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- The maximum utility model achieves the highest out-of-sample return and utility.
- None of the models are consistently better than the 1/N rule in Sharpe ratio.

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Keywords: Cryptocurrency; Portfolio diversification; Out-of-sample performance

JEL classification: G11

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Utilizing the empirical data of ten major cryptocurrencies, this article examines the investablitiy and role of diversification in cryptocurrency market, and evaluates the out-of-sample performance of commonly used asset allocation models across cryptocurrencies. We show that portfolio diversification across different cryptocurrencies can significantly improve the investment results. We also find robust evidence that the maximum utility model dominates the out-of-sample utility, although none of the models can consistently beat the naïve 1/N portfolio in Sharpe ratio.

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1. Introduction

As the sharp increase in the trading volume of Bitcoin recently, the cryptocurrency market has drawn explosive attention from the media, financial industry and government institutions. Bitcoin is the first and largest decentralized cryptocurrency built upon blockchain technology, and has accumulated sufficient data of prices for empirical analyses. Therefore, quite a lot of academic research has been conducted on Bitcoin, such as market efficiency (Urquhart, 2016; Nadarajah and Chu, 2017; Bariviera, 2017; Vidal-Tomás and Ibañez, 2018), price volatility (Dyhrberg, 2016; Katsiampa, 2017), price clustering (Urquhart, 2017), and transaction cost (Kim, 2017). On the other hand, different kinds of cryptocurrencies rise immensely in the last couple of years, resulting in rapid expansion of market dimensions. As of April 2018, the number of cryptocurrencies has exceeded 1,500, with more than 500 having a market capitalization over 10 million dollars. Thus, the research on cryptocurrencies beyond Bitcoin has received considerable impetus in recent literatures.

There are debates on the cryptocurrencies that they are mainly regarded as a new class of assets rather than traditional currencies (Glaser et al., 2014; Baek and Elbeck, 2015). Several important stylized facts of cryptocurrencies found recently are common in financial assets, such as leptokurtosis (Chan et al., 2017), heteroscedasticity

(Gkillas and Katsiampa, 2018) and long-memory (Phillip et al., 2018). Moreover, researches on cryptocurrencies further demonstrate potential possibility of diversification in this emerging market for institutional and retail investors. First, a sufficient amount of cryptocurrencies have large market capitalization at million or billion level, while they usually provide lower transaction costs to individuals than one of the most efficient financial markets (Kim, 2017), which indicates ample liquidity. Second, there are cryptocurrencies that the dynamics are relatively isolated to the others (Corbet et al., 2018), which may offer diversification benefits from investors. Third, the variety of cryptocurrencies is still increasing, and therefore the cryptocurrency market has a growing place in diversification and portfolio management. However, despite the huge growth of the cryptocurrency market, research on the portfolio diversification of cryptocurrencies is rather limited.

In this paper, we analyze the investability and role of diversification in the cryptocurrency market with applying six classical portfolilio selection models through an out-of-sample evaluation method. The out-of-sample method means the parameters involved in the models are estimated via a "rolling window" at each rebalancing date instead of using the entire sample (the in-sample method, which cannot reflect the real investment decisions). We compare the out-of-sample performance of different models using various evaluation criteria to understand which model performs the best in one or more specific aspects. We also analyze the portfolio performance by setting different transaction costs, rebalancing periods, as well as the risk aversion parameters, such that the results are robust and instructive for real investment.

2. Data and Methodology

In this study, the cryptocurrency dataset covers the period from 07-Aug-15 to 09-Apr-18 with 977 trading days in total. The data are available at coinmarketcap.com, consisting of open, high, low and close prices, as well as dollar volume and market capitalization on a daily basis. We collect those cryptocurrencies which begin trading no later than 07-Aug-15[1], and whose market capitalization is larger than 1 billion. To this end, 10 cryptocurrencies, including Bitcoin, Ethereum, Ripple, Litecoin, Stellar, Monero, Dash, Tether, NEM and Verge, are analyzed in this article.

