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EID-based sliding mode investment policy design for fuzzy stochastic jump financial systems

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ABSTRACT

This paper proposes a sliding mode investment policy design for nonlinear stochastic financial systems which can be represented by the well-known Takagi–Sugeno fuzzy model. When modeling the financial systems, it is more important to consider the unpredictable investment changes and worldwide unpredictable events which can be regarded as external disturbances. The equivalent-input-disturbance (EID) approach combined with sliding mode investment policy design is implemented to reject the unpredictable investment changes for having better investment. Moreover, the Luenberger state observer is constructed for the addressed financial system to estimate the unpredictable investment changes and worldwide unpredictable events. More precisely, a sliding mode investment policy design is developed by solving the obtained linear matrix inequality (LMI)-based constrained algorithm. Finally, the obtained results of the addressed fuzzy stochastic financial system are verified through numerical simulation to show efficiency of the proposed sliding mode investment policy design.

1. Introduction

In our day-to-day life, financial market is more important in which individuals exchange money related securities and commodities at low exchanges costs [1,2]. In order to stabilize the real economy of the companies, investors and governments, it is much essential to analyze the dynamical behavior between the financial market and the real economy. In general, the main objective of the companies, investors and governments are to increase the profit and reduce the risk. Recently, significant attention has been paid by the research communities on studies of financial systems due to complex process of business operations. In general, financial systems are influenced by a several economic factors, such as national and international situation changes, the variable interest rate, wrong economy policy, oil price change and unpredictable investment-environmental changes [3]. Moreover, in practice, many economic factors are not always be deterministic due to unpredictable sudden investments, wars and natural disorder. Therefore, in the dynamics of financial control systems these kind of unknown disturbance factors should be taken into consideration. On the other hand, the disturbance rejections in dynamical control systems can be handled by various design methodologies [4–9]. In particular, the sliding mode control (SMC) is one of the most recognized controller to reject the effects of matched disturbances and the modeling error in the dynamical control system [10–15]. Generally, sliding mode control contains two steps; one is to design the sliding surface in which the system has desired properties such as stability, disturbance rejection capability and tracking ability and the next is to design the discontinuous controller such that the system state reach the sliding surface in the finite time [16–19].

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On the other hand, state observer-based disturbance estimation is a successful approach to deal with both matched and unmatched disturbances by means of equivalent-input-disturbance (EID) approach [20]. EID is the input signal that is produced by inner feedback control loop with utilization of internal model principle and that compensates the effect of external disturbance on the controlled output. The disturbance rejection performance of the EID approach is determined by the frequency of the low-pass filter in the EID estimator [21]. Furthermore, EID input does not affect the performance of the sliding mode controller. According to the above discussion, the EID-based SMC approach improves the control performance of both tracking and disturbance rejection. The EID input based sliding mode control acts as two safety layers of protection of the control system from both matched and unmatched external disturbances. On the other hand, T–S fuzzy system approach is widely used to approximate the complex nonlinear systems (see [22–29] and the reference therein). Specifically, it is used to represent nonlinear system as a weighted sum of some simple linear subsystems and then can be stabilized by a model-based fuzzy control. Motivated by the above discussions, in this paper, an EID-based sliding mode investment policy design is developed for a nonlinear stochastic financial system represented by T–S fuzzy model [30–32]. The main contributions of this paper are highlighted as follows:

(1) A novel sliding mode investment policy design is proposed for a nonlinear fuzzy stochastic financial system based on the EID approach, which can effectively attenuate unpredictable investment changes and worldwide unpredictable events.
(2) The system understudy in this paper contains the Poisson jump process to represent the financial random fluctuations such as war, natural disaster, epidemic disease so that the considered model is more comprehensive and realistic.
(3) The proposed control scheme guarantees the achievement of desired target by actively eliminating the effects of unpredictable sudden investments, wars and natural disorder.

Finally, effectiveness of the proposed EID-based sliding mode investment policy scheme is demonstrated through numerical simulation for the nonlinear stochastic financial systems.

2. Problem formulation and preliminaries

In this section, first a linear model of the nonlinear stochastic jump financial system (SJFS) is approximately presented by using the fuzzy approach and then, an EID-based sliding mode investment policy is introduced for the SJFS. For this purpose, consider a nonlinear SJFS and its dynamics can be represented by the following differential equations [33]:

$$\begin{cases} \mathrm{d}x_1(t) = & (x_3(t) + (x_2(t) - \alpha)x_1(t))\mathrm{d}t + f_1(x(t))\mathrm{d}\mathcal{W}(t) + g_1(x(t))\mathrm{d}\mathcal{N}(t, \eta_k), \\ \mathrm{d}x_2(t) = & (1 - \beta x_2(t) - (x_1(t))^2)\mathrm{d}t + f_2(x(t))\mathrm{d}\mathcal{W}(t) + g_2(x(t))\mathrm{d}\mathcal{N}(t, \eta_k), \\ \mathrm{d}x_3(t) = & (-x_1(t) - \gamma x_3(t))\mathrm{d}t + f_3(x(t))\mathrm{d}\mathcal{W}(t) + g_3(x(t))\mathrm{d}\mathcal{N}(t, \eta_k), \end{cases} \tag{1}$$

