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Maximal covering location problem (MCLP) with fuzzy travel times

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ABSTRACT

This paper presents a fuzzy maximal covering location problem (FMCLP) in which travel time between any pair of nodes is considered to be a fuzzy variable. A fuzzy expected value maximization model is designed for such a problem. Moreover, a hybrid algorithm of fuzzy simulation and simulated annealing (SA) is used to solve FMCLP. Some numerical examples are presented, solved and analyzed to show the performance of the proposed algorithm. The results show that the proposed SA finds solutions with objective values no worse than 1.35% below the optimal solution. Furthermore, the simulation-embedded simulated annealing is robust in finding solutions.

1. Introduction and problem description

The term location analysis refers to the modeling, formulation, and solution of a class of problems that can best be described as siting facilities in some given space. The expressions deployment, positioning, and siting are frequently used as synonyms (ReVelle & Eiselt, 2005). Applications of location problems range from gas stations and fast food outlets to landfills and power plants. One of the traditional location problems, which has been well studied since its introduction, is the covering location problem. In a covering location problem, one seeks a solution to cover a subset of customers considering one or more objectives. The covering location problem is often categorized as location set covering problem (LSCP) and maximal covering location problem (MCLP). In a standard MCLP, one seeks location of a number of facilities on a network in such a way that the covered population is maximized. A population is covered if at least one facility is located within a pre-defined distance of it. This pre-defined distance is often called coverage radius. The choice of this distance has a vital role and affects the optimal solution of the problem to a great extent. MCLP is of paramount importance in practice to locate many service facilities such as schools, parks, hospitals and emergency units. The problem was first introduced by Church and ReVelle (1974) on a network and since then, various extensions to the original problem have been made. Normally, MCLP is considered whenever there are insufficient resources or budget to cover the demand of all the nodes. Therefore, the decision maker determines a fixed budget/resource to cover the demands as much as possible.

Uncertainty is ubiquitous in reality and this makes description of many parameters difficult or even impossible. Some examples of uncertainty in real world problems are the estimation of customer demands, travel times, inflation rate, etc. In this paper, we assume that there is not precise information concerning travel times on the arcs of network. In addition, there is not enough data to be used in order to find a statistical distribution. Therefore, demands are estimated based on the knowledge of experts. For example, experts may state their ideas as "about 40 units per day", "between 10 and 20 units weekly", etc. Fuzzy variables are used in these cases to deal with this kind of uncertainty. Travel time is an instance of variables which are difficult to estimate using traditional methods such as probabilistic methods. In most of the cases, there is not enough data to be used to fit a probability distribution of travel times between nodes or probabilistic approach is too costly to be used. On the other hand, based on the expert's judgment; one can easily estimate transportation times. Therefore, we use fuzzy theory in order to model and solve our problem.

In this paper, we present a fuzzy version of MCLP (FMCLP) where travel times are considered to be fuzzy variables. A model based on credibility theory is presented and a hybrid intelligent algorithm is proposed in order to solve this problem. The hybrid algorithm is comprised of a simulation embedded within a simulated annealing procedure.

The rest of the paper is organized as follows: First, a concise literature review of covering problems and related issues is presented. Then, fuzzy variables and basics of credibility theory are discussed. Section 4 is dedicated to description of our problem. The proposed solution algorithm is presented in Section 5 and a numerical example appears in Section 6. Finally, conclusions and outlooks for potential future research are given in Section 7.

2. Literature review

In the literature, there are several methods to solve MCLP including exact, heuristic, and metaheuristic methods. The first

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wave of published location models are deterministic and, thus, do not account for uncertainties, which may exist in covering problems such as the probability that a particular ambulance might be busy at a given time or the fuzzy demands in a network.

