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THEORY/CONCEPTUAL

Complex systems: marketing's new frontier

William Rand[1] · Roland T. Rust[2] · Min Kim[3]

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Abstract
Complex systems approaches are emerging as new methods that complement conventional analytical and statistical approaches for analyzing marketing phenomena. These methods can provide researchers with tools to understand and predict marketing outcomes that emerge at the aggregate level by modeling feedback between heterogeneous agents and agent interaction with various marketing environmental variables. While the benefits of complex systems approaches often come with a high computational cost, steady advances in access to better computational resources has allowed more researchers to adopt complex systems approaches as part of their portfolio of methods. In this paper, we will provide a description of the key concepts, benefits, and tools of complex systems. The goal of this work is to encourage marketing researchers and practitioners who are not familiar with these approaches to consider the adoption of these methods. We end with a discussion of the future research opportunities that this powerful methodology enables.

Keywords Complex systems · Agent-based models · Network science · System dynamics · Chaos theory · Machine learning

Analyzing marketing phenomena is often complex, and two particular aspects often make analysis difficult: (1) interactions between heterogeneous individuals, and (2) many elements of the marketing environment operate simultaneously. Together these features can mean that to truly understand a marketing phenomenon, researchers need to adopt increasingly advanced methods. Complex systems is one framework that can help deal with these difficult aspects of marketing research. The basic idea behind complex systems analysis is that individuals are modeled from the ground up, and the patterns of behavior of the system are observed as a result of the interactions of those individuals.

For instance, individuals in the marketing environment, such as consumers, sellers and distributors, are far from being homogeneous entities and they are not isolated from the influence of other agents. In fact, these individuals, often called *agents*, constantly affect each others' behaviors and choices. These constant interactions gradually lead to various emerging patterns at the aggregate level that are not obtained by simply summing properties of individual agents and elements at the micro-level of the system. For instance, consumers create and share word-of-mouth about goods and services (Berger 2014). The extent to which consumers interact with one another determines the speed and the strength of word-of-mouth, which leads to various adoption patterns of goods and services among the consumer population. Marketing researchers who are interested in experimenting with multiple factors that can affect the word-of-mouth need appropriate tools to capture the aspects of interactions between heterogeneous agents over time.

Another interesting marketing phenomenon driven by social interactions between heterogeneous groups of consumers is the adoption of fashion (Rust 2015). One way to conceptualize fashion is that there are two different groups of consumers: an "in-group" and an "out-group". The "in-group" can be classified as a group of influential consumers such as celebrities, vloggers, or rich. When a certain fashion gains popularity

✉ William Rand
wmrand@ncsu.edu

Roland T. Rust
rrust@rhsmith.umd.edu

Min Kim
minkim@rhsmith.umd.edu

[1] Poole College of Management, North Carolina State University, Box 7229, Raleigh, NC 27695-7229, USA

[2] Center for Excellence in Service at the Robert H. Smith School of Business, University of Maryland, 3451 Van Munching Hall, College Park, MD 20742, USA

[3] Robert H. Smith School of Business, University of Maryland, 3330 Van Munching Hall, College Park, MD 20742, USA

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among the "in-group", it is attractive to both the "in-group" and the "out-group". However, as the "out-group" picks up the fashion initially shared by the "in-group", the "in-group" loses their interest in pursuing the fashion and exhibits disadoption behaviors to differentiate themselves. In this context, one of marketers' questions is how quickly adoption and disadoption behaviors emerge and peak depending on the strength of the two different groups in a market.

Besides heterogeneity, another difficulty in analyzing marketing phenomena is the simultaneous interaction of various disparate variables in the marketing environment. The marketing mix is a composite of various marketing actions, including pricing, advertising, distribution activities, and product improvement. These can have effects that are greater or less than the sum of each of the separate effects when there are variations in more than two simultaneous marketing activities (Lilien et al. 2011). Recent marketing theory has also advocated that businesses focus increasing attention on individual customers and look more at relationship-driven marketing instead of transaction-based marketing. These changes motivate researchers to model relationship-based and repeated-choice environment that can be supplemented by a dynamic and holistic diagnosis of the marketing mix (Rust and Huang 2014).

While conventional analytical and statistical approaches may be capable of analyzing (to some degree) marketing phenomena complicated by the aspects described above, complex systems approaches can be an effective alternative lens to the conventional approaches because these approaches are more suitable to predict emergent outcomes caused by repeated feedback that endogenously occurs between heterogeneous agents, and incorporating simultaneous interactions of various marketing variables. In fact, complex systems analysis is based on an alternative premise that there is no equilibrium or no multiple equilibria. Instead, at the heart of the complex system analysis, there is constant change whose direction and rate can be modeled only by a systems-level approach to modeling.

For example, the recent advent of the structural modeling approach in the marketing literature to some extent tackles the Lucas critique (1976) by explicitly laying out rules and processes of micro decision-making in a model. However, a few limitations of the structural modeling approach exist. First, modeling diversity patterns within structural modeling becomes prohibitive as heterogeneity in agents increases. Second, modeling constant interactions between agents is difficult in the structural modeling approach. On top of that, the computational burden exponentially increases with the number of state variables and state spaces. As such, complex systems can be an alternative to the structural modeling approach if researchers are concerned about the limitations in the structural modeling approach. Also, researchers can implement more reliable and flexible counter-factual analysis with more plausible and realistic assumptions using complex systems approaches.

In some cases, the methods complex systems employ may also come with a high computational cost, due to the need to carry out repeated simulation and classification, but as computational resources become cheaper and more available, researchers can increasingly adopt complex systems approaches as part of their portfolio of methodologies. Thus, the adoption of complex systems as a major component of marketing analysis is not only likely, but also desirable.

This paper intends to facilitate growth in the use of complex systems methods within marketing by:

- Introducing key concepts of complex systems approaches,
- Identifying the scope of problems with which complex systems approaches are appropriate and beneficial in marketing research,
- Exploring the existing marketing applications of complex systems approaches in the marketing literature, and
- Suggesting future research opportunities that can be analyzed using complex system approaches.

The rest of the paper proceeds as follows. Section 2 begins with working definitions and concepts and identifies the scope of marketing problems for which complex systems approaches can provide a useful perspective. Section 3 discusses major methods used for modeling complex systems. Section 4 presents a review of applications of complex systems approaches in marketing literature. Section 5 discusses future research opportunities for which complex systems can provide a better perspective on marketing problems. Section 6 concludes with a discussion about how complex systems can improve marketing research.

The nature and characteristics of complex systems

In this section, we begin with working definitions and characteristics of complex systems, to help researchers identify the scope of complex systems approaches for marketing analysis, and then proceed to discuss its benefits.

A *complex system* consists of a large number of autonomous, interacting agents and elements describing the environment in which the agents interact. Agents can be defined at various levels depending on the scope of analysis: individual consumers, sellers, distributors, or even countries. In the context of marketing, elements, for instance, can be promotion channels or an individual's social ties through which word-of-mouth is circulated.

Complex systems are often characterized by interacting and overlapping latent feedback effects where stochastic impulses can trigger complex emergent patterns of behavior. When small differences in inputs to the system create very different ultimate outcomes that end in a bifurcation, the system is described as chaotic because of its lack of predictability and wide variance in outcomes. We will discuss these chaotic systems in a later section. However, all complex systems, regardless of whether they are chaotic, exhibit a large number of interacting components, and because of the complex nature of these interactions, simulations must be adopted to study these systems. Results of thousands of simulations varying over a large number of exogenous impulses, endogenous and dynamic relationships, and distribution variances are usually needed to generalize outcomes from inputs. One clear contrast between conventional marketing analysis and complex systems analysis is that many complex systems approaches start with models that generate data (such as consumer demand forecasts) and the parameters are then adjusted to fit to data, while the conventional marketing analysis creates the model from the data.

The premise of complex systems is consistent with marketing phenomena since the marketing environment is always changing through feedback processes. Arthur (1994) provides a foundation for criticism of the conventional equilibrium proposition due to the fact that markets are inherently dynamic. For example, he criticizes the notion that actions taken by sellers and buyers generate negative and positive feedbacks that create a predictable equilibrium for prices and market shares. Arthur argues that it is unlikely that a market is in equilibrium because supply curves and demand curves are continuously changing. To incorporate this idea, the dynamic impact of changing supply on demand and changing on supply can be represented by the following equations (Dickson 1995, 1996):

$$\left\{\frac{\partial d_i}{\partial t} : i = 1, \ldots, n\right\}_{t+1} = f\left(\left\{\frac{\partial s_j}{\partial t} : j = 1, \ldots, k\right\}\right)_t,$$

$$\left\{\frac{\partial s_j}{\partial t} : j = 1, \ldots, k\right\}_t = g\left(\left\{\frac{\partial d_i}{\partial t} : i = 1, \ldots, n\right\}\right)_{t-1},$$

where $\partial d_i / \partial t$ is change in demand of buyer i, and $\partial s_j / \partial t$ is changes in the supply of supplier j, and the equations within the brackets {} represents the set of response functions across all n buyers and k sellers. Response heterogeneity across buyers and suppliers is captured in these equations. That is, $\partial d_i / \partial t$ can vary across i (i.e., some buyers adopt preferences faster than others). Similarly, $\partial s_j / \partial t$ can vary across j (i.e., some suppliers learn or adopt preferences faster than others). Thus, the natural interaction of the system produces a complex system of supply and demand that leads to perpetual disequilibria.