We first investigate the performance of individual cryptocurrencies. Table 1 reveals the mean, volatility, annualized return, maximum drawdown, Sharpe ratio and utility of each cryptocurrency, where the utility is defined by the quadratic form:

$$U_k = \hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2, \tag{1}$$

in which $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ are the mean and variance of the *k*-th cryptocurrency, and γ is the risk aversion parameter. We report the results for case $\gamma = 1$ in Table 1. It can be found that most of cryptocurrencies have considerably high return, Sharpe ratio and utility. This stylized fact inspires us to further investigate the investment values of

[1] This is the date when the second largest cryptocurrency, Ethereum, begins trading. Similar consideration was involved in Brauneis and Mestel (2018).

cryptocurrencies. Among the ten cryptocurrencies, Bitcoin performs in the relatively lower reach compared to the newly issued cryptocurrencies, e.g. NEM and Verge. Tether seems quite different from others, and it is much similar to a traditional currency with a fairly flat trend of prices.

Table 1

Historical performance of cryptocurrencies (in % for mean, volatility, annualized return, and maximum drawdown). The mean and volatility (standard deviation) are reported on a daily basis. The annualized return is compound and calculated as (Ending Value/Starting Value) $^{(360/days)}$-1, and the Sharpe ratio and utility reported are simply annualized (multiplied by $\sqrt{360}$ and 360, respectively, to the corresponding daily measure).

Cryptocurrency	Mean	Volatility	Annualized Return	Maximum Drawdown	Sharpe Ratio	Utility
Bitcoin	0.41	4.14	223.61	65.96	1.89	1.18
Ethereum	0.84	7.86	523.97	84.30	2.02	1.90
Ripple	0.78	9.75	353.00	85.90	1.51	1.08
Litecoin	0.53	6.37	238.27	68.42	1.57	1.16
Stellar	0.88	10.22	405.25	82.58	1.64	1.30
Monero	0.85	7.96	635.80	67.69	2.02	1.91
Dash	0.66	6.40	428.19	81.59	1.95	1.63
Tether	0.00	0.64	-0.09	8.66	0.05	0.00
NEM	1.26	11.20	1409.24	88.65	2.14	2.29
Verge	2.52	20.03	1900.24	94.44	2.39	1.85