The literature of covering models is too diverse to be completely studied in this paper. Hereby, just a few interesting papers are addressed. There has been numerous papers and technical manuscripts related to covering problem since its introduction. Resende (1998) studied the performance of GRASP in solving the maximal covering problem. Qu and Weng (2009) studied the problem of multiple allocation hub maximal covering problem. In their work, it is assumed that to reach a destination, it is mandatory to pass one or two hubs in a limited time, cost or distance. The problem in Qu and Weng (2009) is to locate p hubs so that the serviced flows are maximized. The proposed method to solve such a problem is an evolutionary approach based on path relinking. De Assis Correa, Lorena, and Ribeiro (2009) analyzed the probabilistic version of MCLP in which there is one server per center. They used a combination of column generation and covering graph approaches in order to solve this problem. Araz, Selim, and Ozkarahan (2007) considered a multi-objective fuzzy goal programming for covering-based emergency vehicle location model. The objective of Araz et al. (2007) is to maximize the population with backup coverage and increasing the service level by minimizing the total travel distance from locations at a distance larger than a pre-specified distance standard. Berman and Krass (2002) considered partial coverage of customers for a general class of MCLP. The same problem was considered by Karasakal and Karasakal (2004) where they used a Lagrangean Relaxation to solve the problem. To solve a covering problem, Murawski and Church (2009)) proposed a model to assume that the established facilities are fixed and the accessibility of demand nodes are to be improved. Their model is called maximal covering network improvement problem which was formulated as an integer-linear programming problem and a case study in Ghana is surveyed. Shavandi and Mahlooji (2006)) presented a fuzzy location–allocation model for congested systems and called it fuzzy queuing maximal covering location–allocation. They used a genetic algorithm to solve the problem. Batanovic, Petrovic, and Petrovic (2009) suggested maximal covering location problems in networks with uncertainty. They studied problems with equal importance of demand nodes, relative deterministic weights of demand nodes, and linguistic terms for weights of demands. In addition, they proposed suitable algorithms to solve these models. ReVelle, Eiselt, and Daskin (2008) reviewed papers relating median, center and covering problems and the contributions made in these location models. Interested readers may refer to this paper for a thorough review of covering problems.

Table 1 shows some maximal covering location problems which have been studied in the literature. It can be observed that most of the literature is devoted to deterministic cases and heuristics/metaheuristics have been a major tool to solve these problems so far.

Considering the literature review, it is clear that there is no work on maximal covering location problems with fuzzy travel times. Based on such a finding and the fact that uncertainty is present in most of the real world problems, we propose using fuzzy logic in modeling the problem. This paper makes the following significant contributions to the literature. We model MCLP in a fuzzy environment with expected value maximization programming. Moreover, the model is solved using credibility theory and simulated annealing. Finally, a new approach is proposed in order to validate such a problem.

3. Fuzzy variable

The term "Fuzzy variable" was coined by Kaufmann (1975) and then discussed in Zadeh (1975, 1978) and Nahmias (1978). Possibility theory was proposed by Zadeh (1978) and its extensions and developments were followed by Dubois and Prade in many publication such as Dubois and Prade (1988). A modification to possibility theory which is called credibility theory was founded by Liu (2008) and recently studied by many scholars all over the world. Since a fuzzy version of covering problem in credibility space will be considered in this paper, we present a brief introduction to basic concepts and definitions as follows:

Definition 1 (Nahmias (1978)). Let Θ be a nonempty set, and $P(\Theta)$ as the power set of Θ, for each $A \in P(\Theta)$, there is a nonnegative number *Pos(A)*, called as possibility, such that

(i) $\text{Pos}\{\emptyset\} = 0$.
(ii) $Pos\{\Theta\} = 1$.
(iii) $Pos\{U_k A_k\} = sup_k\ Pos\{A_k\}$ for any arbitrary collection $\{A_k\}$ in $P\{\Theta\}$.

The triplet $(\Theta, P(\Theta), Pos)$ is called a possibility space and *Pos* is a possibility measure.

Definition 2 ((Zheng and Liu, 2006)). A fuzzy variable is defined as a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the real line $\mathcal{R}$.

Let us clarify the difference between a fuzzy variable and a random variable. Suppose that a fair coin is tossed, we know that the probability of each side is equal ($Pr(\theta_1) = Pr(\theta_2) = 0.5$). In such a case, one may define a random variable as:

$$\psi(\theta) = \begin{cases} 0 & \text{if } \theta = \theta_1, \\ 1 & \text{if } \theta = \theta_2. \end{cases} \tag{1}$$

If the problem is defined in a credibility space (will be discussed later) and a measure is used called credibility instead of probability ($Cr(\theta_1) = Cr(\theta_2) = 0.5$), then a fuzzy variable can be defined as follows:

$$\xi(\theta) = \begin{cases} 0 & \text{if } \theta = \theta_1 \\ 1 & \text{if } \theta = \theta_2 \end{cases} \tag{2}$$

It is easily recognized that both variables are a mapping to the set of real numbers from a space with known properties and axioms. It should be noted that while probability theory is used to study the behavior of random variables, credibility theory is used in order to study fuzzy variables.

Definition 3 ((Zheng and Liu, 2006)). Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), Pos)$. Then its membership function is derived from the possibility measure *Pos* by:

$$\mu(x) = Pos\{\theta \in \Theta | \xi(\theta) = x\} x \in \mathcal{R}. \tag{3}$$

Definition 4 ((Zheng and Liu, 2006)). Let $(\Theta, P(\Theta), Pos)$ be a possibility space, and A be a set in $P(\Theta)$. Then the necessity measure of A is defined by: $Nec\{A\} = 1 - Pos\{A^c\}$.