This nature is consistent with marketing phenomena. The purpose of marketers is to create disequilibrium. Marketers are paid to improve all aspects of marketing including all aspects of customer relationships, disrupting the current state of the market by shifting market share. Thus, to build a descriptive or normative model that can help marketers make decisions researchers must use disequilibrium models. Models that assume stationarity and equilibrium may not be adequate to capture many components of marketing phenomena. Although the paradigm shift from a state of change to the rate of change has not taken hold in the marketing field, we expect that marketing researchers will appreciate the premises of complex systems and more marketing studies with complex system approaches will come in the future.

Instead of embracing the conventional equilibrium concept, well-defined complex systems exhibit *path dependencies*, such that current and future states or actions affect the path of previous states or actions (Page 2006). For instance, consumers who are loyal to a product tend to exhibit path dependence on their purchase decisions in the future. In a platform market, the growth of each side of agents depends on the past actions of one another. Nonlinear effect occurs as a system advances through *path dependencies* (Dickson 1995, 1996). Chaos theory that we will visit in a later section is useful to deal with very unstable feedback systems subject to rare exogenous jolts that can bifurcate into an unexpected outcome (e.g., black-swan events) (Levy 1994; Taleb 2007).

The following are several other key characteristics that most complex systems exhibit. *Emergence* arises in a complex system when autonomous agents and elements of a system result in aggregate patterns not obtained by simply summing properties of individual agent and elements (Holland 2014). After assigning agents their own properties and behavioral rules, emergence is a consequence of the interactions of the agents in response to the marketing environment. Emergence occurs through a large number of sequences that form loops of recirculating signals and resources (*feedback*) generated in the system. Behaviors of the agents are constantly moderated by surrounding activity (Holland 2014). As a result, emergence can only be realized at the aggregate level (Holland 1998; Miller and Page 2009).

The idea of emergence in complex systems can shed light on the development of the marketing environment. For example, complex system approaches can assess the impact of social influence on product diffusion rates (Delre et al. 2007, 2010) and can track the process of competitive price adjustment between retailers (Ottino-Loffler et al. 2015).

When analyzing complex systems, an interest of researchers is to identify points where emergence begins to occur. A well-defined complex system can reveal *leverage points* and *tipping points*. A leverage point is a place where the complex system can be shifted from one state to another (Rand 2015). Tipping points are a related phenomenon where a small action taken by agents causes changes in the aggregate behavior of agents (Holland 2014). The word leverage point is often used to describe instances where the force is exerted from a controller (e.g., a manger or policymaker), whereas a tipping point may be internal to the agents of the system. Discovering the leverage and tipping points which identify when, and to what extent marketing actions can be the most effective, can be a goal of marketing study.

In order to understand these properties of complex systems, a theory of complex systems and a set of corresponding methods have been developed. In the past, complex systems approaches that were based on simulation were infeasible to implement due to the computational cost of simulating constant feedbacks and interactions in a system. However, thanks to steady advances in data processing and computing power, today a modeler can use complex systems methods much more readily, and there are many advantages to doing so.

For instance, complex systems models can incorporate considerable *heterogeneity* and *diversity* of agents and components in the marketing environment at relatively low modeling cost compared to the conventional approaches. By varying degrees of heterogeneity and diversity, complex systems approaches can produce emergent outcomes that other approaches cannot predict and explain. In addition, complex systems can be modeled at multiple scale levels (Miller and Page 2009), thereby providing predictions of outcomes of marketing phenomenon at different scales of markets.

Consumers and firms are in fact ecological entities; in that they operate not independently but within an overall ecosystem. Many marketing phenomena of research interest are the consequences of *interconnectedness* and *interactions* of agents. Changes in information technology have broadened various channels of communications, such as social media, text messaging, and consumer video blogs, and complicated these patterns of interactions. Complex systems approaches allow various patterns of communication to occur, and these different channels can even operate simultaneously. Agents in a complex systems approach can adapt and evolve over time through continuous feedback with each other. In other words, agents can learn from the environment constantly in a complex system. While the conventional approaches may not capture the *adaptive* and *evolutionary* behavior, complex systems approaches can do a better job of predicting marketing outcomes and can be relatively free from the Lucas critique (Lucas 1976).

Conventional modeling approaches are often based on the linearity assumption that adding up properties of the parts in a system is equal to the corresponding properties of the whole. However, when the interactions between the components exhibit a *non-linear relationship* (i.e., the aggregate outcome is not additive), the predictions on the aggregates based on summations of parts are biased (Holland 2014). Complex systems approaches that study a system as a holistic entity avoid limitations of the linearity assumption.

In many marketing analyses, researchers want to examine whether outcomes will significantly change even after adding or eliminating subcomponents of the system. *Robustness* is characterized in a complex system that maintains its distinctive outcomes even after altering elements in the system (Bankes 2002). Robustness is a desirable property in a marketing analysis because identifying ideal marketing mixes and the level of efforts that maintains the validity of predictions is helpful to make sure that the results guard against unexpected changes and shocks in the marketing environment. As we discuss basic concepts and benefits of the complex systems, we present in the next section several methods that enable researchers to model complex systems.

Methods

In this section, we introduce several complex systems methods for marketing researchers who are interested in analyzing marketing phenomena that exhibit the properties and harness the advantages that we have discussed in the previous section. We introduce five methods: agent-based models, network science, system dynamics, chaos theory, and other computational methods, such as machine learning. These methods are useful to predict the outcomes of interests and to validate theories. Each method has its own strengths and is more suitable for particular problems than the other tools depending on the context. We summarize the advantages and disadvantages of each method in Table 1.

In the advent of cheap and improved computing power, these methods are becoming more easily accessible. Moreover, these methods are not necessarily stand-alone approaches. They can provide more complementary insights to traditional methods, such as statistical analysis, psychological experiments, surveys, and game theory to provide more accurate and reliable analysis.

Agent-based models

Agent-based modeling (ABM) is an increasingly popular method that can help researchers understand and analyze aggregated patterns of marketing phenomena that originate at the individual-agent level. ABM gives researchers the ability to model an arbitrary degree of heterogeneity in agents and to examine consumer and organizational behavior over time. The main strength of ABM is to model emergent phenomena without relying on knowledge of macro-dynamics; the micro-rules of agents will lead to the macro-level dynamics (Bonabeau 2002). In addition, ABM can extend both econometric models and analytical models. It is difficult to incorporate heterogeneity in many existing econometric models while it is not impossible (Midgley et al. 2007). Analytical models often require researchers to compromise assumptions that may not reflect reality correctly due to model tractability (Heinrich and Grabner 2017).

ABM primarily consists of two components: agents and the environment. First, instead of representative agents, ABM specifies multiple types of individual agents who make autonomous decisions based on pre-specified behavioral rules created by the researchers such as response threshold to making purchases. In addition, these behavioral rules can incorporate conditions for

Table 1 Advantages and disadvantages of complex systems methods

Tools	Advantages	Disadvantages	Scalability
Agent-based models	• Models various degrees of heterogeneity in agents and can examine how micro-rules of agents lead to macro-level dynamics.	• Factors that are hard to quantify but crucial to impact behaviors of agents are difficult to incorporated into the agent-based model. • Performance limitations may occur when dealing with many agents.	• ABM is scalable in that increasing the number and complexity of agents is possible by using larger computational systems or even a high-performance computing solution • Scalability and implementation efforts can be significantly impacted by what simulation frameworks or software are adopted by a researcher
Network science	• Models the impact of relations of agents or entities on various marketing outcome variables.	• Data on relations for meaningful analysis are hard to obtain. • If latent network relationships that data may not capture exist, then these may bias analysis.	• Network science calculations can easily become computationally intractable since many of these calculations require examining each node in the network relative to every other node • The researcher may need to invest time in studying theoretical backgrounds to implement efficient algorithms to deal with scalability problems in network analysis
System dynamics	• Models the dynamics in positive and negative feedback mechanisms and examines emergence from the feedback.	• Incorrectly specifying latent feedback effects can undermine predictions. • Dynamics can easily get complicated when there are many factors or variables considered in systems.	• System dynamics software usually supports graphical user interface and is relatively easy to scale
Chaos Theory	• Models unstable feedback systems subject to rare but extreme exogenous jolt that can bifurcate into an unexpected outcome.	• Initial input parameters or conditions can vary depending on data used or methods adopted to derive the initial parameters. Because of the sensitivity to initial conditions, emerging results from a chaos model provide completely different pictures depending on initial conditions.	• Patterns identified by chaotic systems are independent of the scale of a problem. • Sometimes the resolution required to numerically solve a chaotic system can become computationally expensive. • This can be a desirable or undesirable feature of adopting chaos theory depending on phenomena that the researcher wants to capture.
Machine Learning	• Discovers patterns from large-scale data with minimal human intervention.	• Statistical relationships derived from machine learning algorithm do not necessarily identify causal-relationships.	• Many software tools or frameworks are developed and ready-to-use. • Scalability can be dealt with through parallelization. • Modern graphic processing unit (GPU) architectures enable massive scaling.

engaging in interactions with other agents. Second, environments are spaces and conditions that describe where the agents are located and how they are connected. Environments can be geographical locations or social networks between agents. Within the pre-specified environment, each type of agents is allowed to interact with other agents or to respond to the environment in an iterative process. After the model is sufficiently simulated, emergent outcomes from interactions between agents are aggregated and reported as outcomes.

