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A theoretic analysis of key person insurance

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ABSTRACT

As the death or a major accident of a key person will bring a firm with disastrous losses, key person insurance has attracted increasing attention worldwide. But key person insurance is a double-edged sword because it has both positive and negative effects on a firm's performance. Different from prior papers, this study proposes to capture the two opposite effects of key person insurance by using a microeconomic analysis. The novel contribution of this paper is that besides risk-reducing effects of key person insurance, we find that key person insurance reduces the salaries of employees, output and excepted profit of the firm. More importantly, we illustrate that strong ability of the key person will promote the efficiency of employees. So this paper offers a full evaluation of firms' purchase behavior of key person insurance and also develops the theory of key person insurance.

1. Introduction

Key person insurance, also called key man insurance, is an important form of business insurance. It is an insurance policy taken out by a business to protect from financial losses that would arise from the death or extended incapacity of a crucial member of the business. Key person insurance is necessary if the sudden loss of a key executive could have a significant negative effect on a company's operations. In recent years an increasing number of insurance companies have started to offer key person insurance policies.

Key persons have significant effects on a firm's performance in the short-term. For example, in the case of Baidu.com Inc., owner of China's most popular internet search site, the company's shares fell sharply following the death of its chief financial officer (CFO), Shawn Wang, on Dec. 27, 2008. In the two days following his death, the share price fell 4.7%. In another case, when Steve Jobs, the co-founder, chairman and chief executive officer (CEO) of Apple, resigned from the company on Aug. 24, 2011, and then passed away on Oct. 5, 2011, there was a huge shock to the company's stock (the share price fell 6% on the day after his death), which greatly affected the interests of investors. As we know, these sudden events have considerable effects on firms' profits, and key person insurance can protect firms' losses because these losses will be covered by the insurers. Therefore, key person insurance attracts attention from both firms and insurance companies because the risk for the key persons can be eliminated.

Very few papers on key person insurance have been published. However, there is a vast literature on the effects of insurance on economic activities in industries and on the economy. For example, in microeconomics, Wang et al. (2017) and Pieper et al. (2015) addressed the effects of insurance on firms' innovation in improving environmental quality. In industrial economics, competitive relationships between insurance companies and hospitals are discussed by Wang and Nie (2016), and the factors that affect the insurance industry have been confirmed by Biener et al. (2016). In macroeconomics, existing literature focuses on economic growth under insurance (Lee et al., 2016; Courbage and Rey, 2016; Nie, 2007; Eling and Schaper, 2017; Nie et al., 2016; Wang and Nie, 2018). Lee, Chang, Arouri & Lee (2016) recently examined the negative relationship between insurance and growth. Also, Bertrand & Prigent (2016) argued that insurance significantly impacts the equilibrium in portfolios, and they showed that the equilibrium risk-neutral density is equal to the product of a factor corresponding to the total risk tolerance with exogenous insurance constraint.

Because life insurance relates to almost everyone, most of the literature highlights the effects of life insurance on human capital (Israelsen and Yonker, 2017; Nie, 2014; Dineen and Allen, 2016). Below we briefly introduce the related research about the effects of life insurance on human capital.

Many papers highlight the effects of some types of insurance on labor.

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Olsson and Thoursie (2015) analyzed how health insurance affects labor supply. Dillender, Heinrich & Houseman (2016) identified the effects of health insurance on part-time employees, and argued that employer mandates on health insurance increase part-time employment among workers without a college degree. Further, Gatzert & Maegebier (2015) captured the effects of critical illness insurance on human capital supply.

As a special type of life insurance, key person insurance has attracted increasing attention in recent years, but few researchers have focused on key person insurance. Compared with general life insurance, key person insurance is taken out by firms. Chandy, Davidson, Garrison & Worrell (1986) examined the relationship between key man insurance and market reaction. Block, Keenan & Malone (2006) introduced the key person claim in detail and Nicholson and Corbett (1987) examined the market reaction to key person insurance. Furthermore, Owusu et al. (2016) addressed the effects of key person life insurance in Ghana. Johnson, Magee, Nagarajan & Newman (1985) investigated a sample of executives who were corporate founders, and observed significant positive returns associated with the sudden deaths of these executives. Nicholson and Corbett (1987) addressed the results of Chandy et al. (1986) and supported the conclusions of Johnson et al. (1985). In summary, most researchers agree that key person insurance affects firms' performance in the short-term.

