k} \tag{21}$$

$$V_k\ge\frac{1}{r^k}\left(\frac{2d^2\|P\|^2}{\sigma}+\sum_{i=1}^{l}h_i\frac{\|\mathbf{a}^{ki}\|^2}{\sum_{i=1}^{l}h_i}+\sum_{i=1}^{l}h_i\frac{|b^{ki}|^2}{\sum_{i=1}^{l}h_i}\right) \tag{22}$$

$$A_d^TP+PA_d=-\mathbb{I}_n \tag{23}$$

If (21) and (22) are valid and taking into account (23), then V_k is bounded and thus $e_k, \tilde{\mathbf{a}}^{ki}, \tilde{b}^{ki}\in L_\infty$ and $\hat{\mathbf{a}}^{ki}, \hat{b}^{ki}\in L_\infty$ too. Another critical point in the proposed control scheme is the boundedness of the control signal (8). Although $\hat{b}^{ki}$ is bounded, the term $1/\hat{b}^{ki}$ may become unbounded if the adaptation rule generates values very close to zero or equal to zero for the $\hat{b}^{ki}$ parameter. In this case the adaptation rule has to be modified towards a direction that prevents the parameter from approximating zero or becoming zero. In order to accomplish this goal, the following assumptions are necessary: (i) the sign of b^i and a lower bound $b_0^i>0$ for $|b^i|$ are known for all $i=1,\dots,l$ and (ii) the initialization of the $\hat{b}^{ki}$ fulfills the following inequality:

$$\hat{b}^{ki}(0)\ \mathrm{sgn}(b^i)=\left|\hat{b}^{ki}(0)\right|\ge b_0^i \tag{24}$$

The new adaptive law is given in (25) and Theorem 1 ensures that the plant is stable in a certain region and the plant follows the state of the reference model (3):

$$(\dot{\hat{\mathbf{a}}}^{ki})^T = r^kP_s^Te_kx^T-r^k\sigma(\hat{\mathbf{a}}^{ki})^T,$$

$$\dot{\hat{b}}^{ki} = \begin{cases} r^kP_s^Te_ku-r^k\sigma\hat{b}^{ki}, & \begin{cases}\text{if } \left|\hat{b}^{ki}\right|>b_0^i \text{ or} \\ \text{if } \left|\hat{b}^{ki}\right|=b_0^i \text{ and } P_s^Te_ku\mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|\ge0\end{cases}, \text{for all } i,k. \\ 0, & \text{if } \left|\hat{b}^{ki}\right|=b_0^i \text{ and } P_s^Te_ku\mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|<0, \text{ for all } i,k.\end{cases} \tag{25}$$

Remark 1. The region in which the system is stable can be regulated appropriately due to the fact that the positive quantities r^k, d, σ in (22) are chosen by the designer. For example, the upper bound of the modeling error could be reduced if the designer uses more fuzzy rules in the fuzzy model [33]. In this case the stable region for the system would be larger. The same result would be possible if the designer increases the leakage term σ or the learning rate constants r^k. It has to be noted here that using the techniques which are given in [33, Subsection 14.1.3] one could obtain the upper bound of the modeling error for a specific dynamical system.

Theorem 1. *Consider the real plant (1) and the reference model of (3) with the control signal (8) and the adaptation laws of (25). The proposed approach guarantees that in a certain region and for all $i=1,\dots,l$ and $j,k\in\Lambda$, (i) $\hat{\mathbf{a}}^{ki}$, $\hat{b}^{ki}$, $1/\hat{b}^{ki}$, $e_k(t)$, $e_m(t)$ are bounded, and (ii) $e_k(t)$, $\dot{\hat{\mathbf{a}}}^{ji}(t)$, $\dot{\hat{b}}^{ji}(t)$, $e_m(t)$ are $(d^2\|P\|^2+\sigma)$-small in the mean square sense.*