Following Bass (1969), Goldenberg et al. (2009) and Rand and Rust (2011), we illustrate an example of how ABM approach can be used to describe consumer adoption patterns. In this model, agents are consumers. Each agent makes a decision to adopt a product, which is characterized by the probability of adopting a product based on the probability of adopting due to mass media (p) and the probability of adopting due to word-of-mouth effects (q). In this model, each agent has a set of neighboring agents. The probability of adopting the product by an agent is $p + q*\left(\frac{n_a}{n}\right)$, where n_a is the number of innovative neighbors, and n is the total number of neighbors of the agent. In addition, a social relationship between agents describes the environment. After researchers provide initial values for the input parameters, agents are allowed to engage in repetitive interactions until no more agents are left to adopt the product. The major advantage of ABM in this context is that it gives us the ability to model an arbitrary flexibility of heterogeneity in consumers. For example, a model can reflect the effect of agent's social networks on a consumer's adoption decision, which the traditional analytical model approach is difficult to capture. Moreover, in the traditional Bass modeling approach every consumer has to have the same p and q, but the agent-based model gives the researcher the ability to specify a different p and q for every agent. The agent-based model can simulate a much more realistic environment and agent properties, and thus provide a useful tool for researchers and marketers to explain and predict flexible adoption patterns.

Using ABM, marketing researchers can model complex systems to capture marketing phenomena such as procurement of services in a marketplace, the purchase of tickets for events, or the adoption of innovations (Rand and Rust 2011). However, Bazghandi (2012) argues that the agent-based modeling has two main limitations. First, factors that are hard to quantify but crucial to impact behaviors of agents are difficult to be incorporated in the agent-based model. Second, performance limitations may occur when dealing with a large number of agents and thus it may not be designed for extensive simulations.

Scalability is a desirable feature of a system defined as "the ability of a system to accommodate an increasing number of elements or objects, to process growing volumes of work gracefully, and/or to be susceptible to enlargement." (Bondi 2000) In the context of using ABM, scalability is an important issue because a large number of agents need to be modeled to reflect a realistic marketing environment. ABM can be scalable if researchers can add additional computational power or even make use of a high-performance computing solution. However, the scalability can be significantly impacted by what simulation frameworks or software are adopted by a researcher (Lorig et al. 2015). Thus, the researcher needs to ensure that an adopted framework or software supports reasonable scalability given the possibility that the researcher expands the model. Though there may be scalability issues when agent sizes get into the millions, the actual implementation effort required for creating new agent-based models is relatively low thanks to the well-developed frameworks such as NetLogo, which is widely used by researchers. We will examine more applications of ABM in the marketing literature later in this paper.

Network science

Relations among individual agents in marketing environments are often described as discrete types of connections between agents. For instance, consumers form relations with other consumers and influence each other in a discrete setting such as social ties. On the other hand, a supply chain can be depicted as supply relations, in which relationships between upstream and downstream partners with the focal firm are of a dyadic form (Onno et al. 2001). When a researcher is interested in focusing the impact of relations on marketing outcome variables, *network science* may be an appropriate tool to start with.

Network science studies the structure of a collection of nodes (a set of discrete objects), links (relations), and dynamic behaviors of the aggregation of nodes and links (Lewis 2008; Rosenblatt 2013). A few concepts are often employed to characterize networks (Costa et al. 2007; Newman 2010). First, *degree* represents a number of vertices connected to a node. Second, *directedness* implies that an edge runs only in one direction as opposed to *undirectedness* in which the edge goes in both directions. Third, the *geodesic path* is the shortest path connecting vertices with minimum length. *Centrality* is the importance of a node in a network, and a number of measures exist to capture this notion depending on the context (Zafarani et al. 2014). For example, a basic measure of centrality is *degree centrality* that says the importance of a node is the number of connections that it has. However, this may not accurately capture how important a node is because it does not include anything about which nodes the focal node is connected to. *Eigenvector centrality* captures the influence of the neighbors, by examining the relationship between the focal node and all other nodes in the network. In addition, *betweenness centrality* measures the extent that a node plays a role in connecting all pairs of other nodes in the network (Zafarani et al. 2014).

The network science approach is useful in analyzing questions such as propagation of information, vertical and horizontal integration of supply chains, and diffusion of innovation through word-of-mouth marketing. To illustrate marketing applications of network science, consider a model of a firm's efforts in network marketing (Jun et al. 2006). A firm directly sells a product to a small number of consumers, but firms rely on later purchases made by consumers who are referred via their social connections by previous consumers. Thus, the product sales depend on the probability that each consumer will refer her friend on the social network. In this case, each consumer is equivalent to a node and the social acquaintances between consumers can be modeled by links. The firm's problem in network marketing is then translated into setting the optimal amount of referral fees for referrals that can lead to successful sales. A simulation to find optimal referral fees and price of products can be conducted by varying the degree of network link.

Typically, the analytical approach is not tractable in this problem because the profit is based on computations of the contingent profit in all possible networks. When the population of consumers is large, the number of all possible networks grows exponentially. Therefore, using an analytical framework, it is nearly impossible to incorporate all possible networks, which prevents a rigorous and detailed analysis (Jun et al. 2006). However, network analysis has several limitations.

Data on relations are hard to obtain but thanks to systematic data collection technology and social network sites, researchers are now able to access relation data more easily. We think that this will be a blessing rather than a disadvantage in the future.

Network science can easily get complicated as researchers extend the scale of the problem to include larger numbers of nodes in the network. For example, as the number of social media users increases in a social media network, the requirement for data storage for connections between nodes in the network and the requirement for computational resources to analyze the network can exponentially increase. If this is the case the researcher may need to invest time in studying theoretical backgrounds to implement efficient algorithms to deal with scalability problems in network analysis (e.g., Teng 2016).

Another difficulty in network science is that relation data may not capture latent relations that may significantly affect the behaviors of agents. We will explore more applications of network science in the marketing context later in this paper.

System dynamics

System dynamics modeling is a framework to analyze system behaviors as a whole instead of analyzing separated parts in a system, by explicitly describing the dynamics in positive and negative feedback built upon stocks and flows between system components (Forrester 1971; Sterman 2000). A *stock* can be understood as a bathtub that is filled and drained by a *flow* from faucets and drains (Sterman 2000). Mathematically, system dynamics can be represented by a system of differential equations (Borshchev and Filippov 2004). Thus, system dynamics can be a useful approach to understand dynamics created by stocks and flows. The visual presentation of feedback effect using a system dynamics map that has been observed or theorized to exist in marketing helps researchers specify and model complex systems. System dynamic maps represent how factors or variables in the system affect each other in a pictorial format of boxes, circles, and arrows. In general, each circle and box indicates factors or variables in the system, and variables at the tails (heads) of arrows indicate positive causes (effects). The variables at the tails (heads) of arrows with a negative sign indicate negative causes (effects). To illustrate, we provide three examples of feedback maps in marketing contexts: absorptive capacity feedback effects, global learning competition dynamics, and marketing decision routine feedback effects.

First, the dynamic learning capabilities of a firm are an important source of understanding the success of firms in the market. Cohen and Levinthal (1990) define learning capabilities as the ability of a firm to identify the value of new, external information and to integrate and then apply the information. Dell, a late entrant into the PC market, succeeded because it could identify the latest technology and consumer preferences faster than its rival and this developed their learning, which worked in a virtuous circle. Figure 1a illustrates this learning feedback effect. As knowledge about a technology and process ability increases, learning about technology and how to improve processes increases. In turn, as learning increases, knowledge about the technology increases and process ability improves. However, this learning can become localized and a barrier to other types of learning. Learning also slows down as knowledge boundaries are hit (which is represented by an exogenous jolt in the figure).

Second, competitive learning dynamics (e.g., firms deciding between the innovation and imitation of technology) commonly occur in many of today's digital industries. Figure 1b illustrates these dynamics. Existing domestic and global innovation through technology transfer systems increases imitative incentives in industry. However, the increased imitation decreases the advantage of innovators and in turn causes domestic and global innovation to decrease. Most of the time, in most markets, the imitation learning curve is steeper than the innovation learning curve. This implies that imitation is always catching up with innovation, i.e., there are always new innovations entering and so imitators constantly have new products to imitate.

Third, system dynamics provides an insight into path dependence in dynamic marketing environments and can highlight emergent patterns of behavior within these systems (Farris et al. 1998). Farris et al. (1998) provides an excellent example as to how routinized marketing budget rules by marketers can create system dynamics in markets by exploring feedback effects. To illustrate this, Fig. 1c represents marketing decision routine feedback effects. An exogenous trigger increases the manufacturer's marketing effort. This increases the firm's mind share among consumers, which increases the number of consumers searching for and purchasing the brand, which leads to an increase in share of sales. Marketing budgeting rules affect the relationship between current share of sales and next period marketing effort, which leads to the positive feedback-loop in a system.

Feedback maps like these are often used within system dynamics to help guide the exploration of new systems that are being modeled, with past maps serving as archetypes for future systems. We hope that these three examples of feedback maps will help researchers when identifying feedback effects in their marketing environments. Once the system dynamics map is specified and the exact relationship between stocks and flows has been articulated, then an exact simulation of the system can be constructed, by translating the map into a set of deterministic, mathematical equations.

An advantage of using system dynamics over the conventional modeling approaches is to illuminate the feedback in cause and effect relationships. As such, it can provide more reliable forecasts than statistical models for short to mid-term trends (Lyneis 2000). System dynamics is useful for evaluating dynamic marketing mixes over time since dynamic effects of various mixes can be estimated by assessing the flows of the system. There are several system dynamics software packages (e.g., Analytica, AnyLogic, Dynamic Applications). These packages support graphical user interface and it is relatively easy to extend the scale of problems.

However, there are several limitations with system dynamics approaches. First, mis-specifying latent feedback effects can undermine predictions. In fact, it is hard to identify the exact causal relationship of feedback effects due to simultaneity. Thus, researchers should ensure that their system dynamics model reproduces observed patterns and that the model is consistent with theory before simulating any counterfactual scenarios. Second, dynamics can easily get complicated when there are many factors within the system. In this case, dynamic diagrams are difficult to understand. We will introduce more applications of system dynamics to marketing problems later in this paper.

