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On Optimal Audit Mechanisms for Environmental Taxes

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February 17, 2017

Abstract

We consider the auditing problem of an environmental enforcement agency with fixed audit resources: How to decide which firms to audit after having observed the firms' taxable emissions reports. The goal of the agency is to implement the socially efficient emissions level. The audit mechanism is the agency's sole choice variable, while other variables such as the tax rate on emissions and the fine for non-compliance are determined by other governmental actors. The fines and budget of the agency are constrained in such a way that the common random audit mechanism fails to implement socially efficient emissions. Assuming perfect information among the firms, we derive an optimal audit mechanism capable of implementing the socially efficient emissions level. The optimal audit mechanism creates a contest exploiting the strategic interdependencies between the firms, where the probability of winning (not being audited) for each firm depends on costly efforts (their taxable emissions reports).

Keywords: Environmental regulation; information disclosure; regulatory compliance; tournament theory; mechanism design

JEL classification: D62; H83; L51; Q58

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Abstract

We consider the auditing problem of an environmental enforcement agency with fixed audit resources: How to decide which firms to audit after having observed the firms' taxable emissions reports. The goal of the agency is to implement the socially efficient emissions level. The audit mechanism is the agency's sole choice variable, while other variables such as the tax rate on emissions and the fine for non-compliance are determined by other governmental actors. The fines and budget of the agency are constrained in such a way that the common random audit mechanism fails to implement socially efficient emissions. Assuming perfect information among the firms, we derive an optimal audit mechanism capable of implementing the socially efficient emissions level. The optimal audit mechanism creates a contest exploiting the strategic interdependencies between the firms, where the probability of winning (not being audited) for each firm depends on costly efforts (their taxable emissions reports).

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1 Introduction

Economists have frequently proposed unit taxes for industries with externalities, where the tax on emissions equalizes marginal damages and marginal benefits of emissions.[1] Environmental Protection Agencies (EPAs) are typically in charge of enforcing these taxes. The EPAs usually cannot observe emissions directly, and therefore firms are required to self-report their emissions to the EPAs and the EPAs audit as many firms as their budget allows.[2] The economic downturn in the past several years has led to severe cuts in the budgets of many EPAs. For instance, the operating budget of the US Environmental Protection Agency was reduced by 21% from 2010 to 2016.[3] At the same time, the production processes of regulated firms have become increasingly complex resulting in higher auditing costs for these firms. Smaller budgets for EPAs and the rising costs of auditing have amplified the need to identify cost-effective audit mechanisms. Audit mechanisms are strategies applied by the EPA in order to reach its objective of lowering the firms' emissions to the socially efficient level (i.e. where firms' marginal benefits from emissions are equal to the tax rate) by assigning to every firm a particular audit probability. This paper contributes towards the goal of designing an audit mechanism for the EPA that meets that objective while also taking into consideration the EPA's limited audit resources.

In this paper we use a stylized model to design step by step an audit mechanism for EPAs with limited resources that implements the socially efficient emissions level. We consider the EPA to be the designer of the audit mechanism, which can be constructed to make the regulated firms behave in a way the EPA desires. Under the derived optimal audit mechanism, the probability of auditing depends on the relative difference between a firm's emissions report and a reference value for reported emissions (a high emissions level close to the unregulated emissions level) relative to other firms. When a firm increases its emissions report then the firm's assigned audit probability decreases and the audit probabilities of all other firms increase. In essence, the optimal audit mechanism is a contest exploiting the strategic interdependencies between the firms. In this contest, firms compete for a prize (not being audited) by expending costly resources (their taxable emission reports).[4] Higher reported emissions by one firm, relative to the other firms, results in a lower audit probability for that firm and higher audit probabilities for the other firms. The intensiveness of this audit competition will

[1]The development of a corrective tax on emissions is generally attributed to Pigou (1920). Notable contributions include Baumol (1972), Weitzman (1974), Barnett (1980) and Benchekroun and van Long (1998). A recent survey can be found in Sandmo (2008).

[2]Refer to Telle (2013) for a description of the auditing mechanisms of the Norwegian Environmental Protection Agency as a typical example for the practices in many Western countries.

[3]See EPA's Budget and Spending (2016).

[4]See Konrad (2009) for a recent comprehensive survey on contests.

vary according to the EPA's audit budget, the emissions tax and the possible penalties. By "intensiveness of competition" we mean how quickly the audit probabilities per firm change in the reports. The lower the auditing budget or the penalties and the higher the tax rate, the higher will be the intensiveness of the audit competition designed by the EPA.

We utilize a multi-stage game to show that the proposed optimal audit mechanism announced at stage one induces regulated firms to choose socially efficient emissions at stage two, but to under-report their emissions at stage three in order to save on taxes. We find that the emission reports of firms are useful for implementing efficient emissions even though they are not truthful. They can be used to implement and harness strategic effects between the firms. Following Garvie and Keeler (1994), we assume that the EPA does not intend to raise revenue with the emission tax and that self-reported emissions are solely a vehicle to support enforcement, which was a concept first suggested by Kaplow and Shavell (1994). In other words, the EPA is not concerned with tax revenue but only with efficiency in emissions. Thus, from a welfare perspective, it is irrelevant that firms do not report emissions truthfully, and an audit mechanism that induces efficient emissions is welfare optimal.[5]

In our model, the EPA is not informed about firms' emissions and it faces a binding constraint on the number of firms it can afford to audit. Specifically, we focus on audit budgets which are sufficiently small such that random auditing fails to induce efficient emissions. Following Bayer and Cowell (2009), the optimal audit mechanism is designed under the assumption that there is perfect information between the firms. This assumption reflects environmental contexts where firms have more knowledge of each other's emissions than the EPA. For example, a firm is able to estimate the emissions of another firm more precisely than the EPA, as it can use information from its own experience in the production process, which the EPA does not have. We also prove the existence of the equilibrium in this framework.

Finally this paper demonstrates that where firms have no information about each other's emissions and the EPA has limited resources, the random audit mechanism (RAM) is the most efficient audit mechanism to reduce emissions. RAMs assign the same fixed audit probability to symmetric firms regardless of their emission reports. The audit mechanisms in the literature concerning RAMs differentiate the audit probabilities for each firm according to observable characteristics such as firm size, industry and other factors, but not based on firms' self-reported emissions.[6] If firms have the same observable characteristics, they are also assigned

[5]The model does not capture social costs from false reporting although these costs may exist in reality. For instance, truthful reporting on pollution may be required in order to allocate the necessary amount of effort towards cleanup. In those circumstances where false reporting does cause social costs, an audit mechanism that induces efficient emissions but not truthful reporting would not be optimal. We thank an anonymous referee for pointing this out.

[6]See Harford (1987), Kaplow and Shavell (1994), Sandmo (2002), Macho-Stadler and Pérez-Castrillo (2006) and Stranlund et al. (2009).

the same fixed audit probability regardless of their emission reports.[7] Given capped fines and limited auditing resources, RAMs fail to enforce socially efficient emissions if audit budgets are relatively small.[8] However, we will show that where the EPA's budget is limited and firms have no information on each other's emissions, the RAM weakly dominates all other audit mechanisms in terms of reducing emissions.

While the model is presented in the context of environmental taxation, it has wider applicability. In fact, the derived audit mechanism is relevant for any enforcement agency that makes audit decisions after having received imperfect, costly signals from regulated subjects about their compliance efforts. Other applications include the capital requirements for financial institutions, quality control, and the monitoring of corporate social responsibility activities.[9]

The remainder of the paper is organized as follows. Section two discusses the related literature. Section three presents the model with two firms making decisions about their actual emissions as well as the emissions on which they report and pay taxes. Section four summarizes the most important findings from the n-firms version of the model (where $n > 2$), while the n-firms version of the model is presented in detail in the Online Appendix. Section five concludes the paper. All proofs are contained in the Appendix.

2 Related Literature

Bayer and Cowell (2009) and Oestreich (2015) are relevant to this paper in that they suggest audit mechanisms in dynamic models (where the EPA and the regulated firms interact over several stages) based on relative comparisons of firms' self-reported actions. These dynamic models are also referred to as competitive audit mechanisms (CAMs) in the literature. CAMs are audit strategies that allocate more of the available audit resources to the firm with the lower report relative to other firms, while keeping the overall costs for auditing identical to the costs under random auditing. In Bayer and Cowell (2009), firms in an oligopolistic industry are subject to a profit tax. The authors introduce audit mechanisms where the probability of audit of a particular firm depends on that firm's tax report relative to others (assuming perfect information among the firms about each other's profits). The focus is on imperfectly discriminating audit mechanisms, i.e. the allocation of audit resources is influenced

[7] Macho-Stadler and Pérez-Castrillo (2006) write: "We have considered a model where the probability that a firm is audited is independent of the report [..]. We made this reasonable hypothesis because it simplifies the analysis."

[8] For a discussion of restrictions on the magnitude of penalties and fines in the environmental field, see for example Harrington (1988) or Heyes (2001).

[9] See Kotowski et al. (2014) for a current discussion about the wider applicability of auditing models including self-reporting.

to some degree by the firms' reports – but not completely. This type of CAM results in a "double dividend", i.e. firms produce more output and more accurate tax reports. Oestreich (2015) compares the incentives on both emissions and emissions reports under two types of CAMs (with different levels of intensiveness of competition) to the random audit mechanism. Both CAMs lead to more truthful reporting in comparison to the random audit mechanism. However, it is also found that depending on their exact specification, CAMs can induce higher or lower emissions among the regulated firms.[10] In both Bayer and Cowell (2009) and Oestreich (2015), the audit mechanisms are not tailored to achieve, and do not achieve, a socially efficient outcome; but they do improve effort and reporting choices among the firms as compared to random auditing. In contrast, in this paper we step by step derive the optimal audit mechanism that makes firms behave according to the socially efficient outcome. In addition, the optimal audit mechanism does not conform to the simplifying assumptions of the audit mechanisms suggested by Bayer and Cowell (2009) and Oestreich (2015).[11]

There are also several static models in the recent literature on CAMs. Gilpatric et al. (2011) study rank order tournaments, wherein the EPA will audit those firms for which the difference between expected and reported emissions is greatest. The EPA's expectation of firms' emissions is assumed to be subject to error, but on average it is accurate. The authors show that firms report higher emissions under this type of CAM in comparison to the random audit mechanism. The emissions level in Gilpatric et al. (2011) is assumed to be exogenous so the incentives for emissions reductions remain unclear. Cason et al. (2016) endogenize the emissions choices and the reporting choices of firms in a similar model to Gilpatric et al. (2011). They find that reporting is greater under CAMs in comparison to random auditing. They also find that the output (emissions) is the same under both audit mechanisms - that is emissions are independent of the applied audit mechanism. In contrast, the current paper finds, in line with Bayer and Cowell (2009) and Oestreich (2015), that the EPA can design the audit mechanism when audit resources are relatively low, such that the mechanism actually does effectively influence the firms in their choice of emissions, as well as in their choice of reporting. Colson and Menapace (2012) consider a model where the audit intensity is a function of actual and reported emissions. The key assumption is that the regulator has access to informative ambient emission measures for various subgroups of firms. They show that an audit mechanism that reallocates inspection efforts across these groups of firms based

[10] Several authors advocate the use of dynamic enforcement mechanisms that use the information obtained through an audit to assign the agent's probability of future audits (Harford (1987), Harrington (1988), Livernois and McKenna (1999), Heyes and Rickman (1999), Friesen (2003) and Gilpatric et al. (2015)).

[11] Specifically, under the optimal audit mechanism the audit probability of a firm changes in a concave manner in that firm's report, which is a violation of the simplifying assumption D4 by Bayer and Cowell (2009) and also a violation of the ratio form suggested by Oestreich (2015).

on each group's share of under-reported emissions can create strategic interactions among firms resulting in better environmental outcomes.

If the regulator has more information at its disposal, i.e. relatively accurate information about each firm's emissions (as in Gilpatric et al. (2011) and in Cason et al. (2016)) or informative ambient emission measures for a subgroup of firms (as in Colson and Menapace (2012)), the EPA could use this information in order to achieve better results than under random auditing. Information is an important resource; however, there are a number of circumstances wherein the regulator may not have such additional information. For instance, budget cuts may limit the availability of ambient emission measures. Also, relatively accurate estimates of every firm's emissions may not be available especially when it comes to new or changed technology or regulations. The derived optimal audit mechanism herein does not require the EPA to have such additional information, rather the EPA is solely informed by the emission reports of all firms.

3 The Model

Environmental Taxation We derive an optimal audit mechanism for a common externality tax in a framework similar to the one in Macho-Stadler and Pérez-Castrillo (2006). We define an audit mechanism as a strategy for deciding which of the regulated firms to select for an audit. We consider an industry with n firms that create negative externalities as a by-product of their production process. We will call the externality "emissions" and denote per-firm emissions by e_i.

The benefits a firm accrues from causing emissions are captured by a continuously differentiable benefit function $g(e_i)$. This benefit function is assumed to be strictly concave with a maximum at emissions level e^0. Hence, in the absence of regulation, a firm would choose the maximum emissions level that benefits its operating process, i.e. $e_i = e^0$. In order to control pollution, emissions are taxed at rate t. We suppose that t is exogenously given; it is set by other governmental actors. We think of t to induce the efficient per-firm emissions level e^t if firms comply with it and choose emissions according to $g'(e^t) = t$. This paper focuses on the problem of the EPA, which is in charge of enforcing the tax. It is important to note that the optimal audit mechanism is in principle able to enforce *any* tax rate on emissions whether or not this tax rate is set at the appropriate level (i.e. where marginal damages equal marginal benefits from emissions). Under the optimal audit mechanism, firms choose emissions such that their marginal benefits are equal to the tax rate. Thus, the optimal audit mechanism derived herein is a powerful tool in the hands of the EPA.[12]

[12]However, if the tax rate is set too high or too low, the according emissions may be too low or too high

Enforcement Issue The EPA is charged with enforcing the tax system. The agency is at a disadvantage in comparison to the firms as it can only observe emissions after conducting a costly audit. Its operating budget is fixed including the resources allotted to conduct audits. Let K be the number of firms which the agency can afford to audit, where $K \leq n$. For ease of exposition, we present in detail the case of *two firms*, where the agency can *audit one* of them ($n = 2$ and $K = 1$). The case of $n > 2$ and $K \geq 1$ is in the Online Appendix. The key insights from this $n-$firms case are discussed in section four of this paper.

The objective of the agency is to make all firms comply with the environmental tax, that is all firms choose emissions e^t. The choice variable of the agency is the audit mechanism: assigning an individual audit probability p_i to each of the firms.