Proof. As it was mentioned above, the values of $\hat{b}^{ki}$ should be kept away from zero in order to avoid singularities problems. This means that if (24) is valid then the following inequality should be valid too:

$$\dot{\hat{b}}^{ki}\hat{b}^{ki}\ge0 \tag{26}$$

Analyzing (26) it follows that

$$\dot{\hat{b}}^{ki}\hat{b}^{ki}=(r^kP_s^Te_ku-r^k\sigma\hat{b}^{ki})\hat{b}^{ki}=r^kP_s^Te_ku\ \mathrm{sgn}(b_i)\hat{b}^{ki}\ \mathrm{sgn}(b_i)$$
$$-r^k\sigma\ \mathrm{sgn}(b_i)\hat{b}^{ki}\ \mathrm{sgn}(b_i)\hat{b}^{ki}=r^kP_s^Te_ku\ \mathrm{sgn}(b_i)|\hat{b}^{ki}|-r^k\sigma|\hat{b}^{ki}|^2$$
$$=r^k(P_s^Te_ku\ \mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|)|\hat{b}^{ki}|$$

From the above equation it is obvious that if $P_s^Te_ku\ \mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|\ge0$ then $\dot{\hat{b}}^{ki}\hat{b}^{ki}\ge0$ and $|\hat{b}^{ki}|\ge b_0^i$ for all i,k. In addition $\dot{V}_k\le0$ under specific circumstances is given by (22). The case where $P_s^Te_ku\ \mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|<0$ and $|\hat{b}^{ki}|=b_0$ should be examined for the negativity of $\dot{V}_k$. From (25), $\dot{\hat{b}}^{ki}=0$ and $\dot{V}_k$ is given as follows:

$$\dot{V}_k=-e_k^Te_k+\epsilon_f^TPe_k+e_k^TP\epsilon_f-2\sigma\sum_{i=1}^{l}h_i\frac{(\hat{\mathbf{a}}^{ki})^T\tilde{\mathbf{a}}^{ki}}{\sum_{i=1}^{l}h_i}-2\sum_{i=1}^{l}h_i\frac{\tilde{b}^{ki}P_s^Te_ku}{\sum_{i=1}^{l}h_i} \tag{27}$$

For the sake of proof's calculations the term $\dot{V}_k$ of (27) will be denoted as $\dot{V}_k(\dot{\hat{b}}^{ki}=0)$ and the term $\dot{V}_k$ of (17) as $\dot{V}_k(\dot{\hat{b}}^{ki}\ne0)$. It was shown above, that under certain circumstances, $\dot{V}_k(\dot{\hat{b}}^{ki}\ne0)\le0$. If $\dot{V}_k(\dot{\hat{b}}^{ki}=0)\le0$ under the same circumstances, then the modified adaptive law ensures that the time derivative of V_k is negative or equal to zero in every case. From (17) and (27) and using the fact that $P_s^Te_ku\ \mathrm{sgn}(b_i)-\sigma|\hat{b}^{ki}|<0$ it follows that,

$$\dot{V}_k(\dot{\hat{b}}^{ki}=0)-\dot{V}_k(\dot{\hat{b}}^{ki}\ne0)=-2\sum_{i=1}^{l}h_i\frac{\tilde{b}^{ki}P_s^Te_ku}{\sum_{i=1}^{l}h_i}+2\sigma\sum_{i=1}^{l}h_i\frac{\tilde{b}^{ki}\hat{b}^{ki}}{\sum_{i=1}^{l}h_i}$$

$$=-2\sum_{i=1}^{l}h_i\frac{\tilde{b}^{ki}\ \mathrm{sgn}(b_i)P_s^Te_ku\ \mathrm{sgn}(b_i)}{\sum_{i=1}^{l}h_i}+2\sigma\sum_{i=1}^{l}h_i\frac{\tilde{b}^{ki}\hat{b}^{ki}}{\sum_{i=1}^{l}h_i}$$

$$< -2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki}\,\mathrm{sgn}(b_i)|\hat{b}^{ki}|}{\sum_{i=1}^{l} h_i} + 2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki}\hat{b}^{ki}}{\sum_{i=1}^{l} h_i}$$

$$= -2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki}\,\mathrm{sgn}(b_i)\hat{b}^{ki}\,\mathrm{sgn}(b_i)}{\sum_{i=1}^{l} h_i} + 2\sigma \sum_{i=1}^{l} h_i \frac{\tilde{b}^{ki}\hat{b}^{ki}}{\sum_{i=1}^{l} h_i} = 0$$