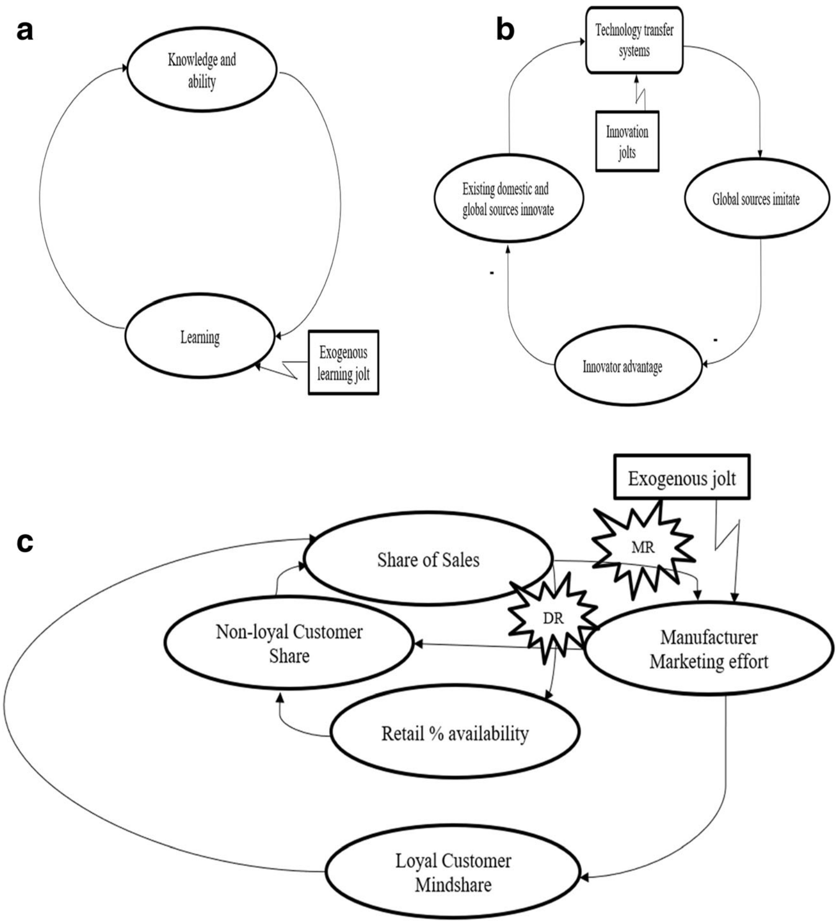

Fig. 1 Examples of maps of feedback dynamics: **a** absorptive capacity feedback **b** the global competitive learning dynamic **c** marketing decision routine (← and ←represent a positive and a negative relationship, respectively. MR represents marketing budgeting decision routine, and DR represents distribution decision routine)

Chaos theory

Chaos is defined as a pattern of outcomes over time that evolve according to a deterministic equation, where those outcomes are extremely sensitive to initial conditions such that no matter how similar two initial conditions are, they will drastically diverge over time. Thus, one of the benefits of complex systems analysis of a potentially chaotic system is to discover the potential for unintended or unexpected consequences (e.g., black swan (Taleb 2007)). Black swan events are, almost by definition, given their unexpected nature difficult to predict, but chaos theory helps us to understand the potential for them to happen.

An implication of chaos theory is that long-term predictions are nearly impossible even if researchers know the rules governing the system's behavior. Many existing marketing models focus on "equilibrium behavior." However, many marketing phenomena exhibit disequilibrium rather equilibrium as pointed out in the previous section. *Chaos theory* provides an understanding of disorderly and unpredictable patterns of chaotic systems, and as such, can shed light into sales, inventories, brand shares, and prices over time (Hibbert and Wilkinson 1994).

For example, Lambkin and Day (1989) build an ecological model of market evolution based on the following equation:

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right),$$

where N is the number of organizations (businesses) in the population of interest, t is the time, K is the upper limit or carrying capacity, and r represents the difference between rates of organizational births and deaths in the population, which is usually assumed to be positive. The emergence of chaos is dependent on the value of r.[1] A stable equilibrium value occurs if $r < 2$, periodic patterns occur for $2 < r < 2.57$, and chaos occurs if $r > 2.57$. Thus, researchers can diagnose whether chaos occurs or not depending on r, which can assist in creating more stable market forecasts, or realizing that forecasting may not be very helpful.

One advantage of adopting chaos theory is that patterns identified by chaotic systems are independent of the scale of a problem because similar patterns emerge by a chaotic system regardless of the system resolution that has been adopted.

[1] For mathematical details, see Appendix A in Hibbert and Wilkinson (1994).

For example, economic time series are known to display this property. Stock prices show a similar pattern, which is independent of whether a researcher observes daily changes over 1 year or minute-by-minute changes over a day (Levy 1994). This can be a desirable feature of adopting chaos theory depending on a phenomenon that the researcher wants to capture. However, sometimes due to the sensitivity of initial conditions of a chaotic system it is necessary to track state variables at a very fine level of resolution which can increase the computational complexity of a numerical solution.

A major limitation of applying chaos theory in complex systems is that initial parameters that are given as inputs (in above example, r) can vary depending on data used or methods adopted to derive the initial parameters. Because of the premise of chaotic theory that the system is sensitive to the initial conditions, emerging results from a chaos model provide completely different pictures depending on initial conditions and parameters, which make them difficult to use for a decision support system, but can be useful for exploring the space of possible outcomes.

Other computational methods: machine learning

What we have described above are the primary methods of complex systems analysis, but there are other methods that are often used in conjunction with these methods. One of these methods, machine learning is often used in complex systems to allow for the analysis of empirical and large-scale data (AKA Big Data). As information technology enables the faster collection of marketing data, the need for fast and reliable data processing methods becomes increasingly important. *Machine learning* can address this problem within the space of big data. Machine learning refers to computer-intensive methods that aim to discover patterns from large-scale data with minimal human intervention. Machine learning can be used for either classification, i.e., to take input data about an object (such as a customer) and then provide a class label (such as the customer is likely to be a high revenue customer) or for regression, i.e., making a particular numerical prediction (such as what the Customer Lifetime Value of a particular customer will be). Machine learning methods include association rules, decision trees, neural networks, genetic algorithms, and many others (Cui et al. 2006). Machine learning can help researchers to identify meaningful specifications of statistical models from large datasets, which can then be used to evaluate how a new policy will affect the decisions of those individuals (Rand 2015). This can complement and inform other forms of complex systems analysis in that it can be used to find patterns in big data, or it can be used to examine the results of complex systems analysis to discover interesting patterns, e.g., using a genetic algorithm to optimize the parameters of an agent-based model.

There are machine learning frameworks ready-to-use for researchers (e.g., R packages for genetic algorithms and decision trees, MLPACK, Keras). Issues in scalability in machine learning get more attention these days because large numbers of datapoints with a large number of attributes are often analyzed. Developing efficient machine learning algorithms for a large-scale data problem is an active research area. For example, the field of machine learning has been actively dealing with scalability by adopting massive parallelization. A *graphics processing unit* (GPUs) is one solution that uses a large number of simple processing units as opposed to a small number of complex processing units and it can enable researchers to extend the scale of the problem.

A major limitation of machine learning is that statistical relationships derived from machine learning algorithms do not necessarily imply a causal relationship.

Existing applications of complex systems in marketing

In this section, we present a survey of marketing applications using the methods introduced in the previous section. Each method has its own strength, and the extent of applicability of different methods is dependent on the context. Our goal is that this section serves to illustrate the wide range of applicability that Complex Systems has in Marketing, and a summary of these applications is available in Table 2.

Diffusion and word-of-mouth

Diffusion is one of the most common applications of the agent-based modeling, and hence complex systems, found in the marketing literature. ABM has advantages over the traditional equation-based approaches, such as the Bass model (Bass 1969), in modeling more realistic market environment characteristics such as heterogeneity on initial perceptions, adoption threshold, and individual responsiveness to information (Garcia 2005; Garcia and Jager 2011). Moreover, agent-based modeling (ABM) has been combined with network science to model the role of influential customers in diffusion (Goldenberg et al. 2009, 2007; Rahmandad and Sterman 2008; Stephen et al. 2010; Dover et al. 2012).