Firms pay taxes on reported emissions r. Taxes on reported emissions may be potentially evaded by the firms. Thus, $e-r$ is the amount of under-reported emissions if $e > r$. Following Garvie and Keeler (1994), we assume that the EPA does not intend to raise money with the emission tax and that firms' self-reported emissions are solely a vehicle to ease enforcement.

The sole objective of the EPA is to make audit decisions to induce efficient emissions subject to its audit constraint. Before making its audit decision, the agency costlessly observes the vector of reported emissions from all firms. After an audit, the agency can observe the actual emissions caused by the firms and potentially levy a linear penalty θ per unit of under-reported emissions where $\theta \geq t$.

A brief overview of the applied multi-stage game is as following:

- In the *first stage*, the EPA announces an audit mechanism that will map emission reports into audit probabilities upon receiving the reports.
- In the *second stage*, firms choose emissions, which are observable to the other firms.
- In the *third stage*, firms choose emission reports.
- In the *fourth stage*, some of the firms are audited according to the announced audit mechanism at the first stage. Fines for potentially under-reported emissions are levied.

The timing of the game is natural as firms would produce emissions first before they account for them and report them to the agency. We follow Bayer and Cowell (2009) in assuming that there is perfect information between the firms, which means firms observe each other's

from a welfare point of view. Thus, an audit mechanism that fully enforces taxes which are either too low or too high would not be 'optimal' from a welfare point of view. Instead, when the tax is too high, an audit mechanism that does not fully enforce the tax may be welfare optimal. In order to call the audit mechanism 'optimal' in this paper, we consider the tax rate to be set at the appropriate level in the Pigouvian tradition so that an audit mechanism that fully enforces the tax on emissions can be called 'optimal'.

emissions. In many environmental contexts firms do have more knowledge of each other's emissions than the agency. For example, a firm is able to estimate the emissions of another firm more precisely than the authority, as it can use information from its own experience in the production process, which the agency does not have. The previous literature tends to neglect this information advantage by implicitly making the extreme assumption that firms have no information about each other's emissions. For simplicity, the current paper considers the opposite extreme, wherein firms are completely informed about each other's emissions.

Finally, the problem of firm i is to choose emissions e_i and reporting r_i to maximize expected profit:[13]

$$\max_{r_i \le e_i} \ \Pi_i = g(e_i) - tr_i - p_i\theta(e_i - r_i) \ \ for \ \ i = 1, 2. \tag{1}$$

Emissions provide benefits to the firm through the benefit function $g(e_i)$ and their cost is determined by tax t, their individual audit probability p_i and penalty θ.

Random Audit Mechanism Several studies in the aforementioned literature assume that the agency allocates equal audit probabilities among symmetric firms, regardless of the reports. We call this audit strategy the random audit mechanism (RAM) and it is used as benchmark throughout, formally: $p_i = 1/2 \ \forall r_1, r_2, \ for \ i = 1, 2.$ We note that the RAM can fully implement taxes on emissions if the expected marginal cost of under-reporting, $\theta/2$, is larger than the tax rate, t. In that case, firms have no beneficial alternative but to truthfully report their emissions. Knowing it is going to pay taxes on all of its emissions, a firm chooses socially efficient emissions. Thus, the agency can fully enforce taxes on all emissions and implement the socially efficient aggregate emissions level if either the audit rate or the fine are sufficiently large.

To reflect the reality of many enforcement agencies (constrained auditing budgets and capped fines), we focus on cases where the relation between tax t and fine θ does not lead to socially efficient emissions when only one firm can be audited and the RAM is applied.

Assumption 1 The relation between tax t and fine θ is given by:

$$\theta/2 < t < \theta.$$

[13] The agency does not reward over-reporting. If a firm is not audited, this firm pays $r_i t$ in taxes. If a firm is audited this firm pays in addition $\max\{\theta(e_i - r_i), 0\}$. Since over-reporting is not rewarded, optimality implies that reported emissions never exceed actual emissions, that is $r_i \le e_i$. Hence, without loss of generality, we can set $\max\{\theta(e_i - r_i), 0\} = \theta(e_i - r_i)$, and restrict the set of reported emissions to be $r_i \le e_i$.

Assumption 1 sets the stage for the interesting case in which the RAM fails to implement efficient emissions, because it is cheaper for a firm to evade taxes t and rather face the expected penalty $\theta/2$. Given Assumption 1, we establish next the reporting and the emissions level which is induced by the RAM.

Proposition 1 *If $\theta/2 < t$, the RAM fails to enforce socially efficient emissions. Instead, the RAM induces zero reporting, i.e.: $r_i = 0$ for $i = 1, 2$ and emissions that are higher in comparison to socially efficient emissions. The emissions per firm under the RAM are denoted by $e^{\theta/2}$, which is implicitly defined by:*

$$g'(e^{\theta/2}) = \theta/2 \quad for \quad i = 1, 2. \tag{2}$$

* * * Figure I about here * * *

Proposition 1 says that both firms report zero emissions so as to evade all tax payments. Instead, they opt for the expected fine for under-reported emissions under the RAM. The expected fine decreases emissions compared to unregulated emissions ($e^{\theta/2} < e^0$), even though the firm pays no taxes on emissions.[14]

Figure 1 illustrates the discussed enforcement problem with emissions per firm on the horizontal axis and marginal benefits (MB) on the vertical axis. Emissions e^t is the socially efficient emissions level for each firm, $e^{\theta/2}$ is the socially inefficient per-firm emissions level which results when the common RAM is used and $e^{\theta-t}$ is shown because it will be important later on.

General Audit Mechanism Since the RAM is not capable of implementing efficient emissions with capped fines and low auditing budgets, more intelligent audit mechanisms are required for these situations.

Definition 1 The audit mechanism is represented by function $p_i : (r_1, r_2) \to [0, 1]$ for $i = 1, 2$, which maps the vector of emission reports into probabilities for each firm of being audited.

Definition 1 introduces function $p_i(r_1,r_2)$ which is called *a decentralized mechanism* in the literature. Roughly formulated, a decentralized mechanism determines an outcome (probability of being audited for each firm) that depends on a vector of costly signals (firms' taxable

[14]Marchi and Hamilton (2006) show that in the case of air emissions in the US chemical industry, the regulated plants often do not accurately report their actual air emissions.

emission reports). The audit mechanism in Definition 1 determines the audit probabilities of each firm based on their emission reports. The expected audit probabilities in turn influence the firms' emissions decisions. The following analysis deals with the question of how the agency would design the audit mechanism $p_i(r_i, r_j)$ such that the mechanism influences the firms to act the way the agency desires, i.e. choosing socially efficient emissions. The audit mechanism is supposed to be *budget-balancing* and *symmetric,* which we define as the following.

Definition 2 A *budget-balancing* audit mechanism is defined by:

$$p_1(r_1, r_2) + p_2(r_1, r_2) = 1 \quad \forall r_1, r_2 \geq 0. \tag{3}$$

The budget of the agency allows for one audit out of the two firms, but the agency has to decide which one. Budget-balancedness excludes the possibility that the agency could audit one firm, but decides not to audit at all. Thus, the audit probabilities for the two firms have to add up to one. This implies that the probability that firm 1 is audited is equal to the probability that firm 2 is not audited and vice-versa.

Definition 3 A *symmetric* audit mechanism is defined by:

$$p_1(r_1,r_2) = p_2(r_2,r_1) \quad \forall r_1, r_2 \geq 0. \tag{4}$$

Symmetry of the audit mechanism implies that the audit probability is identical for each firm, if the vector of reports observed by the audit agency is reversed. In other words, the audit mechanism is "fair" in the sense that the agency does not discriminate systematically against one of the firms for reasons other than their emissions report. One implication of symmetry is that if firms' reports coincide then the audit probability is identical for both firms and both firms are audited with probability 1/2. We can now derive the first Lemma.

Lemma 1 *Any differentiable symmetric audit mechanism that exhausts the budget of the EPA for all r_1, r_2 satisfies at $r_1 = r_2$:*

$$\left.\frac{\partial^2 p_i}{\partial r_i \partial r_j}\right|_{r_i=r_j} = 0 \quad for \quad j \neq i = 1, 2. \tag{5}$$

Lemma 1 says that any mechanism allowing a firm to modify its audit probability through its emissions report cannot make the magnitude of this change contingent on the other firm's report when reports coincide. If the cross-partial derivative in Lemma 1 was not zero, an audit mechanism would either not be budget-balancing or would not be symmetric.

The solution concept applied is the Subgame Perfect Nash Equilibrium (SPNE). The game is solved by way of backwards induction focusing attention on symmetric equilibria.

3.1 Stage 3: Reporting Equilibrium

At this stage firms simultaneously choose emission reports to minimize the total cost of emissions. Throughout this section and the next, we focus on the point of view of firm 1. Firm 1's stage 3 profit-maximization problem given the audit mechanism $p_1(.)$, its own emissions e_1 and its competitor's report r_2, is:

$$\max_{r_1 \leq e_1} \; \Pi_1(p_1(.), e_1, r_1, r_2) = g(e_1) - tr_1 - p_1(r_1, r_2)\theta(e_1 - r_1). \tag{6}$$

Firms pay taxes for their reported emissions and they face expected penalties for their unreported emissions. For any $r_i \leq e_i$, it is better for a firm to declare its actual emissions rather than r_i, if r_i induces an auditing probability of $p_i\theta \geq t$ or $p_i \geq t/\theta$. That means, we can restrict the upper value of the audit probability to t/θ instead of 1. If $p_i = t/\theta$, it follows from the symmetry of the audit mechanism that $p_j = (1 - t/\theta)$. Consequently, without loss of generality, we can restrict auditing probabilities to a range between the lowest value $\underline{p} = (1 - t/\theta)$ and the highest value $\overline{p} = t/\theta$.

Differentiating (6) the first- and second-order conditions for a unique interior reporting solution – denoted by r_1^* – are:

$$\underbrace{p_1(r_1^*, r_2)\theta}_{\substack{\text{direct}\\ \text{MB}}} - \underbrace{\frac{\partial p_1(r_1^*, r_2)}{\partial r_1}\theta(e_1 - r_1^*)}_{\substack{\text{indirect}\\ \text{MB}}} \;=\; \underbrace{t}_{\text{MC}}, \; at \; r_1 = r_1^* \in [0, e_1], \tag{7}$$

$$2\frac{\partial p_1(r_1, r_2)}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1) \;<\; 0, \;\; \forall r_1 \in [0, e_1]. \tag{8}$$

The first-order condition (7) implies that the reporting equilibrium r_1^* can only be in the interior, i.e.: $0 < r_1^* < e_1$, if $\partial p_1/\partial r_1 < 0$ at $r_1 = r_1^*$. That means the agency applies an audit rule that allows firms to lower their assigned audit probability by increasing their emission reports, given their competitor's report. Such audit mechanisms have been suggested in some of the previous literature presented in section 2 "Related Literature". Given that the reporting choice is interior ($0 < r_1^* < e_1$), the first-order condition (7) has a simple "marginal benefit = marginal cost" interpretation: The marginal cost (MC) of reporting is t, i.e. higher reporting results in paying higher taxes. The marginal benefit (MB) of reporting has a *direct effect* and an *indirect effect* on the cost of emissions. First, reporting one more unit of emissions lowers the cost of emissions *directly*, because the amount of under-reported emissions decreases which lowers the expected fine by $p_1\theta$. Second, reporting emissions lowers the cost of emissions *indirectly*, because the audit probability decreases, which lowers the expected fine for the remaining under-reported emissions by $-(\partial p_1/\partial r_1)\theta(e_1 - r_1)$, given that $\partial p_1/\partial r_1 < 0$ in equilibrium. It is the indirect effect that may induce firms to report some of their emissions while they would report zero emissions under the RAM, i.e.: when $\partial p_1/\partial r_1 = 0$.[15]

In the Appendix (Section 6.3) we derive common comparative static results, namely how sensitive the emission reports of both firms are to changes in the emissions by firm 1, i.e.: the values for partials $\frac{\partial r_1}{\partial e_1}$ and $\frac{\partial r_2}{\partial e_1}$. The key insight for the following analysis is the observation that these two partials are solely dependent on the specific design of the audit mechanism. That means, the audit mechanism announced by the EPA influences how strongly a firm strategically changes its emission reports when itself or its competitor changes their emissions.

3.2 Stage 2: Emissions Equilibrium

At this stage firms simultaneously choose emissions while considering how emissions translate into the reporting equilibrium at stage 3. Firm 1's stage 2 profit-maximization problem given the audit mechanism $p_1(.)$ and its competitor's emissions e_2, is:

$$\max_{e_1 \geq 0} \; \Pi_1(e_1, e_2, r_1^*(e_1, e_2), r_2^*(e_1, e_2)) = g(e_1) - tr_1^* - p_1(r_1^*, r_2^*)\theta(e_1 - r_1^*). \tag{9}$$

To determine how emissions change profit, we consider the total derivative of Π_1 with respect to e_1.[16] From the optimization at the reporting stage we know that $\partial \Pi_1/\partial r_1 = 0$. Thus the effect of e_1 on Π_1 through the firm's own reporting choice should be ignored (this is the

[15] Combining the first-order condition and the second-order condition yields:

$$\frac{\partial^2 p_1(r_1^*, r_2)/\partial r_1^2}{(\partial p_1(r_1^*, r_2)/\partial r_1)^2} > -\frac{2}{t/\theta - p_1(r_1^*, r_2)}, \; at \; r_1 = r_1^* \in [0, e_1].$$

The above condition is necessary for a local maximum at $r_1 = r_1^* \in [0, e_1]$, which we note for later use.

[16] The argumentation closely follows Tirole (1988), p. 324.

envelope theorem). Only two terms remain:

$$\frac{d\Pi_1}{de_1} = \underbrace{g'(e_1) - p_1\theta}_{\substack{\text{Direct}\\ \text{effect}}} - \underbrace{\frac{\partial p_1}{\partial r_2}\frac{\partial r_2}{\partial e_1}\theta(e_1 - r_1^*)}_{\substack{\text{Strategic}\\ \text{effect}}}, \text{ for } e_1 \geq 0.$$

By changing e_1, firm 1 has a *direct effect* on its own profit. For instance, higher e_1 may have positive profit implications, if the benefits from emissions in the production process increase more quickly than the expected cost of e_1 regardless of any strategic effects. The *strategic effect* comes from the fact that e_1 not only changes the firm's own reporting behaviour, but also firm 2's reporting behaviour (by $\partial r_2/\partial e_1$). The change in firm 2's reporting behaviour affects the audit probability of firm 1, p_1, which in turn affects the firm's expected fine of unreported emissions (in proportion to $(\partial p_1/\partial r_2)\theta(e_1 - r_1^*)$). The total effect of e_1 on Π_1 is the sum of the direct and strategic effects.