Since $\dot{V}_k(\dot{\hat{b}}^{ki}=0) - \dot{V}_k(\dot{\hat{b}}^{ki}\neq 0) < 0$ and $\dot{V}_k(\dot{\hat{b}}^{ki}\neq 0) \le 0$, it is obvious that $\dot{V}_k(\dot{\hat{b}}^{ki}=0) < 0$. This result means that $\dot{V}_k \le 0$, for all k and $t \ge 0$. Consequently, the function V_k is a Lyapunov function for the error system (11) when the parameters are updated according to (25). This implies that e_k, $\tilde{\mathbf{a}}^{ki}$, $\tilde{b}^{ki}$, $\hat{\mathbf{a}}^{ki}$, $\hat{b}^{ki} \in L_\infty$ thus x, $\hat{x}_k$, u, $e_m \in L_\infty$ too. Also, V_k is bounded from below and non-increasing with time and the following equation stands:

$$\lim_{t\to\infty} V_k(e_k(t), \quad \tilde{\mathbf{a}}^{ki}(t), \quad \tilde{b}^{ki}(t)) = V(\infty) < \infty \tag{28}$$

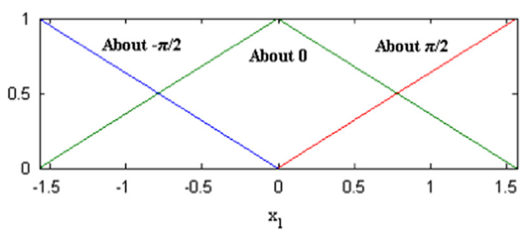

Fig. 2. The membership functions of the fuzzy model.

The inequality (20) implies that

$$\dot{V}_k \le (\sigma r^k \lambda_{(max)}(P) - (1/2))\|e_k\|^2 + 2d^2\|P\|^2 + \sigma \sum_{i=1}^{l} h_i \frac{\|\mathbf{a}^{ki}\|^2}{\sum_{i=1}^{l} h_i}$$

$$+\sigma \sum_{i=1}^{l} h_i \frac{|b^{ki}|^2}{\sum_{i=1}^{l} h_i} \tag{29}$$

Integrating both sides we obtain

$$\int_0^t \|e_k\|^2\, d\tau \le \frac{V_k(0)-V_k(t)}{c} + \frac{c_1}{c}\int_0^t (d^2\|P\|^2+\sigma)\, d\tau \tag{30}$$

where $V_k(0) = V_k(e_k(0), \tilde{\mathbf{a}}^{ki}(0), \tilde{b}^{ki}(0))$ and

$c = -\sigma r^k \lambda_{max}(P) + \frac{1}{2} > 0$

and

$$c_i = \sum_{i=1}^{l} h_i \frac{\|\mathbf{a}^{ki}\|^2 + |b^{ki}|^2}{\sum_{i=1}^{l} h_i}$$

and

$c_1 = \max\{2, c_i\}$

Inequality (30) implies that $e_k \in \mathbf{S}(d^2\|P\|^2+\sigma)$ where $\mathbf{S}(k) = [x : [0,\infty) \to R^n \mid \int_t^{t+T} x^T(\tau)x(\tau)\, d\tau \le g_1 + g_2 kT, T, g_1, g_2 \ge 0]$. Applying the control signal $u(t) = u_j(t)$ in the model $\mathcal{M}_j$ we obtain $\dot{\hat{x}}_j(t) = A_d\hat{x}_j(t)$, and taking into account Eq. (4), the time derivative