Although key person insurance is critical for the development of some firms in reality, a systematic theory that can be used to guide practice is absent. On the one hand, nearly all the existing research about key person insurance does not establish a necessary theory or capture the effects of key person insurance on employees, firms' risk or expected profits by considering the ability of the key person. These factors are very important both in theory and for decision-makers. On the other hand, empirical data about key person insurance are very scarce, which results in difficulty to carry out an empirical study. So this article aims to establish a theory about key person insurance, and tries to identify the effects of key man insurance on employees and firm's performance.

Interestingly, by establishing a theoretic model, we argue that key person insurance reduces the salaries of employees, expected profits and the risks for insured firms. Moreover, the choice of a firm on whether to buy key man life insurance is supported by decision-makers.

The main contributions of this article lie in the following three areas. First, we establish a microeconomic theory about key person insurance. The theory of key person insurance will attract the attention of scholars. Second, based on our theoretic model, the effects of key person insurance are captured. The model in this article supports further research in key person insurance. Finally, in the application of the model, this article supports a theory about decision-makers and key person insurance. Moreover, our conclusions will help governments to regulate key person insurance.

The rest of this article is organized as follows: The model is established in Section 2. In the model, both the key person and the employees are considered. Section 3 analyzes in detail the benchmark model without key person insurance. Section 4 addresses key person insurance and compares the results with those in Section 3. The primary results are explained in this section. Conclusions are discussed in the final section.

2. Model of key person insurance

We focus on key person insurance of a risk-averse firm. This firm would choose such an insurance scheme to shield itself from the uncertainties associated with the loss of services provided by its key persons.

The model of a firm with key person insurance is established. The state of the key person in the firm is θ, where $\theta \in \{0, 1\}$. In this article, the state reflects the marginal effects of the key person on each employee in production, or the ability of the key person. The price of the firm's final product is assumed to be $p = 1$.

Employees: We assume that there are N identical employees in the firm, and will consider a representative employee below. The effort level of an employee is denoted by e. The production of this employee is $(1 + \theta)e$ and the costs incurred by the employee for exerting effort e are $\frac{e^2}{2}$. The salary of this employee is $S_0 + \tau(1 + \theta)e$, where $S_0 > 0$ and $\tau > 0$. S_0 is the employee's reservation salary. τ represents the marginal human capital cost of a unit of output, and $\tau(1 + \theta)e$ is the reward for the employee's effort. In practice, τ may be the salary incentive intensity for the key person, which is consistent with the work situation associated with the pay (annual salary and incentives) of individuals (Joshi et al., 2006). This type of compensation model is utilized by many firms, and is also employed in this article. Therefore, the utility of the employee is

$$u(e, \theta) = S_0 + \tau(1 + \theta)e - \frac{e^2}{2} \tag{1}$$

In (1), $S_0 + \tau(1 + \theta)e$ is the employee's salary, and $\frac{e^2}{2}$ is the costs incurred by the employee for exerting effort e. It is easy to see that the optimal effort level for the employee is $e^* = \tau(1 + \theta)$. In equation (1), the key person improves the marginal products of each employee.

Key person: We assume that other key persons in this firm are fixed, and will focus on one key person. To simplify the problem, we assume that the objective of the key person is consistent with the firm's profits or revenues. Therefore, the key person aims to maximize the following function

$$U(\theta, \tau) = \varsigma[N(1 - \tau)(1 + \theta)e - NS_0 - c_0 N(1 + \theta)e], \tag{2}$$

where $1 > c_0 > 0$ is the marginal cost to sell the product, and $c_0 N(1 + \theta)e$ is the total cost to sell all outputs. The constant $\varsigma \in (0, 1)$ is the proportional gain of the key person to the firm's revenues. Because we assume that the objective of the key person is consistent with the firm's profits (revenues), the firm is not addressed in this article. Moreover, if the salary of the key person is proportional to the firm's revenues, this assumption is rational.

When considering key person insurance, we assume that the probability of "good" state is α, and the probability of "bad" state is $1 - \alpha$, where $0 < \alpha < 1$ and $\alpha > 0.9$. Namely, the probability of "good" state is much larger than that of "bad" state. The expected objective function of the key person is

$$EU(\theta, \tau, \alpha) = F(\theta, \tau, \alpha) = \varsigma[\alpha N(1 - \tau)(1 + \theta)e + (1 - \alpha)N(1 - \tau)e - NS_0]. \tag{3}$$

In function (3), when the key person is in "bad" state, he/she has no effect on employees. In "good" state, the key person affects both the marginal production of each employee and the profits of the firm. Before analyzing the model, we make the following assumption.

Assumption: α is close to 1, c_0 is sufficiently large, and ς is very small.