where $x_1(t),\ x_2(t)$ and $x_3(t)$ are the interest rate, investment demand and price index, respectively; the positive constants $\alpha,\ \beta$ and γ are the saving amount, per-investment cost and elasticity of demands of commercials, respectively; $x(t) = [x_1(t)\, x_2(t)\, x_3(t)]^T$; $f_i : \mathbb{R}^3 \to \mathbb{R},$ and $g_i : \mathbb{R}^3 \to \mathbb{R},\ (i = 1, 2, 3)$ are nonlinear Borel measurable continuous functions and they satisfy the Lipschitz condition; the stochastic functions $\mathcal{W}(t)$ and $\mathcal{N}(t, \eta_k)$ represent the one-dimensional standard Wiener process and marked Poisson jump process, respectively in which η_k denotes a financial emergencial incident; Moreover, it is well-known that the SJFS (1) is affected by several factors, such as unpredictable investment changes, worldwide unpredictable events and international situation like war or natural disaster, which can be regarded as a stochastic external disturbance, namely $w(t)$.

According to this fact, the controlled system corresponding to the SJFS (1) can be expressed as follows:

$$\mathrm{d}x(t) = (h(x(t)) + Bu(t) + w(t))\mathrm{d}t + f(x(t))\mathrm{d}\mathcal{W}(t) + g(x(t))\mathrm{d}\mathcal{N}(t, \eta_k), \tag{2}$$

where $h(x(t)) = \begin{bmatrix} x_3(t) + (x_2(t) - \alpha)x_1(t) \\ 1 - \beta x_2(t) - (x_1(t))^2 \\ -x_1(t) - \gamma x_3(t) \end{bmatrix}$ and $u(t)$ is the appropriate investment policy to regulate the financial system (1) and B is a constant matrix. For more details about the formulation of stochastic nonlinear jump financial system (2), one can refer the paper [33].

Further, by incorporating the fuzzy model approach, the ith fuzzy rule of the nonlinear stochastic jump diffusion financial system (2) can be expressed by

System rule i: if z^1 is G^{i1} and $\cdots$ and z^g is G^{ig} then

$$\begin{aligned} \mathrm{d}x(t) = & [A_i x(t) + Bu(t) + w(t)]\mathrm{d}t + D_i x(t)\mathrm{d}\mathcal{W}(t) + \sum_{k=1}^{m} E_i(\eta_k)x(t)\mathrm{d}\mathcal{N}(t, \eta_k), \\ y(t) = & C_i x(t), \quad i = 1, 2, \ldots, l, \end{aligned} \tag{3}$$

where l is the number of fuzzy rules; $y(t)$ is the output vector; G^{ij} $(j = 1, 2, \ldots, g)$ are the fuzzy sets; $A_i,\ B,\ C_i,\ D_i$ and $E_i(\eta_k)$ are constant matrices for $k = 1, 2, \ldots, m$; and $z(t) = [z^1(t), \ldots, z^g(t)]$ represents the premise variable of G^{ij}.

The overall inferred fuzzy stochastic jump system can be formulated as

$$\begin{aligned} \mathrm{d}x(t) = & \sum_{i=1}^{l} h_i(z(t)) \left([A_i x(t) + Bu(t) + w(t)]\mathrm{d}t + D_i x(t)\mathrm{d}\mathcal{W}(t) + \sum_{k=1}^{m} E_i(\eta_k)x(t)\mathrm{d}\mathcal{N}(t,\eta_k) \right), \\ y(t) = & \sum_{i=1}^{l} h_i(z(t))C_i x(t), \end{aligned} \tag{4}$$

where $h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{l} w_i(z(t))}$ with $w_i(z(t)) = \prod_{j=1}^{g} G^{ij}(z(t))$. In order to estimate the state and disturbance of the system (4), we construct the following Luenberger state observer:

$$\begin{aligned} \mathrm{d}\hat{x}(t) = & \sum_{i=1}^{l} h_i(z(t)) \left(A_i\hat{x}(t) + Bu^d(t) + L_i(y(t) - \hat{y}(t))\right) \mathrm{d}t, \\ \hat{y}(t) = & \sum_{i=1}^{l} h_i(z(t))C_i\hat{x}(t), \end{aligned} \tag{5}$$

where $\hat{x}(t)$, $\hat{y}(t)$ and L_i are the state, output and gain matrices of the observer, respectively.

Now, the fuzzy investment policy design can be described by

$$u^d(t) = \sum_{i=1}^{l} h_i(z(t))K_i(\hat{x}(t) - x_d(t)), \tag{6}$$

where $x_d(t)$ represents the desired reference state and K_i $(i = 1, 2, \ldots, l)$ are the feedback control gain matrices to be determined.