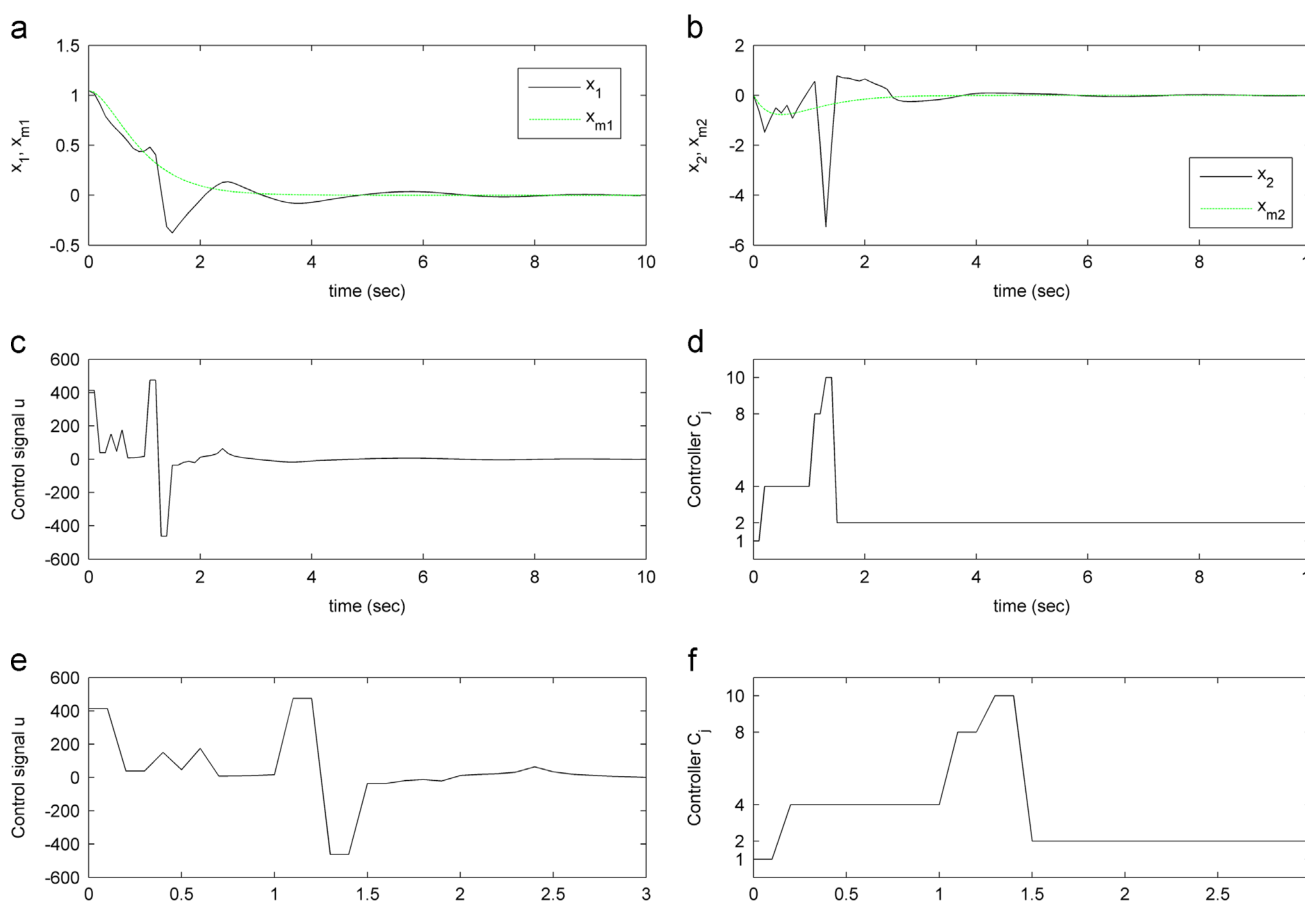

Fig. 3. (a) and (b) State response; (c) control signal; (d) controllers switching sequence; (e) and (f) zoom out of control signal and controllers switching sequence respectively on the time interval [0, 3], using ten adaptive identification models, $\sigma = 0.05$ and $r^k = 1$.

of the identification error for $\mathcal{M}_j$ is given as follows:

$$\dot{e}_j(t)=\dot{x}(t)-\dot{\hat{x}}_j(t)=A_d e_j(t)+\frac{\sum_{i=1}^{l} h_i(x(t))((A_i-A_d)x(t)+B_i u(t))}{\sum_{i=1}^{l} h_i(x(t))}+\epsilon_f \tag{31}$$

The time derivative of the reference model error, $e_m=x-x_m$, is given as follows:

$$\dot{e}_m(t)=\dot{x}(t)-\dot{x}_m(t)=A_d e_m(t)+\frac{\sum_{i=1}^{l} h_i(x(t))((A_i-A_d)x(t)+B_i u(t))}{\sum_{i=1}^{l} h_i(x(t))}+\epsilon_f \tag{32}$$

From (31) and (32) one obtains

$$\dot{e}_j(t)-\dot{e}_m(t)=A_d(e_j(t)-e_m(t)) \tag{33}$$

Eq. (11) implies that $\dot{e}_k \in L_\infty$. Using (33) and the fact that $e_k \in \mathbf{S}(d^2\|P\|^2+\sigma)$, it follows that $e_m \in \mathbf{S}(d^2\|P\|^2+\sigma)$ too. This means that e_k, e_m are $(d^2\|P\|^2+\sigma)$-small in the mean square sense. □

Remark 2. From (30) it is obvious that when using the σ-modification method, the L_2 property, which is a requirement in order to achieve $e_m \to 0$, cannot be guaranteed even if the modeling error is equal to zero. Consequently, the steady state errors may not be zero for the sake of robustness. This problem can be solved by using a switching σ-modification method [6] which requires the knowledge of an upper bound of the unknown parameters.