Using (3) and (7), the first-order necessary condition can be written as:

$$\underbrace{g'(e_1)}_{MB} = \underbrace{p_1\theta + \frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1}(t - p_1\theta)}_{MC}, \; for \; e_1 \geq 0. \tag{10}$$

We define the left-hand side of (10) as marginal benefit (MB) of emissions and the right-hand side as marginal cost (MC) of emissions. Note that the condition for efficient emissions ($MC = t$) only holds if the tax is equal to the expected fine, i.e.: $t = p_1\theta$, or if $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} = 1$. The first condition, $t = p_1\theta$, also implements truthful reporting, which in turn implements efficient emissions. However, it is not feasible for the agency to induce $p_1\theta = t$ and $p_2\theta = t$, because the audit mechanism has to be budget-balancing, i.e. $p_1 + p_2 = 1$ or $2t/\theta = 1$, which contradicts the Assumption that $\theta/2 < t$.

The second condition,

$$(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} = 1, \tag{11}$$

is remarkable for the designer of the audit mechanism, because we know from the analysis of the reporting stage, that the three partials on the left-hand side of this condition solely depend on the specific design of the audit mechanism. Hence, the agency can design the audit mechanism in a way that influences the choice of emissions favourably, but it has to figure out which is the optimal way.

In contrast, if firms are uninformed about the other firms' emissions, the report of one firm cannot react to a change in the emissions of the other firm. Thus $\frac{\partial r_2}{\partial e_1} = 0$ and $g'(e_i) = p_i\theta$, i.e. the firm equalizes marginal benefits from emissions $g'(e_i)$ to marginal cost, $p_i\theta$. Hence, emissions e_i are a function of marginal cost, $p_i\theta$. Macho-Stadler and Pérez-Castrillo (2006) for instance, make the standard assumption that the marginal incentive of the marginal cost to reduce emissions is decreasing, i.e.: the function $e_i(p_i\theta)$ is convex. We find the following result for this case.

Proposition 2 *Given firms are uninformed about each other's emissions and the function $e_i(p_i\theta)$ is convex [both assumptions are in Macho-Stadler and Pérez-Castrillo (2006)], the RAM (or any other audit mechanism that induces $p_1 = p_2 = 1/2$ at $r_1 = r_2$) induces the lowest feasible emissions level in the industry. However, aggregate emissions are larger than efficient emissions in that case.*

To the best of our knowledge, Proposition 2 is a new result in the literature. It supports the common approach of using the RAM for symmetric firms among policies of continuous audit mechanisms for this particular information structure. In contrast, if firms do have information about each other's emissions, the agency can design more intelligent audit mechanisms which induce and harness strategic effects between the firms.

3.3 Stage 1: Designing the Optimal Audit Mechanism

At this stage the agency announces the audit mechanism in order to induce its desired behaviour among the regulated firms. Anticipating firms' strategic emissions and reporting behaviour at stage 2 and stage 3 respectively, this section derives a candidate for the optimal audit mechanism.

Insights from the Reporting Stage Since firms are symmetric, we conjecture that there is a symmetric SPNE.[17] It follows that emission reports and audit probabilities coincide in this case as well. We note that when reports coincide, we have $\frac{\partial p_1}{\partial r_1} = \frac{\partial p_2}{\partial r_2}$, which follows from Definition 3. Thus, in order to fulfill condition (11), the agency is solely concerned with designing the audit mechanism such that it induces $\partial r_2/\partial e_1 = 1$ at $r_1 = r_2$. That means the optimal situation for the agency in equilibrium may occur when firm 1's emission increases are strategically responded to by firm 2 by increasing its report by the same amount. This implicitly resembles a model where firm 2 tells the agency about the increasing emissions of firm 1, although here firms are solely asked to report on their own emissions.

Using (3), (4), (7), and evaluating $\partial r_2/\partial e_1$ at symmetry we obtain:[18]

[17] We show below that this symmetric equilibrium exists under certain conditions.
[18] See the Proof of Theorem 1 for details.

$$\left.\frac{\partial r_2}{\partial e_1}\right|_{r_1=r_2} = \frac{1}{(2+(\frac{t}{\theta}-\frac{1}{2})\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2})^2 - 1}. \tag{12}$$

Insights from the Emissions Stage Setting the right-hand side of equation (12) equal to one and solving for the relevant characteristic of the optimal audit mechanism leads to:[19]

$$\left.\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2}\right|_{r_1=r_2} = -\frac{2-\sqrt{2}}{t/\theta - 1/2}. \tag{13}$$

The left-hand side of the partial differential equation (13) is the ratio of the second derivative of the audit function $p_1(r_1, r_2)$ to its squared first derivative with respect to reporting.[20]

Let:

$$c \equiv \frac{2-\sqrt{2}}{t/\theta - 1/2}, \tag{14}$$

and solving the expression in (13) for $p_1(r_1, r_2)$ yields:[21]

$$p_1(r_1, r_2) = \kappa + \frac{1}{c}\ln(R - r_1), \;\; at \;\; r_1 = r_2 = r^*, \tag{15}$$

where κ and R are constants of integration. We explain the choice of R first and the choice of κ thereafter.

Reference Value for Reported Emissions Reference value R is an emissions level chosen by the EPA that the EPA uses as a point of reference for comparing firms' reported emissions.

[19] *The steps in detail:* setting the right-hand side of (12) equal to one is equivalent to:

$$\left.\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2}\right|_{r_1=r_2} = \begin{cases} -\frac{\sqrt{2}+2}{t/\theta-1/2}, \\ -\frac{2-\sqrt{2}}{t/\theta-1/2}. \end{cases}$$

Note, we can neglect the smaller fraction on the right-hand side of the above equation because it violates the condition in footnote 15. Recall, this condition is necessary to hold at equilibrium to ensure a local maximum at the reporting stage.

[20] The normalization of the second derivative by the first derivative is regularly found in economic modelling in order to determine functional forms. Consider, for example, the Arrow-Pratt measure of absolute risk-aversion, $A(w) = -\frac{\partial^2 u/\partial w^2}{\partial u/\partial w}$, where $u(w)$ is the von Neumann-Morgenstern utility function of an agent and w is its wealth.

[21] *The steps in detail*: taking the indefinite integral on both sides of equation (13) yields:

$$-\frac{1}{\partial p_1(r_1, r_2)/\partial r_1} = c(R - r_1), \;\; \text{at} \;\; r_1 = r_2 = r^*,$$

where R is an arbitrary constant of integration. Rearranging and taking the indefinite integral again on both sides yields (15).

The emissions level chosen by the EPA to be a reference value for reported emissions depends on the parameters of the model. The gap between the reference value and reported emissions influences the assigned audit probability to the reporting firm. The larger the gap, the larger the audit probability and the smaller the gap, the smaller the audit probability. We use $R = e^{\theta - t}$ in the following analysis, defined by $g'(e^{\theta-t}) = \theta - t$ and illustrated in Figure 1.[22] A refinement is suggested in footnote 25.

Implications of Symmetry and Budget-balancing Next, we explain the choice of κ in the derived audit function (15). Recall, the audit mechanism is defined to be symmetric and budget-balancing. In order to make the audit function in (15) symmetric and budget-balancing, it is required that $\kappa = \frac{1}{2} - \frac{1}{c}\ln(R - r_2)$. Thus:

$$p_1(r_1, r_2) = \frac{1}{2} + \frac{1}{c}\ln(\frac{R - r_1}{R - r_2}), \ \ at\ r_1 = r_2 = r^*. \tag{16}$$

Audit mechanism (16) is a derived and specific functional form that maps reported emissions into audit probabilities in such a way that it gives firms an incentive to choose efficient emissions. Recall that, by construction the optimal audit mechanism satisfies the necessary first-order condition to induce $e_1 = e_2 = e^t$ for all firms, i.e.: $g'(e_i) = t$ for $i = 1, 2$.

Limits for the Audit Probabilities Finally, we need to discuss the design of the optimal audit mechanism when reports do not coincide. Recall that we have restricted auditing probabilities to the set $[(1 - t/\theta), t/\theta]$ without loss of generality. If $p_1(r_1, r_2) \geq t/\theta$ then firm 1 is induced to report truthfully, i.e.: $r_1 = e_1$ and also to choose efficient emissions, i.e.: $e_1 = e^t$. Thus, increasing the audit probability beyond t/θ cannot improve the outcome for the agency, i.e.: $p_1(r_1, r_2) = t/\theta$ if $p_1(r_1, r_2) \geq t/\theta$ or if $\frac{1}{2} + \frac{1}{c}\ln(\frac{R-r_1}{R-r_2}) \geq t/\theta$ or if $r_1 \leq R - (R - r_2)\exp(2 - \sqrt{2})$ (where exp(.) denotes the natural exponential function). Equivalently, it is never optimal for the agency to increase the audit probability beyond t/θ for firm 2. The symmetry of the audit mechanism implies the following for the audit probability of firm 1: $p_1(r_1, r_2) = 1 - t/\theta$ if $p_1(r_1, r_2) \leq 1 - t/\theta$ or if $\frac{1}{2} + \frac{1}{c}\ln(\frac{R-r_1}{R-r_2}) \leq 1 - t/\theta$ or if $r_1 \geq R - (R - r_2)\exp(-(2 - \sqrt{2}))$. The next section presents our candidate for the optimal audit mechanism based on the analysis above.

[22] The lowest possible audit probability is $\underline{p}$ which results in marginal cost of emissions $\underline{p}\theta = \theta - t$. With marginal cost $(\theta - t)$, a firm's profit-maximizing emissions are $e^{\theta-t}$. Hence, the agency can be certain that $r < R$, if $R = e^{\theta-t}$. On the one hand, this reference value is small enough to encourage positive reporting levels. On the other hand this value is high enough to deter firms from reporting close to this value in order to minimize their assigned audit probability.

3.3.1 The Optimal Audit Mechanisms

Informed by the analysis above, a conjecture for the optimal audit mechanism for both firms is given by:

$$p_i(r_i, r_j) = \begin{cases} \underline{p} & \textit{if } r_i > R - (R - r_j)\exp(-(2-\sqrt{2})) \\ \overline{p} & \textit{if } r_i < R - (R - r_j)\exp(2-\sqrt{2}) \\ \frac{1}{2} + \frac{1}{c}\ln(\frac{R - r_i}{R - r_j}) & \textit{otherwise,} \end{cases} \tag{17}$$

where $\overline{p} = t/\theta$ and $\underline{p} = 1 - t/\theta$, c is a positive constant (depending on the magnitude of the tax and the penalty) defined in (14) and $R = e^{\theta - t}$ ensures that $R - r_i > 0$.

The derived audit mechanism is quite simple because the probability of auditing mainly depends on the relative differences between the two reports and a reference value R for reported emissions. Since the mechanism is based on a common *ln*-function, it has a natural interpretation: When firm i decreases the difference between its emission report and the reference value by one percent, then the firm's assigned audit probability decreases by $1/c$ percentage points. It is interesting to note that if the expected penalty under the RAM $\theta/2$ is equal to the emissions tax t, then $1/c = 0$ that is the optimal audit mechanism generalizes into the RAM for this special case. If $\theta/2 < t$ as per Assumption 1 then $1/c$ is positive. In fact, the larger the difference between t and $\theta/2$, the larger is $1/c$. In other words, the smaller the relative audit budget of the agency (measured by the difference between t and $\theta/2$), the larger the "intensiveness of competition" induced by the optimal audit mechanism. By intensiveness of competition we mean how quickly the audit probabilities per firm change in the reports.

Figure 2 illustrates the audit probabilities for both firms under the proposed optimal audit mechanism. Audit probabilities $p_1(r_1, r_2^*)$ and $p_2(r_1, r_2^*)$ are measured on the vertical axis dependent on r_1 which is measured on the horizontal axis. Report r_2 is fixed at the equilibrium reporting level r_2^*. If the reports coincide, the audit probabilities for both firms are 1/2. Increasing r_1 results in a lower audit probability for firm 1 and in a higher audit probability for firm 2. It can be shown that the slope of p_1 at $r_1 = r_2$ determines the level of reporting in equilibrium and the ratio of the curvature to the slope determines the level of emissions in equilibrium.

* * * Figure II about here * * *

Recall, the optimal audit mechanism fulfills by construction the necessary condition for the implementation of the efficient emissions level in the industry. In the following we work towards establishing a sufficient condition for the existence of outcome $e_1 = e_2 = e^t$ as a SPNE.

3.3.2 Reporting under the Optimal Audit Mechanism

The next Proposition establishes the reporting behaviour of firms under the proposed optimal audit mechanism.

Proposition 3 *The best response function of firm 1 in terms of reporting is given by:*

$$r_1(e_1, r_2) = \begin{cases} 0 & \text{if } r_2 < R - \frac{R}{\exp(2-\sqrt{2}-e_1/R)} \\ e_1 & \text{if } r_2 > R - \frac{R-e_1}{\exp(2-\sqrt{2})} \\ r_1^{int}(e_1, r_2) & \text{otherwise,} \end{cases} \tag{18}$$

where the interior reporting best response function $r_1^{int}(e_1, r_2)$ is increasing in e_1 and in r_2 as implicitly defined by the first-order condition for a profit-maximizing reporting choice:

$$\frac{e_1 - r_1^{int}}{R - r_1^{int}} + \ln\left(\frac{R - r_1^{int}}{R - r_2}\right) - 2 + \sqrt{2} = 0, \text{ at } r_1 = r_1^{int}. \tag{19}$$

* * * Figure III about here * * *

Figure 3 illustrates the best reporting response functions with the report of firm 1 on the vertical axis and the report of firm 2 on the horizontal axis. When firms' reports are close together (both are near the 45°-line) then the audit probabilities are in the interior, i.e.: $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$, which is the situation within the white cone. In this case, both firms report some of their emissions, while none of the firms reports truthfully. In Figure 3the curve $BR_2[e_2 : fix]$ is the best response function of firm 2 holding e_2 fixed $e_2 = e^t$. The three curves $BR_1[.]$ present the best response function of firm 1 for smaller, equal and larger e_1 in relation to e_2. All three illustrated SPNE are marked with black dots. Note, none of the SPNE are outside of the white cone. Proposition 4 links the reporting behaviour of Proposition 3 to the assigned audit probabilities of both firms.