α being close to 1 means that the probability of "good" state is much greater than that of "bad" state. This is an important condition for an insurance company. A large c_0 depicts fierce competition in this industry. A small ς indicates that the salary of the key person is only a small part of the firm's revenues. In reality these assumptions are rational.

The timing of decisions is as follows. In the first stage, the insurance company determines the insurance premium. In the second stage, the firm determines whether to buy key person insurance or not. In the final stage, the key person decides the marginal salaries of employees, and the employees determine their effort levels (see Fig. 1).

3. The benchmark model

Here we address the benchmark model without key person insurance. On the basis of the formulation $e^* = \tau(1 + \theta)$, (1) and (2), the key person can select the parameter τ to maximize the objective function. Function (2) is restated as

$$U(\theta, \tau) = \varsigma\left[N(1 - \tau)\tau(1 + \theta)^2 - NS_0 - c_0 N\tau(1 + \theta)^2\right]. \tag{4}$$

Apparently, the function (4) is concave in τ, and a unique solution exists, which satisfies the first-order conditions. Therefore, when the key

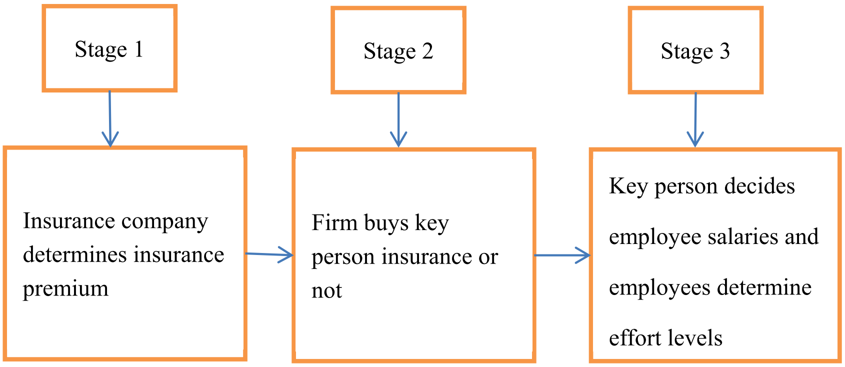

Fig. 1. Timing of decisions.

person is in "good" state, we have the corresponding solution meeting the following optimal conditions

$$\frac{\partial U(\theta,\tau)}{\partial \tau}=N(1+\theta)^2(1-2\tau-c_0)=0$$

and

$$\tau^{*,1}=\frac{1-c_0}{2}. \tag{5}$$

Equation (5) has no relationship with the state of the key person because we assume that the objective of the key person is consistent with the firm's profits or revenues. The effects of this key person's state on revenue are the same as those on the key person's salary. Therefore, the incentive intensity has no relationship with the state of the key person.

The utility of the key person and the corresponding profits of the firm are given as

$$U^{*,1}=\varsigma\left[\frac{N(1-c_0)^2(1+\theta)^2}{4}-NS_0\right],$$
$$\pi^{*,1}=(1-\varsigma)\left[\frac{N(1-c_0)^2(1+\theta)^2}{4}-NS_0\right]. \tag{6}$$

The corresponding output is

$$q^{*,1}=\frac{N\left(1-c_0^2\right)(1+\theta)^2}{4}. \tag{7}$$

By (6) and (7), the effects of the key person's state are a quadratic function on output, firm profit and the utility of the key person. This is based on the fixed price. Under market power, the effects of this key person's state are not consistent.

When this key person is in the "bad" state, his/her effects on employees and firms are exactly $\theta=0$. Then, we have the following result

$$\tau^{*,2}=\frac{1-c_0}{2}. \tag{8}$$

The corresponding key person's utility and the profits of the firm are

$$U^{*,2}=\varsigma\left[\frac{N(1-c_0)^2}{4}-NS_0\right]$$
$$\pi^{*,2}=(1-\varsigma)\left[\frac{N(1-c_0)^2}{4}-NS_0\right]. \tag{9}$$

The corresponding output is

$$q^{*,2}=\frac{N\left(1-c_0^2\right)}{4}. \tag{10}$$

The expected objective function of this key person and the expected profits are

$$EU^*=\alpha\varsigma N\frac{(1-c_0)^2}{4}(1+\theta)^2+(1-\alpha)\varsigma N\frac{(1-c_0)^2}{4}-\varsigma NS_0,$$
$$E\pi^*=\alpha(1-\varsigma)N\frac{(1-c_0)^2}{4}(1+\theta)^2+(1-\alpha)(1-\varsigma)N\frac{(1-c_0)^2}{4}-(1-\varsigma)NS_0. \tag{11}$$

For the risk of this firm, we describe the risk by variance, and we obtain the following results

$$D^1(\alpha)=\alpha\left|E\pi^{*,1}-E\pi^*\right|+(1-\alpha)\left|E\pi^{*,2}-E\pi^*\right|$$
$$=\alpha(1-\alpha)(1-\varsigma)N\frac{1-c_0^2}{4}\left(2\theta+\theta^2\right). \tag{12}$$

(12) is based on the definition of variance to combine the above model. Equation (12) shows that the risk increases with the state of the key person and the number of employees. Furthermore, the probability of a "good" state has important effects on the risk. High probability of a "good" state implies low risk.