Definition 5 ((Zheng and Liu, 2006)). Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), Pos)$. Then the set

$$\xi_\alpha = \{\xi(\theta) | \theta \in \Theta,\ Pos\{\theta\} \geqslant \alpha\}. \tag{4}$$

Is called the α-level set of ξ.

Definition 6 (*(Zheng and Liu, 2006)*). Let $(\Theta, P(\Theta), Pos)$ be a possibility space, and A be a set in $P(\Theta)$. Then the credibility measure of A is defined by: $Cr\{A\} = \frac{1}{2}(Pos\{A\} + Nec\{A\})$ which is a self-dual measure. (Possibility and Necessity measures lack the self-duality property.)

Table 1
The literature review matrix.

Paper	Parameter type			Solution type			Solution procedure
	Deterministic	Probabilistic	Fuzzy	Exact	Heuristic	Metaheuristic	
Galvao, Espejo, and Boffey (2000)	√				√		LR
Aytug and Saydam (2002)		√				√	GA
Berman and Krass (2002)	√				√		LR
Espejo, Galvao, and Boffey (2003)	√				√		LR
Karasakal and Karasakal (2004)	√				√		LR
Barbas and Marin (2004)	√				√		DC
Younies and Wesolowski (2004)	√				√		N/A
Shavandi and Mahlooji (2006)		√				√	GA
Araz et al. (2007)			√	√			N/A
Curtin et al. (2007)	√			√			N/A
Berman and Huang (2008)	√					√	TS/LR
Plastria and Vanhaverbeke (2008)	√				√		IE
ReVelle et al. (2008)	√					√	HC
Batanovic et al. (2009)			√		√		N/A
Canbolat and Massow (2009)	√					√	SA
Murawski and Church (2009)	√			√			N/A
Qu & Weng (2009)	√					√	PR
Ratick, Osleeb, and Hozumi (2009)	√			√			N/A

IE, intelligent enumeration; PR, path relinking; SA, simulated annealing; HC, heuristic concentration; DC, decomposition; LR, Lagrangean relaxation; CG, column generation; GA, genetic algorithm; TS, tabu search.

If the membership function $\mu(u)$ of ξ is given as μ (u is an event), then the possibility, necessity, credibility of the fuzzy event $\{\xi \geqslant r\}$ can be represented by:

$$Pos\{\xi \geqslant r\} = \sup_{u \geqslant r} \mu(u), \tag{5}$$

$$Nec\{\xi \geqslant r\} = 1 - \sup_{u<r} \mu(u), \tag{6}$$

$$Cr\{\xi \geqslant r\} = \frac{1}{2}(Pos\{\xi \geqslant r\} + Nec\{\xi \geqslant r\}). \tag{7}$$

Considering Eq. (7), the credibility of a fuzzy event is defined as the average of its possibility and necessity. The credibility measure is self-dual. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. Now let us consider an example of a trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, r_4)$. From the definitions of possibility, necessity and credibility, it is easy to obtain:

$$Pos\{\xi \geqslant r\} = \begin{cases} 1 & \text{if } r \leqslant r_3 \\ \frac{r_4 - r}{r_4 - r_3} & \text{if } r_3 \leqslant r \leqslant r_4 \\ 0 & \text{if } r \geqslant r_4, \end{cases} \tag{8}$$

$$Nec\{\xi \geqslant r\} = \begin{cases} 1 & \text{if } r \leqslant r_1 \\ \frac{r_2 - r}{r_2 - r_1} & \text{if } r_1 \leqslant r \leqslant r_2 \\ 0 & \text{if } r \geqslant r_2, \end{cases} \tag{9}$$

$$Cr\{\xi \geqslant r\} = \begin{cases} 1 & \text{if } r \leqslant r_1 \\ \frac{2r_2 - r_1 - r}{2(r_2 - r_1)} & \text{if } r_1 \leqslant r \leqslant r_2 \\ \frac{1}{2} & \text{if } r_2 \leqslant r \leqslant r_3 \\ \frac{r_4 - r}{2(r_4 - r_3)} & \text{if } r_3 \leqslant r \leqslant r_4 \\ 0 & \text{if } r \geqslant r_4. \end{cases} \tag{10}$$