Proposition 4 *The audit mechanism in (17) induces a unique and pure strategy reporting equilibrium at stage 3 of the game for any $e_i \in (0, e^{\theta-t})$. In any of these equilibria the audit probability is in the interior, i.e.: $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$.*

One implication of the Proposition is that for any profitable combination of emissions, no reporting equilibrium would lead to the scenario in which one of the firms is assigned the lowest possible audit probability $\underline{p}$. This means that with regard to Figure 3there are no SPNE along the upper left and upper right envelopes of the white cone.

Proposition 5 *The audit mechanism in (17) induces a symmetric reporting equilibrium at $e_1 = e_2 = e^t$ given by:*

$$r^* = \frac{e^t - R(2-\sqrt{2})}{\sqrt{2}-1}. \tag{20}$$

Reporting is zero if $e^t < R(2-\sqrt{2})$, where $R = e^{\theta - t}$.

The Proposition shows that equilibrium reporting r^* is decreasing in R and that equilibrium reporting is never truthful, given that $R = e^{\theta-t} > e^t$. Equilibrium reporting is positive if $e^t > (2-\sqrt{2})e^{\theta-t}$ and we recall that the functioning of the proposed audit mechanism relies on an reporting equilibrium which is non zero. We note that under-reporting of emissions is needed to generate the strategic effect. The strategic effect relies on firms changing their emission reports when one of the firms changes their emissions. In other words, under the optimal audit mechanism, it is not possible to achieve efficient emissions and truthful reporting in equilibrium. We recall that firms report zero emissions under the RAM when audit resources are low. Hence, the equilibrium level of reporting under the optimal audit mechanism is higher in comparison to the level of reporting under the RAM when audit resources are low.

The smaller the reference value for reported emissions, R, the larger is the reporting level in equilibrium. However, the smaller R, the larger is the possibility that the symmetric SPNE does not exist. The suggested value $R = e^{\theta-t}$ guarantees the existence of the symmetric SPNE and a positive reporting level given the sufficient condition stated below (footnote to Theorem 1).

The next Proposition offers some important insights into how the optimal audit mechanism works. Thereafter, we can state our main result.

Proposition 6a *Under the optimal audit mechanism, the reports of both firms increase (decrease) when one of the firms increases (decreases) its emissions. The firm that changes its emissions chooses a larger change in its reported emissions than the other firm, i.e.:*

$$\frac{\partial r_1^*}{\partial e_1} > \frac{\partial r_2^*}{\partial e_1} > 0, \ \forall \ e_1, e_2 \in [0, e^{\theta-t}],$$

whenever reports are positive.

Proposition 6b *The audit probability of firm 1, $p_1(r_1^*(e_1, e_2), r_2^*(e_1, e_2))$ decreases in e_1 and increases in e_2:*

$$\frac{\partial p_1}{\partial e_2} > 0 > \frac{\partial p_1}{\partial e_1}, \forall\ e_1, e_2 \in [0, e^{\theta - t}].$$

Proposition 6a says that if firm 1 increases its emissions by one unit, which is consequently responded to by a one-unit increase in firm 2's emissions report, then firm 1 also finds it worthwhile to increase its own emissions report by more than firm 2. This makes it rather unattractive for firm 1 to increase emissions. Proposition 6b shows that the audit probability of a firm decreases in its own emissions, but increases in its competitor's emissions. Figure 4illustrates the insights of Proposition 6a and 6b. The Figure provides an illustration of $r_1(e_1, e^t)$ and $r_2(e_1, e^t)$ under the proposed optimal audit mechanism with e_1 on the horizontal axis (while e_2 is fixed at $e_2 = e^t$) and the equilibrium reporting choices $r_1(e_1, e^t)$ and $r_2(e_1, e^t)$ on the vertical axis. If firm 1 unilaterally deviates upwards from $e_1 = e_2 = e^t$ to $e_1 > e^t$ then $r_2(e_1, e^t)$ increases by the exact same amount.

* * * Figure IV about here * * *

Why the Optimal Audit Mechanism Works In the equilibrium of the game, the two firms choose efficient emissions and they are both assigned an audit probability of 1/2 by the EPA. If a firm deviates upwards and chooses higher emissions (as it would under the RAM) this firm would benefit directly from its increased emissions. The (marginal) cost of these increased emissions is endogenously determined by the behaviour of both firms at the reporting stage.

At the reporting stage, the marginal benefit from reporting higher emissions increases for the deviating high-emissions firm, so it subsequently increases its report. Proposition 6b says that increasing emissions decreases its own audit probability and increases the audit probability of the low-emissions firm. As a strategic reaction, the non-deviating low-emissions firm will also increase its reported emissions because, given its increased audit probability, the marginal benefit from reporting higher emissions has increased as well. In fact, by design, the optimal audit mechanism induces the low-emissions firm to increase its report by exactly the same amount as the increase in emissions by the high-emissions firm. In other words, under the optimal audit mechanism, firm 1's emission increases are strategically responded to by firm 2 by increasing its reported emissions by the exact same amount.

As a result, the high-emissions firm finds itself forced to increase its report even more than the low-emissions firm to win the reporting competition, by which we mean that the high-emissions firm ends up with a lower audit probability. This is what Proposition 6a says. Thus, the high-emissions firm increases its reported emissions overproportionately and faces increased tax payments. The outcome of the reporting competition is that the high-emissions firm is assigned an audit probability less than 1/2 and the low-emissions firm is assigned an audit probability greater than 1/2. That is, the high-emissions firm has a lower expected fine for its unreported emissions.

To conclude, higher emissions result in higher benefits and a lower expected fine for under-reported emissions. These two benefits are offset by an overproportionate increase in reporting and hence higher tax payments. At the margin, the optimal audit mechanism leads by design to a marginal cost of emissions that is exactly equal to tax t. Thus, firms choose the efficient level of emissions in equilibrium.

We can now establish our main result:

Theorem 1 *If the audit mechanism satisfies budget-balancedness and symmetry, then the audit mechanism in (17) induces a symmetric pure strategy emissions equilibrium, where emissions are socially efficient, i.e.:* $e_1 = e_2 = e^t$*, implicitly defined by* $g'(e^t) = t$.[23]

Theorem 1 is proven in the Appendix. The Theorem makes an important contribution to the literature. First, it shows that it is possible to enforce socially efficient emissions among regulated firms even if the expected cost of non-compliance using random auditing is lower than the expected cost of compliance.

Second, we explicitly derived an audit mechanism for a specific enforcement problem which many EPAs around the globe may face. This is in contrast to some of the previous literature where the audit mechanisms presented were assumed to be exogenous to the enforcement agency (Bayer and Cowell (2009) and Oestreich (2015)). That means, the previous audit mechanisms were not tailored to a particular auditing issue. They were suggested in order to

[23]A sufficient condition for the existence of the equilibrium is that $g'(e)$ is sufficiently steep such that a positive reporting equilibrium occurs and such that a pure strategy emissions equilibrium exists. Specifically, this condition ensures that $MB(e_1)$ intersects $MC(e_1)$ exactly once from above at $e_1 = e^t$. In other words, the marginal benefits from causing emissions have to decline quickly enough in e. Formally: there exists some $m > 0$ such that if $|g''(e)| > m$ for all $e \in [0, e^0]$, then $e^t > (2 - \sqrt{2})e^{\theta - t}$ and a pure strategy emissions equilibrium exists. Parameter m remains unspecified. We would expect this steepness condition to hold regularly in environmental tax systems because of the well-established Weitzman Proposition which states that *if the aggregate marginal benefit function is steep relative to the aggregate marginal damages function,* then a price measure tends to be the preferred policy instrument over a quantity measure to regulate emissions (see for instance Kolstad (2011), pp. 310). We also note that the emission tax in the current paper is a price measure.

improve efforts and reporting choices among the firms as compared to random auditing, and they did not achieve a socially efficient outcome.

Third, while it has been argued elsewhere that emission reports are not useful for the EPA when they are not truthful, we find that the reports can be used to implement efficient behaviour even though they are not truthful.[24] They can be used to implement and harness strategic effects between the firms in order to achieve better outcomes for the environment.

Finally, Theorem 1 strengthens the idea of implementing audit mechanisms that use as a basis the difference between reported emissions and a reference value for reported emissions. For instance, Gilpatric et al. (2011) suggest an audit mechanism that is based on the difference between the report of a firm and a reference value. In their paper, the reference value is the EPA's noisy signal about a firm's emissions. In the current paper, the EPA does not have such information but instead uses R as reference value for reported emissions. If the EPA had more prior information about the emissions of firms (before conducting an audit) we conjecture that the EPA could improve the proposed audit mechanism in two ways: a) this mechanism may be able to induce higher reporting levels, and b) this mechanism may be able to enforce efficient emissions for a wider parameter set than the one suggested in the current context.[25]

3.3.3 Illustrative Example

Suppose the two firms in an industry gain marginal benefits from emissions equal to $g'(e_i) = 1 - e_i$, for $i = 1, 2$ resulting in unregulated emissions per firm of $e^0 = 1$. Suppose further that the socially optimal tax to regulate emissions in this industry is $t = 1/2$ resulting in efficient emissions per firm of $e^t = 1/2$. Say the EPA has audit resources to inspect only *one* of the *two* firms, and the penalty for under-reported emissions is $\theta = 2/3$. We note that the expected penalty under the RAM is $\theta/2 = 1/3$ which is lower than the tax rate. Hence, under the RAM, firms report zero emissions and produce higher than socially efficient emissions, specifically: $r_i^{RAM} = 0$ and $e_i^{RAM} = 2/3$, for $i = 1, 2$.

In contrast, the proposed optimal audit mechanism is capable of enforcing socially efficient emissions (using the refinement suggested in footnote 25, i.e.: $R = 2/3$ and $c \approx 2.34$). For illustrative purposes, let the audit agency announce the optimal audit mechanism and consider

[24] For instance, Colson and Menapace (2012) write: "In the Macho-Stadler and Pérez-Castrillo [2006] model, the auditing probability is by construction unaffected by firms' actions because the enforcement agency has no useful information on which the inspection probability can be conditioned on."

[25] Note, the proposed audit mechanism can be refined if $g'(.)$ is linear, i.e.: $g'''(.) = 0$. In this particular case, reference value $R = e^{\theta/2}$, implicitly defined by $g'(e^{\theta/2}) = \theta/2$, can be applied to successfully deter deviations from $e_1 = e_2 = e^t$ and thus the audit mechanism in (17) can induce efficient emissions for a larger set of parameter values in comparison to reference value $R = e^{\theta - t}$.

the following reports (off-equilibrium) by the firms: $r_1 = 0.3$ and $r_2 = 0.4$. Consequently, the audit mechanism assigns audit probabilities $p_1 \approx 0.64$ and $p_2 \approx 0.36$ to the firms and one of them will be audited based on those probabilities.

In the symmetric SPNE under the optimal audit mechanism, it is profit maximizing for the two firms to choose efficient emissions $e_i^t = 1/2$ and to report according to (20), i.e.: $r_i \approx 0.26$, for $i = 1, 2$. In conclusion, the audit agency achieves its objective in terms of emissions with the help of the optimal audit mechanism.

4 The n-Firms Case

This section summarizes the analysis of the n-firms case, where $n > 2$. The main insight and intuition of the n-firms case is similar to the two-firms case above while the notation is more complex. The detailed analysis is in the attached online Appendix where we derive step by step the optimal audit mechanism for n firms.

Let K denote the subset of firms the agency can afford to audit ($K \geq 1$). We begin by defining the audit ratio as $k \equiv K/n$.[26] We use the same Definitions of the audit mechanism as in the case of two firms, namely the audit mechanism is defined to be budget-balancing[27] and symmetric[28]. Furthermore, the budget of the EPA is assumed to be insufficient to implement efficient emissions with the common random audit mechanism (RAM) which is established by Assumption I.

Assumption I *The relation between tax t, fine θ and the fraction of firms that can be audited k is given by:*

$$k\theta < t < \theta.$$

An implication of Assumption I is that it is cheaper for a firm to evade tax t and rather

[26] For instance, in Canada's largest province Ontario, the operating budget of the Ministry of the Environment (MOE) allows for approximately 5,000 inspections each year while MOE is responsible for at least 125,000 facilities (ECO 2007, pp. 23–24). Accordingly, the audit ratio in Ontario is approximately 4%.

[27] A *budget-balancing audit mechanism* is defined by:

$$\sum_{i=1}^{n} p_i(\mathbf{r}) = K, \tag{21}$$

where r denotes the vector of all n emission reports, i.e. $(r = r_1, ..., r_n)$.

[28] A *symmetric audit mechanism* is defined by:

$$p_i(r_1, .., r_i, .., r_j, .., r_n) = p_j(r_1, .., r_i, .., r_j, .., r_n)\ \forall r_1, .., r_i, .., r_j, .., r_n \geq 0 \tag{22}$$

and p_i is unchanged if we switch r_j and r_k where $j, k \neq i$.

face the expected penalty $k\theta$ under the RAM. The derived optimal audit mechanism (from the point of view of firm 1) for n firms is given by:

$$p_1(r_1, ..., r_n) = \begin{cases} \underline{p} & \text{if } p_1 \leq \underline{p}, \\ \overline{p} & \text{if } p_1 \geq \overline{p}, \\ k + \dfrac{1}{c(n-1)} \ln\left(\dfrac{(R-r_i)^{n-1}}{\prod_{j\neq i}^{n}(R-r_j)}\right) & \text{otherwise.} \end{cases} \tag{23}$$

where $\overline{p} = t/\theta$ and $\underline{p} = K - (n-1)t/\theta$, c is a positive constant determined below, and R is the reference value for reported emissions such that $R - r_i > 0$ $\forall r_i \in (0, e_i)$. It is sufficient, yet not necessary to set $R = e^0$ to achieve $R - r_i > 0$, assuming that unregulated emissions e^0 are known to the enforcement agency from the time without emissions regulation.[29] The value for constant c is:

$$c \equiv \frac{2-N}{t/\theta - k}, \text{ where: } N = \frac{n - 2 + \sqrt{n^2 + 4n - 4}}{2\,(n-1)}$$

and we note that N is decreasing in a convex manner in the number of firms n such that $N = \sqrt{2}$ when $n = 2$ and $N \to 1$ when $n \to \infty$.

Equivalent to the case with two firms, the optimal audit mechanism requires the reporting equilibrium to be positive.

Theorem I *If the audit mechanism satisfies budget-balancedness and symmetry, then the audit mechanism in (23) induces a symmetric pure strategy emissions equilibrium, where emissions are socially efficient, i.e.:* $e_1 = ... = e_n = e^t$, *implicitly defined by* $g'(e^t) = t$.[30]

The Theorem is proven in the online Appendix. The main implication of the Theorem is that as long as the EPA can afford to audit one firm, then regardless of the number of firms in the industry, we are able to construct an audit mechanism that induces efficient emissions for all firms at least for some parameters of the model, while the RAM would fail to enforce efficient emissions.