Also, we consider the integral sliding mode surface for fuzzy system (4) in the following form

$$s(t) = G(\hat{x}(t) - \hat{x}(0)) - G\int_0^t \left(\sum_{i=1}^{l} h_i(z(s))\Big(A_i\hat{x}(s) + BK_i(\hat{x}(s) - x_d(s))\Big)\right) \mathrm{d}s, \tag{7}$$

where G is the real constant matrix. In particular, G is chosen such that the matrix GB is non-singular such that system (4) in the sliding mode is stochastically stable. Taking the time derivative on (7), we have

$$\dot{s}(t) = G(\dot{\hat{x}}(t)) - G\sum_{i=1}^{l} h_i(z(t)) \left(A_i\hat{x}(t) + BK_i(\hat{x}(t) - x_d(t))\right), \tag{8}$$

Moreover, based on the sliding mode control theory, the equivalent control is obtained when $s(t) = 0$ and $\dot{s}(t) = 0$. Then, the equivalent investment policy design for nonlinear stochastic fuzzy financial system can be described by

$$u_{eq}(t) = (GB)^{-1}G\sum_{i=1}^{l} h_i(z(t)) \left(BK_i(\hat{x}(t) - x_d(t)) - L_iC_i(x(t) - \hat{x}(t))\right). \tag{9}$$

Substituting (9) into (5), the observer dynamics under the sliding mode surface (7) can be written as

$$\begin{aligned} \mathrm{d}\hat{x}(t) = & \sum_{i=1}^{l} h_i(z(t)) \left(A_i\hat{x}(t) + K_i(\hat{x}(t) - x_d(t)) + (I - (GB)^{-1}G)L_iC_i(x(t) - \hat{x}(t))\right) \mathrm{d}t, \\ \hat{y}(t) = & \sum_{i=1}^{l} h_i(z(t))C_i\hat{x}(t). \end{aligned} \tag{10}$$

The estimated disturbance can be expressed as in [21] by $\hat{w}(t) = \sum_{i=1}^{l} h_i(x(t))B^{+}L_iC_i(x(t) - \hat{x}(t)) + u^d(t) - u(t)$. In order to select the angular frequency band for the EID estimation, we consider the low-pass filter $F(s)$ such that $|F(j\omega)| \approx 1$, for all $\omega \in [0,\ \omega_r]$, where ω_r is the highest angular frequency. The state-space equation of the low-pass filter $F(s)$ is chosen as

$$\begin{aligned} \mathrm{d}x_F(t) = & \ (A_F x_F(t) + B_F\hat{w}(t))\mathrm{d}t, \\ \tilde{w}(t) = & \ C_F x_F(t). \end{aligned} \tag{11}$$

Incorporating EID disturbance estimation output $\tilde{w}(t)$ with the sliding mode fuzzy investment policy, we can have the following improved investment policy $u(t) = u^d(t) - \tilde{w}(t)$. Take the signals $x_d(t)$ and $w(t)$ as zeros and denote $x_\delta(t) = x(t) - \hat{x}(t)$. Further, by considering augmented state $\psi(t) = [\hat{x}(t)\ x_\delta(t)\ x_F(t)]^T$ and from (4), (10) and (11), the augmented nonlinear stochastic financial system can be expressed as follows:

$$\mathrm{d}\psi(t) = \sum_{i=1}^{l} h_i(z(t)) \left[\mathcal{A}_i\psi(t)\mathrm{d}t + \mathcal{D}_i\psi(t)\mathrm{d}\mathcal{W}(t) + \sum_{k=1}^{m} \mathcal{E}_i(\eta_k)\psi(t)\mathrm{d}\mathcal{N}(t,\eta_k)\right], \tag{12}$$

$$\mathcal{A}_i = \begin{bmatrix} A_i + K_i & (I-(GB)^{-1}G)L_iC_i & 0 \\ & A_i - L_iC_i & -BC_F \\ 0 & B_FB^{+}L_iC_i & A_F + B_FC_F \end{bmatrix}, \ \mathcal{D}_i = \begin{bmatrix} 0 & 0 & 0 \\ D_i & D_i & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{E}_i(\eta_k) = \begin{bmatrix} 0 & 0 & 0 \\ E_i(\eta_k) & E_i(\eta_k) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In order to prove the main result, the following lemma is required.

Lemma 2.1 ([9]). *For any given matrix $C_i = UM_iV \in \mathbb{R}^{q\times n}$ $(q < n)$ with full row rank – that is, $rank(C_i) = q$ and a symmetric matrix P of order n – there exists a matrix $\hat{P}$ of order n satisfying $C_iP = \hat{P}C_i$ if and only if P can be described as $P = V\begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}V^T$, where $P_{11} \in \mathbb{R}^{q\times q}$ and $P_{22} \in \mathbb{R}^{(n-q)\times(n-q)}$.*

3. Main results

This study aims to solve the sliding mode investment policy problem of the stochastic financial system (2) by using a sliding mode investment policy design with EID approach. Firstly, a suitable sliding surface function is designed so that the dynamics restricted to the switching surface has the desirable property of asymptotic stability. The following theorem provides a set of sufficient conditions for how to design fuzzy sliding mode investment policy such that the fuzzy stochastic nonlinear financial systems in Eq. (4) can be solved.