4. The fuzzy maximal covering location problem (FMCLP)

Assume that there is a network $G = (N, A)$ where N and A represent nodes and arcs respectively. There is a weight associated with each node which represents its demand. The travel times on the arcs of the network are uncertain which are presented as fuzzy variables. Before introducing the problem formulation and our solution approach, decision variables and parameters are presented as follows:

Problem parameters

- h_j Demand of node j (the weight of jth node)
- P Number of facilities to be located
- t_{ij} Travel time between nodes i and j
- R Coverage radius

$$a_{ij} = \begin{cases} 1 & \text{if node } j \text{ is covered by the facility in node } i \text{ or } t_{ij} \leqslant R \\ 0 & \text{otherwise} \end{cases}$$

Decision variables

$$X_j = \begin{cases} 1 & \text{if node } j \text{ is covered by a facility} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_i = \begin{cases} 1 & \text{if a facility is established in node } i \\ 0 & \text{otherwise} \end{cases}$$

Now the problem of fuzzy MCLP (MCLP) could be stated as follows:

$$Max\ E\left(\sum_j h_j X_j\right), \tag{11}$$

$$\sum_i a_{ij} Y_i - X_j \geqslant 0 \quad \forall j, \tag{12}$$

$$\sum_i Y_i = P, \tag{13}$$

$$X_j, Y_{ij} \in \{0, 1\}. \tag{14}$$

Objective function (11) states that the total value of covered demand is to be maximized. Constraint (12) states that the demand of node j is covered if there is at least one located facility with a distance less than the coverage radius. Constraint (13) holds that there are P facilities to be located. Finally, constraint (14) imposes binary restriction on decision variables.

Solution of large instances of the maximal covering location problem with high percentage coverage is cumbersome using exact methods (ReVelle, Scholssberg, & Williams, 2008). Exact methods are handicapped to render an optimal solution for all instances of MCLP within a reasonable amount of time. Therefore, using heuristic or metaheuristic methods to solve MCLP is reasonable. There are exact methods such as Branch and Bound which do not search the solution space exhaustively. Despite this fact, due to the rapid increase in the number of possible solutions of MCLP, even these methods are unfit to solve larger instances.

5. Proposed solution algorithm

Although exact methods are capable of solving MCLPs of small and medium size, our experiments show that runtimes needed to solve MCLP instances of larger size balloon to as much as more than 24 h. This shows that exact methods are unable to solve real-world problems of MCLP and one may use heuristics or metaheuristics. In this section of the paper, we will propose and elaborate a simulated annealing approach to solve MCLP. We will show how well this procedure works on the crisp version of the problem. Then, a fuzzy simulation will be embedded within simulated annealing to solve the fuzzy version of MCLP.

5.1. Overall structure of the solution procedure

In this paper, a simulation-embedded simulated annealing (SA) is used to solve FMCLP. The simulation is embedded within SA in order to estimate the expected value of the covered demand. Moreover, SA is utilized to search the solution space effectively. Arostegui, Kadipasaoglu, and Khumawala (2006) evaluated using different heuristics in different location problems. They found that although Tabu Search (TS) shows the best performance, SA has shown good performance for many location problems. In this paper, SA shows promising results to solve FMCLP.

5.2. Fitness evaluation and simulating the expected value of a solution

Eq. (12) shows that the value of X is determined by the value of Y. Since the value of Y depends on the travel time between the demand node and the facility, it can easily be observed that X may be considered as a fuzzy variable. Therefore, to determine the expected value of (11), one might use fuzzy simulation. In order to solve a fuzzy programming model like our model which is generally stated as $U : x \to E[f(x, \xi), g_j(x) j = 1, 2, \ldots, m]$, Liu (2008) presented the algorithm to estimate the expected value as shown in Fig. 1.

From now on, we will call the above algorithm as simulate. The algorithm to solve FMCLP is shown in Fig. 2:

5.3. Simulated annealing

5.3.1. General framework

Metaheuristics are often categorized into two distinct groups: population-based and local search methods. Simulated annealing (SA) is a local search procedure that is capable of searching the space stochastically and tries to escape from being trapped in local minima. SA avoids local minima by accepting worse solutions during search with a monotonically decreasing probability. It has been applied to various combinatorial optimization problems as well as real world problems. Vehicle routing, scheduling and facility location are some problems in which SA has shown its ability. SA was first introduced by Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and popularized by Kirkpatrick Gelatt and Vecchi (1983).