The impact of the number of firms in the industry is interesting. We keep the audit ratio $k = K/n$ constant and investigate changes in the mechanism in (23) when K and n change

[29] This assumption comes without loss of generality. More consistent with the two-firms case would be to set $R = e^{\theta K-(n-1)t}$. This is because $e^{\theta K-(n-1)t}$ is the profit-maximizing emissions level corresponding to the lowest possible audit probability $\underline{p}$ with related expected penalty $\underline{p}\theta$. To simplify the presentation we use $R = e^0$.

[30] A sufficient condition is that $g'(e)$ is sufficiently steep such that a positive reporting equilibrium occurs and such that a pure strategy emissions equilibrium exists. However, if n is relatively large and K is relatively small, solely very steep private benefit functions $g'(.)$ would ensure efficient emissions.

by the same factor. We observe that $1/c$ is decreasing when n and K increase by the same factor. That is, the "intensiveness of competition" induced by the optimal audit mechanism is decreasing in the number of firms. Thus, the more firms there are in the industry (keeping the audit ratio constant), the less intensiveness of competition is necessary in the audit contest in order to induce efficient emissions.

4.0.4 Illustrative Example

Suppose there are three firms in an industry and each of them gain marginal benefits from emissions equal to $g'(e_i) = 1 - e_i$, for $i = 1, 2, 3$ resulting in unregulated emissions per firm of $e^0 = 1$. The socially optimal tax to regulate emissions in this industry is assumed to be $t = 1/4$ resulting in efficient emissions per firm of $e^t = 3/4$; the penalty for under-reported emissions is assumed to be $\theta = 1/2$. Say the EPA has audit resources to inspect *one* of the *three* firms, i.e.: $n = 3$, $K = 1$ and $k = 1/3$. We note that in this case the expected penalty under the RAM is $k\theta = (1/3)(1/2) = 1/6$ which is lower than the tax rate. Hence, under the RAM, firms report zero emissions and produce higher than socially efficient emissions, specifically: $r_i^{RAM} = 0$ and $e_i^{RAM} = 5/6 > e^t$, for $i = 1, 2, 3$.

In contrast, the proposed optimal audit mechanism is capable of enforcing socially efficient emissions. The optimal audit mechanism takes on the following form in this particular example (from the point of view of firm 1):

$$p_1(r_1, r_2, r_3) = \begin{cases} 0 & \text{if } p_1 \leq 0, \\ 1/2 & \text{if } p_1 \geq 1/2, \\ \frac{1}{3} + \frac{1}{2c}(\ln \frac{1-r_1}{1-r_2} + \ln \frac{1-r_1}{1-r_3}) & \text{otherwise.} \end{cases}$$

where $\overline{p} = t/\theta = 1/2$ and $\underline{p} = kn - (n-1)t/\theta = 0$, constant $c \approx 4.32$, audit ratio $k = 1/3$ and reference value $R = 1$.

For illustrative purposes, let the audit agency announce the optimal audit mechanism and consider the following reports (off-equilibrium) by the firms: $r_1 = 0.30$, $r_2 = 0.40$ and $r_3 = 0.50$. Consequently, the audit mechanism assigns audit probabilities $p_1 \approx 0.39$, $p_2 \approx 0.34$ and $p_3 \approx 0.27$ to the firms and *one* out of the *three* firms will be audited based on those probabilities. We note in passing that $\sum p_i = 1$ as the audit mechanism has to be budget-balancing.

In the symmetric SPNE, it is profit maximizing for all three firms to choose efficient emissions $e_i^t = 3/4$ and to under-report their emissions according to $r_i \approx 0.11$, for $i = 1, 2, 3$. To conclude, the audit agency achieves its objective in terms of emissions with the help of the optimal audit mechanism.

5 Conclusion

We have derived the optimal audit mechanism for EPAs with limited audit resources that can meet their objective of lowering the firms' emissions to the socially efficient level. The fines and budget of the EPA are constrained in such a way that the common random audit mechanism (RAM) fails to implement the socially efficient emissions level. In terms of policy implications, the insights gained in this paper question the common practice of EPAs to keep their auditing mechanisms confidential. This paper makes the case that publicly announced audit mechanisms can induce strategic behaviour among firms, which can improve the effectiveness of auditing efforts. In our model, we abstract from elements of the environmental enforcement issue that are complementary to our analysis. Some of these are as follows.

While the optimal audit mechanism induces firms to choose socially efficient emissions, it does not induce truthful reporting when audit resources are low. This may be a limitation of the optimal audit mechanism especially if there are social costs attached to untruthful emission reports. However, while it has been argued elsewhere that emission reports are not useful for the EPA because they are not truthful, we contradict this notion and find that the reports can be used to implement efficient behaviour even though they are not truthful. They can be used to implement and harness strategic effects between the firms in order to achieve better outcomes for the environment. Considering the social cost of untruthful reporting could be an avenue for future research.

In addition, in our model firms are assumed to be symmetric given the common practice by EPAs of sorting firms for auditing purposes according to observable characteristics, such as industry and size. As for potential unobservable factors, such as firm heterogeneity in technology, it is possible to show that the optimal audit mechanism induces less aggregate emissions in the industry in comparison to the socially efficient emissions level. However, it is not immediately clear that under the optimal audit approach a solution which implements efficient emissions could be found. In any case, the option of playing the firms off against each other provides the EPA with an extra auditing tool. Where it proves more beneficial to use that tool, the agency could choose to do so. Where it does not, the EPA could simply revert to random auditing.[31]

Finally, the optimal audit mechanism is designed under the assumption that firms have

[31]Some environmental protection agencies do announce that they use reported emissions in their auditing strategies (see for instance EPA Victoria (2011) for a technical report about the auditing procedure at the Australian province Victoria).

perfect information about each other's emissions. This assumption represents environmental contexts where firms have more knowledge of each other's emissions than the EPA; however, it may not reflect other contexts, such as industries with many small-sized firms. While this structure of perfect information is common in the current literature, it would be a valuable extension to derive an optimal audit mechanism in a framework of imperfect information among the EPA and the firms. That is, EPAs and firms would obtain upfront a noisy signal about firms' emissions, but the signal to the firms about other firms would be more accurate than the signal to the EPA. This extension would more realistically reflect the notion that firms have more information about other firms' emissions from their own production processes, and it is left for future research.

6 Appendix

6.1 Proof of Proposition 1

At stage 3, since $-tr_i - \theta/2(e_i - r_i)$ is decreasing in r_i, firms choose $r_i = 0$. At stage 2, firms adjust their emissions according to the marginal expected fine, i.e.: $g'(e^{\theta/2}) = \theta/2$. Since $g(.)$ is strictly concave and $\theta/2 < t$, we have $e^t > e^{\theta/2}$.

6.2 Proof of Lemma 1

First, we have the following set of implications from (3):

$$\begin{aligned} \frac{\partial p_1}{\partial r_1} + \frac{\partial p_2}{\partial r_1} &= 0 \;\; \forall r_1, r_2 \geq 0, & (24) \\ \frac{\partial^2 p_1}{\partial r_1^2} + \frac{\partial^2 p_2}{\partial r_1^2} &= 0 \;\; \forall r_1, r_2 \geq 0, & (25) \\ \frac{\partial^2 p_1}{\partial r_1 \partial r_2} + \frac{\partial^2 p_2}{\partial r_1 \partial r_2} &= 0 \;\; \forall r_1, r_2 \geq 0. & (26) \end{aligned}$$

Second, we have the following set of implications from (4) when reports coincide ($r_1 = r_2$):

$$\begin{aligned} \left.\frac{\partial p_1}{\partial r_1}\right|_{r_1=r_2} &= \left.\frac{\partial p_2}{\partial r_2}\right|_{r_1=r_2} & (27) \\ \left.\frac{\partial^2 p_1}{\partial r_1^2}\right|_{r_1=r_2} &= \left.\frac{\partial^2 p_2}{\partial r_2^2}\right|_{r_1=r_2} & (28) \\ \left.\frac{\partial^2 p_1}{\partial r_1 \partial r_2}\right|_{r_1=r_2} &= \left.\frac{\partial^2 p_2}{\partial r_1 \partial r_2}\right|_{r_1=r_2} & (29) \end{aligned}$$

Finally, a mechanism that exhausts the budget of the regulator satisfies (26). Any symmetric mechanism satisfies (29) at $r_1 = r_2$. Equation (26) and (29) can both hold if and only if (5) is true.

6.3 Comparative Statics: $\frac{\partial r_1}{\partial e_1}$ and $\frac{\partial r_2}{\partial e_1}$

Totally differentiating the system of first-order conditions for firm 1 (7) and firm 2 yields:

$$\begin{bmatrix} 2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1) & \frac{\partial p_1}{\partial r_2} - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(e_1 - r_1) \\ \frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(e_2 - r_2) & 2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2) \end{bmatrix} \begin{pmatrix} dr_1 \\ dr_2 \end{pmatrix} = \begin{bmatrix} \frac{\partial p_1}{\partial r_1} & 0 \\ 0 & \frac{\partial p_2}{\partial r_2} \end{bmatrix} \begin{pmatrix} de_1 \\ de_2 \end{pmatrix}. \quad (30)$$

Applying Cramer's rule to system (30) leads to:

$$\frac{\partial r_1}{\partial e_1} = \frac{\frac{\partial p_1}{\partial r_1}(2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2))}{|D|}, \tag{31}$$

$$\frac{\partial r_2}{\partial e_1} = \frac{-\frac{\partial p_1}{\partial r_1}(\frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(e_2 - r_2))}{|D|}, \tag{32}$$

where $|D|$ is:

$$\begin{aligned} |D| \;=\; & [2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1)][2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)] \\ & -[\frac{\partial p_1}{\partial r_2} - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(e_1 - r_1)][\frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_1 \partial r_2}(e_2 - r_2)]. \end{aligned} \tag{33}$$

Note, if the set of second-order conditions (8) hold for both firms and if determinant $|D|$ is positive for all $r_1 \in [0, e_1]$ and $r_2 \in [0, e_2]$, both conditions imply global uniqueness of the equilibrium (see Nikaido (1968, ch. 7)). In addition these conditions satisfy the Routh-Hurwitz conditions for reaction function stability.[32] We will show in the proof of Proposition 4 that the presented optimal audit mechanism indeed satisfies these conditions.

6.4 Proof of Proposition 2

Given that efficient emissions in the industry are unattainable for the agency, it aims to make audit decisions in order to minimize emissions. Thereby it solves the following program:

$$\min_{p_1, p_2} e_1(p_1\theta) + e_2(p_2\theta) \;\; s.t. \;\; p_1 + p_2 = 1$$

Given that $e_i(p_i\theta)$ is convex for both firms, the best the agency can do is to set $p_1 = p_2 = \frac{1}{2}$, which is exactly what the RAM does or any other audit mechanism that implements equal audit probabilities in equilibrium. Choosing any $p_1 \neq p_2$ is not optimal, because any linear combination of a convex function is above that function, i.e. the agency would end up with higher emissions.

6.5 Proof of Proposition 3

Firm 1's stage 3 problem is to choose some $r_1 \in [0, R]$ to maximize:

$$\Pi_1 = \begin{cases} -tr_1 - \underline{p}\theta(e_1 - r_1) & \textit{if } p_1 \leq \underline{p} \textit{ (large } r_1\textit{)} & \textit{Case (i)} \\ -tr_1 - p_1\theta(e_1 - r_1) & \textit{if } p_1 \in (\underline{p}, \overline{p}) \textit{ (intermediate } r_1\textit{)} & \textit{Case (ii)} \\ -tr_1 - \overline{p}\theta(e_1 - r_1) & \textit{if } \overline{p} \leq p_1 \textit{ (small } r_1\textit{)} & \textit{Case (iii).} \end{cases}$$

[32] A concise account of the Routh-Hurwitz problem can be found in Coppel (1965). According to the Routh-Hurwitz conditions, a 2×2 real matrix A is stable if and only if $tr(A) < 0$ and $\det(A) > 0$.

We analyze each of the three cases (i)-(iii) separately:

Case (i): The first case, $p_1 \leq \underline{p}$, applies whenever $r_1 \geq R - (R - r_2)\exp(\sqrt{2} - 2)$. This case necessitates that $e_1 > r_1 \geq R - (R - r_2)\exp(\sqrt{2} - 2)$. On this range, profit Π_1 is decreasing in r_1, i.e.: $\frac{\partial \pi_1}{\partial r_1} = 2(\frac{\theta}{2} - t) < 0$. Hence, it can never be optimal to report some $r_1 > R - (R - r_2)\exp(\sqrt{2} - 2)$. Aside, at $r_2 = 0$, it is never optimal to report some $r_1 > R(1 - \exp(\sqrt{2} - 2)) \approx 0.44R$.

Case (ii): The second case, $p_1 \in (\underline{p}, \overline{p})$, applies when $\frac{1}{2} + \frac{1}{c}\ln(\frac{R-r_i}{R-r_j}) \in (\underline{p}, \overline{p})$. In this case, firm 1 chooses $r_1^{int}(e_1, r_2)$ given e_1 and r_2 to satisfy the first-order condition under the proposed candidate for the optimal audit mechanism. Hence, $r_1^{int}(e_1, r_2)$ is implicitly defined by $\frac{\partial \Pi_1}{\partial r_1} = 0$ which is:

$$\frac{e_1 - r_1^{int}}{R - r_1^{int}} + \ln(\frac{R - r_1^{int}}{R - r_2}) - 2 + \sqrt{2} = 0, \ at\ r_1 = r_1^{int}.$$

The second-order condition for concavity of the profit function in the firm's report is:

$$-\frac{1}{(R - r_1)^2}(2R - e_1 - r_1) < 0, \ \forall r_1 \in [0, e_1]. \tag{34}$$

The second-order condition holds with certainty as long as $R > e_1$, which is guaranteed with $R = e^{\theta - t}$.

Note, the interior part of the best response function r_1^{int} is increasing in a convex manner in r_2. This can be seen by totally differentiating (19) and solving for $\partial r_1/\partial r_2$ and $\partial^2 r_1/\partial r_2^2$ respectively:

$$\begin{aligned}\frac{\partial r_1}{\partial r_2} &= \frac{(R - r_1)^2}{(R - r_2)(2R - e_1 - r_1)} > 0, \\ \frac{\partial^2 r_1}{\partial^2 r_2} &= \frac{(R - r_1)^2}{(R - r_2)^2(2R - e_1 - r_1)} > 0.\end{aligned}$$

Note, the firm chooses $r_1 = 0$ if $r_2 < R - \frac{R}{\exp(2-\sqrt{2}-e_1/R)}$ (can be derived from (19)).