Theorem 3.1. *Let some scalars $\lambda_k > 0$ $(k = 1, 2, \ldots, m)$ be given. If the following LMIs constrained sliding mode investment policy problem can be solved, for symmetric positive definite matrix $\mathcal{X} = diag\{X, X, X\}$ and appropriate dimensioned matrices W_i, Y_i, such that the following LMIs, for all $i = 1, 2, \ldots, l$,*

$$\begin{bmatrix} \Theta_{1i} + \Theta_{1i}^T + \sum_{k=1}^{m}\lambda_k(\Theta_{3i}(\eta_k) + \Theta_{3i}^T(\eta_k)) & \Theta_{2i}^T & \Theta_{3i}^T(\eta_1) & \cdots & \Theta_{3i}^T(\eta_m) \\ * & -\mathcal{X} & 0 & 0 & 0 \\ * & * & -\lambda_1^{-1}\mathcal{X} & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & -\lambda_m^{-1}\mathcal{X} \end{bmatrix} < 0, \tag{13}$$

where

$$\Theta_{1i} = \begin{bmatrix} A_iX + Y_i & (I-(GB)^{-1}G)W_iC_i & 0 \\ 0 & A_iX - W_iC_i & -BC_FX \\ 0 & B_FB^{+}W_iC_i & A_FX + B_FC_FX \end{bmatrix}, \ \Theta_{2i} = \begin{bmatrix} 0 & 0 & 0 \\ D_iX & D_iX & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{3i}(\eta_k) = \begin{bmatrix} 0 & 0 & 0 \\ E_i(\eta_k)X & E_i(\eta_k)X & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ with } Y_i = K_iX \text{ and } W_i = L_i\hat{X},$$

then, the sliding mode investment policy problem for the nonlinear fuzzy stochastic financial system (4) can be solved.

Proof. In order to solve the nonlinear fuzzy stochastic financial system (4) under fuzzy sliding mode investment policy it is enough to consider closed-loop system (12). Let $V(\psi(t)) = \psi^T(t)\mathcal{P}\psi(t)$, where $\mathcal{P} = \mathrm{diag}\{P, P, P\}$, be the Lyapunov function for the stochastic nonlinear financial fuzzy system. Based on Ito–Levy formula [33], by taking the stochastic derivative of $V(\psi(t))$ along the solution of the system (12), we obtain

$$\begin{aligned} dV(\psi(t)) \le & \sum_{i=1}^{l} h_i(z(t))\left[V_\psi^T(\psi(t))\mathcal{A}_i\psi(t) + \frac{1}{2}\psi^T(t)\mathcal{D}_i^TV_{\psi\psi}(\psi^T(t))\mathcal{D}_i\psi(t)\right]dt + \sum_{i=1}^{l} h_i(z(t))V_\psi^T(\psi(t))\mathcal{D}_i \\ & \times \psi(t)d\mathcal{W}(t) + \sum_{i=1}^{l}\sum_{k=1}^{m} h_i(z(t))\left\{V(\psi(t) + \mathcal{E}_i(\eta_k)\psi(t)) - V(\psi(t))\right\}d\mathcal{N}(t,\eta_k) \\ \le & \sum_{i=1}^{l} h_i(z(t))\left\{\psi^T(t)\left[\mathcal{P}\mathcal{A}_i + \mathcal{A}_i^T\mathcal{P} + \mathcal{D}_i^T\mathcal{P}\mathcal{D}_i\right]\psi(t)\right\}dt + 2\sum_{i=1}^{l} h_i(z(t))\psi^T(t)\mathcal{P}\mathcal{D}_i\psi(t)d\mathcal{W}(t) \\ & + \sum_{i=1}^{l} h_i(z(t))\sum_{k=1}^{m}\left\{\psi^T(t)\left[\mathcal{E}_i^T(\eta_k)\mathcal{P}\mathcal{E}_i(\eta_k) + \mathcal{P}\mathcal{E}_i(\eta_k) + \mathcal{E}_i^T(\eta_k)\mathcal{P}\right]\right\}d\mathcal{N}(t,\eta_k). \end{aligned}$$

By applying mathematical expectation on both sides of the above inequality and utilizing the properties of stochastic variables, we have

$$\mathbb{E}\{dV(\psi(t))\} \le \sum_{i=1}^{l} h_i(z(t))\psi^T(t)\Big[\mathcal{P}\mathcal{A}_i + \mathcal{A}_i^T\mathcal{P} + \mathcal{D}_i^T\mathcal{P}\mathcal{D}_i + \sum_{k=1}^{m}\lambda_k\left\{\mathcal{E}_i^T(\eta_k)\mathcal{P}\mathcal{E}_i(\eta_k)\right.$$
$$\left. +\mathcal{P}\mathcal{E}_i(\eta_k) + \mathcal{E}_i^T(\eta_k)\mathcal{P}\right\}\Big]\psi(t)dt. \tag{14}$$