There is an analogy between SA and the annealing process used in metallurgical industry. SA mimics the annealing process starting from an initial solution. In each iteration of SA, the algorithm looks for a new solution in a neighborhood of the current solution. Then, the fitness of the new solution is compared with that of the current solution in order to determine if an improvement has attained. If an improvement is attained, the new solution becomes the current solution. If the solution has a worse fitness, it is accepted as the current solution with a probability determined using the Boltzmann function Exp. $(-\Delta/kT)$, where k is the predetermined constant and T is the current temperature. A chance of acceptance is given to worse solutions in order to avoid restricting moves to only those leading to better solutions. This mechanism avoids premature convergence (being trapped in local minima) to a great extent.

In the following sections, the proposed SA of this paper will be discussed in detail, including solution representation, neighborhood generation, fitness evaluation and tuning SA parameters.

5.3.2. Encoding scheme and initial solution generation

An effective encoding scheme has significant impact on the performance of SA. A possibility to encode the solutions of our problem is to use binary representation to show establishment of facilities in candidate nodes. The procedures of encoding and decoding a candidate solution are illustrated by applying them to an example. Consider a problem with six candidate nodes to establish a facility in which two facilities are to be located. A feasible solution may look like [1 0 0 0 1 0] which shows that two facilities are to be located at 1st and 5th nodes. It should be noted that using the solution vector, travel times matrix and covering radius, one can easily find the set of covered nodes.

5.3.3. Cooling schedule and stopping criterion

Generally, a few cooling schedules are used such as linearly, exponentially or hyperbolic cooling schedules. The acceptance probability of worse solutions in SA is highly dependent on the cooling schedule. Therefore, how to update temperatures plays a pivotal role in the quality of solutions in simulated annealing.

In this paper, we used the methodology proposed in Crama and Schyns (2003). We set the initial temperature in a way that during the first L steps of the algorithm, there is a pre-defined acceptance probability for moves that lead to worse solutions. This probability should be relatively high and is represented as χ. We set the value of χ to be 0.8 and run the algorithm for 100 steps without rejecting any moves. Then the average increase of the objective function over this phase is computed and shown by Δ. Finally, T_0 is set equal to $T_0 = \frac{-\Delta}{\ln \chi}$ where ln is the logarithm operand. Furthermore, the scale factor equals 0.95. In this paper, the algorithm stops whenever the current temperature reaches T_0.

5.3.4. Neighborhood structure

In order to search for better solutions, we define the set $N(X)$ to be the set of solutions neighboring a solution X. In each iteration, the next solution Y is generated from $N(X)$ by a 2-opt, reorder or shuffle move as follows. These moves guarantee the feasibility of the generated neighbors by fixing the number of located facilities. A 2-opt move is carried out when the values of two elements in a solution is substituted with each other. In our problem, the number of open facilities must be fixed. Therefore, a 2-opt move always substitutes two elements with different values. To show the performance of the encoding scheme, take the example in Fig. 3. It

Step 1. Set $e = 0$.
Step 2. Randomly generate θ_k from the credibility space (Θ, P, Cr), write $\nu_k = (2Cr\{\theta_k\}) \wedge 1$ and produce $\xi_k = \xi(\theta_k), k = 1,2,\ldots,N$ respectively. Equivalently, randomly generate ξ_k and write $\nu_k = \mu(\xi_k)$ for k = 1, 2, …, N, where μ is the membership function of ξ.
Step 3. Set two numbers $a = f(x,\xi_1) \wedge f(x,\xi_2) \wedge \ldots \wedge f(x,\xi_N)$ and $b = f(x,\xi_1) \vee f(x,\xi_2) \vee \ldots \vee f(x,\xi_N)$.
Step 4. Randomly generate r from [a, b].
Step 5. If $r \geq 0$, then $e \leftarrow e - Cr\{f(x,\xi \geq r)\}$
Step 6. If $r < 0$, then $e \leftarrow e - Cr\{f(x,\xi : r)\}$
Step 7. Repeat the fourth to sixth steps for N times.
Step 8. $U_1(x) = a \vee 0 + b \wedge 0 + e.(b - a)/N$

Fig. 1. The outline of the simulation based on credibility theory.

```
% Get the values of T0: Initial temperature; α: The scale factor, MaxItr: Number of iterations
t=T0
x=initialize a solution
xbest=x
fbest=inf
while(stopping criteria is not met)
        for Itr=1:MaxItr
                s=Local Search(x)
                f(s)=Simulate(x)
                if(f(s)>f(x))
                        x=s
                        fbest=f(x)
                        xbest=x
                else
                        if(random<=exp(-(f(x)-f(s))/t)
                        x=s
                end if
                t=αT0
        end for
end while
```

Fig. 2. The outline of our simulation-embedded simulated annealing.

is shown that the facility in node 3 is closed and a facility is opened in node 6. Shuffle is associated with flipping the solution from a random position. Fig. 3 shows the performance of shuffle on a sample solution. Finally, in a reorder move, the elements of l consecutive bits are reordered randomly as shown in Fig. 3. It should be noted that while 2-opt and reorder moves are used to intensify the search, in order to diversify the search, one may use shuffle.