Note also, from case (i) we know that $r_1 = R - (R - r_2)\exp(\sqrt{2} - 2)$ in case $r_1^{int} > R - \exp(\sqrt{2} - 2)(R - r_2)$. Combining the latter with (19) gives $\overline{r}_2$, where $\overline{r}_2 = R + \frac{R - e_1}{(3-2\sqrt{2})\exp(\sqrt{2}-2)}$, which is the required reporting level of firm 2 that would lead to $p_1 \leq \underline{p}$ and $p_2 \geq \overline{p}$. Note, that $\overline{r}_2$ is outside of the rational action space of firm 2, i.e.: $\overline{r}_2 \notin [0, e^{\theta - t})$, since $R = e^{\theta - t}$ and $\overline{r}_2 > e^{\theta - t}$. Resulting, the best response function of firm 1 never leads to the existence of case (i), where $p_1 \leq \underline{p}$. Hence, there cannot be an equilibrium where $p_1 \leq \underline{p}$ and $p_2 \geq \overline{p}$. Since firms are symmetric, there can also be no equilibrium where $p_2 \leq \underline{p}$ and $p_1 \geq \overline{p}$. Consequently, in all reporting equilibria we have $p_1 \in (\underline{p}, \overline{p})$.

Case (iii): The third case, $\overline{p} \leq p_1$, applies whenever $r_1 \leq R - (R - r_2)\exp(2 - \sqrt{2})$. On this range, profit Π_1 does not change in r_1, i.e.: $\frac{\partial \Pi_1}{\partial r_1} = 0$. We assume $r_1 = e_1$ in this case in order to simplify the exposition, but without loss of generality. Aside, $e_1 \leq R - (R - r_2)\exp(2 - \sqrt{2})$ is equivalent to $\frac{R - r_2}{R - e_1} < \frac{1}{\exp(2-\sqrt{2})} \approx 0.56$. The latter expression shows that this case necessitates that $r_2 > e_1$.

6.6 Proof of Proposition 4

Recall from the proof of Proposition 3 (case (ii)) that every $e_i \in (0, e^{\theta-t})$ for $i = 1, 2$ leads to reporting equilibria that cause $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$.

Recall next that $r_1 = 0$ if $r_2 \leq R - \frac{R}{\exp(2-\sqrt{2}-e_1/R)}$. In this case, the best response of firm 2 is implicitly defined by its best-response function $\frac{e_2-r_2}{R-r_2} + \ln(\frac{R-r_2}{R}) - 2 + \sqrt{2} = 0$. The equivalent argument holds for $r_2 = 0$. In all other cases $r_1, r_2 > 0$.

Under the optimal audit mechanism, determinant (33) is positive whenever $p_i \in (\underline{p}, \overline{p})$. To show this, plugging in the partials of the optimal mechanism in (36) yields:

$$[-\frac{2}{c(R-r_1)} + \frac{e_1 - r_1}{c(R-r_1)^2}][-\frac{2}{c(R-r_2)} + \frac{e_2 - r_2}{c(R-r_2)^2}] - [\frac{1}{c(R-r_2)}][\frac{1}{c(R-r_1)}] > 0,$$

which is equivalent to:

$$[-2 + \underbrace{\frac{e_1 - r_1}{R - r_1}}_{(+)\in(0,1)}][-2 + \underbrace{\frac{e_2 - r_2}{R - r_2}}_{(+)\in(0,1)}] > 1.$$

Second, the set of second-order conditions (8) hold for both firms whenever $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$. To show this, plugging in the partials of the optimal mechanism in (36) yields:

$$-2 + \underbrace{\frac{e_i - r_i}{R - r_i}}_{(+)\in(0,1)} < 0, \; for \; i = 1, 2.$$

Both conditions imply global uniqueness of the reporting equilibrium (see Nikaido (1968, ch. 7). Also the Routh-Hurwitz conditions for reaction function stability are satisfied.

6.7 Proof of Proposition 5

First, combining the interior best response functions r_1^{int} and r_2^{int} implicitly defined by (19) yields:

$$\frac{e_1 - r_1}{R - r_1} + \frac{e_2 - r_2}{R - r_2} = 4 - 2\sqrt{2}, \; for \; r_1, r_2 \in [0, R). \tag{35}$$

Evaluating (35) at $e_1 = e_2 = e^t$ and $r_1 = r_2 = r^*$ leads to the candidate r^*. Second, the profit function is concave in r_1 as shown above for $p_i \in (\underline{p}, \overline{p})$. Recall, every $e_i \in (0, e^{\theta-t})$ for $i = 1, 2$ leads to reporting equilibria that cause $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$. Third, plugging r^* into (8) leads to a negative value, i.e. there is a local maximum at r^*. Since this local maximum is the only stationary point, it has to be a global maximum.

6.8 Proof of Proposition 6a

Consider the following partial derivatives of the optimal mechanism:

$$\begin{aligned}\frac{\partial p_i}{\partial r_i} &= -\frac{1}{c(R-r_i)} < 0 \ \ \forall r_1, r_2 \in [0, R), & \frac{\partial p_i}{\partial r_j} &= \frac{1}{c(R-r_j)} > 0 \ \ \forall r_1, r_2 \in [0, R), \\ \frac{\partial^2 p_i}{\partial r_i^2} &= -\frac{1}{c(R-r_i)^2} < \ \ \forall r_1, r_2 \in [0, R), & \frac{\partial^2 p_i}{\partial r_i \partial r_j} &= 0 \ \ \forall r_1, r_2 \in [0, R),\end{aligned} \tag{36}$$

The proof of the Lemma is straight forward to see when substituting the partials in (36) into (31) and (32). Recall, every $e_i \in (0, E)$ for $i = 1, 2$ leads to reporting equilibria that cause $p_i \in (\underline{p}, \overline{p})$ for $i = 1, 2$.

6.9 Proof of Theorem 1

Overview A SPNE induces a Nash Equilibrium (NE) in every stage of the original game. We will prove that $e_1 = e_2 = e^t$ is the outcome of a NE of stage 2 of the game under the optimal audit mechanism. Specifically, given the sufficient condition that $g'(.)$ is sufficiently steep, we will prove that:

$$\Pi_i(e^t, e^t, r_i^*(e^t, e^t), r_j^*(e^t, e^t)) \geqq \Pi_i(e_i, e^t, r_i^*(e_i, e^t), r_j^*(e_i, e^t)) \forall e_i \in (0, e^0) \text{ for } i = 1, 2 \text{ and } i \neq j, \tag{37}$$

establishing the existence of the symmetric NE.

We recall that the first-order necessary condition for a profit maximum, $\partial \Pi_1 / \partial e_1 = 0$, can be rewritten as equation (10): $g'(e_1) = p_1 \theta + (\frac{\partial p_2}{\partial r_2} / \frac{\partial p_1}{\partial r_1}) \frac{\partial r_2}{\partial e_1}(t - p_1 \theta)$ with a common marginal benefit (MB) equal marginal cost (MC) interpretation. Both, $MB(e_1)$ and $MC(e_1)$ are functions of e_1 holding e_2 fixed at $e_2 = e^t$. We will prove that $\Pi_1(e_1)$ has a global maximum at $e_1 = e^t$ with the help of these $MB(e_1)$ and $MC(e_1)$ functions.

The proof progresses in two main steps. First, we will prove that under the optimal audit mechanism $MB(e_1)$ intersects $MC(e_1)$ from above at $e_1 = e_2 = e^t$. Whenever $MB(e_1)$ intersects $MC(e_1)$ from above, a local maximum is identified. Second, we will show that $e_1 = e_2 = e^t$ is the only stationary point of $\Pi_1(e_1)$, given that $g'(.)$ is sufficiently steep. That is, $MB(e_1)$ intersects $MC(e_1)$ exactly once at $e_1 = e_2 = e^t$. If $e_1 = e_2 = e^t$ is a local maximum of $\Pi_1(e_1)$ and in addition it is the only stationary point of $\Pi_1(e_1)$, it follows that $e_1 = e_2 = e^t$ has to be a global maximum of $\Pi_1(e_1)$.[33] Figure 5 illustrates.

* * * Figure V about here * * *

[33] The fact that $MB(e_1)$ crosses $MC(e_1)$ once from above, is the same as saying that $MB(e_1) > MC(e_1)$ for $e_1 < e^t$ and $MB(e_1) < MC(e_1)$ for $e_1 > e^t$, where e^t is the point where they cross. Thus, $\Pi_1(e_1)$ has exactly one maximum, at $e_1 = e^t$. That is, $e_1 = e^t$ must be the optimal choice.

1. Proof that Π_1 has a local maximum at $\mathbf{e}_1 = \mathbf{e}^t$ First, we analyze $MB = g'(e_1)$. MB is strictly decreasing in e_1, because $g(.)$ is strictly concave in e_1 which is illustrated in Figure 5.

Second, we analyze the shape of MC for $e_1 \in [0, e^0]$ in three steps: (a) we analyze the shape of $\frac{\partial r_2}{\partial e_1}$ under the optimal audit mechanism, (b) we analyze the shape of $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1}$ and (c) we analyze the entire shape of the MC function.

(a) Using (7) and (24), the expression for (32) $\frac{\partial r_2}{\partial e_1}$ can be manipulated as following [setting $\frac{\partial^2 p_1}{\partial r_1 \partial r_2} = 0$, which is the case in general at $r_1 = r_2$ (recall, Lemma 1) and which is always the case under the proposed optimal audit mechanism]:

$$\begin{aligned}
\frac{\partial r_2}{\partial e_1} &= \frac{-\frac{\partial p_1}{\partial r_1}\frac{\partial p_2}{\partial r_1}}{[2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1)][2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)] - [\frac{\partial p_1}{\partial r_2}\frac{\partial p_2}{\partial r_1}]} \\
&= \frac{-\frac{\partial p_1}{\partial r_1}\frac{\partial p_2}{\partial r_1}}{[2\frac{\partial p_1}{\partial r_1} + \frac{\frac{\partial^2 p_1}{\partial r_1^2}}{\frac{\partial p_1}{\partial r_1}}(\frac{t}{\theta} - p_1)][2\frac{\partial p_2}{\partial r_2} + \frac{\frac{\partial^2 p_2}{\partial r_2^2}}{\frac{\partial p_2}{\partial r_2}}(\frac{t}{\theta} - p_2)] - [\frac{\partial p_2}{\partial r_2}\frac{\partial p_1}{\partial r_1}]} \\
&= \frac{-\frac{\partial p_2}{\partial r_1}/\frac{\partial p_2}{\partial r_2}}{[2 + \frac{\frac{\partial^2 p_1}{\partial r_1^2}}{(\frac{\partial p_1}{\partial r_1})^2}(\frac{t}{\theta} - p_1)][2 + \frac{\frac{\partial^2 p_2}{\partial r_2^2}}{(\frac{\partial p_2}{\partial r_2})^2}(\frac{t}{\theta} - p_2)] - 1}
\end{aligned} \tag{38}$$

Aside, at the symmetric equilibrium $e_1 = e_2$ and $r_1 = r_2$. In this case, using (24) and (27) $-\frac{\partial p_2}{\partial r_1} = \frac{\partial p_1}{\partial r_1} = \frac{\partial p_2}{\partial r_2}$ and the numerator of (38) equals one. Also in this case, $p_1 = p_2 = \frac{1}{2}$ and $\frac{\partial^2 p_1}{\partial r_1^2} = \frac{\partial^2 p_2}{\partial r_2^2}$ using (27) and (28) respectively. Thus, (38) can be written as (12) in the main text.

Using (3) and (36), (38) is:[34]

$$\frac{\partial r_2}{\partial e_1} = \frac{\frac{R - r_2}{R - r_1}}{[2 - c(\frac{t}{\theta} - p_1)][2 - c(\frac{t}{\theta} - p_2)] - 1} \tag{39}$$

$$= \frac{R - r_2}{R - r_1}\frac{1}{c^2(p_1 - p_1^2) - \frac{c^2}{4} + 1}. \tag{40}$$

(b) Given (36), we note that:

[34]The denominator in (39) $[2 - c(\frac{t}{\theta} - p_1)][2 - c(\frac{t}{\theta} - p_2)] - 1$ can straight forwardly be manipulated with the help of (3) to $c^2(p_1 - p_1^2) + q$, where q is some constant. We know that at $r_1 = r_2$ it follows that $p_1 = \frac{1}{2}$ and the value of the denominator has to equal one, because $\frac{\partial r_2}{\partial e_1} = 1$ at $r_1 = r_2$. Thus $c^2(p_1 - p_1^2) + q = 1$ at $r_1 = r_2$ or $\frac{c^2}{4} + q = 1$ or $q = 1 - \frac{c^2}{4}$ which yields the denominator in (40).

$$\frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}} = \frac{-\frac{1}{c(R-r_2)}}{-\frac{1}{c(R-r_1)}} = \frac{R-r_1}{R-r_2}. \tag{41}$$

Given (41) we find:

$$\frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1} = \frac{1}{c^2(p_1-p_1^2)-\frac{c^2}{4}+1}.$$

Let $z \equiv \frac{R-r_1}{R-r_2}$ and we note that $z=1$ if and only if $r_1=r_2$. Then the interior part of the proposed audit mechanism is $p_1=\frac{1}{2}+\frac{1}{c}\ln z$ and $\frac{\partial r_2}{\partial e_1}$ in (40) can be written as:

$$\begin{aligned} \frac{\partial r_2}{\partial e_1} &= \frac{1}{z}\frac{1}{1+c^2(p_1-p_1^2-\frac{1}{4})} \\ &= \frac{1}{z(1+c^2(\frac{1}{2}+\frac{1}{c}\ln z-(\frac{1}{2}+\frac{1}{c}\ln z)^2-\frac{1}{4}))} \\ &= \frac{1}{z-z(\ln z)^2} \end{aligned} \tag{42}$$

Thus:

$$\frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1} = \frac{1}{1-1(\ln z)^2}$$

* * * Plot I about here * * *

A plot of $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} = \frac{1}{1-1(\ln z)^2}$ is shown in Plot 1.