By pre- and post-multiplying (14) by $\mathcal{P}^{-1} = \mathcal{X}$ and using the singular value decomposition relation on $C_i = UM_iV$, the term C_iX can be equivalently rewritten as $\hat{X}C_i$ with the help of Lemma 2.1 where $\hat{X} = UM_iX_{11}^{-1}M_i^{-1}U^{-1}$, and denoting $Y_i = K_iX$ and $W_i = L_i\hat{X}$, we can get

$$\mathbb{E}\{dV(\psi(t))\} \le \sum_{i=1}^{l} h_i(z(t))\psi^T(t)\left[\Omega_i\right]\psi(t)dt,$$

where

$$\Omega_i = \Theta_{1i} + \Theta_{1i}^T + \Theta_{2i}^T\mathcal{P}\Theta_{2i} + \sum_{k=1}^{m}\lambda_k\left\{\Theta_{3i}^T(\eta_k)\mathcal{P}\Theta_{3i}(\eta_k) + \Theta_{3i}(\eta_k) + \Theta_{3i}(\eta_k)^T\right\}, \tag{15}$$

Θ_{qi} $(q = 1, 2, 3)$ are defined as in statement of Theorem 3.1. Further, by the virtue of Schur complement, Ω_i is equivalent to left hand side of LMI (13). Moreover, if the set of LMIs (13) hold, then $\mathbb{E}\{dV(\psi(t))\} < 0$. Therefore, sliding mode investment policy problem for nonlinear fuzzy stochastic financial system (12) is solvable, which completes the proof.

Now, we are in position to synthesize a sliding mode investment policy design, by which the trajectories of the fuzzy nonlinear financial system (12) can be driven onto the pre-specified switching surface $s(t) = 0$ in a finite time and then are maintained there for all subsequent time. The following theorem will provide the sufficient condition for the sliding mode fuzzy investment policy $u^d(t)$ can be driven onto the predefined sliding mode surface $s(t)$ in a finite time and maintain a sliding mode motion thereafter.

Theorem 3.2. *For the fuzzy nonlinear financial stochastic system (4), assume that the LMIs-constrained in Theorem 3.1 is feasible. Then, under the designed sliding surface given in (7), where G is chosen such that GB is non-singular, the sliding mode reaching condition can be satisfied with the following investment policy design:*

$$u^d(t) = \sum_{i=1}^{r} h_i(z(t))\Big[K_i(\hat{x}(t) - x_d(t)) - \beta_i(t)sgn(s(t))\Big], \tag{16}$$

where $\beta_i(t) = \lambda + \eta_i(t, x(t)) + \|(GB)^{-1}G\|(\|BK_i\hat{x}(t)\|) + \|L_iC_i(x(t) - \hat{x}(t))\|$ *and* $sgn(\cdot)$ *is the signum function.*

Proof. To prove the reachability condition, we select G such that GB is non-singular. Now, we consider the following Lyapunov function

$$\mathcal{V}(t) = \frac{1}{2}s^T(t)(GB)^{-1}s(t). \tag{17}$$

From (7), it can be easily obtained that

$$\dot{s}(t) = -(GB)^{-1}G\sum_{i=1}^{l} h_i(z(t))\big[BK_i(\hat{x}(t) - x_d(t)) + L_iC_i(\hat{x}(t) - x(t))\big] + \sum_{i=1}^{l} h_i(z(t))(u^d(t) - K_i(\hat{x}(t))). \tag{18}$$

Taking the time derivative of $\mathcal{V}(t)$ and using the above equation, we can get

$$\begin{aligned}\dot{\mathcal{V}}(t) &= s^T(t)(GB)^{-1}\dot{s}(t)\\ &= -s^T(t)(GB)^{-1}G\sum_{i=1}^{l} h_i(z(t))\big[BK_i(\hat{x}(t)) + L_iC_i(x(t) - \hat{x}(t))\big] + s^T(t)\sum_{i=1}^{l} h_i(z(t))(u^d(t) - K_i(\hat{x}(t)))\\ &\le \|s(t)\|\sum_{i=1}^{l} h_i(z(t))\Big[\|(GB)^{-1}G\|(\|BK_i\hat{x}(t)\|) + \|L_iC_i(x(t) - \hat{x}(t))\| + \eta_i(t, x(t))\Big]\\ &\quad + s^T(t)\sum_{i=1}^{l} h_i(z(t))(u^d(t) - K_i(\hat{x}(t))).\end{aligned} \tag{19}$$

By substituting (16) into (19), we can obtain $\dot{\mathcal{V}}(t) \leq -\lambda\|s(t)\| < 0, \quad \forall \|s(t)\| \neq 0$. Thus, the system trajectories of (4) converge to the predefined sliding surface in finite time and are restricted to the surface itself for all subsequent time, thereby completing the proof.