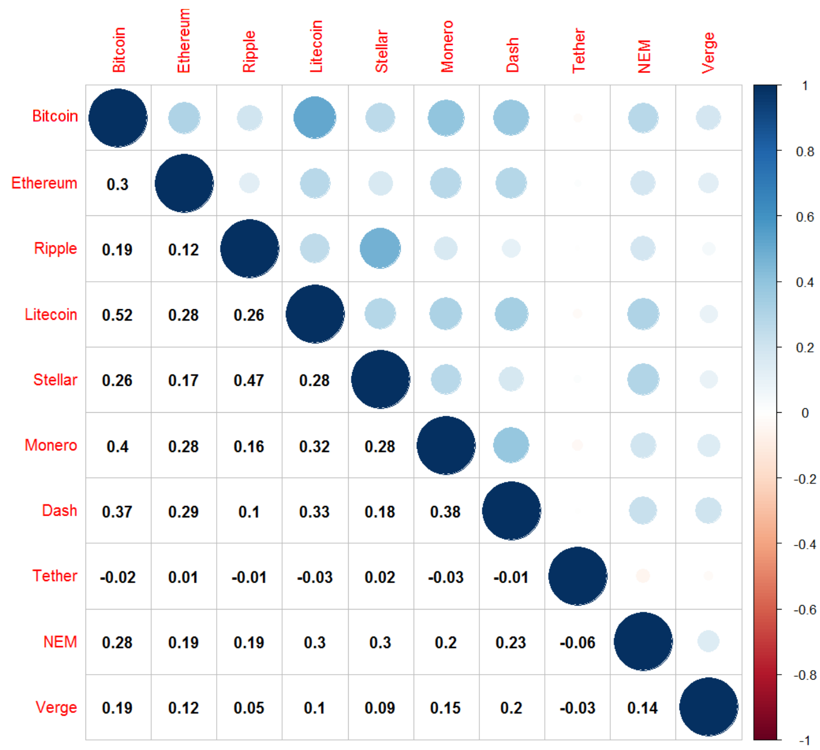

Fig. 1. Correlation matrix of cryptocurrencies

Fig. 1 shows the correlation matrix of the cryptocurrencies. It can be observed the that highest correlation among the pairs is between Bitcoin and Litecoin at 0.52, while most of the others are at a relatively low level for less than 0.3, especially for the last three, i.e. Tether, NEM and Verge. Moreover, Tether is even negative or nearly uncorrelated to the others, which coincides with the analysis from Table 1 that it is more like a traditional currency while the others are much more speculative and fluctuated. The relatively low correlations among different cryptocurrencies indicate we may benefit by combining several cryptocurrencies into a portfolio.

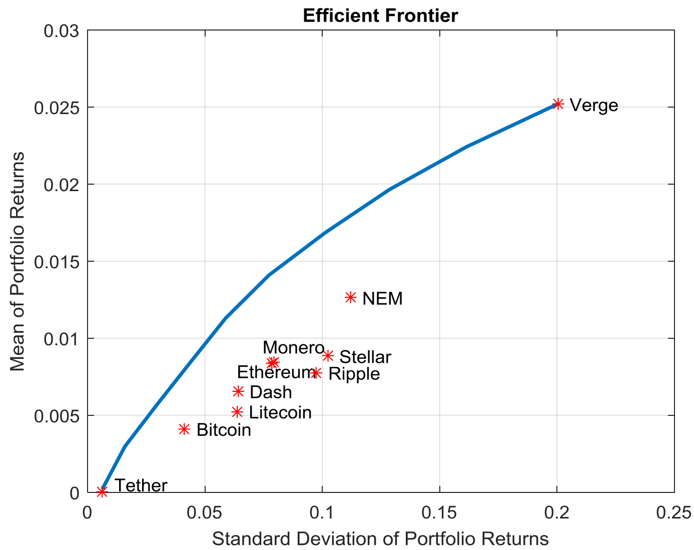

Fig. 2. Efficient frontier of cryptocurrency portfolios

To see the combination of the cryptocurrencies, Fig. 2 exhibits the efficient frontier of the portfolios. We plot Fig. 2 under the short-selling constraints, such that the Tether and Verge are located on the efficient frontier, representing the lowest volatility and highest expected return among all the cryptocurrencies, respectively. The maximum Sharpe ratio and maximum utility can be achieved on the efficient frontier in Fig. 2, through reasonable asset allocation across the cryptocurrencies. Note that Fig. 2 displays the in-sample results only, as we estimate the mean and covariance matrix based on the entire sample, which is infeasible for real investment. For example, when we stand at date 1-Jan-2016, we can only obtain the data no later than 1-Jan-2016, but the in-sample method uses data before and after that date, which is unrealistic in real investment, and hence the performance under the in-sample method is generally viewed as a theoretical ideal case rather than the realistic case. Thereafter, we will focus on the out-of-sample results of the portfolio models, where the expected return and covariance matrix are estimated via a rolling window procedure discussed in Section 3.

The portfolio models[2] considering in this article are: 1/N equal weighted rule (EW), minimum variance (MV), risk parity (RP), Markowitz (MW), maximum Sharpe ratio (MS), and maximum utility (MU). The detailed settings of those models are listed in Table 2.

[2] More details about the portfolio models can be found in Markowitz (1952), DeMiguel et al. (2009), Chaves et al. (2011) and Hatemi-J and El-Khatib (2015), etc.

Table 2
List of various asset allocation models.