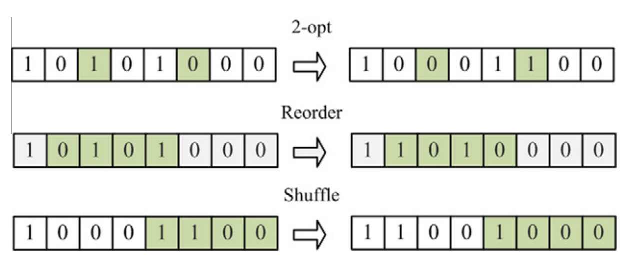

Fig. 3. Neighborhood search structures.

6. Numerical examples

6.1. Test problems

To generate the test problems, we have used the following procedure: The locations of the nodes in the test problems are randomly generated using a uniform distribution between 0 and 30 for x and y-coordinates. The distances between the nodes are then defined as their Euclidean distances. Populations on the nodes are randomly generated using a uniform distribution between [0, 100]. This procedure is used to generate sets of crisp problems with 50, 100, 200, 500 and 900 nodes. Moreover, travel times are generated randomly as a function of the distance between nodes. In other words, travel times are generated as triangular fuzzy variables which are represented as (a, b, c). The first, second and third parameters of these fuzzy variables are set to be equal to the distance between the two nodes with some noise. These noises are set to be 10, 30 and 50 for a, b, c, respectively. It is remarkable that triangle inequality do not necessarily hold for our problem due to the random noises added to the travel times.

6.2. Computer specifications

Both fuzzy simulation and simulated annealing were programmed using MatLab 2009a and run on a high-level computer system. In addition, LINGO 8 was used in order to solve the crisp version of the problem on a computer with the same specification.

6.3. Results, validation, and discussions

To validate the results, we have implemented a couple of analysis. First, using the crisp version of the problem (with deterministic travel times), we solved problems using various combinations of SA parameters. Table 2 shows various levels of SA used in this step. Using each of the 12 combinations, the problem was solved 10 times and best, average, and worst fitness are reported in Table 3. The results show that the proposed SA performs well and the best results are obtained using more runs in each temperature, cooling the schedule slowly, and using an exponential type of scheduling. Although these results were almost expected, even before running the algorithm, we have used these results to validate our proposed algorithm.

Then, to assess the performance of SA from another aspect, the crisp version of the problem was coded using LINGO 8 and the same problem was solved with the proposed SA. Table 4 depicts that although the proposed SA was unable to find optimal solutions in larger instances, there are only negligible gaps compared to optimal solutions. Furthermore, runtimes of SA are very small. Therefore, SA may be used to get solutions in cases where exact solutions are unable to reach a solution. It should be noted that in all of the cases considered in Table 4, the coverage radius equals 6.

Moreover, we show how robust our algorithm works using various SA parameters. The results of the numerical example are compared in Table 5 using various settings for our hybrid algorithm. To compare these results, an index is used called error ratio (ER) which corresponds with the last three columns of Table 5. The error ratio is calculated as:

$$\text{Error ratio} = \frac{\text{Fitness} - \text{Fitness}^*}{\text{Fitness}^*} \times 100\%, \qquad (15)$$

Table 2
Various settings to solve the problem with simulated annealing.

Factor	Types
Cooling schedule	(a) Linear (b) Exponential
Number of iterations per temperature	(a) 10 (b) 20 (c) 50
Scale factor	(a) Rapid (100 iterations to reach the termination temperature) (b) Slow (200 iterations to reach the termination temperature)

Table 3
Fitness results in 10 runs (coverage percentage).

Cooling schedule	Linear						Exponential					
Num of iterations	10		20		50		10		20		50	
Cooling rate	Slow	Rapid	Slow	Rapid	Slow	Rapid	Slow	Rapid	Slow	Rapid	Slow	Rapid
Best	0.4049	0.3848	0.3697	0.3802	0.393	0.384	0.372	0.374	0.385	0.410	0.443	0.365
Average	0.3857	0.3687	0.3562	0.3643	0.390	0.375	0.361	0.351	0.372	0.355	0.399	0.354
Worst	0.3729	0.3377	0.3478	0.3538	0.371	0.365	0.342	0.309	0.358	0.323	0.384	0.338

Table 4
Comparison of solutions obtained from exact solution and simulated annealing.