Remark 3.3. It should be mentioned that the proposed the fuzzy rule-dependent EID-based sliding mode investment policy design is more general and appropriate than the conventional EID-based investment policy and sliding mode investment policy. By restricting $G = 0$ then the proposed fuzzy EID-based sliding mode investment policy design becomes conventional EID-based investment policy design.

In particular, the conventional sliding mode investment policy design is deduced by the following corollary. The proof of the following corollary is similar to that of proof of Theorem 3.1. Before providing the corollary, estimation error dynamics between the considered fuzzy system (4) and estimated system (10) can be calculated as

$$\mathrm{d}x_\delta(t) = \sum_{i=1}^{l} h_i(z(t))\left(\Big((A_i - (I-(GB)^{-1}G)L_iC_i)x_\delta(t) + w(t)\Big)\mathrm{d}t + D_ix(t)\mathrm{d}\mathcal{W}(t) + \sum_{k=1}^{m} E_i(\eta_k)x(t)\mathrm{d}\mathcal{N}(t,\eta_k)\right)\mathrm{d}t. \tag{20}$$

Corollary 3.4. *For given some positive scalars λ_k $(k = 1, 2, \ldots, m)$, the investment policy design along with the fuzzy state estimator system (20) and the sliding mode dynamics (7) stochastically stabilizes the nonlinear stochastic fuzzy Poisson jump financial system in Eq. (4) if there exist positive definite matrix $\mathbb{X} = diag\{X, X\}$ and some appropriate dimension matrices W_i and Y_i such that the following LMIs hold:*

$$\tilde{\Omega}_i < 0, \quad \text{for all } i = 1, 2, \ldots, l, \tag{21}$$

where

$$\tilde{\Omega}_i = \begin{bmatrix} \tilde{\Theta}_{1i} + \tilde{\Theta}_{1i}^T + \sum_{k=1}^{m}\lambda_k(\tilde{\Theta}_{3i}(\eta_k) + \tilde{\Theta}_{3i}^T(\eta_k)) & \tilde{\Theta}_{2i}^T & \tilde{\Theta}_{3i}^T(\eta_1) & \cdots & \tilde{\Theta}_{3i}^T(\eta_m) \\ * & -\mathbb{X} & 0 & 0 & 0 \\ * & * & -\lambda_1^{-1}\mathbb{X} & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & -\lambda_m^{-1}\mathbb{X} \end{bmatrix}$$

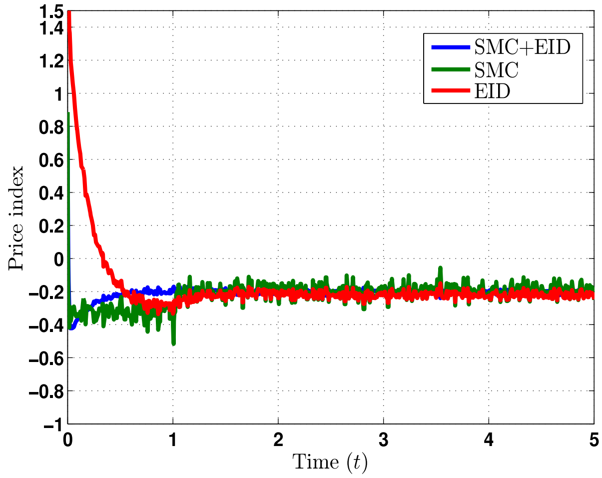
with $\tilde{\Theta}_{1i} = \begin{bmatrix} A_iX + Y_i & (I-(GB)^{-1}G)W_iC_i \\ 0 & A_iX - W_iC_i \end{bmatrix}$, $\tilde{\Theta}_{2i} = \begin{bmatrix} 0 & 0 \\ D_iX & D_iX \end{bmatrix}$ *and* $\tilde{\Theta}_{3i} = \begin{bmatrix} 0 & 0 \\ E_i(\eta_k)X & E_i(\eta_k)X \end{bmatrix}$.

Remark 3.5. It should be noted that in the sliding mode control approach, one particular surface is designed to drive the system states on it. Once the system states reach the sliding surface, the controller keeps the states on the close neighborhood of the sliding surface. That is, the closed-loop response becomes totally insensitive to some particular uncertainties, disturbance and nonlinearity.