(c) The entire expression for MC can now be manipulated as following:

$$\begin{aligned} g'(e_1) &= p_1\theta + \frac{1}{1-1(\ln z)^2}(t-p_1\theta) \\ &= (\frac{1}{2}+\frac{1}{c}\ln z)\theta + \frac{1}{1-1(\ln z)^2}(t-(\frac{1}{2}+\frac{1}{c}\ln z)\theta) \\ &= \frac{2\theta\ln^3 z + c\theta\ln^2 z - 2ct}{2c\left(\ln^2 z - 1\right)} \end{aligned}$$

At $z=1$, we have $MC=t$. The first derivative of the MC curve w.r.t. z shows that there are two stationary points on the relevant interval for z, where $z \in [0,5567; 1.7964]$:[35]

$$\begin{aligned} \frac{\partial}{\partial z}(\frac{2\theta\ln^3 z + c\theta\ln^2 z - 2ct}{2c\left(\ln^2 z - 1\right)}) &= 0 \\ \frac{1}{2z}\frac{\ln z}{\left(\ln^2 z - 1\right)^2}(\theta-2t)\left(\frac{1}{2}\sqrt{2}+1\right)\left(-\ln^3 z + 3\ln z + 2\sqrt{2}-4\right) &= 0, \end{aligned}$$

[35] The relevant interval for z will be explained below.

which is solved by $z = 1$ and $z = \exp(\sqrt{2}-1) \approx 1.51$. The MC curve has a minimum in z at $z = 1$, because

$$\frac{\partial}{\partial z \partial z}\left(\frac{2\theta \ln^3 z + c\theta \ln^2 z - 2ct}{2c\left(\ln^2 z - 1\right)}\right)\Bigg|_{z=1} = 2t - \theta > 0,$$

and a maximum at $z = \exp(\sqrt{2}-1) \approx 1.51$, because

$$\frac{\partial}{\partial z \partial z}\left(\frac{2\theta \ln^3 z + c\theta \ln^2 z - 2ct}{2c\left(\ln^2 z - 1\right)}\right)\Bigg|_{z=e^{\sqrt{2}-1}} = (\theta - 2t)\frac{3}{8}\exp(2 - 2\sqrt{2})\left(\sqrt{2}+2\right) < 0.$$

We have shown that MC has a minimum at $e_1 = e_2$ while $g'(e_1)$ intersects the MC curve from above establishing a local maximum at $e_1 = e_2 = e^t$.

2. Proof that $\mathbf{e}_1 = \mathbf{e}^t$ is the only stationary point of $\mathbf{\Pi}_1$ We consider (a) downwards deviations and (b) upwards deviations of firm 1, given that $e_2 = e^t$.

(a) We consider downward deviations of firm 1, i.e. $e_1 < e^t$, given that $e_2 = e^t$. We analyze first the case when $e_1 \leq \underline{e}_1$, where $\underline{e}_1$ is defined as the largest e_1 (given $e_2 = e^t$) that induces firm 1 to choose $r_1 = 0$ at stage 3. Note, considering downward deviations of e_1 from $e_1 = e^t$, firm 1 is always first to report zero emissions. In this case, $MC = t$, because firm 1 chooses to report zero emissions and rather faces the highest possible expected fine, which equals t. Thus, $g'(e_1) > t$ in this interval.

Next, we analyze the case when $\underline{e}_1 < e_1 < e^t$. With $\underline{e}_1 < e_1 < e^t$ the equilibrium in the reporting stage is interior, and $r_1 < r_2$. Thus $z > 1$. Also, $p_1 > p_2$, but since we are in the interior, it must also be the case that $p_1 < \overline{p} = \frac{t}{\theta}$, or $\frac{1}{2} + \frac{t/\theta - 1/2}{2-\sqrt{2}} \ln z < \frac{t}{\theta}$ or $z < \exp(2-\sqrt{2}) \approx 1.7964$. Thus $1 < z < \exp(2-\sqrt{2})$. For $1 < z < \exp(2-\sqrt{2}) \approx 1.7964$, we can see from Plot 1 that $1 < (\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1}$. Consider MC from (10): $MC(e_1) = p_1\theta + (\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1}(t - p_1\theta)$ and we note that $MC(e_1) = t$ if and only if $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} = 1$ or $(t - p_1\theta) = 0$. Since the reporting equilibrium is in the interior when $\underline{e}_1 < e_1 < e^t$, the second condition is not satisfied. We also know the first condition is not satisfied. Thus, $MC(e_1) > t$ whenever $\underline{e}_1 < e_1 < e^t$. At $e_1 = e^t$, $MC = t$ and we have shown above that $MC(e^t)$ is a minimum. Further we have seen that MC has a maximum at $z = \exp(\sqrt{2}-1) \approx 1.51$, which may lie in this interval. If this interval contains the MC maximum, its value is at most $1.0303t$ which is around 3% lager than t. Thus, at marginally to the left of e^t, $g'(e_1) > MC(e_1)$ can be guaranteed when $g'(e_1)$ is sufficiently steep.

(b) We consider upwards deviations of firm 1, i.e. $e_1 > e^t$, given that $e_2 = e^t$. Let $\overline{e}_1$ be the lowest e_1 (given $e_2 = e^t$) that leads to audit probabilities $p_1 = \underline{p}$ and $p_2 = \overline{p}$. When $\overline{e}_1 > e_1 > e^t$, the reporting equilibrium is in the interior, and we have $p_1 > \underline{p}$ or $\frac{1}{2} + \frac{t/\theta - 1/2}{2-\sqrt{2}} \ln z > 1 - \frac{t}{\theta}$ or $z > \exp(\sqrt{2}-2) \approx 0.5567$. Thus $\exp(\sqrt{2}-2) < z < 1$. For $\exp(\sqrt{2}-2) \approx 0.5567 < z < 1$, we can see from Plot 1 that $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} > 1$ (or more to the point $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} \neq 1$). Since $MC(e^t) = t$ is a minimum and there are no further stationary points in this interval, it follows that $MC(e_1) > t$ for all $\overline{e}_1 > e_1 > e^t$. Thus $g'(e_1) < MC(e_1)$ on this interval.

When $\overline{e}_1 \leq e_1 \leq e^0$, we have $p_1 = \underline{p} = 1 - \frac{t}{\theta}$ leading to $MC = \theta - t$ and it must be true that $MC \geq g'(e_1)$.

In conclusion, on the interval of emissions for which the reporting equilibrium is in the interior (by which we mean that $p_1 \in (\underline{p}, \overline{p})$) there is $g'(e_1)$ and $MC(e_1)$ intersecting only once at $e_1 = e^t$ given $e_2 = e^t$ and $g'(e_1)$ is sufficiently steep. Recall, any reporting equilibrium for $e_i \in (\underline{e}_i, e_i^{\theta - t})$ for $i = 1, 2$ leads to the interior case $p_1 \in (\underline{p}, \overline{p})$. There are no further intersections of MB and MC when reporting is not interior.

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Till Requate, Co-Editor
Journal of Environmental Economics and Management

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MS. JEEM-D-16-00263R2
Revised Submission: "On Optimal Audit Mechanisms for Environmental Taxes"

Dear Till,

Please find attached my revised manuscript "On Optimal Audit Mechanisms for Environmental Taxes" for potential publication in the *Journal of Environmental Economics and Management*. I am delighted to read that the two Reviewers were satisfied with my responses to their comments. I am also grateful for your detailed and constructive suggestions, as they have helped me to improve the paper further. I realize that your thorough review took considerable time and effort and I truly thank you for it.

I have carefully addressed all of your comments in this resubmission of my manuscript. Please find below my itemized responses to your comments (quotes from your report are in italics):

1. *To start with, you have Lemmas, Propositions and a Theorem. By definition, a Lemma is an auxiliary result which is of minor interest itself but serves mainly as a technical step or intermediate result to prove a result of major interest. As far as I can see only Lemma 1 is used in the proof of Theorem 1.*

   In order to address your comment, I reviewed all Lemmas, Propositions and Theorems of the paper. I agree that several of the Lemmas in the previous version of the manuscript were not solely auxiliary results which served to prove results of major interest, but they were of interest in themselves. After consideration, I renamed Lemma 2, 3, 4, 5a and 5b as Propositions. I changed the wording throughout the paper accordingly.

2. *The axioms: "Axiom" is a strong word. As you write it, your axioms are pure definitions of balanced budgets and symmetry. I suggest you coin them as definitions, and then in your results you can simply say, "if p(..) satisfy budget balancedness and symmetry (or so), then ...." If you want to keep the Axiom version you have to formulate them in a different way. Axiom 1: "The audit mechanism satisfies budged balancedness, i.e. ...."*

I followed your suggestion and coined previous Axioms 1 and 2 as Definitions 2 and 3 instead (page 9 of the revised manuscript). In addition, previous Axioms I and II are coined as Definitions II and III (page 1 and 2 of the revised online Appendix). The wording in the revised manuscript has been updated accordingly. For instance, Theorem 1 (on page 20) for the two firm case and Theorem I (on page 23) for the n firms case now both begin with "If the audit mechanism satisfies budget balancedness and symmetry, then the audit mechanism [..]."

3. *Another example for confusing organization is the discussion about the sufficient condition for existence of SPNE. On page 20 you have the paragraph "Sufficient condition". At that point the reader does not know where you are heading at. In Theorem 1 "sufficient steepness" is itself a sufficient condition for existence of equilibrium. In that paragraph "Sufficient condition" you set up another stronger sufficient condition for the sufficient condition "g' being sufficiently steep". Then, in Theorem 1 you give already a condition for existence. But then in section 3.3.3 you discuss it again. This is overkill and confusing. I think you can formulate the sufficient condition for "g' being sufficiently steep" as a footnote to the Theorem. Do you really need 3.3.3?*

   I agree that it is a good idea to formulate and discuss the sufficient condition for the existence of the SPNE "$g'(e)$ is sufficiently steep" in a footnote to Theorem 1 (page 22, footnote 23). I deleted the discussion of the sufficient condition at other places in the manuscript including section 3.3.3 of the original manuscript.

4. *I found the structure of the proof of Theorem 1 not sufficiently clear. Can you give a short roadmap at the beginning of the proof about what steps are necessary to prove the Theorem? Then on page 39, line 10 from below you write: "… the slope of MC in this interval is likely relatively flat…" An argument like this ("likely") should not be part of a proof. Is it or is it not?*

   In order to make the proof of Theorem 1 (page 31) as clear as possible, I included an overview in the beginning of the proof explaining what steps are necessary to prove the Theorem. The wording of the proof has been updated as well. The word "likely" has been deleted. In fact, the sufficient condition, "$g'(e)$ is sufficiently steep", ensures that MB and MC do not cross regardless of the slope of MC in the relevant interval.

5. *Your Conclusions section is a) much too long, and b) redundant. Conclusions should not repeat a summary of the results. This is what the abstract is for. You should discuss the limitations of your analysis and maybe indicate some open questions and paths for further research.*

   The Conclusions section of the manuscript (beginning on page 25) has been condensed significantly. I have deleted the summary of the results and the Conclusions now focus on the limitations of the analysis and indicate some avenues for further research.

6. *You should ask a native speaker, possibly a professional, to edit English writing. There are several sentences that sound like a word by word translation from German.*

I have sent the manuscript to a professional editor to be edited for English writing (*www.editperfect.ca*). As a result, the wording of several sentences has been improved without changing any of the content of the paper.

7. *Equation (6) you did not introduce small "pi", Can't you just use and differentiate Capital Pi? The result will be the same.*

   Following your suggestion, in the revised manuscript I use and differentiate Π_i in equation (6) to denote the profit function of firm i. As you say, the result is the same.

8. *Avoid equation numbers in the footnote (equation 14). You may refer to it as "on the right hand side of the above equation".*

   I removed equation number (14) in footnote 19 and also equation number (9) in footnote 15 of the original manuscript. I updated the wording accordingly when I refer to these equations. For instance, in footnote 19, line 3, of the revised manuscript I refer to the equation with, "on the right hand side of the above equation."

9. *Paragraph "reference value": I was confused. You write a lot here, but you should explain immediately what you mean by the reference value. Reference value of what? You also write "One sufficient, but not necessary value". There are sufficient and necessary conditions but not values.*

   On page 14, paragr "Reference Value for Reported Emissions", of the revised manuscript, I define "reference value" as follows: "Reference value R is an emissions level chosen by the EPA that the EPA uses as a point of reference for comparing firms' reported emissions. The emissions level chosen by the EPA to be a reference value for reported emissions depends on the parameters of the model." I further explain that "the gap between the reference value and reported emissions influences the assigned audit probability to the reporting firm. The larger the gap, the larger the audit probability and the smaller the gap, the smaller the audit probability." The following discussion about the reference value has been significantly condensed.

   I erased the phrase "One sufficient, but not necessary value". You are right that there are no necessary values.

10. *Do you need an extra section "Using Axioms 1 and 2"? If you think you need it, better coin it as "Implications of symmetry and budget balancing"*

    Reviewer 2 had recommended to break down section 2 into several subsections so that this section is "easier to digest for the reader". From this perspective, the subsection in question seems useful. I followed your suggestion and have changed the title of this subsection to "Implications of symmetry and budget-balancing" which I think more clearly reflects the content of this subsection to the reader (page 15, third paragraph).

11. *Page 16, line 7: better write "induce e_1=e_2=e^t for all firms" (in TeX-notation).*

    I changed the phrase to "induce $e_1 = e_2 = e^t$ for all firms" (page 15, last line of the main text).

12. *Page 16, paragr: "Limits for the Audit Probabilities", line 7: why introducing the Euler number as late as here? You have used it already earlier in your paper.*

    In the original manuscript, I introduce the Euler number as e^y where y is the exponent of the Euler number (page 16, paragr: "Limits for the Audit Probabilities", line 7). Earlier in the paper, I denote e^t as the emissions level which is implicitly defined by $g'(e^t) = t$. I can see how this chosen notation may cause confusion to the reader. In order to resolve this issue, I changed the notation in the revised manuscript for the Euler number from e^y to $\exp(y)$ where $\exp(.)$ denotes the natural exponential function or Euler's number (page 16, paragr: "Limits for the Audit Probabilities", line 7). As a result, the differentiation between the Euler number and the emissions level is more straight forward to the reader.

13. *Section 3.3.1. First sentence sounds strange: what is "an informed conjecture"?*

    Agreed. I changed the wording of the first sentence as follows: (page 16, paragr: "The Optimal Audit Mechanism", line 1): "Informed by the analysis above, a conjecture for the optimal audit mechanism for both firms is given by: [..]."

14. *2nd line up: Better: "Recall that, by construction, the optimal audit mechanism satisfies ..."*

    I agree that your suggestion does sound better. I changed the wording accordingly (please refer to my response to your next comment).