#	Model	Abb.	Objective function	Type	Constraints
1	1/N rule	EW			
2	Minimum variance	MV	$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$	Minimize	$\mathbf{w}'\mathbf{1}=1, \mathbf{w}\geq\mathbf{0}$
3	Risk parity	RP	$\sum_{i=1}^{N}\left[w_i(\boldsymbol{\Sigma}\mathbf{w})_i-\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}/N\right]^2$	Minimize	$\mathbf{w}'\mathbf{1}=1, \mathbf{w}\geq\mathbf{0}$
4	Markowitz	MW	$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$	Minimize	$\mathbf{w}'\boldsymbol{\mu}\geq\mu_0$, $\mathbf{w}'\mathbf{1}=1, \mathbf{w}\geq\mathbf{0}$
5	Maximum Sharpe	MS	$\mathbf{w}'\boldsymbol{\mu}/\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}$	Maximize	$\mathbf{w}'\mathbf{1}=1, \mathbf{w}\geq\mathbf{0}$
6	Maximum utility	MU	$\mathbf{w}'\boldsymbol{\mu}-\frac{\gamma}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$	Maximize	$\mathbf{w}'\mathbf{1}=1, \mathbf{w}\geq\mathbf{0}$

Notice that in Table 2, if we remove the short-selling constraints, there would be additional 5 models (except for the 1/N rule). To save space, we only present the results for the constrained models in Section 3 (while we did the same procedures for the unconstrained models, which did not generate better performance). In the following empirical analysis, μ_0 in the Markowitz model is set to be the corresponding mean under 1/N rule, and the risk aversion parameter γ is 1 in the maximum utility model (results for other values of γ are separately discussed as a robustness check in Table 4).

3. Empirical Results

In this section, we compare the out-of-sample performance of the asset allocation models listed in Table 2. The weights of different models are calculated in a rolling window procedure via the following steps: 1) Utilize the nearest M=360 days' data before and on the rebalancing time t to estimate the parameters in the optimization, i.e. $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$; 2) Solve the corresponding optimization problem and obtain the weights at t; 3) Rebalance the weights by conducting steps 1) and 2) after the holding period τ (also called rebalancing period).

For example, our dataset is ranging from 07-Aug-15 to 09-Apr-18, and suppose the rebalancing period is $\tau=30$. The first rebalancing date is 01-Aug-16, and we use the starting 360 days' returns (from 07-Aug-15 to 01-Aug-16) to estimate the parameters of the models and solve the weights on 01-Aug-16, which will last for the following $\tau=30$ days (from 02-Aug-16 to 31-Aug-16). The second rebalancing date is

31-Aug-16, and we use the nearest 360 days' return (from 7-Sep-15 to 31-Aug-16) to estimate the parameters of the models and solve the weights on 31-Aug-16, which will last for the following $\tau = 30$ days (from 1-Sep-16 to 30-Sep-16). The third rebalancing date is 30-Sep-16, and the followings are routine repeated.

Once obtaining the weights, we can investigate the backtesting performance of different models through the portfolio return $R_{t+h} = \mathbf{w}_t' \mathbf{r}_{t+h}, h = 1, \cdots, \tau$, where t is at each rebalancing time, R_{t+h} is the portfolio return, and $\mathbf{r}_{t+h}$ comprises the returns of cryptocurrencies. In other words, we utilize the information never after day t to determine the weights $\mathbf{w}_t$, and compute the portfolio returns of the future τ days based on weights $\mathbf{w}_t$.

Table 3 presents the empirical performance of portfolio models with the rebalancing period τ =30. Comparing to the individual results in Table 1 for each cryptocurrency, we observe that the Sharpe ratio and utility of portfolios are greatly increased, implying that diversification across cryptocurrencies does enhance the investment results. The minimum variance model achieves the smallest out-of-sample volatility and maximum drawdown, but performs less attractive in Sharpe ratio and utility. Moreover, the maximum Sharpe model does not maximize the out-of-sample Sharpe ratio, while the maximum utility model attains the highest out-of-sample return and utility. It is worth noting that none of the sophisticated models beat the naïve 1/N rule in the criterion of Sharpe ratio. This might be due to the estimation error of $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$, also discussed in DeMiguel et al. (2009) for stock markets.

Table 3

Empirical performance of portfolio models (in % for mean, volatility, annualized return, maximum drawdown, and turnover) with the rebalancing period being τ =30 and the transaction cost being 1% multiplied by the spot turnover at each rebalancing day. The model parameters are estimated using the nearest one year data (M=360). The annualized measures (annualized return, Sharpe ratio and utility) are computed as the same method presented in Table 1.