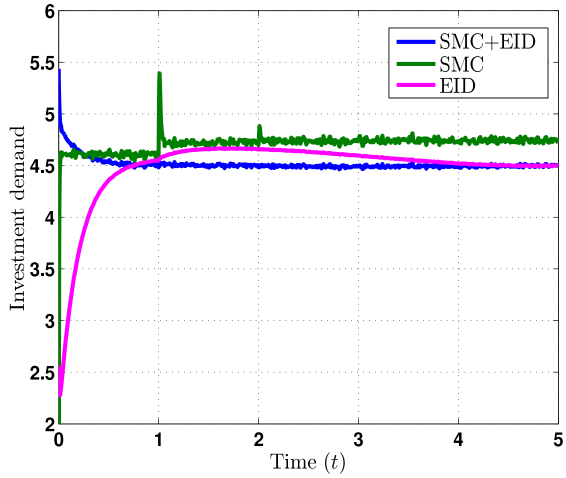
4. Numerical simulations

In order to illustrate the performances of the proposed EID-based sliding mode investment policy design, we borrow the nonlinear stochastic jump diffusion financial system parameter values provided in [33]. Let $z(t) = [z^1(t), z^2(t)]$ and assume that each $z^a(t)\,(a = 1, 2)$ has eight operational points, which are given in Appendix. Therefore, the fuzzy system (4) contains totally 64 fuzzy rules and is assumed to be affected by continuous stochastic noise and six discontinuous Poisson jumps $\Delta = \{\eta_1, \eta_2, \ldots, \eta_6\}$. Further, additional matrix values of the stochastic fuzzy system (4) are provided in Appendix. Let us choose $G = I_3$ to reduce the computational complexity and consider the filter parameters as $A_F = -101I_3$, $B_F = 100I_3$ and $C_F = I_3$. With these considered parameter values, by solving the LMIs in (13) in Theorem 3.1, the feasible solution can be obtained and the calculated feedback controller and observer gains are not provided here due to the page constraint. For the simulation purposes, we choose the initial state conditions of the fuzzy plant and observer as $\hat{x}(t) = x(t) = [0.47\ 1.44\ 0.51]^T$. Further, the reference input and disturbance inputs are chosen as $x_d(t) = [-0.2\ 4.5\ 0.1]^T$ and $w(t) = 0.01\sin(t)$, respectively.

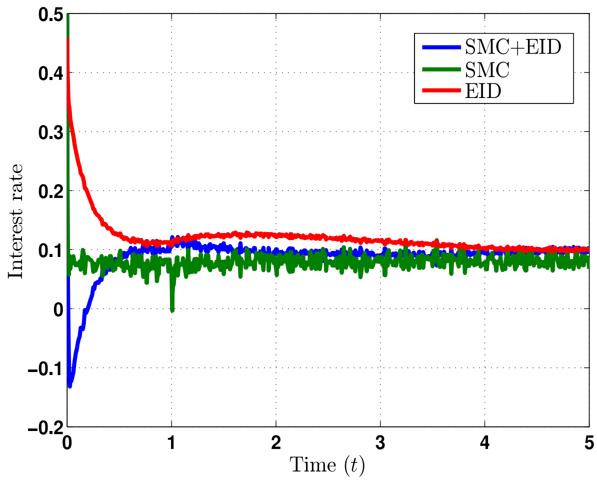
In order to show the advantage of the proposed investment policy design, the trajectories of the considered fuzzy stochastic jump system (4) based on the conventional sliding mode investment policy design and the conventional EID-based investment policy design and the proposed EID-based sliding mode investment policy design are provided in Fig. 1. Specifically, Fig. 1 shows the state trajectories of the interest rate, investment demand and price index. From Fig. 1, it is concluded that the EID-based sliding mode investment policy design can achieve the desired interest rate, investment demand and price index even in the presence of unpredictable investment changes or worldwide events such as nature disaster and war. Fig. 2 shows the investment policy of the considered financial model. Further, Fig. 3 shows the open-loop response of the considered system in which it can be seen that the desired target cannot be achieved. From the



(a) Interest rate



(b) Investment demand



(c) Price index

Fig. 1. State trajectories of the closed-loop system with Poisson jump.

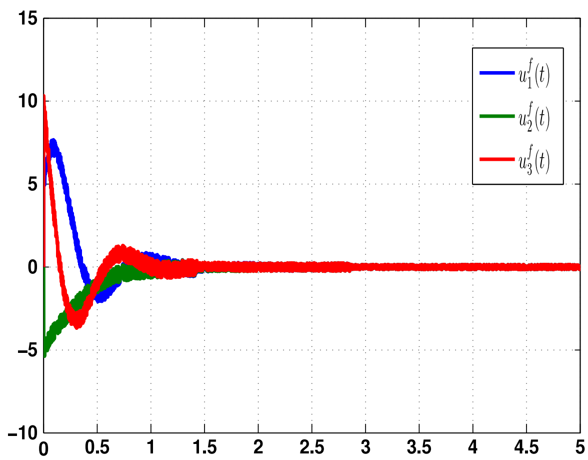


Fig. 2. Investment policy.