Here, the equilibrium without key person insurance is achieved. For the equilibrium, by simple calculations we reached the following conclusions.

Proposition 1. High θ improves the utility of employees and firm profits and lowers the risk.

Remarks: The conclusions in Proposition 1 are consistent with reality. Therefore, firms have intentions to pursue key persons with a strong ability to improve their profits and competitiveness. Because key persons with a strong ability promote the efficiency of employees, the revenues of firms are also improved. Moreover, by equation (12), a healthy key person reduces the risk. Therefore, to reduce risk, good health of the key person attracts high attention.

The policy implication for a firm is to hire a key person with strong abilities and who is in a good health state in order to improve revenues and reduce risk. Hence, ability and state of health are important for key persons in firms, and this is highly consistent with reality.

4. Situation with key person insurance

In this section, key person insurance is modelled. We assume that the impact of this key person on production is known to the company. We also assume that the insurance premium is proportional to the firm's revenues, which is exactly $\psi N(1-\tau)\tau(1+\theta)^2$, where $\psi\in(0,1)$. When the key person is in a "bad" state, the firm receives a claim for the loss. We will discuss the parameter of the insurance premium $\psi\in(0,1)$. Moreover, in general, the ability of the key person cannot be observed while the output can be informed. Therefore, we consider the insurance premium given by $\psi N(1-\tau)\tau(1+\theta)^2$. We then analyze the model above using backward induction.

In the third stage, by using function (13), we discuss the key person insurance in which the firm's revenues for the key person in a "bad" state is equal to those for the key person in a "good" state. Thus, the corresponding utility of the key person is

$$U(\theta,\tau)=\varsigma\left[N(1-\tau)\tau(1+\theta)^2-\psi N(1-\tau)\tau(1+\theta)^2-NS_0-c_0N\tau(1+\theta)^2\right]. \tag{13}$$

Therefore, for the key person in a "good" state, we have the corresponding solution satisfying

$$\frac{\partial U(\theta,\tau)}{\partial\tau}=N(1+\theta)^2[(1-2\tau)(1-\psi)-c_0]=0$$

and

$$\tau^{*,3}=\frac{1}{2}-\frac{c_0}{2(1-\psi)}. \tag{14}$$

Equation (14) also has no relationship with the state of the key person because we assume that the objective of the key person is consistent with the firm's profits or revenues. The key person's utility and the corresponding profits of the firm are

$$U^{*,3}=\varsigma\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4}-NS_0\right],$$
$$\pi^{*,3}=(1-\varsigma)\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4}-NS_0\right]. \tag{15}$$

The corresponding output is given by

$$q^{*,3}=\frac{N\left[1-\frac{c_0^2}{(1-\psi)^2}\right](1+\theta)^2}{4}. \tag{16}$$

When the key person is in a "bad" state, his/her effects on both the employees and the firm are exactly $\theta=0$. Then, we have the following result

$$\tau^{*,4}=\frac{1}{2}-\frac{c_0}{2(1-\psi)}. \tag{17}$$

The utility of the key person and the corresponding profits of the firm are

$$U^{*,4}=\varsigma\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}-NS_0\right].$$
$$\pi^{*,4}=(1-\varsigma)\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}-NS_0\right]+I. \tag{18}$$

I indicates the insurance indemnity, which comes from the insurance company when the key person is in a "bad" state.

We assume that $\pi^{*,4}=\pi^{*,3}$, which means that with the key person insurance, the firm's revenues when the key person is in a "good" state are equal to those when the key person is in a "bad" state. Thus, we hope to achieve the threshold value for a firm to take part in key person insurance.

The corresponding output is

$$q^{*,4}=\frac{N\left[1-\frac{c_0^2}{(1-\psi)^2}\right]}{4}. \tag{19}$$

The expected objective function of the key person and the expected profits of the firm are given as

$$EU^{**}=\alpha\varsigma N\frac{(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4}+(1-\alpha)\varsigma N\frac{(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}-\varsigma NS_0,$$
$$E\pi^{**}=(1-\varsigma)\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4}-NS_0\right]. \tag{20}$$

In the second stage, the firm determines whether to buy life insurance or not. For the key person, the firm is willing to pay for insurance if

$$\left[\frac{N(1-\psi)\left(1-\frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4}-NS_0\right]>\left[\frac{N(1-c_0)^2}{4}-NS_0\right]. \tag{21}$$

According to inequality (21), we came to the following conclusion.