Delre et al. (2007, 2010) examined the role of timing and targeting of promotions on product adoption. Their ABM model differs from existing models by using more realistic agents who have both individual preferences and social influence and adopting more realistic networks, which are scale free, along with cost constraints. In addition, it allows consumers to be modeled with heterogeneous weights for the links they have (i.e., some friends have a larger influence on

Table 2 Marketing applications with complex systems

Tools	Diffusion and WOM	Promotion strategy	Competitive strategy
Agent-based models	• Goldenberg et al. (2009); Rahmandad and Sterman (2008); Stephen et al. (2010); Dover et al. (2012); Goldenberg et al. (2007); Delre et al. (2007); Goldenberg et al. (2010); Fuentes (2015); Goldenberg et al. (2004); Libai et al. (2005); Goldenberg et al. (2011a); Goldenberg et al. (2001b); Goldenberg et al. (2002); Zhang et al. (2011)	• Delre et al. (2007);	• Tay and Lusch (2005); Midgley et al. (1997); Heppenstall et al. (2006); Heinrich and Grabner (2017); Ottino-Loffler et al. (2015)
Network science	• Delre et al. (2007); Goldenberg et al. (2009); Trusov et al. (2013);	• Goel et al. (2015); Watts and Dodds (2007); Hill et al. (2006); Haenlein (2011); Haenlein (2013); Cui et al. (2006); Jun et al. (2006); Goldenberg et al. (2012);	• Netzer et al. (2012)
System dynamics	• Pagani and Fine (2008);	• Farris et al. (1998); Dickson et al. (2001); Nicholson and Kaiser (2008); Pavlov et al. (2008); Lin and Liu (2008);	• Dickson et al. (2001)
Chaos theory			• Hibbert and Wilkinson (1994)
Machine learning			• Netzer et al. (2012)

purchases than others) and permits links to be directional (i.e., not all influence effects are bidirectional), which can be flexibly modeled in ABM framework. Goldenberg et al. (2010) use the cellular automata modeling that allows analyzing network effects in consumer demand and show that the presence of network externalities may hold initial growth on new products because potential customers wait for early adopters, who provide them with more utility before they adopt. Haenlein and Libai (2013) examine the benefit of targeting customers with high lifetime value, or "revenue leaders." They show that targeting revenue leaders can create high value by accelerating adoption of other customers who have similarly high CLV. This is due to social networks composed of others who are similar to them. Using ABM modeling of a seeding program for a new product that is able to track the nonlinear processes inherent in the spread of influence in social networks, they suggested that the distribution of CLV in the population and the seed size play a crucial role in determining which seeding approach performs better. Goldenberg et al. (2009) examine the role of two types of hubs in diffusion and adoption: innovative hubs (innovators) and follower hubs (those who adopt early because of exposure to other adopters) while previous literature does not clearly distinguish between two. They use ABM since it can easily incorporate an environment where each consumer has a different number of neighbors who made adoption. They found that innovative hubs have a greater impact on the speed of the adoption process. On the other hand, follower hubs have a greater impact on total number of adoptions.

Trusov et al. (2013) propose a combination of Bayesian statistics and agent-based modeling by demonstrating that stochastic relationships simulated in complex systems through agent-based modeling can serve as informative priors for Bayesian inference. They show that incorporating network structure and agent-based modeling can improve predictions of future diffusion dynamics of products across subsets of the same network.

A related form of modeling to ABM is Cellular Automata (CA). It is a discrete model based on cells. Each cell can take on a state from a set of states, with binary states being the most common form of CA modeling. The cells are placed on a regular grid. Then, given an initial condition for the cellular automata, the future state will be an update of the grid according to local rules (Fuentes 2015), i.e., at each time step, each cell examines its own state and the states of its neighbors and then decides how to change on the basis of those states. Compared to ABM the interaction structure between cells in CA is more limited and the decision rules are derived from only the state of the focal cell and its neighbors. CA is often used to model the diffusion of innovations because of its parsimony, but it still generates a wide variety of dynamics and growth patterns (Goldenberg et al. 2004; Libai et al. 2005). Word of mouth is the ideal example to which cellular automata can be applied because the local interactions are types of interpersonal interactions. Using stochastic cellular automata, Goldenberg et al. (2001b) examine how the aggregate level effect of strong social ties (communications within an individual's own personal group) and weak social ties (less personal communications that an individual makes with a wide set of other acquaintances and colleagues) emerge through word-of-mouth. They found that the influence of weak ties is at least as strong as the influence of strong ties, and external marketing efforts such as advertising are only effective at the early stage of introduction of the new product. Goldenberg et al. (2001a) demonstrate that even when the common assumption of homogeneity in the consumers' communication behavior is relaxed that all adopters have equal effect on all other potential adopters, the Bass model behaves well and the heterogeneity does not affect aggregate-level results significantly. Goldenberg et al. (2002) use CA to examine cross-market communications in innovative products in the consumer electronics industry.

Pagani and Fine (2008) study adoption of 3G (third generation) wireless services by customers. The system dynamics approach enabled them to examine how various factors, which can be depicted by feedback mechanisms in system dynamics, can drive adoption of new technology. They specify causal loop diagrams that incorporate several dynamic forces such as network investment, user population, entry of service innovators, price competition, and positive network externalities arising from a larger user population. Their context fits with the complex systems approach because these dynamic forces are difficult to incorporate in other frameworks, especially network externalities, but are easily modeled using a system dynamics approach with a strong feedback mechanism. Their complex systems study provides an assessment of which future scenarios are likely in the wireless market and what dynamic triggers make them plausible.

Zhang et al. (2011) examine factors that can affect the diffusion of alternative fuel vehicles. They formulate an agent-based model to allow interdependencies between three key agents in the auto industry: manufacturers, consumers, and government agencies. They focus on the role of a technology push, word of mouth, and a government push on the speed of the diffusion. One advantage of using ABM in this context is to incorporate the interactions between multiple agents when the decision of one agent can affect the decision of another agent. The novelty of their model is that it combines optimization models of the manufacturing agents with a consumer choice (conjoint) model that allows them to model heterogeneity in consumer preference using consumer data. Without an ABM it would be difficult to incorporate these diverse modeling frameworks within one model. In the end, their model provides a better understanding of how to formulate an agent-based model to estimate factors that can impact the diffusion of other eco-innovations.

Promotion strategy

Timing and efficacy of different promotional activities such as seed marketing and mass media campaigns are an important question. Delre et al. (2007) use ABM to simulate these promotional activities and suggest that diffusion dynamics are affect by them.

Pricing is one of the fundamental promotion strategies in marketing, and system dynamics can shed light on a counterintuitive pricing strategy that may not be explained by conventional economic theory (Dickson et al. 2001). For example, in the early 1930s, firms facing a decrease in sales counterintuitively raised prices. This phenomenon led the federal government to investigate for a price conspiracy but the cause was attributed to the use of a cost plus pricing rule. As sales fell, the average cost of products and services rose. The cost-plus rule was applied to the higher costs. Then, prices were increased, which further reduced market sales (Dickson et al. 2001). Thus, a complex systems analysis can help us understand very counterintuitive events. Figure 2 represents this feedback effect.

System dynamics can measure the effectiveness of promotions that are often complicated by other marketing efforts. Nicholson and Kaiser (2008) examine dynamic market impacts of generic dairy advertising using a stock-flow-feedback simulation model. They specify several market drivers and model the impact of generic advertising expenditures. Their results provide new intuition about selective advertising strategies. Pavlov et al. (2008) study email marketing, with an emphasis on the dynamics of email marketing infrastructure, using a system dynamics model. They show that filtering may have an unintended consequence of increasing the amount of spam because the filtering would give spammers the equivalent of the information that they would need to target their messages. Lin and Liu (2008) use a system dynamics model for a policy experimentation, to examine the impact of price, network advertisement memory length, and customer sales effectiveness in an online shopping store, and found that these are key parameters that can increase the performance of the site’s revenue.

Network-based marketing is a set of marketing tactics that utilizes links between consumers to increase sales (Goel et al. 2015). Watts and Dodds (2007) find that large cascades of influence are driven not by influentials but by a critical mass of easily influenced individuals. Hill et al. (2006) study consumer networks using direct interactions such as communications between consumers. They show that network neighbors who are linked to a prior customer are more likely to adopt the service than baseline groups.

Much of social network in marketing literature focuses on how customer acquisition or adoption is made. However, recent social network studies focus on its effect on revenue and usage. Haenlein (2011) examines the relationship between social relationships between customers of a mobile phone provider and revenue data. They find a substantial degree of positive relationship between network correlations and revenue at the customer level. That is, high (or low) revenue customers tend to be related to other high revenue (or low revenue) customers.

Haenlein (2013) examines the relationship between the social network and the customer retention process using data on customers of a mobile phone provider and shows that a customer is substantially likely to defect from a provider if other customers to whom the customers is socially related have previously defected from the provider.

In a marketing context, types of network structures are distinct and combinations of different network structures can facilitate content exploration and usage. For example, many social networking sites such as YouTube have dual-network

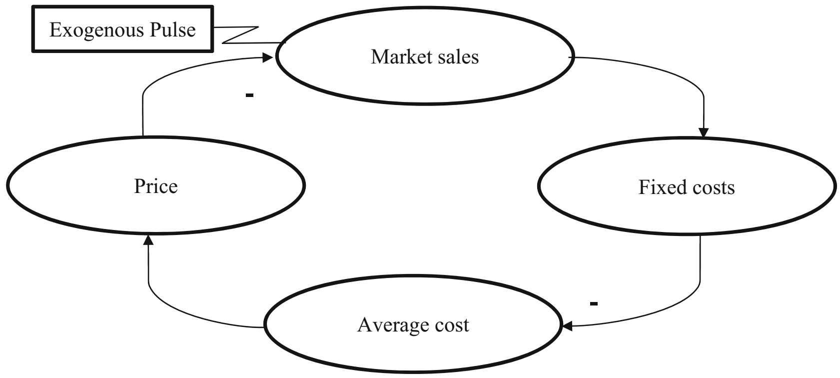

Fig. 2 Cost plus pricing feedback effect (← and ←represent a positive and a negative relationship, respectively)

structures: the product network and the social network. In the product network, each product (e.g., video) is connected by links to other products. In the social network, users with relationships (e.g., friends, collaborators, colleagues) are linked. Goldenberg et al. (2012) examine the role of the dual-network structure on facilitating content exploration using data on more than 700,000 videos and users from the YouTube dual network. They found that the dual network structures can lead to an increase in consumers' ratings of the content and to higher overall satisfaction.

Cui et al. (2006) model consumer responses to direct marketing by adopting a machine learning method: Bayesian networks trained by evolutionary programming. Evolutionary computation is a set of computational methods that simulates the natural evolution process based on Darwinian principles. They show that the method has notable advantages over more conventional methods, based on the accuracy of prediction, transparency of procedures, and interpretability of results. Their results suggest that machine learning can be a robust tool for modeling consumer response and assisting management decision-making.