Remark 3. The matrices A, B of the linear plants that are used in the consequent part of the fuzzy rules derive from the nonlinear plant to be controlled. There are two main approaches for constructing fuzzy models [33]: (i) to identify the nonlinear system by using input-output data and (ii) to use the nonlinear system dynamic equations, in case these are known, even uncertainly. In this paper, the second approach is utilized. This approach is mainly based on the "sector nonlinearity" method. The aim of this method is to find a global or a local sector such that $\dot{x}(t)=f(x(t)) \in [a_1, a_2]\, x(t)$ where $f(x(t))$ is a nonlinear function and $a_1, a_2 \in R$. This sector is used for the construction of the linear models in fuzzy rules. When the reduction of the number of fuzzy rules that describe a system is necessary the "local approximation in fuzzy partition spaces" method is used. The objective of this method is to approximate the nonlinear terms by judiciously chosen linear terms. When T–S multiple models are engaged in the control scheme this method offers a reduction in the complexity of the whole control procedure.

6. Simulation studies

In this section, the proposed robust control algorithm is applied on a nonlinear plant with unknown parameters. Its efficiency is demonstrated and the results are discussed in detail.

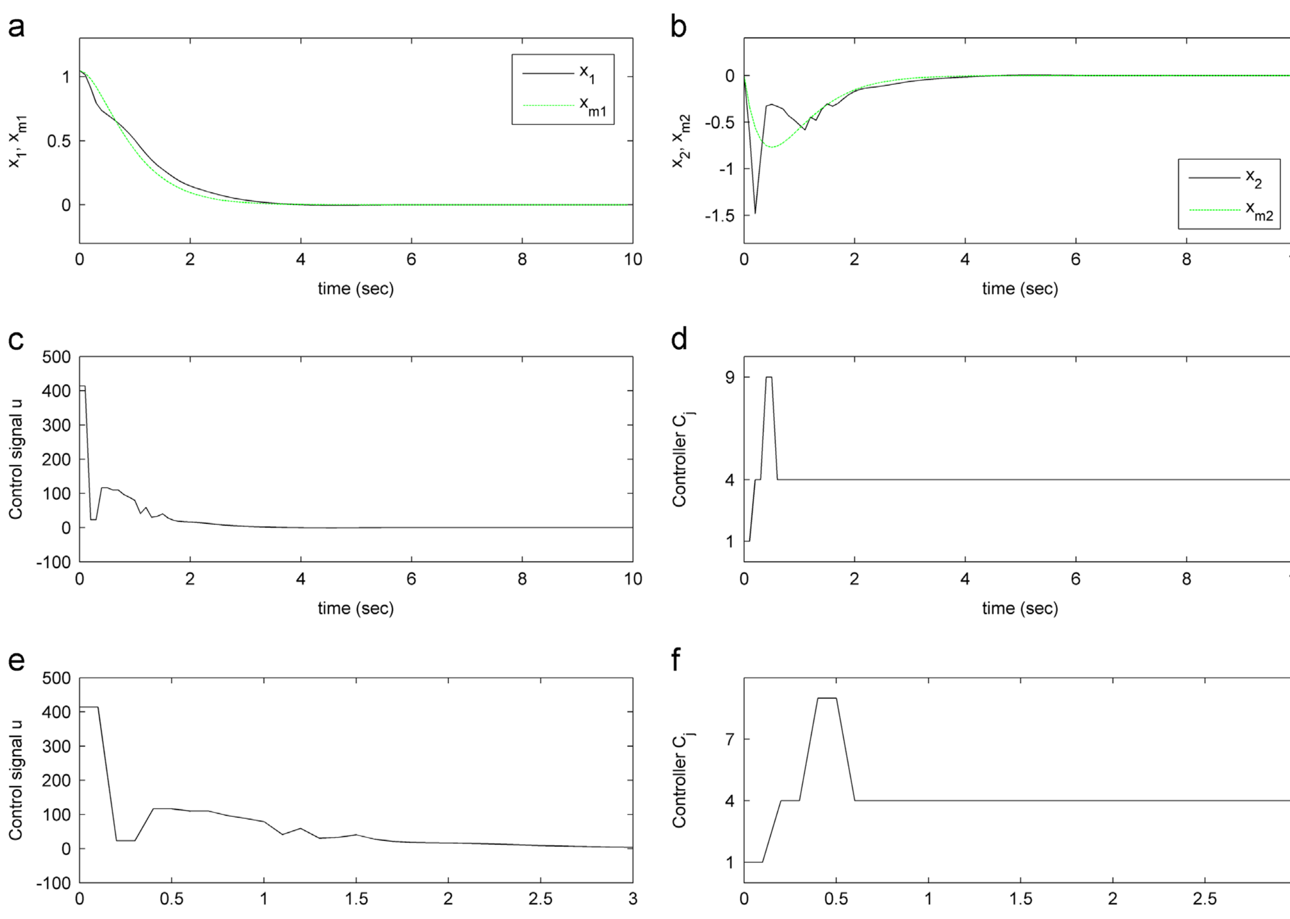

Fig. 4. (a) and (b) State response; (c) control signal; (d) controllers switching sequence; (e) and (f) zoom out of control signal and controllers switching sequence respectively on the time interval [0, 3], using ten adaptive identification models, $\sigma=0.08$ and $r^k=2$.

Consider the following inverted pendulum system (34) which is a highly nonlinear system and is widely used as a benchmark control problem [38,17]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\ \sin(x_1) - amlx_2^2\ \sin(2x_1)/2 - a\ \cos(x_1)u}{4l/3 - aml\ \cos^2(x_1)} \tag{34}$$

where x_1 denotes the angle (in radians) of the pendulum from the vertical, x_2 is the angular velocity, $g = 9.8\ \text{m/s}^2$ is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pole, u is the control signal applied to the cart and $a = 1/(m+M)$. The nonlinear system (34) can be approximated using the following two fuzzy rules:

Rule 1: IF $x_1(t)$ is about 0 THEN $\dot{x}(t) = A_1x(t) + B_1u(t)$
Rule 2: IF $x_1(t)$ is about $\pm\pi/2$ THEN $\dot{x}(t) = A_2x(t) + B_2u(t)$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ a_2^1 & a_1^1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ b^1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ a_2^2 & a_1^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ b^2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

and $\beta = \cos(88^{\circ})$. Here, $x_1 \in (-\pi/2,\ \pi/2)$ and the membership functions used are depicted in Fig. 2.

The unknown parameters $\mathcal{E}_{AB}$ lie in the three dimensional compact space Ξ:

$$\Xi = \{0.1 \le l \le 1.1,\ \ 0.6 \le m \le 5,\ \ 5 \le M \le 15\}$$

The objective of the robust controller is to force the system (34) to follow as closely as possible the reference model (3) with the following stable state matrix:

$$A_d = \begin{bmatrix} 0 & 1 \\ -5 & -5 \end{bmatrix}$$

From the Lyapunov Eq. (13), we obtain

$$P = \begin{bmatrix} 1.10 & 0.10 \\ 0.10 & 0.12 \end{bmatrix}, \quad P_s = [0.10\ \ 0.12]^T$$

and $\lambda_{max}(P) = 1.1101$. For the purposes of this particular control problem we used a scheme with ten adaptive fuzzy T–S models $\{\mathcal{M}_k\}_{k=1}^{10}$ along with their corresponding controllers $\{\mathcal{C}_k\}_{k=1}^{10}$. The initialization of these models is made in such a way that their parameters $\hat{\mathbf{a}}^{ki}$, $\hat{b}^{ki}$ are uniformly distributed in the three dimensional region Ξ. The values for the real plant are given as follows: l=0.8 m, m=2 Kg and M=12 Kg.

Two simulations for the same plant but with different design parameters took place in order to indicate the crucial role of design parameters values in the controller's performance. More specifically, the design parameters for the first simulation are $\sigma = 0.05$, $r^k = 1$ and for the second simulation are $\sigma = 0.08$, $r^k = 2$. In both simulations, the values $a_c = 6$, $b_c = 1$, $\lambda = 0.01$ are used for the cost criterion (6), $T_{min} = 0.1$ s and h=0.01. Also the lower bounds for the b^i's are $b_0^1 = 0.02$ and $b_0^2 = 0.001$. The initial states for the real plant, the estimation models and the reference model are $x(0) = \hat{x}_k(0) = x_m(0) = [\pi/3\ 0]^{\mathrm{T}},\ \forall k$.