Proposition 2. Firms will buy the key person insurance only if the effects of the key person satisfy $\theta\geq\frac{1-c_0}{\sqrt{(1-\psi)^2+c_0-2(1-\psi)}}$.

Remarks: By inequality (21), we give the threshold value of the key person insurance based on whether the firm is willing to buy key person insurance or not. Firms would not like to buy key person insurance when the key person has weak abilities or has little effect on employee production. Insurance premiums and key person's abilities have a crucial effect on the key person's purchasing behavior.

On the one hand, firms have intention to buy key person insurance for key persons with strong abilities. On the other hand, high costs deter this purchasing activity. Proposition 2 gives the threshold condition for the decision-makers.

In the first stage, the insurance company determines the insurance premium $\psi\in(0,1)$. Under a competitive insurance market, we have the following formulation to determine the price of key person insurance

$$\psi(1-\tau)\tau(1+\theta)^2=\frac{(1-\alpha)(1-\psi)(1-\varsigma)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}(2\theta+\theta^2). \tag{22}$$

(22) is the condition that the revenues are equal to the pay for the insurance company. This condition is generally employed to address the competitive insurance market. The price of key person insurance is equal to the expected value of insurance compensation.

From equations (15) and (18) and $\pi^{*,3}=\pi^{*,4}$, we can obtain that the insurance indemnity $I=\frac{(1-\psi)(1-\varsigma)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}(2\theta+\theta^2)$and the expected values of insurance compensation are equal to $\frac{(1-\alpha)(1-\psi)(1-\varsigma)\left(1-\frac{c_0}{1-\psi}\right)^2}{4}(2\theta+\theta^2)$.

We describe the risk of the firm by variance, and immediately obtain the following results

$$D^2(\alpha) = \alpha\left|\pi^{*,3} - E\pi^{**}\right| + (1-\alpha)\left|\pi^{*,4} - E\pi^{**}\right| = 0. \quad (23)$$

Obviously, the expected risk of this firm is cut down to zero by the key person insurance. On the basis of the above equilibrium, we conclude the following

Proposition 3. Key person insurance reduces the salaries of employees, outputs, the expected profits of firms, along with risk.

Proof. See Appendix. ■

Remarks: Under key person insurance, a firm's profits are jointly shared by the firm and the insurance company, and the profits of the firm are reduced. This reduction demonstrates that both the outputs and the salaries of employees deviate from the equilibrium or optimal level. Therefore, key person insurance decreases the salaries of employees, and the outputs and profits of firms. When insurance companies settle claims, the profits of firms under a “bad” state do not reduce. Moreover, key person insurance reduces the risk of the firm.

For infant firms or small and medium size enterprises (SME), it is very important to reduce risk, and the government should encourage key person insurance to maintain the development of SMEs.

The outputs and profits under key person insurance are compared with those without key person insurance in Figs. 2 and 3.

Note: According to (5) and (14), we find that the relationship between ψ and c_0 is $1-\psi \le c_0 \le 1$. According to Figs. 2 and 3, key person insurance reduces output and firms' profits, which is consistent with Proposition 2.

Proposition 2 points out both the disadvantages (the reduced effects on profits and salaries of employees) and the advantages (decreased risk) of key person insurance. Actually, a risk loving firm may not pursue key person insurance. In summary, key person insurance can efficiently eliminate the risk from uncertainty incurred by the key person, which seems consistent with the conclusions in Wang et al. (2017).

5. Concluding remarks

This article captures the effects of key person insurance on the salaries of employees, the outputs, the profits and the risk of firms. Interestingly, we argue that key person insurance decreases the salaries of employees and the profits of firms. Importantly, key person insurance reduces the risk of firms coming from the uncertainty of key persons.