15. *Same line: why "a necessary condition"? Which one? Each implication of any set of conditions is a necessary condition. So saying "it satisfies a(!) necessary condition" is not informative at all.*

    Given comment 14 and 15, I updated the wording of the fourth paragraph on page 15 as follows: "Audit mechanism (16) is a derived and specific functional form that maps reported emissions into audit probabilities in such a way that it gives firms an incentive to choose efficient emissions. Recall that, by construction, the optimal audit mechanism satisfies the necessary first-order condition to induce $e_1 = e_2 = e^t$ for all firms, i.e.: $g'(e_i) = t$ for $i = 1, 2$."

16. *Page 18, line 3: "Lemma 3 tells us about..." Tells us what?*

    This sentence has been clarified (page 17, paragr: "Reporting under the Optimal Audit Mechanism", line 1). It now reads: "The next Proposition establishes the reporting behaviour of firms under the proposed optimal audit mechanism".

17. *Page 18: 1st line after Lemma 4: Avoid "Lemma 4 is remarkable".*

    I deleted this sentence.

18. *Delete paragraph "Sufficient condition" (see above). Mention that condition in a foot-note. It was also not clear whether Lemma5a and 5b still belong to that paragraph. I guess not.*

I deleted the paragraph "Sufficient condition" (please refer to my response to your comment 3). I state and discuss the sufficient condition in footnote 23.

Lemma5a and 5b (Proposition 6a and 6b in the revised manuscript) do not belong to this paragraph which has been clarified (page 20).

19. *Page 22: You should mention that the Theorem is proven in the appendix. It sounds as if it is a simple implication of the Lemmas, which is not the case.*

    In the revised manuscript, underneath Theorem 1 (page 20) I state that the Theorem is proven in the Appendix. Similarly, I state under Theorem I (page 23) that the Theorem is proven in the online Appendix.

20. *Page 23: e1=e2=e^t cannot be a SPNE. It can be the outcome of a SPNE. A SPNE is a mapping of outcomes or choices into a set of actions.*

    I updated the wording throughout the revised manuscript in accordance with your advice. For instance, on page 17, line 10, the main text reads: "[..] for the existence of outcome $e_1 = e_2 = e^t$ as a SPNE."

21. *Section 3.3.4 does not buy us much. I think you can delete it. You may briefly mention that in the conclusions.*

    I deleted section 3.3.4. The main insights from this section about asymmetries between the firms are briefly mentioned in the Conclusions section on page 25, second paragraph.

22. *3.3.5 is also a bit repetitive and comes rather late. Strip it down to the essentials.*

    The content of 3.3.5 "Why the Optimal Audit Mechanism Works" has been condensed. This paragraph now appears earlier in the revised manuscript on page 20, paragr 4.

23. *Skip 3.3.6.*

    I skipped section 3.3.6. The refinement is briefly summarized in footnote 25 on page 23.

24. *Section 4: I think symmetry should be formally defined. At least on a footnote.*

    Symmetry of the audit mechanism is formally defined in the online Appendix (page 2, Definition III) for the detailed analysis of the n firms case. I agree that it is a good idea to mention this definition of symmetry in the main text as well. I now define symmetry in footnote 28 on page 24 of the revised manuscript. For completeness, budget-balancedness is defined in footnote 27 using the same wording as in the online Appendix.

I trust that these responses adequately address all of your comments. Please do not hesitate to contact me again in case you have further suggestions, questions or comments.

I look forward to hearing from you soon.

Sincerely,

Marcel Oestreich

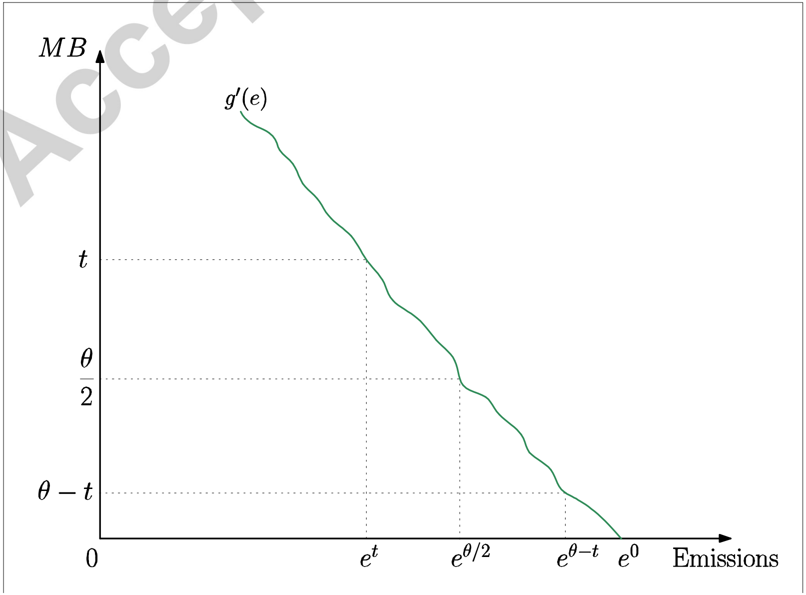


Figure 1: Illustration of the enforcement problem with emissions per firm on the horizontal axis and marginal benefits (MB) on the vertical axis. The socially efficient emissions level for each firm is e^t while $e^{\theta/2}$ is the higher and socially inefficient per-firm emissions level which results when the common RAM is used.

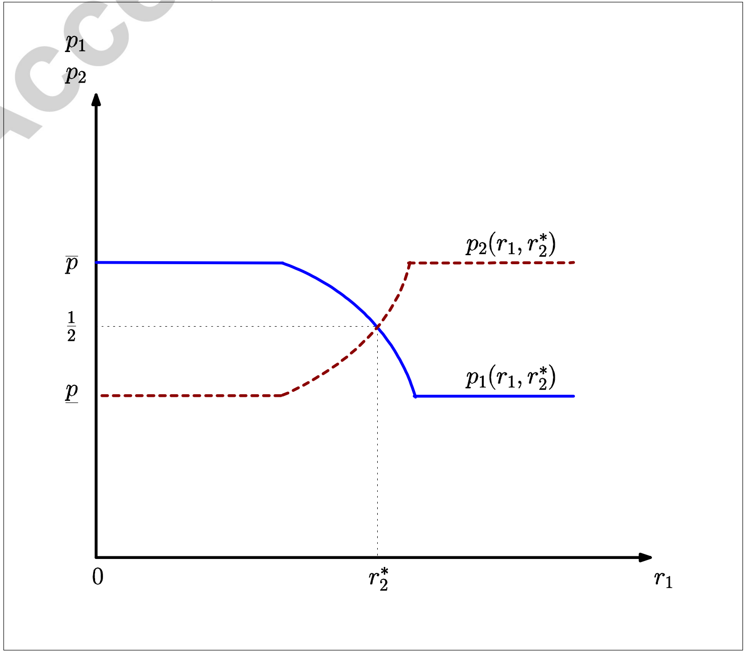


Figure 2: Sketch of $p_1(r_1, r_2^*)$ and $p_2(r_1, r_2^*)$ under the proposed optimal audit mechanism depending on r_1 with r_2 fixed at $r_2 = r_2^*$.

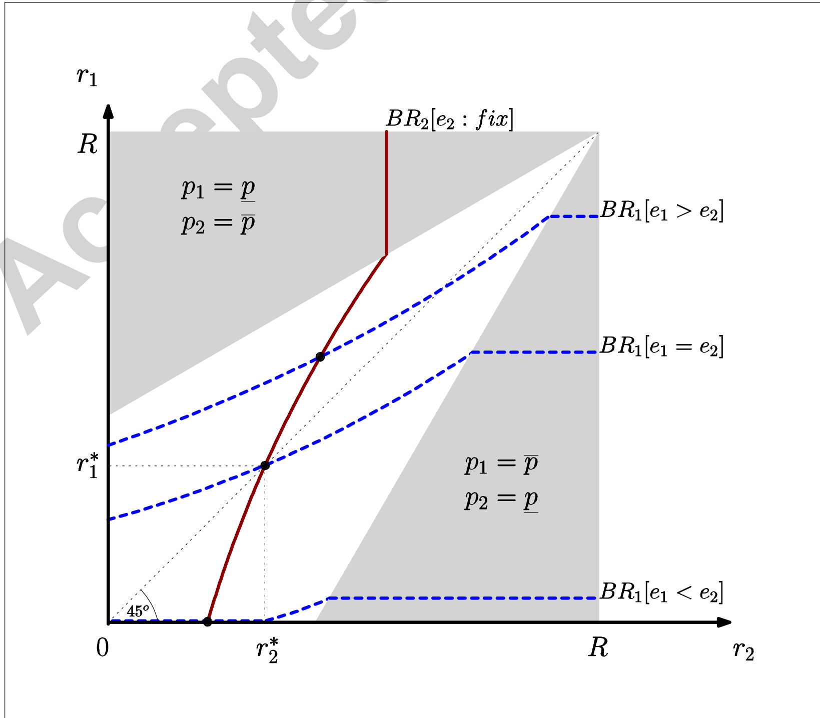


Figure 3: Sketch of the best reporting response functions for various levels of emissions e_1 with e_2 fixed at $e_2 = e^t$. The curve $BR_2[e_2 : fix]$ is the best response function of firm 2 holding e_2 fixed at $e_2 = e^t$. The curve $BR_1[.]$ is the best response function of firm 1 for smaller, equal and larger e_1 in relation to e_2. All three illustrated SPNE are marked with black dots.

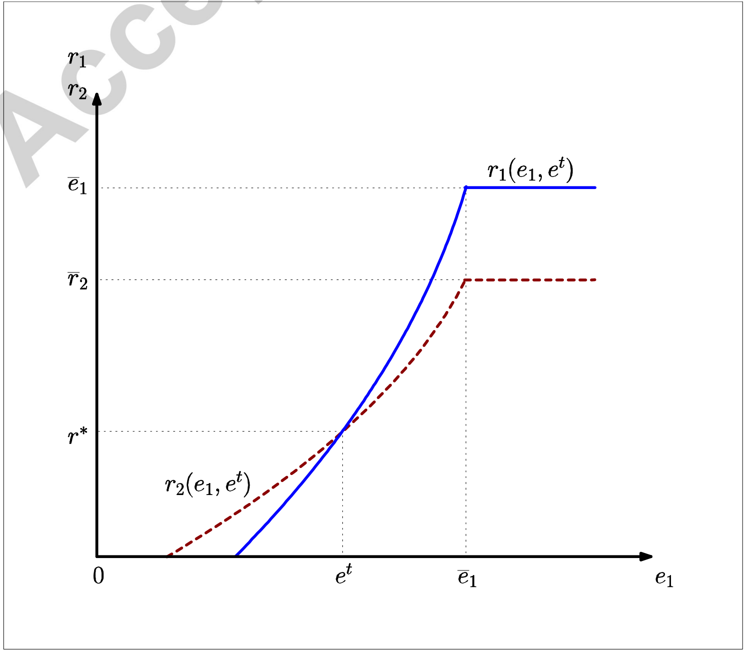


Figure 4: Sketch of $r_1(e_1, e^t)$ and $r_2(e_1, e^t)$ under the proposed optimal audit mechanism depending on e_1 with e_2 fixed at $e_2 = e^t$.

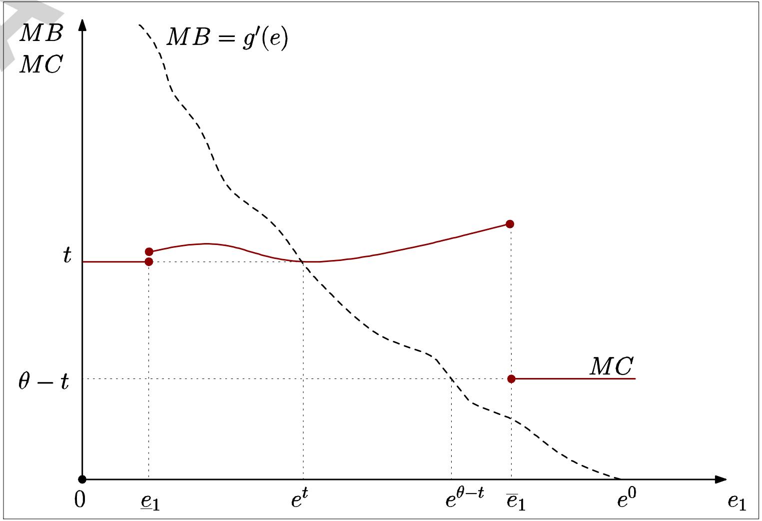
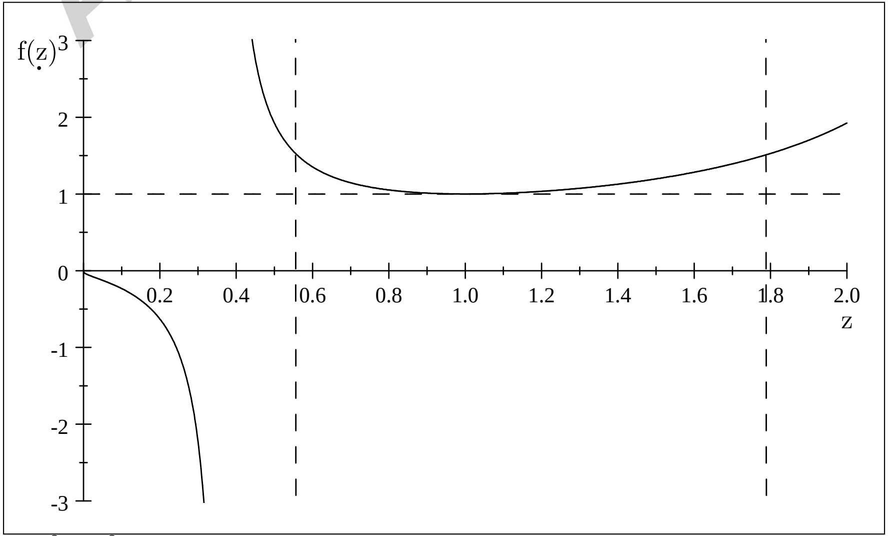


Figure 5: Marginal benefit (MB) and marginal cost (MC) of emissions under the proposed audit mechanism for firm 1 with e_2 fixed at $e_2 = e^t$.



Plot 1: Plot of $(\frac{\partial p_2}{\partial r_2}/\frac{\partial p_1}{\partial r_1})\frac{\partial r_2}{\partial e_1} = f(z) = \frac{1}{1-1(\ln z)^2}$. The dashed vertical lines mark the relevant interval for z: $[0,5567;1.7964]$.