Competitive strategy

ABM has been used to model competition in an oligopolistic market to test Hunt's General Theory of Competition (HGTC), which incorporates the evolutionary processes of competition (Tay and Lusch 2005). Midgley et al. (1997) show that genetic algorithms can be used to refine strategies in oligopolistic markets characterized by asymmetric competition.

Heppenstall et al. (2006) study retail location decisions using ABM. By integrating ABM and geographic information systems (GIS), the model can reproduce the spatial pattern observed in the real market and is able to accurately predict the long-term profitability of individual retailers.

Hibbert and Wilkinson (1994) investigate the equilibrium level of marketing effort in the brand competition context using chaos theory. A brand's marketing effort affects its own relative attractiveness and its competitors' relative attractiveness. Hibbert and Wilkinson show that key parameters such as price and advertising elasticity determine whether complex dynamics of equilibrium or chaotic behaviors occur or not. Their analysis demonstrates that market dynamics can be sensitive to changes in the value of parameters in the model and that researchers can predict whether equilibrium or chaos can emerge.

Two-sided markets, such as the credit card and video game markets, have been at the center of attention for both marketers and marketing researchers, thanks to the emerging digital economy. Network externalities, i.e., the idea that users of one side can benefit from the network of users on the other side, characterize the two-sided market. Network externalities are perfectly consistent with feedback dynamics. As production and sales increases, the user network of buyers/sellers increases. As the usage network increases in size, the utility of the product or service increases, which in turn increases demand. Increased demand affects production and sales. Figure 3 illustrates these feedback effects.

Heinrich and Grabner (2017) criticize that the conventional analytical models of the two-sided markets by Rochet and Tirole (2003, 2006) impose strong assumptions that are necessary for not only tractability but also for a unique and stable equilibrium. For example, equal probability of interaction between any buyer and seller agent is imposed in a model. They view these assumptions as unrealistic to describe the characteristics of two-sided markets in a real world. They formulate an agent-based model to relax these assumptions and show that unrealistic features of models significantly change the dynamics, which differs from prediction derived from the conventional model.

Competitive price setting behaviors by firms can be affected by consumer search. These relations and dynamics can be modeled with a system dynamics approach. For example, as a new low-price supermarket enters the market, variance in prices increases. Greater variance in prices increases benefits of consumer search and thus increases the amount of consumer search. Supermarkets with high prices lower their prices when observing greater consumer search. This reduces the variance in prices. In turn, this reduces the benefit of consumer

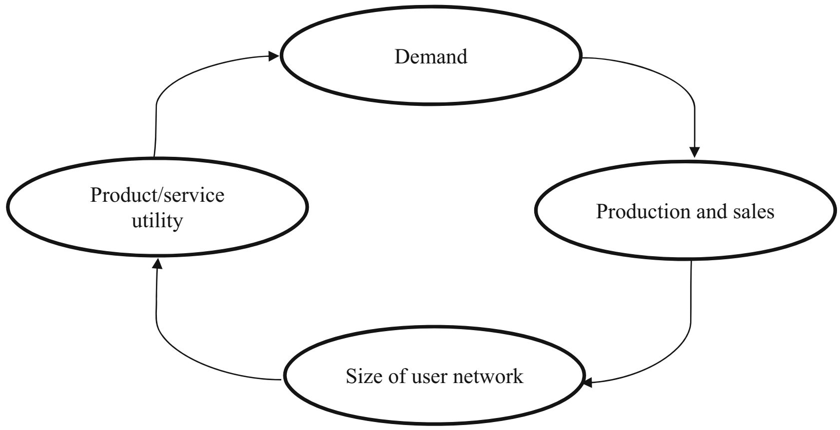

Fig. 3 Network feedback effect (← represents a positive relationship)

search, and consumer search is reduced. This is noticed and some supermarkets raise their prices thus increasing the variance of prices in the market and so forth. Figure 4 represents this feedback effects.

Recently it has been shown that consumer-generated content contains valuable information to predict sales (e.g., Chevalier and Mayzlin 2006), enormous and rich data in terms of volume and velocity restricts researchers to analyzing contents manually due to high costs. Recently, there have been more opportunities to analyze such data with the advent of text mining techniques, a branch of machine learning. For example, semantic network analysis can be combined with text-mining approach. Netzer et al. (2012) use the combination of two approaches to produce market-structure perceptual maps among sedan cars and diabetes drugs. They compare a market structure based on user-generated content data with a market structure obtained from conventional sales and survey-based data. They show that the market structure obtained from the consumer-generated content is similar to the market structure obtained from the conventional data.

Future opportunities

We have discussed several notable applications of complex systems approaches in marketing and yet there are more places where application of complex systems approaches may be fruitful, but that have not been fully explored yet.

Consumer behavior

As we have seen in the previous section, complex system approaches have the potential to predict not only individual-specific behaviors but also emergent behaviors at the macro level. For instance, how consumer activities in stores such as navigation patterns and responses to promotions impact profit is a suitable direction for applications of complex system approaches since these methods allow researchers to incorporate rules of consumers who respond to a store environment based on the various levels of susceptibility. For instance, the store environment could be customized to induce people of different psychographics to explore the store in different ways. In addition, it could also be helpful for the entire shopping environment, e.g., identifying and predicting bottleneck areas in a mall could be beneficial for increasing the profitability of the mall because they can adopt effective targeting strategies based on these areas.

Traditionally, consumer behavior is viewed as decisions made by an individual consumer. However, even an individual consumer's thoughts and beliefs may usefully be thought of as arising from multiple agents. Minsky (1988) calls this the "society of mind." Thus, an individual decision might be thought of as an outcome of the strategic interaction of

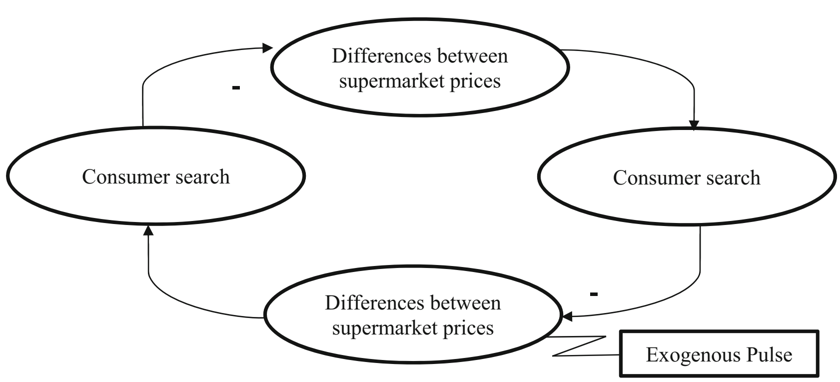

Fig. 4 Consumer search and price feedback effect (← and ↚ represent a positive and a negative relationship, respectively)

multiple selves (Ding 2007). Ding posits that there are two types of intra-person agents: an efficiency agent and an equity agent. While the efficiency agent attempts to maximize the total utility for the selves that consist of a person's mind, the equity agent is concerned about the equity of utilities across the selves. Standard consumer utility theory is a subset case of the game, including only the role of the efficiency agent (Ding 2007). He illustrates variety-seeking behavior using intra-person games and shows that the model can provide accurate predictions of an individual tendency toward variety seeking. The theory provides an understanding of a consumer's decision process, such that a decision is driven by the intra-person games. We envision that testing the implications of the new perspective may be an interesting application of agent-based modeling.

Behavioral economics, which adopts more reasonable and relaxed assumptions about individual behaviors such as bounded rationality, provides many hypotheses that are tested at the individual level by conducting lab experiments. However, consequences and implications from aggregations of individuals are not yet explored fully due to limitations of conducting large-scale experiments involving a diverse pool of subjects. Complex systems approaches can be useful methods for complementing lab results by examining patterns of behaviors that are not necessarily determined by a composite of a handful of single agents but rather through examining the emergent behavior of larger populations. This approach has been explored in the organizational behavior literature (Smith and Rand, in press), but could also be applied to Marketing. For example, capturing the nature of emergent behaviors that arises from social interactions between individuals is a useful factor in predicting the donation activities as a whole. Therefore, using complex systems approaches we could ask a question as to how much individuals who have a heterogeneous level of altruistic and image-motivation for pro-social activity will engage in a donation campaign and how they affect each other's donation activity.

Supplier networks and channels

Competitive analyses of integrating manufacturer and retailer relations using conventional approaches are traditionally limited to a small number of firms. This is a limitation in reflecting and predicting an accurate picture of the competitive environment in reality. Complex systems approaches can specify flexible cost structures and bargaining power. In other words, firms can be modeled to adopt a variety of possible strategies. For instance, peer-to-peer platform businesses are now prevalent across a wide range of industries due to the recent evolution of information technology, such as mobile apps. Efficient supply management of these markets is driven by cross-network externalities between end users who are different in their influence and responsiveness. Using complex system approaches, researchers can ask questions such as what is the impact on the behavior of one side of agents if there are changes in promotion or price structures on the other side of agents?

Competitive strategy

Complex systems is a useful framework for examining inter-firm relationships, since as many heterogeneous firms as needed can be flexibly modeled. In addition, it can incorporate complex learning models of firms that enable researchers to examine a richer space of firm strategies than that of the equation-based modeling. Thus, researchers can focus on the effects of those strategies on long-term competition and cooperation between firms (Rand and Rust 2011).