# of Nodes	P^*	LINGO		Simulated annealing				Gap (%)	Gap (%)	Gap (%)
		Optimal solution	Time (s)	Worst fitness	Average fitness	Best fitness	Time (s)			
50	1	417.5	<1	417.5	417.5	417.5	1	0.00	0.00	0.00
50	2	826.7	<1	826.7	826.7	826.7	1	0.00	0.00	0.00
100	2	1777.5	<1	1777.5	1777.5	1777.5	2	0.00	0.00	0.00
100	5	3298.9	<1	3298.9	3298.9	3298.9	2	0.00	0.00	0.00
200	3	5710.3	1	5710.3	5710.3	5710.3	5	0.00	0.00	0.00
200	8	10,325.7	1	10325.7	10325.7	10325.7	5	0.00	0.00	0.00
500	10	24,717.9	30	24498.7	24536.1	24717.9	69	0.89	0.74	0.00
500	15	25,068.7	104	25068.7	25068.7	25068.7	71	0.00	0.00	0.00
900	10	44,793.4	299	44190.1	44413.7	44685.1	263	1.35	0.85	0.24
900	15	45,003.2	207	45003.2	45003.2	45003.2	251	0.00	0.00	0.00
								Worst	Average	Best

* P: Number of facilities to locate.

Table 5
Comparing solutions of fuzzy version using various settings.

CS	NIT	NTB	Fitness			Error ratio (%)		
			Best	Average	Worst	Best	Average	Worst
Linear	20	100	0.7362	0.7362	0.7362	0.57	0.57	0.57
Linear	20	200	0.7398	0.7381	0.7362	0.08	0.31	0.57
Linear	50	100	0.7404	0.7401	0.7401	0.00	0.04	0.04
Linear	50	200	0.7404	0.7399	0.7395	0.00	0.07	0.12
Exponential	20	100	0.7391	0.7389	0.7388	0.18	0.20	0.22
Exponential	20	200	0.7398	0.7391	0.7377	0.08	0.18	0.36
Exponential	50	100	0.7398	0.7395	0.7362	0.08	0.12	0.57
Exponential	50	200	0.7398	0.7398	0.7398	0.08	0.08	0.08

CS, cooling schedule; NIT, number of iterations per temperature; NTB, number of temperatures between T_0 and T_f; Coverage radius = 15; P (number of facilities to locate) = 2.

where, Fitness* is the best fitness found among the runs. It is shown that ER does not exceed 0.57% using various settings of solution procedure. Therefore, the algorithm is stable and the proposed approach is effective to solve the problem considered in this paper.

7. Conclusion and future research areas

Locating facilities on a network to cover demands fully or partially has been a challenging problem for many years. While a large number of publications concerning covering problems are available in which parameters are considered to be deterministic, there are few papers discussing stochastic or fuzzy covering problems. In this paper, we have attempted to model and solve a maximal covering location problem (MCLP) in which travel times are considered to be fuzzy variables. A fuzzy formulation of the problem was presented and fuzzy simulation was used to estimate the expected coverage. To solve the problem, the fuzzy simulation was embedded inside a simulated annealing approach and the performance of the proposed methodology was shown using a set of test problems.

We believe that our model is a move forward to model and solve covering location problems in uncertain and especially fuzzy environments. This paper contributes to covering location literature in the following respects: (a) A fuzzy chance constrained programming model of the problem is given. (b) A simulation-embedded simulated annealing is proposed to solve the problem. (c) Proposal of a validation approach for the problem.

It may be possible to consider the same problem for other location problems such as *p*-median or center. Moreover, there is the possibility of using other solution methods such as genetic algorithm to solve the same model. Finally, the problem may be considered in an environment where fuzzy and random variables coexist. For example, demands may be considered to be random variables.

References

Araz, C., Selim, H., & Ozkarahan, I. (2007). A fuzzy multi-objective covering-based vehicle location model for emergency services. *Computers and Operations Research, 34*, 705–726.

Arostegui, M. A., Kadipasaoglu, S. N., & Khumawala, B. M. (2006). An empirical comparison of tabu search, simulated annealing, and genetic algorithms for facilities location problems. *International Journal of Production Economics, 103*, 742–754.

Aytug, H., & Saydam, C. (2002). Solving large-scale maximum expected covering location problems by genetic algorithms: A comparative study. *European Journal of Operational Research, 141*, 480–494.

Barbas, J., & Marin, A. (2004). Maximal covering code multiplexing access telecommunication networks. *European Journal of Operational Research, 159*, 219–238.