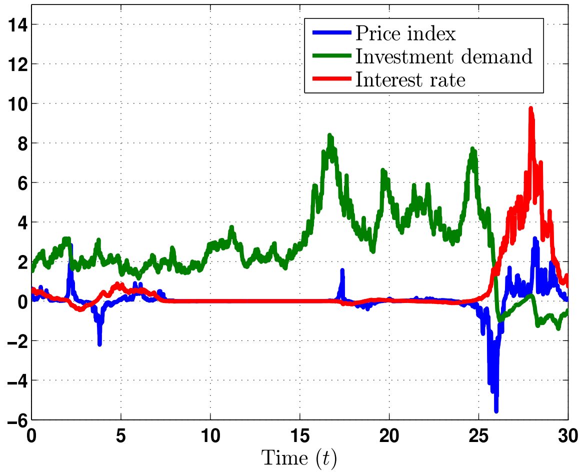


Fig. 3. State trajectories of the open-loop system.

simulation results, it can be concluded that the proposed EID based estimator block effectively rejects the disturbances such as unpredictable investment changes and worldwide unpredictable events. The result reveals that based on the proposed approach, an investor can analyze the financial status and can calculate the better investment with increasing profit in financial market in the presence of unpredictable investment changes and worldwide unpredictable events.

Remark 4.1. It should be noted that the authors in [33] investigated the multi objective H_2/H_∞ investment policy for nonlinear stochastic jump diffusion financial systems via T–S fuzzy model interpolation method. The H_2/H_∞ investment policy requires the information of the interest rate, investment demand and price index at each instants which may difficult to get always due to unpredictable investment changes or worldwide events. In this study, the proposed EID-based sliding mode investment policy effectively estimate the interest rate, investment demand and price index exactly right after the initial time. Moreover, the integral sliding-mode technique offers the advantage of forcing the system to be on sliding phase from the first moment and helps to attain the desired target quickly. In addition, the EID-based disturbance estimator is used to estimate the unpredictable investment changes and worldwide unpredictable events on the financial systems and explains how the deregulation policies leading to the crisis could be pursued each and every instant which helps to attains desired target in an optimal sense.

5. Conclusion

In this paper, we have discussed the fuzzy nonlinear stochastic jump model to describe the dynamical behavior of financial system in the presence of unpredictable investment changes and worldwide unpredictable events. An EID-based sliding mode investment policy is proposed to achieve the desired interest rate, investment demand and price index with better investment cost and increase profit under both the continuous and discrete random jump input. The proposed EID-based sliding mode investment policy could solve the robustness of a nonlinear stochastic jump diffusion financial system to achieve a desired interest rate, investment demand and price index even in the presence of unpredictable investment changes or worldwide events such as nature disaster and war. At last, an example with simulation is provided to confirm the performance of the proposed fuzzy EID-based sliding mode investment policy design for nonlinear stochastic jump diffusion financial systems.

Appendix

The fuzzy operation points are chosen as in [33]. The eight operation points of z^1 are given that $z^{11} = -1.5,\ z^{12} = -1.11,\ z^{13} = -0.72,\ z^{14} = -0.33,\ z^{15} = 0.07,\ z^{16} = 0.46,\ z^{17} = -0.85,\ z^{18} = 1.24$, and the eight operation points of z^2 are given that $z^{21} = -1.5,\ z^{22} = -1.11,\ z^{23} = -0.72,\ z^{24} = -0.33,\ z^{25} = 0.07,\ z^{26} = 0.46,\ z^{27} = -0.85,\ z^{28} = 1.24$, Further the index term i is represented in terms of (j,k), $j = 1,2,\ldots,8$, $k = 1,2,\ldots,8$. The state matrices of the inferred stochastic jump fuzzy model [33] at z^{1j} and z^{2k} operation points are given by $A_i = [a^{jk}]_{3\times3}$ with $a^{jk}_{1,1} = -\alpha + z^{2j} + \frac{z_d + x_d y_d - \alpha x_d}{z^{1i}}$, $a^{jk}_{1,2} = x_d$ $a^{jk}_{1,3} = 1$, $a^{jk}_{2,1} = z^{1i} - 2x_d$, $a^{jk}_{2,2} = -\beta + \frac{1-\beta y_d - x_d^2}{z^{2j}}$, $a^{jk}_{2,3} = 0$, $a^{jk}_{3,1} = -1 - \frac{x_d + c z_d}{z^{1i}}$, $a^{jk}_{3,2} = 0$, $a^{jk}_{3,3} = -\gamma$ and $D_i = \begin{bmatrix} 0.3a^{jk}_{1,1} & 0.3x_d & 0.3 \\ 0.1a^{jk}_{2,1} & 0.1a^{jk}_{2,2} & 0 \\ 0.2a^{jk}_{3,1} & 0 & -0.2\gamma \end{bmatrix}$. Further, $E_i(\eta_1) = \text{diag}\{0.21,\ 0,\ 0\}$, $E_i(\eta_2) = \text{diag}\{-0.3,\ 0,\ 0\}$, $E_i(\eta_3) =$

$\mathrm{diag}\{0,\ 0.05,\ 0\}$, $E_i(\eta_4) = \mathrm{diag}\{0,\ -0.05,\ 0\}$, $E_i(\eta_5) = \mathrm{diag}\{0,\ 0,\ -0.01\}$ and $E_i(\eta_6) = \mathrm{diag}\{0,\ 0,\ 0.15\}$. Moreover, the control input position vectors are assumed to be common $B = I_3$. The measured output matrix C_i is chosen as $C_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

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