Model	Mean	Volatility	Annualized Return	Maximum Drawdown	Sharpe Ratio	Utility	Turnover
EW	1.09	5.33	2890.70	67.67	3.88	3.41	5.00
MV	0.06	0.82	22.19	8.66	1.37	0.20	6.87
RP	1.03	5.12	2392.57	65.24	3.81	3.23	6.53
MW	0.81	4.84	1088.60	66.48	3.16	2.48	32.38
MS	0.72	4.48	812.41	61.59	3.03	2.21	40.95
MU	1.47	8.85	5034.35	81.88	3.14	3.87	39.58

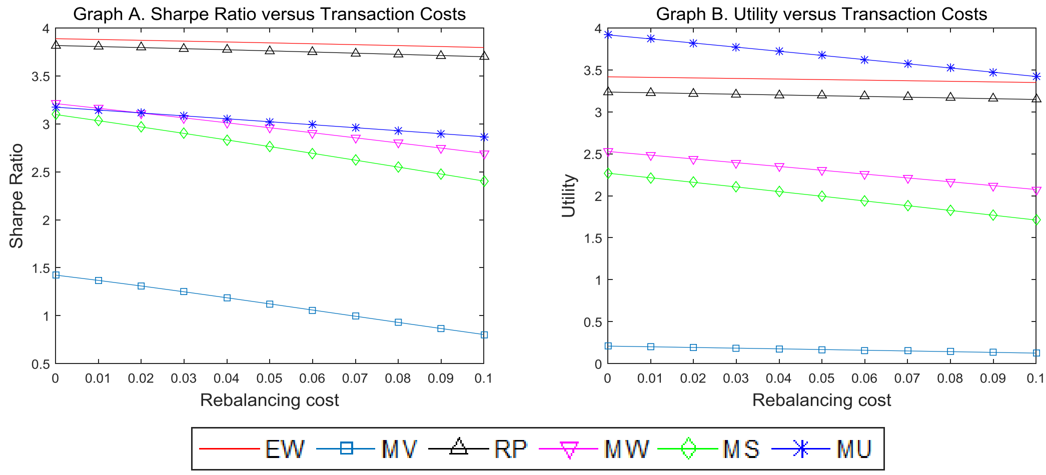

Fig. 3 Portfolio Sharpe ratio (Panel A) and utility (Panel B) versus transaction cost

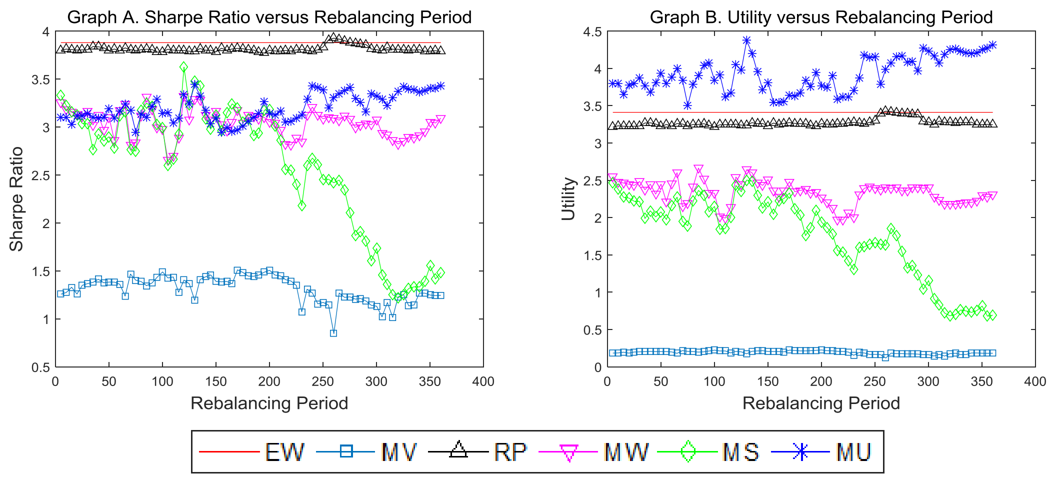

Fig. 4 Portfolio Sharpe ratio (Panel A) and utility (Panel B) versus rebalancing period