One of the challenges in modeling strategic games as the underlying framework for analyzing competitive strategy is the presence of multiple equilibria. The multiplicity of equilibria prevents a model from yielding unique predictions. This makes it difficult to formulate probability statements for the outcomes of the model and difficult to build a likelihood function in estimation. Incorporating learning into models may be one of the solutions to this problem (Borkovsky et al. 2015). As such firms can engage in an adaptive process through which they learn how to obtain an equilibrium state (Fudenberg and Levine 1998). As we have seen in the previous section, complex system approaches can assign adaptive and evolutionary behaviors at a relatively low cost of modeling. By allowing firms to learn to move toward an equilibrium, the complex system approaches may provide a useful tool to understand detailed processes of selection mechanism and outcomes of multiple equilibria. For instance, Huang et al. (2016) examine firms' optimal innovation process in terms of two emergent properties such as speed and quality in a digital technology market. They adopt ABM since they assume that many heterogenous firms exist in a market whose innovation decisions affect each other's decisions and thus lead to a nonlinear emergent market environment. It would not be possible to incorporate heterogeneity and interactions of firms in innovation dynamics using the conventional approaches based on an equilibrium game-theoretic framework such as Ericson and Pakes (1995), which are prone to multiple equilibria as researchers allow more complex structures. Using complex systems approaches is a huge advantage and improvement in testing implications of two crucial factors on market and firm dynamics that should not be assumed away at the expense of model tractability and computational costs. Their results suggest that the optimal innovation speed is rarely as fast as possible. However, rather it varies depending on the nature of the market and the interactions between firms and the market: market structure (i.e., number of firms), speed of technological epoch change, demand uncertainty, IT capability of firms, average innovation speed of firms in the market, and attractiveness of outside options. These factors together affect the speed and quality of innovation processess in a nonlinear manner, which complex system approaches can flexibly handle compared to other conventional approaches. These advantages of complex systems over conventional approaches have the potential to solve many problems in the competitive strategy domain.

Opportunities in teaching

The potential of the complex systems approach can be developed more by increased efforts in education using example applications for marketing practice. A traditional curriculum in marketing education tends to inculcate students with the traditional equilibrium framework that systems are deterministic. This trend makes it difficult for students to accept complex systems concepts. Thus, exposing complexity concepts to students early on can be an effective strategy to promote complex systems (Sakowski and Tóvolli 2015). There are a couple of practical solutions. First, complex systems researchers should continue efforts to provide discussions of methods and managerial interpretation, which can lead to greater acceptance of the methods in both research and practice. Organizing research groups who specialize in this aspect may be a stepping-stone toward this purpose.

Second, education about complex systems needs to put more emphasis on the visualization of how a model of complex systems works, along with the technical details in implementation, because it makes results and implications of models easier to communicate and understand (Rand and Rust 2011).

Third, many of the methods of complex systems require computational literacy. Thus, it makes sense to develop interdisciplinary courses and curricula that incorporate both marketing concepts and computational fields, such as computer science, in order to study complex systems in marketing (Sakowski and Tóvolli 2015).

Industry applications

The complex systems framework is not just a toolkit for academic research and education. It can be and has been applied to industry as well. Researchers should be working closely with industry to identify their problems. Fortunately, the industry is increasingly adopting complex system approaches to solve their business problems. For example, Concentric[2] and ThinkVine[3] are companies that sell software analyzing and forecasting market dynamics using agent-based modeling. One of their contributions to complex systems research is to develop and design an easy user interface that marketers can apply to their own problems. Procter & Gamble Co. actively applies ABM to establish various business strategies. They analyze marketing strategy using complex systems models for predicting market shares of products (North and Macal 2007). They also save $300 million annually using agent-based modeling that simulates individual components of the supply system, including stores, drivers, and logistics to maximize the overall efficiency of the supply network (Anthes 2003). MasterCard used system dynamics to discover that co-branding, a partnership between a credit card brand and company, is an effective marketing strategy to increase their market share compared to increasing advertising alone (Cooper 2016). Procensol, a U.K based consulting firm, applied chaos theory to devise a better business process management (Chiang 2015). Facebook and Twitter are using network science to understand their users and provide better targeting options for their advertisers. Machine learning is in use by a large number of marketing firms. Firms are very interested in applying the toolbox of complex systems to help understand their problems. In the future, we expect an increasing amount of work in which researchers and industry collaborates to increase literacy and adoption of complex systems approaches.

[2] http://concentricmarket.com/how-it-works/
[3] http://www.thinkvine.com/technology

Conclusion

This paper presents a survey of complex system approaches in marketing research. We discuss key concepts, introduce major methods with applications, and envision future opportunities. Complex system approaches promise a new frontier in marketing research because marketing efforts are becoming more centered on individual customers and their relationships, instead of a transaction-based mass market. While we do not propose that all marketing questions can be handled with a complex systems perspective, complex systems approaches can be effective complementary tools or an alternative lens to conventional approaches. We expect that the steady advances in data processing and computational resources will allow more researchers to adopt complex systems approaches, thereby providing important new insights on marketing phenomena and strategies for both researchers and practitioners in the future.

References

Anthes, G. (2003). P&G saves millions with supply network. Retrieved July 22, 2016 from: http://www.itworldcanada.com/article/pg-saves-millions-with-supply-network/20112.

Arthur, B. W. (1994). *Increasing returns and path dependency in the economy.* Ann Arbor: University of Michigan Press.

Bankes, S. C. (2002). Agent-based modeling: a revolution? *Proceedings of the National Academy of Sciences of the United States of America, 99*(Suppl 3), 7199–7200.

Bass, F. M. (1969). A new product growth for model consumer durables. *Management Science, 15*(5), 215–227.
Bazghandi, A. (2012). Techniques, advantages and problems of agent based modeling for traffic simulation. *International Journal of Computer Science Issues, 9*(1), 115–119.
Berger, J. (2014). Word of mouth and interpersonal communication: a review and directions for future research. *Journal of Consumer Psychology, 24*(4), 586–607.
Bonabeau, E. (2002). Agent-based modeling: methods and techniques for simulating human systems. *Proceedings of the National Academy of Sciences, 99*(suppl 3), 7280–7287.
Bondi, B. A. (2000). Characteristics of scalability and their impact on performance. *Proceedings of the 2nd international workshop on Software and performance, ACM.*
Borkovsky, R. N., Ellickson, P. B., Gordon, B. R., Aguirregabiria, V., Gardete, P., Grieco, P., et al. (2015). Multiplicity of equilibria and information structures in empirical games: challenges and prospects. *Marketing Letters, 26*(2), 115–125.
Borshchev, A., & Filippov, A. (2004). From system dynamics and discrete event to practical agent based modeling: reasons, techniques, tools. *Proceedings of the 22nd international conference of the system dynamics society* (Vol. 22).
Chevalier, J. A., & Mayzlin, D. (2006). The effect of word of mouth on sales: online book reviews. *Journal of Marketing Research, 43*(*3*), *345–354*.
Chiang, M. (2015). Chaos Theory in Business Change. Retrieved August 21, 2017 from: https://www.procensol.com/blog/chaos-theory-business-change/.
Cohen, W. M., & Levinthal, D. A. (1990). Absorptive capacity: a new perspective on learning and innovation. *Administrative Science Quarterly, 35*(1), 128–152.
Cooper, K. (2016). System Dynamics Case Repository: MasterCard. Retrieved August 21, 2017 from: http://cases.systemdynamics.org/credit-card-market-simulation/.
Costa, L. d. F., Rodrigues, F. A., Travieso, G., & Boas, P. R. V. (2007). Characterization of complex networks: a survey of measurements. *Advances in Physics, 56*(1), 167–242.
Cui, G., Wong, M. L., & Lui, H. (2006). Machine learning for direct marketing response models: Bayesian networks with evolutionary programming. *Management Science 52*(4), 597–612.
Delre, S. A., Jager, W., Bijmolt, T. H. A., & Janssen, M. A. (2007). Targeting and timing promotional activities: An agent-based model for the takeoff of new products. *Journal of Business Research, 60*(8), 826–835.
Delre, S. A., Jager, W., Bijmolt, T. H. A., & Janssen, M. A. (2010). Will it spread or not? the effects of social influences and network topology on innovation diffusion. *Journal of Product Innovation Management, 27*(2), 267–282.
Dickson, P. R. (1995). Review of "Increasing returns and path dependency in the economy". *Journal of Marketing, 59*(3), 97–99.
Dickson, P. R. (1996). The static and Dynamic Mechanics of Competition: a comment on Hunt and Morgan's comparative theory of competition. *Journal of Marketing, 60*(4), 102–106.
Dickson, P. R., Farris, W. P., & Verbeke, J. W. (2001). Dynamic strategic thinking. *Journal of the Academy of Marketing Science, 29*(3), 216–237.
Ding, M. (2007). A theory of intraperson games. *Journal of Marketing, 71*(2), 1–11.
Dover, Y., Goldenberg, J., & Shapira, D. (2012). Network traces on penetration: uncovering degree distribution from adoption data. *Marketing Science, 31*(4), 689–712.
Ericson, R., & Pakes, A. (1995). Markov-perfect industry dynamics: A framework for empirical work. *Review of Economic Studies, 62*(1), 53–82.
Farris, P., Verbeke, W., Dickson, P. R., & Nierop, V. E. (1998). Path dependencies and the long-term effects of routinized marketing decisions. *Marketing Letters, 9*(3), 247–268.
Forrester, J. W. (1971). Counterintuitive behavior of social systems. *Theory and Decision, 2*(2), 109–140.
Fudenberg, D., & Levine, D. K. (1998). *The theory of learning in games*. Cambridge: MIT Press.
Fuentes, M. (2015). Methods and methodologies of complex systems. In B. A. Furtado, P. A. M. Sakowski, & M. H. Tóvoli (Eds.), *Modeling complex systems for public policies* (pp. 55–71). Brasília: IPEA.
Garcia, R. (2005). Uses of agent-based modeling in innovation/new product development research. *Journal of Product Innovation Management, 22*(5), 380–398.
Garcia, R., & Jager, W. (2011). Agent-based modeling of innovation diffusion. *Journal of Product Innovation Management, 28*(2), 148–151.
Goel, S., Anderson, A., Hofman, J., & Watts, D. J. (2015). The structural virality of online diffusion. *Management Science, 62*(1), 180–196.
Goldenberg, J., Libai, B., & Muller, E. (2001a). Using complex system analysis to advance marketing theory development: modeling heterogeneity effects on new product growth through stochastic cellular automata. *Academy of Marketing Science Review, 9*, 1–19.
Goldenberg, J., Libai, B., & Muller, E. (2001b). Talk of the network: a complex systems look at the underlying process of word-of-mouth. *Marketing Letters, 12*(3), 211–223.
Goldenberg, J., Libai, B., & Muller, E. (2002). Riding the saddle: how cross-market communications can create a major slump in sales. *Journal of Marketing, 66*(2), 1–16.
Goldenberg, J., Libai, B., & Muller, E. (2010). The chilling effects of network externalities. *International Journal of Research in Marketing, 27*(1), 4–15.
Goldenberg, J., Oestreicher-Singer, G., Reichman, S. (2012). The quest for content: how user-generated links can facilitate online exploration. Journal of Marketing Research, 49(4), 452–468.
Goldenberg, J., Garber, T., Muller, E., & Libai, B. (2004). From density to destiny: using spatial dimension of sales data for early prediction of new product success. *Marketing Science, 23*(3), 419–428.
Goldenberg, J., Libai, B., Moldovan, S., & Muller, E. (2007). The NPV of bad news. *International Journal of Research in Marketing, 24*(3), 186–200.
Goldenberg, J., Han, S., Lehmann, D. R., & Hong, J. W. (2009). The role of hubs in the adoption process. *Journal of Marketing, 73*(2), 1–13.
Haenlein, M. (2011). A social network analysis of customer-level revenue distribution. *Marketing Letters, 22*(1), 15–29.
Haenlein, M. (2013). Social interactions in customer churn decisions: The impact of relationship directionality. *International Journal of Research in Marketing, 30*(3), 236–248.
Haenlein, M., & Libai, B. (2013). Targeting revenue leaders for a new product. *Journal of Marketing, 77*(3), 65–80.
Heinrich, T. & Grabner, C. (2017). Beyond equilibrium: Revisiting two-sided markets from an agent-based modeling perspective. Working paper.
Heppenstall, A., Evans, A., & Birkin, M. (2006). Using hybrid agent-based systems to model spatially-influenced retail markets. *Journal of Artificial Societies and Social Simulation, 9*(3).
Hibbert, B., & Wilkinson, I. F. (1994). Chaos theory and the dynamics of marketing systems. *Journal of the Academy of Marketing Science, 22*(3), 218.
Hill, S., Provost, F., & Volinsky, C. (2006). Network-based marketing: identifying likely adopters via consumer networks. *Statistical Science, 21*(2), 256–276.
Holland, J. H. (1998). *Emergence: from chaos to order*. Redwood City: Addison-Wesley.
Holland, J. H. (2014). *Complexity: a very short introduction*. Oxford: Oxford University Press.
Huang, M.-H., Rust, R. T., & Rand, W. (2016). Don't do it right, do it fast? Speed and quality of innovation as an emergent process. Working paper.
Jun, T., Kim, J.-Y., Jun Kim, B., & Choi, M. Y. (2006). Consumer referral in a small world network. *Social Networks, 28*(3), 232–246.