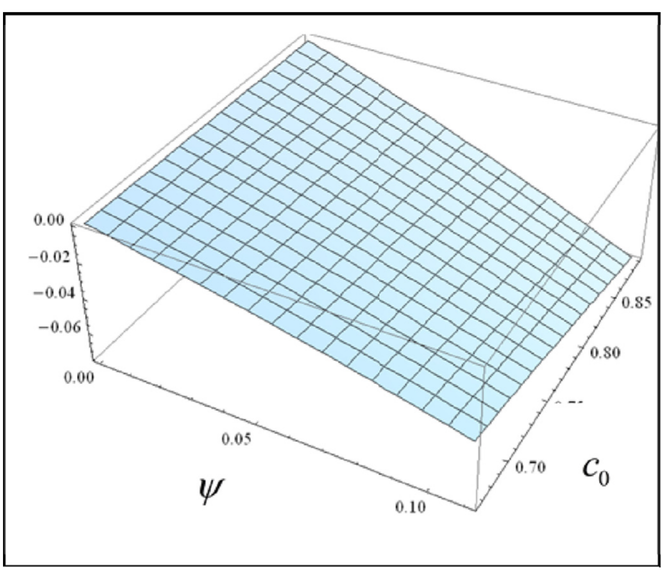

Fig. 2. Numerical simulation of $Eq^* - Eq^{**}, \psi \in \left(0, \frac{2}{9}\right), c_0 \in \left(\frac{7}{9}, \frac{8}{9}\right)$ and $\alpha = 0.999, \theta = \frac{1}{5}, \zeta = \frac{1}{100}$.

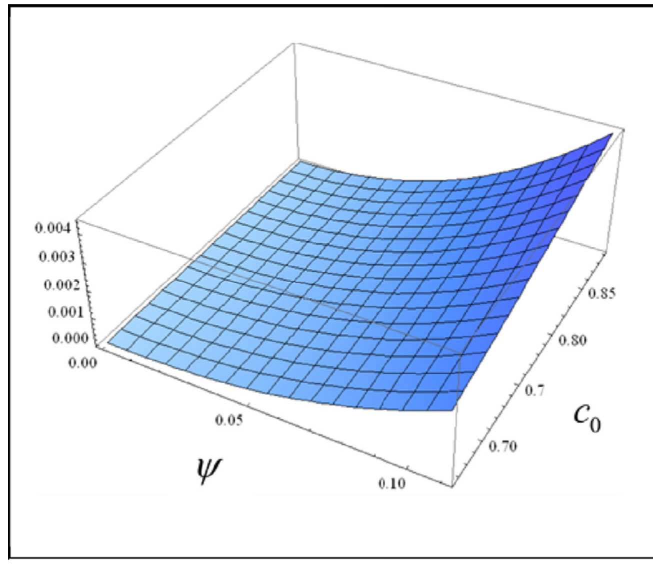

Fig. 3. Numerical simulation of $E\pi^* - E\pi^{**}, \psi \in \left(0, \frac{2}{9}\right), c_0 \in \left(\frac{7}{9}, \frac{8}{9}\right)$ and $\alpha = 0.999, \theta = \frac{1}{5}, \zeta = \frac{1}{100}$.

This article develops the theory of key person insurance, which fills in an existing gap in this topic, and supports the theory of the decision-makers of firms and insurance companies. Moreover, we assume that this theory, along with a competitive insurance market, can easily extend to other situations. By considering market power, the effects of key person insurance are amplified, which we do not investigate in this article.

The main limitations of this study are that we ignore the principal-agent problem, or we assume that the objective of the key person is consistent with the firm and only one key person is taken into account. Moreover, because of lack of data, we cannot launch an empirical research. Therefore, some topics for further research arise. On the one hand, under firm theory, a key person's objective may not be consistent with the firm's profits, which makes it interesting to investigate the framework of firm theory. On the other hand, it is interesting to consider multiple key persons and the corresponding effects of key person insurance for further study. In addition, to reduce the risk, governments may subsidize key person insurance. It is also very important to address this type of subsidy, such as in Nie et al. (2017), Chen et al. (2017).

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Appendix

Proof of Proposition 3.

By considering equations (5) and (14), $\psi \in (0,1)$ implies the relationship of $\tau^{*,3} = \frac{1}{2} - \frac{c_0}{2(1-\psi)} < \tau^{*,1} = \frac{1}{2} - \frac{c_0}{2}$. Similarly, we have $\tau^{*,4} < \tau^{*,2}$. For the output, we have

$$q^{*,3} = \frac{N\left[1 - \frac{c_0^2}{(1-\psi)^2}\right](1+\theta)^2}{4} < q^{*,1} = \frac{N\left(1 - c_0^2\right)(1+\theta)^2}{4} \quad \text{and} \quad q^{*,4} < q^{*,2}.$$

Thus, according to the definition of the expected utility of employees and the expected output, key person insurance reduces both the (expected) utility of employees and the (expected) output. For profits, we also have the following formulation

$$\pi^{*,3} = (1-\varsigma)\left[\frac{N(1-\psi)\left(1 - \frac{c_0}{1-\psi}\right)^2(1+\theta)^2}{4} - NS_0\right] < \pi^{*,1} = (1-\varsigma)\left[\frac{N(1-c_0)^2(1+\theta)^2}{4} - NS_0\right].$$