The results of the proposed control scheme for both cases are depicted in Figs. 3 and 4. In Figs. 3(a) and (b) and 4(a and b), the states of the real plant (34) and the reference model (3) are given. The dashed line in the above figures depicts the reference model's state. In Figs. 3(c) and 4(c), the respective control signals are depicted. In Figs. 3(d) and 4(d), the switching sequence of the controllers is given and finally Figs. 3(e and f) and 4(e and f) zoom out crucial parts of the previous figures. We choose the initial controller randomly, due to the fact that the initial $J_k(t)$ is equal to zero for all the systems, and thus $\mathcal{C}_1$ is the controller that provides the control signal to the system in both cases. In the first case where $\sigma = 0.05$ and $r^k = 1$, the controller stabilizes the system and leads it to the reference model's state after about 10 s, with some small oscillations around the vertical position where the state $x_1 = 0$ and some larger oscillations for x_2 as it can be seen in Fig. 3(a and b). A small burst in the control signal can be noticed at about 1.1 s where there is change in the dominant controller. Five controllers ($\mathcal{C}_1$, $\mathcal{C}_4$, $\mathcal{C}_8$, $\mathcal{C}_{10}$, $\mathcal{C}_2$) are used and finally the controller $\mathcal{C}_2$ undertakes the control of the system (Fig. 3(c–f)). The T_{min} and hysteresis h tools along with the performance index (6) ensure smoothness and stability for the signals. In the second case where $\sigma = 0.08$ and $r^k = 2$, i.e. we increase the values of the design parameters, the results are better, concerning the performance of the controller. The states x_1, x_2 follow the states of the reference model at about four seconds in a smoothly way. The control signal has a small burst at about 0.5 s and takes smaller values than that of the first case. Three controllers ($\mathcal{C}_1$, $\mathcal{C}_4$, $\mathcal{C}_9$ are used and finally the controller $\mathcal{C}_4$ undertakes the control of the system (Fig. 4(c–f)). Consequently the simulation results indicate that the values of σ, r^k, d (the last one is reduced only if the precision of the fuzzy model is increased) play an important role to the performance of the system. From (22) it is obvious that an increase in σ or r^k implies that the requirements for the negativity of $\dot{V}_k$ are more relaxed and thus the stable region becomes larger. This fact offers a better performance to the controlled system. On the other hand, very large values for the leakage term σ or the learning rates r^k may lead to instabilities. The role of the designer is to define the best possible values for these crucial parameters in order to achieve a satisfactory control result.

7. Conclusions

A new control scheme which incorporates robustness in a multiple T–S adaptive models based control architecture is proposed in this paper. The necessity of using fuzzy control theory in order to control nonlinear systems is usually associated with the appearance of modeling errors caused by the process of fuzzy models formulation. In addition, when adaptive control techniques, which are used due to the unknown parameters of the nonlinear systems, coexist with the modeling errors, it is very possible for the system to encounter instability problems. The significance of this issue led us to design robust adaptive laws for the identification T–S models which are based on the σ-modification method. These adaptive laws which are extracted from a stability analysis ensure that the system will track the state of given reference model and that the control signal will stay away from singularities. Due to the use of the leakage term σ, the identification and modeling errors are proven to be $(d^2\|P\|^2 + \sigma)$-small in the mean square sense. Also, the more precise is the fuzzy modeling procedure the less is the upper bound of the modeling error. By changing the values of the design constants r^k, σ, one can modify the stability region of the system and thus the performance of the proposed controller. The theoretical results are confirmed by simulations which use a well known benchmark nonlinear plant, whose parameters are assumed to be unknown.

Future research will focus on (i) a new multiple models architecture which will be composed of different kinds of identification models and (ii) imposing new methods for the initialization of the parameters of the T–S identification models, in order to reduce the computational burden and improve the control performance.

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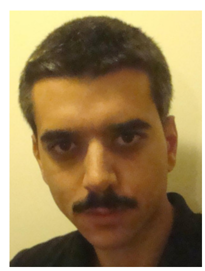

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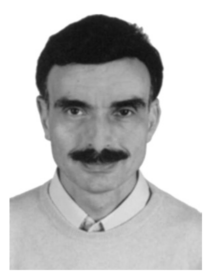

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