Batanovic, V., Petrovic, D., & Petrovic, R. (2009). Fuzzy logic based algorithms for maximum covering location problems. *Information Sciences, 179*, 120–129.

Berman, O., & Huang, R. (2008). The minimum weighted covering location problem with distance constraints. *Computers and Operations Research, 35*, 356–372.

Berman, O., & Krass, D. (2002). The generalized maximal covering location problem. *Computers and Operations Research, 29*, 563–581.

Canbolat, M. S., & Massow, M. V. (2009). Planar maximal covering with ellipses. *Computers and Industrial Engineering, 57*, 201–208.

Church, R., & ReVelle, C. (1974). The maximal covering location problem. *Papers of the Regional Science Association, 32*, 101–118.

Crama, Y., & Schyns, M. (2003). Simulated annealing for complex portfolio selection problems. *European Journal of Operational Research, 150*, 546–571.

Curtin, K. M., Hayslett, K., & Qiu, F. (2007). Determining optimal police patrol areas with maximal covering and backup covering location models. *Networks and Spatial Economics*. doi:10.1007/s11067-007-9035-6.

De Assis Correa, F., Lorena, L. A. N., & Ribeiro, G. M. (2009). A decomposition approach for the probabilistic maximal covering location–allocation problem. *Computers & Operations Research, 36*, 2729–2739.

Dubois, D., & Prade, H. (1988). *Possibility theory: An approach to computerized processing of uncertainty*. New York: Plenum.

Espejo, L. G. A., Galvao, R. D., & Boffey, B. (2003). Dual-based heuristics for a hierarchical covering location problem. *Computers and Operations Research, 30*, 165–180.

Galvao, R. D., Espejo, L. G. A., & Boffey, B. (2000). A comparison of Lagrangean and surrogate relaxations for the maximal covering location problem. *European Journal of Operational Research, 124*, 377–389.

Karasakal, O., & Karasakal, E. K. (2004). A maximal covering location model in the presence of partial coverage. *Computers and Operations Research, 31*, 1515–1526.

Kaufmann, A. (1975). *Introduction to the theory of fuzzy subsets* (Vol. I). New York: Academic Press.

Kirkpatrick, S., Gelatt, C. D., Jr., & Vecchi, M. P. (1983). Optimization by simulated annealing. *Science, 220*, 671–680.

Liu, B. (2008). *Uncertainty theory* (3rd ed.). <http://orsc.edu.cn/liu/ut.pdf>.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equations of state calculations by fast computing machines. *Journal of Chemical Physics, 21*, 1087–1092.

Murawski, L., & Church, R. L. (2009). Improving accessibility to rural health services: The maximal covering network improvement problem. *Socio-Economic Planning Sciences, 43*, 102–110.

Nahmias, S. (1978). Fuzzy variables. *Fuzzy Sets and Systems, 1*, 97–110.

Plastria, F., & Vanhaverbeke, L. (2008). Maximal covering location problem with price decision for revenue maximization in a competitive environment. *OR Spectrum*. doi:10.1007/s00291-008-0158-z.

Qu, B., & Weng, K. (2009). Path relinking approach for multiple allocation hub maximal covering problem. *Computers and Mathematics with Applications, 57*, 1890–1894.

Ratick, S. J., Osleeb, J. P., & Hozumi, P. (2009). Application and extension of the Moore and ReVelle hierarchical maximal covering model. *Socio-Economic Planning Sciences, 43*, 92–101.

Resende, M. G. C. (1998). Computing approximate solutions of the maximum covering problem with GRASP. *Journal of Heuristics, 4*, 161–177.

ReVelle, C. S., Eiselt, H. A., & Daskin, M. S. (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research, 184*, 817–848.

ReVelle, C. S., & Eiselt, H. A. (2005). Location analysis: A synthesis and survey. *European Journal of Operational Research, 165*, 1–19.

ReVelle, C., Scholssberg, M., & Williams, J. (2008). Solving the maximal covering location problem with heuristic concentration. *Computers and Operations Research, 35*, 427–435.

Shavandi, H., & Mahlooji, H. (2006). A fuzzy queuing location model with a genetic algorithm for congested systems. *Applied Mathematics and Computation, 181*, 440–456.

Younies, H., & Wesolowski, G. O. (2004). A mixed integer formulation for maximal covering by inclined parallelograms. *European Journal of Operational Research, 159*, 83–94.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences, 8*, 199–251.

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems, 1*, 3–28.

Zheng, Y., & Liu, B. (2006). Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm. *Applied Mathematics and Computation, 176*, 673–683.