Notice that among the portfolio models in Table 2, we only need to estimate the covariance matrix for MV and RP, while the expected return requires to be estimated for MW, MS and MU, and hence the latter models lead to much higher turnover rate. The reported turnover is computed by the averaged $Turnover_t$ at each rebalancing time *t*, where

$$Turnover_t = \sum_{j=1}^{n} \left| w_{j,t} - w_{j,t-\tau} \right|. \tag{2}$$

The higher turnover results in a higher transaction cost. Fig. 3 illustrates how the transaction cost influences the portfolio performance. The EW and RP are nearly not affected by the transaction cost due to their low turnover rates reported in Table 3. But the Sharpe ratio of MV is still sensitive to the transaction cost even though it has relatively low turnover. This is caused by its low expected return, which is further distorted by the expensive cost of the first trading. Nevertheless, the assessment of the model performance illustrated from Fig. 3 does not change compared to Table 2.

To further demonstrate the robustness of our results, Fig. 4 plots the portfolio performance versus different rebalancing periods τ. The results are still similar to the findings in Table 3. Note that RP performs analogously to the 1/N rule, but none of the models are consistently better than the naïve 1/N portfolio in Sharpe ratio. If we consider the utility as the evaluation criterion, then MU stably dominates the performance.

Table 4

Out-of-sample utility with different selection of risk aversion parameters. The results reported are annualized by multiplying 360 to the daily utility, and the MU model is calculated by the corresponding γ in each column. The other settings are the same as in Table 3.

	γ =0	γ =0.1	γ =0.2	γ =0.5	γ =1	γ =2	γ =5	γ =10	γ =10^6
EW	3.92	3.87	3.82	3.67	3.41	2.90	1.37	-1.19	-5.11×10^5
MV	0.21	0.21	0.21	0.21	0.20	0.19	0.15	0.08	-1.21×10^4
RP	3.70	3.65	3.61	3.46	3.23	2.75	1.34	-1.03	-4.73×10^5
MW	2.91	2.86	2.82	2.69	2.48	2.06	0.79	-1.32	-4.22×10^5
MS	2.58	2.54	2.50	2.39	2.21	1.85	0.77	-1.03	-3.61×10^5
MU	10.55	8.70	7.73	5.45	3.87	2.40	0.83	0.08	-1.17×10^4

As the utility defined as equation (1) relies on the risk aversion parameter γ, Table 4 illustrates the out-of-sample utility of the portfolio models according to different γ. For the two extreme cases $\gamma = 0$ and $\gamma = \infty$, the MU model becomes the maximum expected return and minimum variance (MV) model respectively. As a result, we can observe that in Table 4, for $\gamma = 10$, the performance of MU has been very close to MV, while the other models fall into negative utility under such circumstance, and the distance between MU (or MV) and the others is more amplified for the case of $\gamma = 10^6$. Table 4 also shows MU outperform the others for most cases, and the results are more significant for relatively small γ. Since when $\gamma = 0$ the MU model can be related to a momentum strategy, the large value of MU reveals that there are considerable momentum effects in the cryptocurrencies, which might be hardly captured by the naïve 1/N portfolio.

4. Conclusion

This article examines the investability and role of diversification in cryptocurrencies as an alternative asset class, and further demonstrates whether the portfolio selection theory can benefit the cryptocurrency market. It is found that diversification among the cryptocurrencies can significantly enhance the Sharpe ratio and utility. By comparing the out-of-sample performance of six classical asset allocation models, we show that the minimum variance model is less risky with the smallest maximum drawdown, the maximum utility model possesses higher return and utility, but most of the models cannot beat the naïve 1/N rule under the Sharpe ratio criterion. These findings can help investors make more informed decisions.

On the other hand, the complexities of the cryptocurrency market are far from fully explored. For instance, the results in this article indicate that the estimation error in mean and covariances may offset the gains from optimal diversification, raising a problem about how to improve the estimation with sufficiently considering the stylized facts of cryptocurrencies. We leave a more detailed analysis of estimation risk in future research.

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