Considering the participation condition about key person insurance in the second stage, we have the formulation $\pi^{*,4} = \pi^{*,3} > \pi^{*,2}$ Here, we compare the expected profits under two cases. The expected profits of this firm without key person insurance are given as

$$E\pi^* = \alpha(1-\varsigma)N\frac{(1-c_0)^2}{4}(1+\theta)^2 + (1-\alpha)(1-\varsigma)N\frac{(1-c_0)^2}{4} - (1-\varsigma)NS_0.$$

The expected profits with key person insurance are

$$E\pi^{**} = (1-\varsigma)\left[\frac{N(1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2(1+\theta)^2}{4} - NS_0\right].$$

Therefore, from equation (22), we have

$$\psi\left[\frac{1}{2} + \frac{c_0}{2(1-\psi)}\right]\left[\frac{1}{2} - \frac{c_0}{2(1-\psi)}\right](1+\theta)^2 = \frac{(1-\psi)(1-\alpha)(1-\varsigma)\left(1 - \frac{c_0}{1-\psi}\right)^2}{4}(2\theta + \theta^2), \tag{A1}$$

$$E\pi^* - E\pi^{**} = \frac{(1-\varsigma)N}{4}\left[\alpha(1-c_0)^2(1+\theta)^2 + (1-\alpha)(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2(1+\theta)^2\right]$$

$$= \frac{(1-\varsigma)N}{4}\left\{\alpha(1+\theta)^2\left[(1-c_0)^2 - \left(1 - \frac{c_0}{(1-\psi)}\right)^2\right] + (1-\alpha)\left[(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2(1+\theta)^2\right]\right\}$$

$$= \frac{(1-\varsigma)N}{4}\left[\alpha(1+\theta)^2 + (1-\alpha)\right]\frac{c_0\psi}{1-\psi}\left[2 - \frac{c_0(2-\psi)}{1-\psi}\right] - \psi N\left[\frac{1}{2} + \frac{c_0}{2(1-\psi)}\right]\left[\frac{1}{2} - \frac{c_0}{2(1-\psi)}\right](1+\theta)^2$$
$$+ \frac{(1-\varsigma)N}{4}\psi\left(1 - \frac{c_0}{1-\psi}\right)^2\left[\alpha(1+\theta)^2 + (1-\alpha)\right]$$
$$= \frac{\psi N}{4(1-\psi)^2}\left\{c_0(1-\varsigma)\left[\alpha(1+\theta)^2 + (1-\alpha)\right][2 - 2\psi - c_0(2-\psi)] + \left[(1-\psi)^2 - c_0^2\right](1-\varsigma)\left[\alpha(1+\theta)^2 + (1-\alpha)\right] - \left[(1-\psi)^2 - c_0^2\right](1+\theta)^2\right\} \geq 0.$$

$$E\pi^* - E\pi^{**} = \frac{(1-\varsigma)N}{4}\left[\alpha(1-c_0)^2(1+\theta)^2 + (1-\alpha)(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2(1+\theta)^2\right]$$

$$= \frac{(1-\varsigma)N}{4}\left\{\alpha(1+\theta)^2\left[(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2\right] + (1-\alpha)\left[(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{(1-\psi)}\right)^2(1+\theta)^2\right]\right\}$$

$$= \frac{(1-\varsigma)N}{4}\left[\alpha(1+\theta)^2\left(c_0^2 + \psi - \frac{c_0^2}{1-\psi}\right)\right]$$
$$+ \frac{(1-\varsigma)N}{4}(1-\alpha)\left[(1-c_0)^2 - (1-\psi)\left(1 - \frac{c_0}{1-\psi}\right)^2 - (1-\psi)\left(1 - \frac{c_0}{1-\psi}\right)^2(2\theta + \theta^2)\right]$$
$$= \frac{(1-\varsigma)N}{4}\left[\alpha(1+\theta)^2 + (1-\alpha)\right]\underbrace{\left(c_0^2 + \psi - \frac{c_0^2}{1-\psi}\right)}_{\geq 0} - \underbrace{N\psi\left[\frac{1}{2} + \frac{c_0}{2(1-\psi)}\right]\left[\frac{1}{2} - \frac{c_0}{2(1-\psi)}\right](1+\theta)^2}_{\geq 0}$$

The fourth equality above comes from (A1). The sign of $E\pi^* - E\pi^{**}$ can be seen in Fig. 2 with the numerical simulation.