Lambkin, M., & Day, G. S. (1989). Evolutionary processes in competitive markets: beyond the product life cycle. *Journal of Marketing, 53*(3), 4–20.

Levy, D. (1994). Chaos Theory and Strategy: Theory, Application, and Managerial Implications. *Strategic Management Journal, 15*, 167–178.

Lewis, T. G. (2008). *Network science: theory and applications*. Hoboken: Wiley.

Libai, B., Muller, E., & Peres, R. (2005). The role of seeding in multi-market entry. *International Journal of Research in Marketing, 22*(4), 375–393.

Lilien, G. L., Kotler, P., & Moorthy, K. S. (2011). *Marketing models*. New Delhi: PHI Learning.

Lin, J. H., & Liu, H. C. (2008). System dynamics simulation for Internet marketing. *System Integration, 2008 IEEE/SICE International Symposium on* 83–88.

Lorig, F., Dammenhayn, N., Müller, D. J., & Timm, I. J. (2015). Measuring and Comparing Scalability of Agent-Based Simulation Frameworks. *German Conference on Multiagent System Technologies*.

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy, 1*, 19–46.

Lyneis, J. M. (2000). System dynamics for market forecasting and structural analysis. *System Dynamics Review, 16*(1), 3–25.

Midgley, D. F., Marks, R. E., & Cooper, L. C. (1997). Breeding competitive strategies. *Management Science, 43*(3), 257.

Midgley, D., Marks, R., & Kunchamwar, D. (2007). Breeding competitive strategies. *Journal of Business Research, 60*(8), 884–893.

Miller, J. H., & Page, S. E. (2009). *Complex adaptive systems: an introduction to computational models of social life*. Princeton: Princeton University Press.

Minsky, M. (1988). *Society of mind*. New York: Simon and Schuster.

Netzer, O., Feldman, R., Goldenberg, J., & Fresko, M. (2012). Mine your own business: Market-structure surveillance through text mining. *Marketing Science, 31*(3), 521–543.

Newman, M. (2010). *Networks: an introduction*. Oxford: Oxford University Press.

Nicholson, C. F., & Kaiser, H. M. (2008). Dynamic market impacts of generic dairy advertising. *Journal of Business Research, 61*(11), 1125–1135.

North, M. J., & Macal, C. M. (2007). *Managing business complexity: discovering strategic solutions with agent-based modeling and simulation*. Oxford: Oxford University Press.

Onno, O., Trienekens, J., & Beers, G. (2001). Chain and network science: a research framework. *Journal on Chain and Network Science, 1*(*1*), 1–6.

Ottino-Loffler, B., Stonedahl, F., Veetil, V. P., & Wilensky, U. (2015). A General Model of Spatial Competition. Working Paper.

Pagani, M., & Fine, C. H. (2008). Value network dynamics in 3G–4G wireless communications: a systems thinking approach to strategic value assessment. *Journal of Business Research, 61*(11), 1102–1112.

Page, S. E. (2006). Path dependence. *Quarterly Journal of Political Science, 1*(1), 87–115.

Pavlov, O. V., Melville, N., & Plice, R. K. (2008). Toward a sustainable email marketing infrastructure. *Journal of Business Research, 61*(11), 1191–1199.

Rahmandad, H., & Sterman, J. (2008). Heterogeneity and network structure in the dynamics of diffusion: comparing agent-based and differential equation models. *Management Science, 54*(5), 998–1014.

Rand, W. (2015). Complex systems: concepts, literature, possibilities and limitations. In B. A. Furtado, P. A. M. Sakowski, & M. H. Tóvoli (Eds.), *Modeling complex systems for public policies* (pp. 37–54). Brasília: IPEA.

Rand, W., & Rust, R. T. (2011). Agent-based modeling in marketing: guidelines for rigor. *International Journal of Research in Marketing, 28*(3), 181–193.

Rochet, J. C., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association, 1*(4), 990–1029.

Rochet, J. C., & Tirole, J. (2006). Two-sided markets: a progress report. *The RAND Journal of Economics, 1*(4), 990–1029.

Rosenblatt, M. (2013). Network analysis in marketing: a test of tie strength among networks of franchise operations. *Proceedings of the Northeast Business & Economics Association*, 205–208.

Rust, R. T. (2015). The ups and downs of fashion. *Global Fashion Management Conference, 6*, 910–910.

Rust, R. T., & Huang, M.-H. (2014). The service revolution and the transformation of marketing science. *Marketing Science, 33*(2), 206–221.

Sakowski, P. M., & Tóvolli, M. H. (2015). Complex approaches for education in Brazil. In B. A. Furtado, P. A. M. Sakowski, & M. H. Tóvoli (Eds.), *Modeling complex systems for public policies* (pp. 315–335). Brasília: IPEA.

Stephen, A. T., Dover, Y., & Goldenberg, J. (2010). A comparison of the effects of transmitter activity and connectivity on the diffusion of information over online social networks. Working Paper.

Sterman, J. D. (2000). *Business dynamics: systems thinking and modeling for a complex world*. Boston: Irwin/McGraw-Hill.

Taleb, N. N. (2007). *The black swan: the impact of the highly improbable*. New York: Random House.

Tay, N. S. P., & Lusch, R. F. (2005). A preliminary test of Hunt's general theory of competition: using artificial adaptive agents to study complex and ill-defined environments. *Journal of Business Research, 58*(9), 1155–1168.

Teng, S. H. (2016). Scalable algorithms for data and network analysis. *Foundations and Trends in Theoretical Computer Science, 12*(1-2), 1–274.

Trusov, M., Rand, W., & Joshi, Y. V. (2013). Improving prelaunch diffusion forecasts: using synthetic networks as simulated priors. *Journal of Marketing Research, 50*(6), 675–690.

Watts, D. J., & Dodds, P. S. (2007). Influentials, networks, and public opinion formation. *Journal of Consumer Research, 34*(4), 441–458.

Zafarani, R., Abbasi, M. A., & Liu, H. (2014). *Social media mining: an introduction*. Cambridge: Cambridge University Press.

Zhang, T., Gensler, S., & Garcia, R. (2011). A Study of the Diffusion of Alternative Fuel Vehicles: An Agent-Based Modeling Approach. *Journal of Product Innovation Management, 28*(2), 152–168.