Therefore, key person life insurance reduces the expected profits of firms. Notably, after comparing equation (23) with (12), key person insurance definitely reduces risk. Conclusions have been achieved and the proof is complete. ∎

References

Bertrand, P., Prigent, J.L., 2016. Equilibrium of financial derivative markets under portfolio insurance constraints. Econ. Modell. 52, 278–291.

Biener, C., Eling, M., Wirfs, J.H., 2016. The determinants of efficiency and productivity in the Swiss insurance industry. Eur. J. Oper. Res. 248 (2), 703–714.

Block, A., Keenan, M., Malone, C., 2006. Steps to getting key person claims paid. Risk Manag. 53 (2), 28–31.

Chandy, P.R., Davidson, W.N., Garrison, S., Worrell, D.L., 1986. Key man insurance: an empirical investigation of market reaction in large firms. J. Insur. Issue. Practic. 9 (1), 73–89.

Chen, Y.H., Wen, X.W., Nie, P.Y., Wang, B., 2017. Agricultural pollution and regulation: how to subsidize agriculture? J. Clean. Prod. 164, 258–264.

Courbage, C., Rey, B., 2016. On ambiguity apportionment. J. Econ. 118 (3), 265–275.

Dillender, M.O., Heinrich, C.J., Houseman, S.N., 2016. Health insurance reform and part-time work: evidence from Massachusetts. Lab. Econ. 43, 151–158.

Dineen, B.R., Allen, D.G., 2016. Third party employment branding: human capital inflows and outflows following “best places to work” certifications. Acad. Manag. J. 59 (1), 90–112.

Eling, M., Schaper, P., 2017. Under pressure: how the business environment affects productivity and efficiency of European life insurance companies. Eur. J. Oper. Res. 258 (3), 1082–1094.

Gatzert, N., Maegebier, A., 2015. Critical illness insurances: challenges and opportunities for insurers. Risk Manag. Insur. Rev. 18 (2), 255–272.

Israelsen, R.D., Yonker, S.E., 2017. Key human capital. J. Financ. Quant. Anal. 52 (1), 175–214.

Johnson, W.B., Magee, R.P., Nagarajan, N.J., Newman, H.A., 1985. An analysis of the stock price reaction to sudden executive deaths: implications for the managerial labor market. J. Account. Econ. 7 (1–3), 151–174.

Joshi, A., Liao, H., Jackson, S.E., 2006. Cross-level effects of workplace diversity on sales performance and pay. Acad. Manag. J. 49 (3), 459–481.

Lee, C.C., Chang, C.H., Arouri, M., Lee, C.C., 2016. Economic growth and insurance development: the role of institutional environments. Econ. Modell. 59, 361–369.

Nicholson, J.E., Corbett, R.B., 1987. Key man insurance and market reaction: a comment. J. Insur. Issue. Practic. 10 (1), 53–61.

Nie, P.Y., 2007. Selection games in economics. Appl. Econ. Lett. 14 (3), 223–225.

Nie, P.Y., 2014. Effects of capacity constraints on mixed duopoly. J. Econ. 112 (3), 283–294.

Nie, P.Y., Chen, Y.H., Yang, Y.C., Wang, X.H., 2016. Subsidies in carbon finance for promoting renewable energy development. J. Clean. Prod. 139, 677–684.

Nie, P.Y., Wang, C., Yang, Y.C., 2017. Comparison of energy efficiency subsidies under market power. Energy Pol. 110, 144–149.

Olsson, M., Thoursie, P.S., 2015. Sickness insurance and spousal labour supply. Lab. Econ. 33, 41–54.

Owusu, D.A., Appiah, S.K., Omari-Sasu, A.Y., Berimah, R.T., 2016. Pricing and profit testing of “key man” life policy of insurance companies in Ghana. Int. J. Financ. Account. 5 (4), 202–208.

Pieper, T.M., Smith, A.D., Kudlats, J., Astrachan, J.H., 2015. The persistence of multifamily firms: founder imprinting, simple rules, and monitoring processes. Enterpren. Theor. Pract. 39 (6), 1313–1337.

Wang, C., Nie, P.Y., 2016. Effects of asymmetric medical insurance subsidy on hospitals competition under non-price regulation. Int. J. Equity Health 15 (1), 184.

Wang, C., Nie, P.Y., 2018. A note on effects of rational bubble on portfolios. Phys. A Stat. Mech. Appl. 492, 50–56.

Wang, C., Nie, P.Y., Peng, D.H., Li, Z.H., 2017. Green insurance subsidy for clean innovation. J. Clean. Prod. 148, 111–117.